1985 General Insurance Convention

CASH MECHANICS OF PROPORTIONAL TREATIES

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CONTENTS

1.0 INTRODUCTION

1.1 When the Underwriter is presented with a slip covering reinsurance of a primary carrier on the basis of a proportional treaty he has no great scope for negotiation.

1.2 In the case of quota share treaties the number of underlying insurances is likely to be large, sometimes very large indeed, and there is unlikely to be much fluctuation in the underwriting results due to the effects purely of stochastic variation. In the case of small cedant companies the number of underlying insurances will be smaller but the fluctuations in results not all that large.

1.3 In the case of surplus or fac-oblig treaties the variation in results may be larger and will depend in large measures on the retention of the cedant office and on whether there has been selection against or in favour of the reinsurer.

1.4 The emergence of claims of a special type, however, may well influence the results somewhat more greatly. A winter of bad weather could be one such cause. Such factors may possibly be covered by catastrophe excess loss protections, which may be inbuilt to the treaty.

1.5 The main factors the Underwriter will bear in mind in deciding whether to accept the Risk are:-

(a) General underwriting results for primary carriers overall on that class of business world-wide and, in particular, within the country concerned.

(b) The primary carrier's own record and its managerial capability.

(c) Any legislation in the country concerned that might bear on underwriting results;pressures of consumerism, social attitudes and attitudes of the courts.

(d) Whether legislation requires the retention of premium reserves and how outstanding losses are to be covered.

(e) Inflation rates, strength of the currency, delays in settlement.

(f) The standing of the Broker bringing in the business, how valuable is his portfolio to the Underwriter, what other business he brings in, both for the same Reassured and overall.

1.6 All these are factors over which the Underwriter has no control. He must assess them and take a decision accordingly as to whether to reject the proposal outright or to go on to the next stage. This stage includes the negotiation of terms over which he can have some control:

(g) The actual detailed terms and conditions of the treaty; in particular, any exclusions.

(h) Whether the treaty is protected by an excess loss protection for joint account.

(i) The amount of commission allowed to the cedant office (there is usually less scope for negotiations over the rate of brokerage allowed to the Broker).

(j) The rate of interest allowed on premium reserves.

(k) (Perhaps) Whether O/S losses are to be covered by loss reserves retained or by a Letter of Credit (which in turn may make a considerable difference to the rate of interest earned on funds allocated for that purpose).

1.7 The analysis that follows tracks the effect of varying decisions in regard to points (i) to (k) above. In many portfolios of reinsuring offices the proportional treaties are smaller in number than facultative reinsurances or non-proportional treaties but are large in terms of premium income. The scope for profit is at best marginal as the reinsuring office is seeking to show a margin allowing for brokerage (from about 1 1/2% to 2 1/2%) on top of original rates which may themselves be "thin" in a highly competitive market. In such cases, the underwriter will often console himself with the thought that there will be considerable premium flow in the early stages to generate interest income. It would come as a considerable shock to him, then, if he saw demonstrated that there is likely to be a negative cash flow throughout. Hence the analysis is set out in such a way as to lead to the introduction of a relatively straight-forward computer program which could be used by an underwriter, by means of simple input, to obtain a picture of the expected cash flow under the treaty in terms of the conditions being suggested to him.

2.0 Computer Model

2.1 The model is designed to accept the following input data:

		Symbol
(i)	Ultimate Premiums	UP
(ii)	Ultimate Claims	UC
(iii)	Commission and brokerage percentage	CB%
(iv)	Quarterly interest rate on reserves retained	r
(v)	Quarterly market rate of intrest	i
(vi)	Loss reserves retained factor	LRR
(vii)	Premium reserves retained factor	PRR
(viii)	Cumultative premium development factor at quarter j	WPj
(ix)	Cumulative paid claims development factor at quarter j	PCj
	Cumultative notified claim development factor at quarter j	NCj
(xi)	Time lag of cash settlement from the quarter end	t

2.2 Given this data it is possible to generate the quarterly development of premiums, paid loss and notified claims by applying the quarterly cumulative patterns respectivly to ultimate premiums and claims as follows :-

Written Premium during quarter j = UP X (WPj - WPj -1)

Paid Claims during quarter j = UC X (PCj - PCj - 1)

Notified Claims during quarter j = UC X (NCj - NCj - 1)

2.3 The loss reserves retained are a function of the known case reserves (the outstanding losses) prevailing at the quarter end. It is normal for this relationship to be 100% of the known case reserves although this can vary from 0% to 150%. This variation is accommodated within the loss reserves retained factor (LRR). A similar rationale applies to the premium reserves where it is common to have a reserve of 25% of the previous calendar year's premium. Again this can be anything from 0% upwards.

