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GIRO Convention

23-26 September 2008
Hilton Sorrento Palace

The Case of the Credulous Actuary: Rediscovering the Importance of Judgment

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The Case of the Credulous Actuary: Rediscovering the Importance of Judgment

Outline

▪ Introduction	Mark Graham
▪ Analyses and results	Alex Glencross
▪ Recommendations and issues	Mark Graham
▪ Discussion	

Introduction

- Why?
- How?

Prior hypothesis

Long held beliefs that:

- [Actuaries] consistently give too much credibility to data
- Necessary consequence is tendency to over-fit and, hence, underestimation of uncertainty

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Methodology (1)

- Pick a "true" distribution, e.g. LogNormal (μ,σ)
- Sample a sequence of 100 data points from that distribution
- Calculate the sample mean, sample standard deviation and corresponding 99.5th %ile as each data point is added to the sequence, e.g. after 10 data points, after 100 data points
- Repeat through 10,000 simulations

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Methodology (2)

- End up with 10,000 simulations of 100 sequential data points from the same distribution
- Determine distribution of sample means, standard deviations and 99.5th %iles at each data point in the sequence
- Provides real insight into how much information is really contained in the data

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Analyses and results

- Equity returns
- Large loss experience
  - Frequency
  - Severity
  - Aggregate claims

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Recent example (or something like it)

- Eleven years of equity returns
- Mean: 5.3%
- StDev: 15.0%

- Would you define a distribution with these parameters?

Year	Return
1997	24.7%
1998	14.6%
1999	17.8%
2000	-10.2%
2001	-16.2%
2002	-24.5%
2003	13.6%
2004	7.5%
2005	16.7%
2006	10.7%
2007	3.8%

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Basic Model: methodology

- First define our 'true' distribution
  - Equity-type asset returns
  - Mean return: 10%, Standard Deviation: 16%
  - 0.5th Percentile: -49%
- Repeatedly sample ten years of observations
- Look at errors between 'true' values and those implied by each approximation
  - Mean, Standard Deviation, 0.5th Percentile

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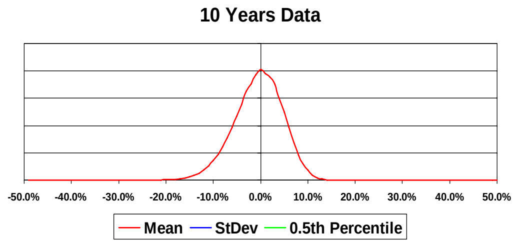
Equity Returns Example
Distribution of Parameter Errors



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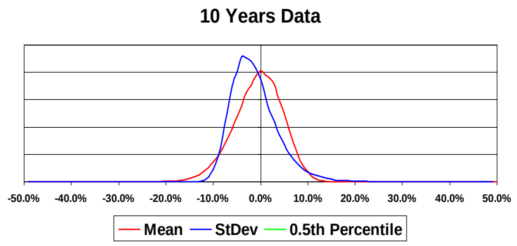
Equity Returns Example
Distribution of Parameter Errors



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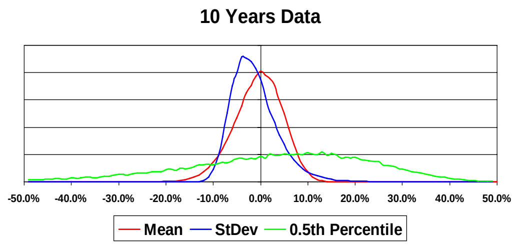
Equity Returns Example
Distribution of Parameter Errors



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Basic Model: Points to note

- Distribution of standard deviation is skewed
  - P(Error < 0) = 0.62
  - So there's a decent chance of systematic under-estimation of standard deviations
- However, errors are not significantly greater than for estimating the mean
- Skewed distribution of 0.5th percentile
  - P(Understate extreme value) = 0.61

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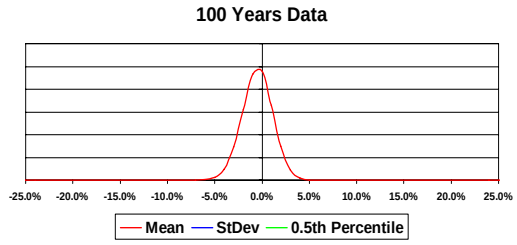
Equity Returns Example
Distribution of Parameter Errors



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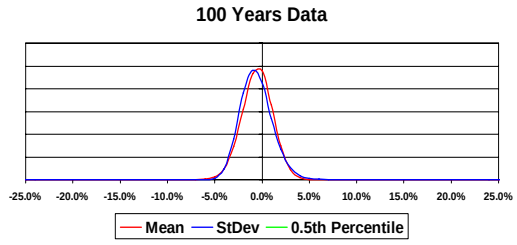
Equity Returns Example
Distribution of Parameter Errors