2.4 Given these restrictions on cash it is possible to simulate the quarterly cash flow of a proportional treaty as:

Premium

plus

Interest on Loss & Premium Reserves Retained

less

Commission & Brokerage

less

Paid Losses

less

Change in Premium Reserves Retained

less

Change in Loss Reserves Retained

2.5 The generated monetary receipt are then lagged in accordance with normal market practice, let's say two quarters, and net present valued back to inception using the quarterly market discount rate i. The underwriter can then assess the true profit or loss in current monetary terms.

3.0 Main Results

3.1 Appendix 1 at the end of this paper gives an example of a typical set of results. The model has proved invaluable in measuring the impact on cash flow and its net present value of varying certain input variables whilst keeping others constant. Numerous linear relationships have been uncovered. From these a predictive theory of proportional treaty cash mechanics has been developed. The results of this research to date can be summarised as follows:

3.2 Result 1

Given a fixed development pattern of premiums and claims, fixed interest on reserves retained and a constant combined ratio;

(i) the change in the undiscounted total cash flow is directly proportional to the change in commission and brokerage. The gradient of change is constant and equal to:

$$r.LRR.\sum_{\text{All } j}(NC_{j-1} - PC_{j-1})$$

(ii) the change in the discounted total cash flow is directly proportional to the change in commission and brokerage. The gradient of change is constant and equal to:

$$v^t\left[r.\,LRR.\sum_{ALL\,j} v^j.(NC_{j-1}-PC_{j-1})+\sum_{ALL\,j} v^j.(WP_j-WP_{j-1})-\sum_{ALL\,j} v^j.(PC_j-PC_{j-1})-\sum_{ALL\,j} v^j.(NC_j-NC_{j-1})\right]$$

(iii) the gradient of the discounted cashflow is geometrically affected by the time lag t of cash settlement.

3.3 Result 2

Providing the following are constant,

(a) loss ratio

(b) commission & brokerage

(c) interest on reserves retained

(d) premium and claim patterns

(e) loss reserves retained percentage

(i) the change in the undiscounted cash flow is directly proportional to the change in premium reserves retained factor. The gradient of change is constant and equal to:

$$r.\sum_{ALL\,j}(WP_{j-1}-WP_{j-5}).\,UP$$

(ii) the change in the discounted total cash flow is inversely proportional to the change in premium reserves retained factor. The gradient of change is constant and equal to:

$$v^{t}\left[r.\sum_{All\,j} v^{j}\left(WP_{j-1} - WP_{j-5}\right) - \sum_{All\,j} v^{j}\left(WP_{j} - WP_{j-1} + WP_{j-5} - WP_{j-4}\right)\right].\,UP$$

A full expose of the theory behind these results is given in appendices 2 and 3 respectively.

4.0 General Observations

4.1 General observations of the work conducted to date are best illustrated in graphical form. Results 1 and 2 described earlier can be seen on graphs 1 and 2 respectively.

4.2 Another observation is the effect of changing the interest payable on reserves retained. This is demonstrated in graph 3. Here again, we can clearly see how the underwriter when reviewing the undiscounted cash receipt can easily overstate the true profitability of his account.

4.3 Lastly, we investigated the undiscounted and discounted effect on cash of changing the combined ratio (graph 4). Here again, the monetary restrictions of proportion treaties cause the true profit or loss always to be less than that observed from historical undiscounted receipts.

4.4 The model is a simple but powerful tool allowing any underwriter to assess, given a set of assumptions, the undiscounted and discounted profit or loss. Furthermore, the model provides an easy way of interpreting the break even loss ratio associated with a proportional treaty, an essential indicator for management. This knowledge is critical if underwriters are to insist on the inclusion or exclusion of clauses which maximise cash flow and hence profit.