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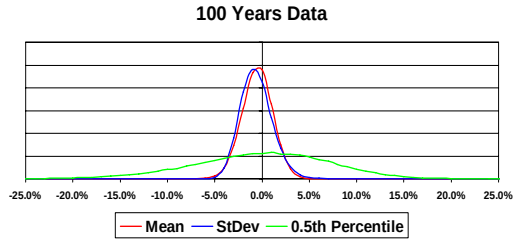
Equity Returns Example
Distribution of Parameter Errors



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Frequency / Severity Example

- Another simple model
  - Poisson Frequency / LogNormal Severity
  - Mean Frequency 5
  - Mean Severity 100
  - StDev Severity 100
- Sample 10 years of data and estimate parameters and percentile points

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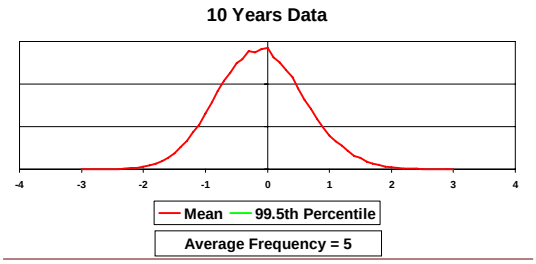
Frequency / Severity Example
Distribution of Parameter Errors
(Frequency)



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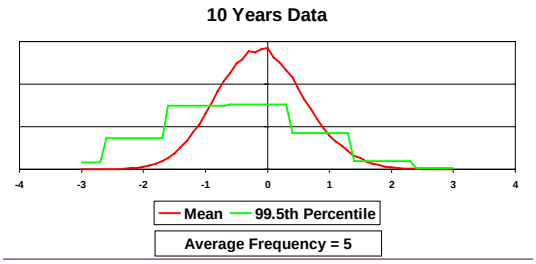
Frequency / Severity Example Distribution of Parameter Errors (Frequency)



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Frequency / Severity Example Distribution of Parameter Errors (Frequency)

- No need for StDev
  - Square root of mean
- Estimate of mean is unbiased and unskewed
- Not so for estimate of 99.5th percentile
  - Biased: average error is -0.4
  - Not particularly skewed

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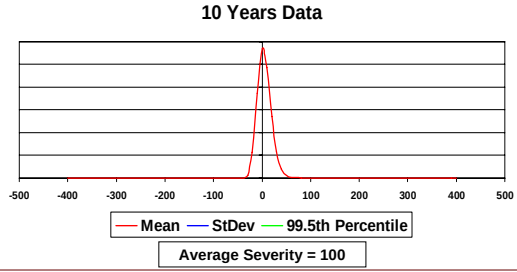
Frequency / Severity Example Distribution of Parameter Errors (Severity)



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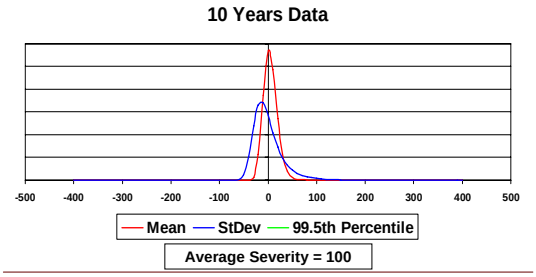
Frequency / Severity Example Distribution of Parameter Errors (Severity)



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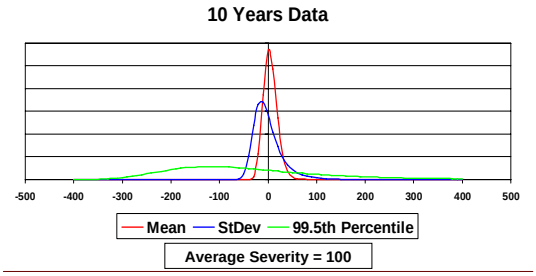
Frequency / Severity Example Distribution of Parameter Errors (Severity)



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Frequency / Severity Example Distribution of Parameter Errors (Severity)

- Mean is still unbiased and unskewed
- StDev is unbiased and skewed
  - Skewness = 0.6
- 99.5th percentile is biased and skewed
  - 'True' value = 603
  - Average error = -33.4
  - Skewness = 2.1

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Technical Problems

- Equity Return example used a single distribution
  - Invert distribution to find percentile points
- Aggregate claims is more difficult
  - No closed form that we can invert
- Simulation approach used instead

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Technical Problems (2)