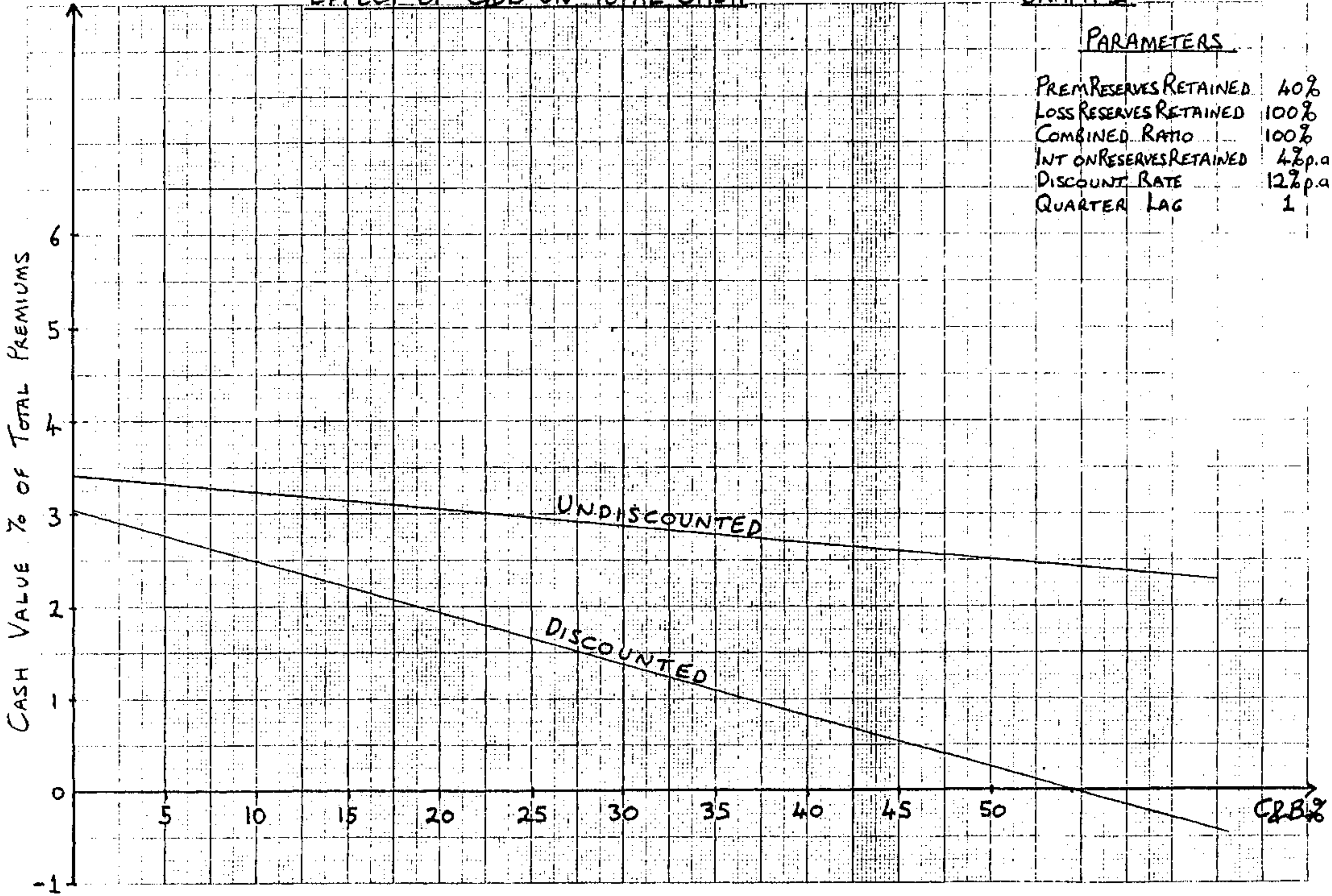
5.0 Conclusion

5.1 We have only just started to uncover the mysteries surrounding the cash mechanics of proportional treaties. Much work still remains to be done. We need to consider the implications of letters of credit, the allocation of administrative costs both direct and indirect, the influence of premium and claim portfolio transfers, together with further theoretical analysis behind our general observations.

5.2 The paper has been written with a view to stimulate an interest and understanding of the monetary implications of proportional treaties. We look forward to an active and lively discussion at this years GIRO conference.