- So for each of the 10,000 simulations…
  - do another 10,000 simulations
  - estimate the 99.5th percentile
- Quickly run out of processing power
  - Managed it on 4,000 simulations before it crashed…

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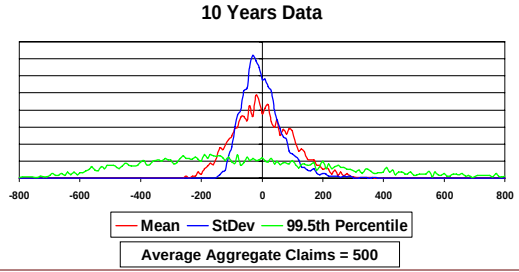
Frequency / Severity Example Distribution of Parameter Errors (Aggregate Claims)



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Technical Problems (3)

- To avoid simulation error…
  - Approximate aggregate distribution by a shifted Gamma distribution
  - *Shift + Gamma(alpha, beta)*
  - Can solve *shift, alpha* and *beta* to match the first three cumulants of the aggregate claims distribution
- Provides very good fit in the tail
  - Removes need for extra dimension of simulations

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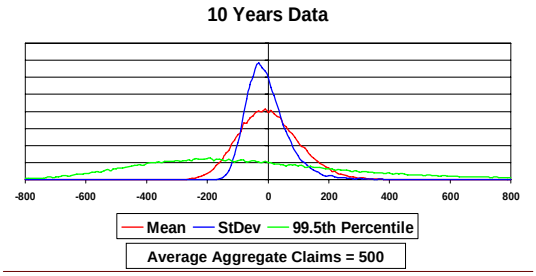
Frequency / Severity Example Distribution of Parameter Errors (Aggregate Claims)



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Frequency / Severity Example Distribution of Parameter Errors (Aggregate Claims)

- Mean unbiased, but slightly skewed
  - Skewness = 0.4
- StDev marginally biased, and more skewed
  - Average error = -7
  - Skewness = 1.7
- 99.5th percentile is biased and skewed
  - Average error = -25
  - Skewness = 3.6

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Frequency / Severity Example

- True mean of aggregate distribution is 500
  - Estimates of the mean carry a range of errors of approximately 500
  - Estimates of 99.5th percentile carry a range of approximately 1500
- Error in each estimate will be correlated
  - Underestimate capital and overestimate profit

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Frequency / Severity Example
Return on Capital

- Suppose premium received = 600
- 'True' statistics
  - mean aggregate claims = 500
  - 99.5th percentile aggregate claims = 1575
  - mean profit = 100
  - Capital = 1075
- 'True' mean Return on Capital = 9.3%

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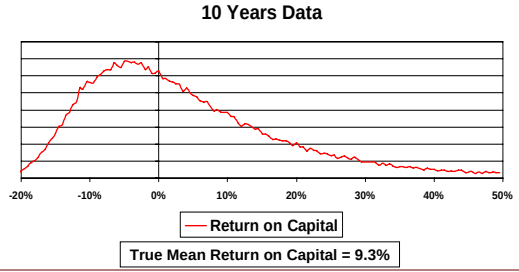
Frequency / Severity Example
Distribution of Parameter Errors
Return on Capital



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Frequency / Severity Example
Distribution of Parameter Errors
Return on Capital

- Average error = 6.0%
- Skewness = 93

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Issues

- Understates problem
- Why over rely on data vs. judgment?
- Poorly understood
- How to respond?

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Understates problem

- Investigation assumed known distribution
  - Actual distribution unknown and unknowable
  - Much modelling based on best fit *distribution*, rather than best fit *parameters* for a given distribution
- Investigation assumed stationary distribution
  - Certainly non-stationary, in practice

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Why over rely on data vs. judgment?

- Actuarial nature
- Lack of understanding
  - Of statistical limitations of data
  - Of underlying exposures
- External influences
  - Fear of being sued
  - Fair value accounting
  - Financial mathematics

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Poorly understood

- Little or no actuarial literature
- FRC Discussion Paper “Promoting Actuarial Quality” makes no mention of understanding the credibility of data as a driver of actuarial quality
- 10 sigma events
- EC/CEIOPS expectations of data in Solvency II

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How to respond? (1)

- Invest time in understanding underlying exposures
- Invest time in understanding the limitations of your data
- Use judgment, informed by data, to parameterise
- Use emerging data to test parameters
- Test over varying time periods, due to non-stationarity
- Do not expect every parameter to pass every test (1:20 fail at 95% confidence interval)

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How to respond? (2)

- Your parameters will be wrong but…
  - … judgment can ensure consistency between related exposures (different asset classes, different business types) …
  - … and parameters can be adjusted to reflect changes in the underlying exposure much earlier than would be the case if you wait for the data to emerge

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