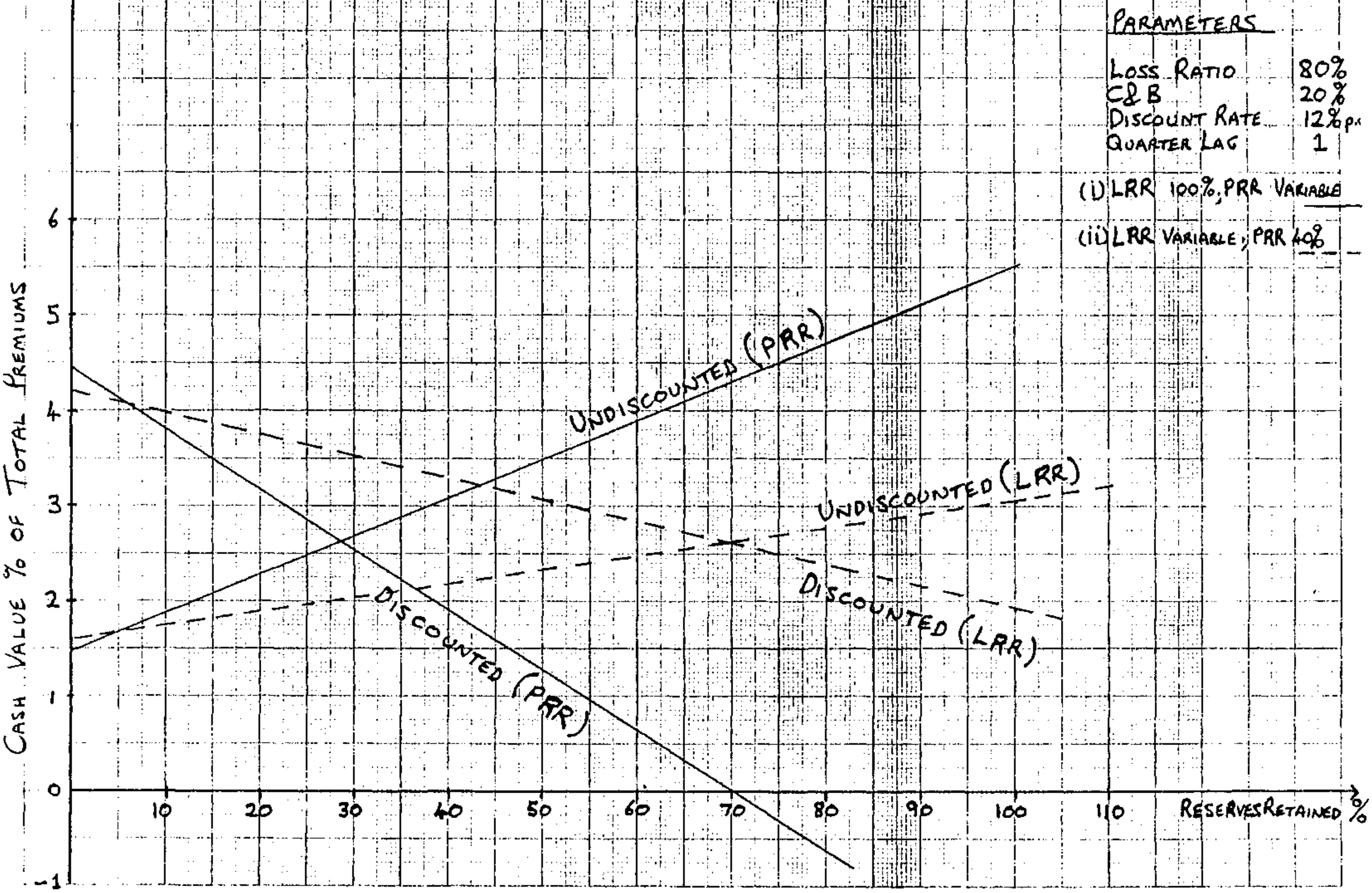
EFFECT OF C&B ON TOTAL CASH

GRAPH 1



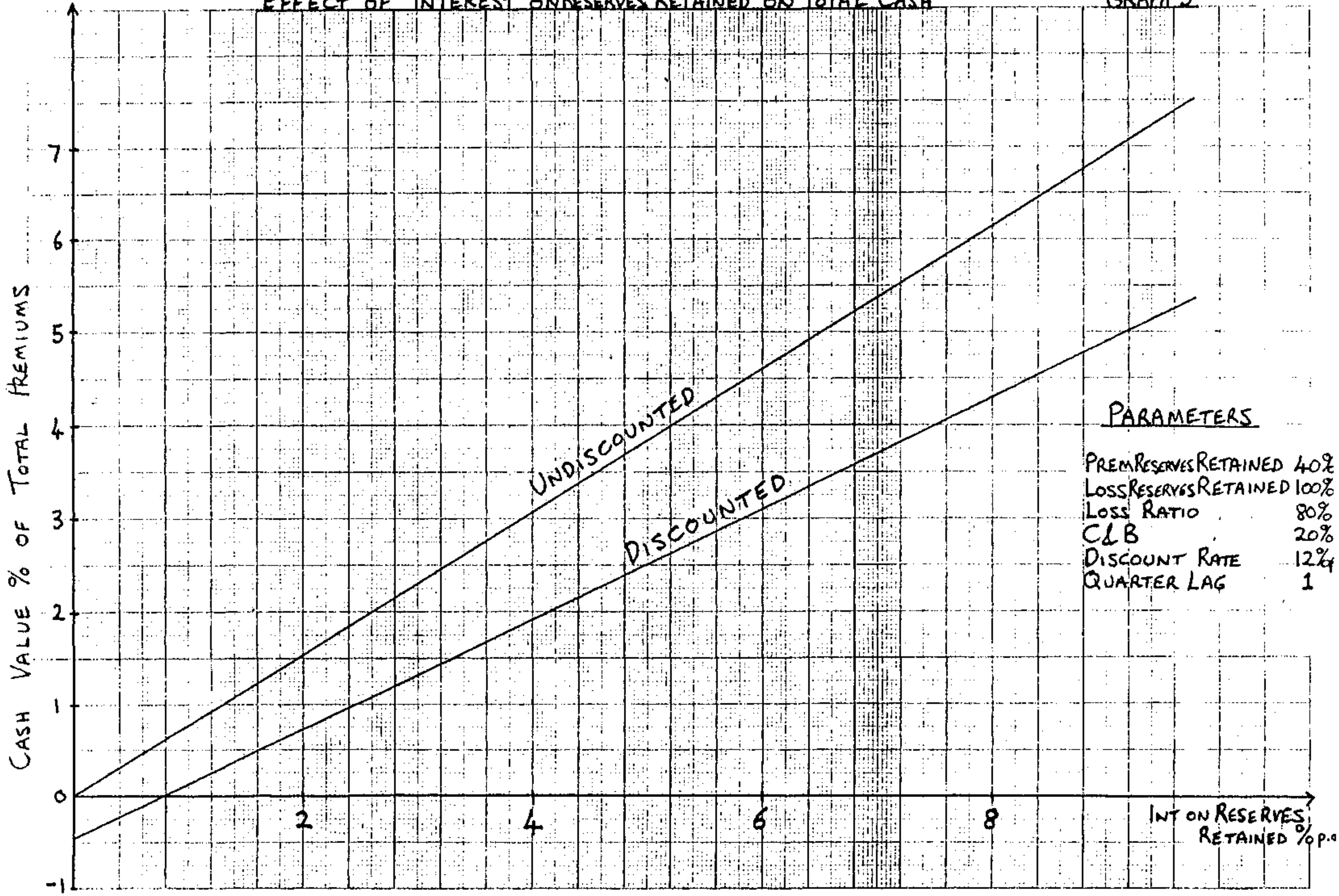
EFFECT OF PREMIUM AND LOSS RESERVES RETAINED ON TOTAL CASH

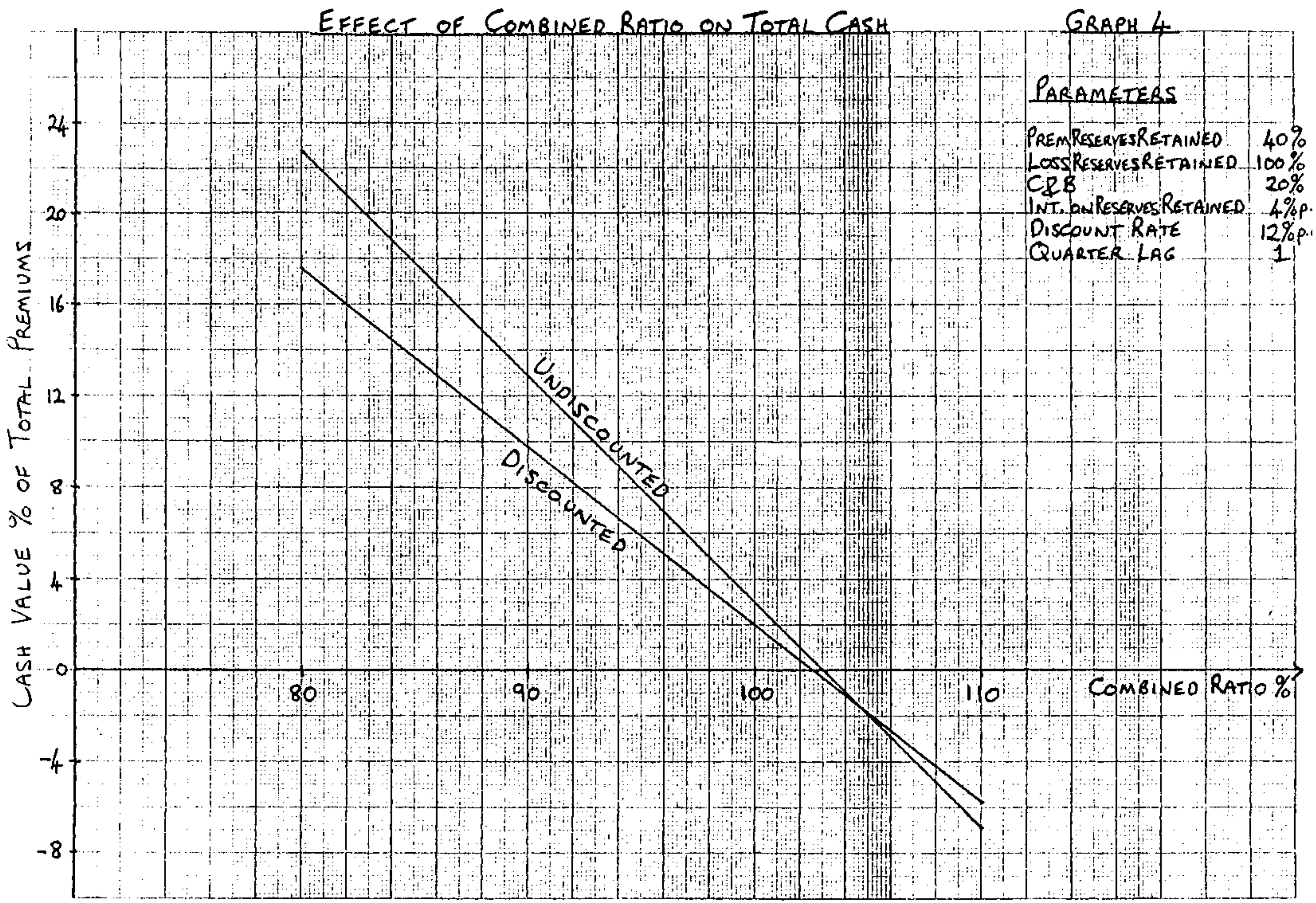
GRAPH 2



EFFECT OF INTEREST ON RESERVES RETAINED ON TOTAL CASH

GRAPH 3





EFFECT OF COMBINED RATIO ON TOTAL CASH
GRAPH 4
PARAMETERS
PREM RESERVES RETAINED 40%
LOSS RESERVES RETAINED 100%
C&B 20%
INT. ON RESERVES RETAINED 4% p.
DISCOUNT RATE 12% p.
QUARTER LAG 1
CASH VALUE % OF TOTAL PREMIUMS
24
20
16
12
8
4
0
-4
-8
80
90
100
110
COMBINED RATIO %
UNDISCOUNTED
DISCOUNTED

PROPORTIONAL TREATY : CASH FLOW ANALYSIS

APPENDIX I

INPUT DATA

ULTIMATE PREMS	10000	P.R.R.%	50
ULTIMATE CLAIMS	7000	L.R.R.%	100
C&E%	40	DISC'NT%	3
INT ON R/RETN%	3		

OUTPUT DATA

DEVELOPMENT PATTERNS %

QUARTER	PREMS	PAID	NOTIFIED	PREMIUM	CHANGE PREM RES	INT ON R/RETAIN	C&D	PAID	CHANGE LOSS R/R	QUARTERLY CASH FLOW	PRESENT VALUE	2 PERIOD LAG	2 PERIOD PRES VAL
1	4	1	2	400	200	0	160	35	105	-100	-97	-	
2	9	1	4	500	250	9	200	35	105	-81	-76	-	0
3	15	3	7	600	300	20	240	105	105	-130	-119	-100	-92
4	25	5	12	1000	500	32	400	175	175	-218	-194	-81	-72
5	40	10	23	1500	550	52	600	350	420	-368	-317	-130	-112
6	55	19	42	1500	500	81	600	630	700	-849	-711	-218	-183
7	67	30	55	1200	300	117	480	770	140	-373	-303	-368	-299
8	75	43	65	800	-100	131	320	910	-210	11	8	-849	-670
9	83	52	75	800	-350	121	320	630	70	251	193	-373	-286
10	90	62	81	700	-400	113	280	700	-280	513	382	11	8
11	95	70	86	500	-350	92	200	560	-210	392	283	251	181
12	98	75	90	300	-250	76	120	350	-70	226	158	513	360
13	100	80	93	200	-300	66	80	350	-140	276	188	392	267
14	100	85	94	0	-350	53	0	350	-280	333	220	226	149
15	100	88	95	0	-250	34	0	210	-140	214	137	276	177
16	100	90	96	0	-150	22	0	140	-70	102	84	333	207
17	100	92	97	0	-100	16	0	140	-70	46	28	214	129
18	100	94	98	0	0	11	0	140	-70	-60	-35	102	60
19	100	96	99	0	0	8	0	140	-70	-62	-35	46	26
20	100	97	100	0	0	6	0	70	0	-64	-35	-60	-33
21	100	98	100	0	0	6	0	70	-70	6	3	-62	-33
22	100	99	100	0	0	4	0	70	-70	4	2	-64	-33
23	100	100	100	0	0	2	0	70	-70	2	1	6	3
24	100	100	100	0	0	0	0	0	0	0	0	4	2
25	100	100	100	0	0	0	0	0	0	0	0	2	1
26	100	100	100	0	0	0	0	0	0	0	0	0	0
27	100	100	100	0	0	0	0	0	0	0	0	0	0
28	100	100	100	0	0	0	0	0	0	0	0	0	0
29	100	100	100	0	0	0	0	0	0	0	0	0	0
30	100	100	100	0	0	0	0	0	0	0	0	0	0
31	100	100	100	0	0	0	0	0	0	0	0	0	0
32	100	100	100	0	0	0	0	0	0	0	0	0	0
33	100	100	100	0	0	0	0	0	0	0	0	0	0
34	100	100	100	0	0	0	0	0	0	0	0	0	0
35	100	100	100	0	0	0	0	0	0	0	0	0	0
36	100	100	100	0	0	0	0	0	0	0	0	0	0
37	100	100	100	0	0	0	0	0	0	0	0	0	0
38	100	100	100	0	0	0	0	0	0	0	0	0	0
39	100	100	100	0	0	0	0	0	0	0	0	0	0
40	100	100	100	0	0	0	0	0	0	0	0	0	0
41	100	100	100	0	0	0	0	0	0	0	0	0	0
42	100	100	100	0	0	0	0	0	0	0	0	0	0
43	100	100	100	0	0	0	0	0	0	0	0	0	0
44	100	100	100	0	0	0	0	0	0	0	0	0	0
45	100	100	100	0	0	0	0	0	0	0	0	0	0
46	100	100	100	0	0	0	0	0	0	0	0	0	0
47	100	100	100	0	0	0	0	0	0	0	0	0	0
48	100	100	100	0	0	0	0	0	0	0	0	0	0
TOTALS				10000	0	1073	4000	7000	0	73	-255	73	-241

APPENDIX 2

Given a fixed development pattern of premiums and claims, fixed interest on reserves retained, and a constant combined ratio, a change in commission and brokerage is proportional to

(i) The change in undiscounted cash flow

(ii) The change in discounted cash flow

PROOF:

Let: WPj = Cumulative Premium Development Factor at time j

PCj = Cumulative Paid Claim Development Factor at time j

NCj = Notified Claim Development Factor at time j

UP = Total Ultimate Premiums

UC = Total Ultimate Claims

TCB = Total Ultimate Commission of Brokerage

r = Interest on Reserves Retained (Quarterly Rate)

i = Commercial Rate of Interest (Quarterly Rate)

t = Time Lag of Cash Settlement from Quarter End.

Now: $\sum_j$ Prem T'put = UP; $\sum_j$ Change Prem.Reserves Retained = 0

$\sum_j$ Paid Loss = $\sum_j$ PCj.UC =UC = Ultimate Claims

$\sum_j$ Changes Loss Reserves Retained = 0.

$\sum_j$ TCB.(WPj - WPj) = $\sum_j$ C&B T'put = TCB

Undiscounted Total Cash Flow = $\sum_j$ [Prem T'put - (change in P.R.R)

(UTCF) + Int. on reserves retained.

- C&B T'put - paid loss T'put

- change in L.R.R.]

= UP + Int. on reserves retained

- TCB-UC

(i) Given a constant combined ratio we can anticipate the effect on the undiscounted cash flow of varying the Commission and Brokerage whilst keeping premiums, interest and development patterns fixed.

Therefore:

$$UP1 = UP2$$

$$TCB1 + UC1 = TCB2 + UC2$$

$$\Longrightarrow TCB1 - TCB2 = UC2 - UC1$$

$$LRR1 = LRR2$$

$$PRR1 = PRR1$$

Undiscounted Total Cash Flow Changes

$$
\begin{aligned}
&= UTCF2 - UTCF1 \\
&= (\cancel{UP} + INT2 - TCB2 - UC2) - \\
&\quad (\cancel{UP} + INT1 - TCB1 - UC1) \\
&= INT2 - INT1 + \cancel{(TCB1 - TCB2)} - \\
&\quad \cancel{(UC2 - UC1)} \\
&= INT2 - INT1 \\
&= r.\sum_j (WPj-1 - WPj-5).UP.PRR + r.\sum_j (NCj-1 - PCj-1).UC2.LRR \\
&- r.\sum_j (WPj-1 - WPj-5).UP.PRR - r.\sum_j (NCj-1 - PCj-1).UC1.LRR \\
&= r.(UC2 - UC1).LRR.\sum_j (NCj-1 - PCj-1) \\
&= r.(TCB1 - TCB2).LRR.\sum_j (NCj-1 - PCj-1)
\end{aligned}
$$

Now r = Constant; $\sum_j (NCj-1 - PCj-1)$ is constant; LRR is constant.

Hence: The change in the undiscounted cash flow is proportional to the change in commission & brokerage.

The gradient of change is constant and equal to

$$r.LRR.\sum_{\text{All } j}(NC_{j-1} - PC_{j-1})$$

(ii) The change in the discounted (net present value) total cash flow is: (DTCF)

$$
\begin{aligned}
DTCF2 - DTCF1 = {} & \cancel{\sum_j v^{t+j}. UP_2 .(WP_j - WP_{j-1})} - \sum_j v^{t+j}. \cancel{PRR_2 (WP_j - WP_{j-4} - WP_{j-1} + WP_{j-5})}. UP_2 \\
& + DINT_2 - \sum_j TCB_2 . v^{t+j}. (WP_j - WP_{j-1}) \\
& - \sum_j v^{t+j}. UC_2 .(PC_j - PC_{j-1}) - \sum_j v^{t+j}. LRR_2 .(NC_j - NC_{j-1}).UC_2 \\
& - \cancel{\sum_j v^{t+j}. UP_1 .(WP_j - WP_{j-1})} + \sum_j v^{t+j}. \cancel{PRR_1 .(WP_j - WP_{j-4} - WP_{j-1} + WP_{j-5})} UP_1 \\
& - DINT_1 + \sum_j TCB_1 . v^{t+j}. (WP_j - WP_{j-1}) \\
& + \sum_j v^{t+j}. UC_1 .(PC_j - PC_{j-1}) + \sum_j v^{t+j}. LRR_1 .(NC_j - NC_{j-1}).UC_1 \\
= {} & DINT_2 - DINT_1 + (TCB_1 - TCB_2). \sum_j v^{t+j}. (WP_j - WP_{j-1}) \\
& + (UC_1 - UC_2). \left[\sum_j v^{t+j}. (PC_j - PC_{j-1}) + \sum_j v^{t+j}. (NC_j - NC_{j-1})\right] \\
= {} & r. \cancelto{0}{(PRR_2 - PRR_1). \sum_j v^{t+j}. (WP_{j-1} - WP_{j-5})}. UP \\
& + r. LRR .(UC_2 - UC_1). \sum_j v^{t+j}. (NC_{j-1} - PC_{j-1}) \\
& + (TCB_1 - TCB_2). \sum_j v^{t+j}. (WP_j - WP_{j-1}) \\
& + (UC_1 - UC_2). \left[\sum_j v^{t+j}. (PC_j - PC_{j-1}) + \sum_j v^{t+j}. (NC_j - NC_{j-1})\right]
\end{aligned}
$$

Now : $TCB_1 - TCB_2 = UC_2 - UC_1$

THEREFORE:

$$
\begin{aligned}
DTCF_2 - DTCF_1 = (TCB_1 - TCB_2). v^t. \Big[& r. LRR. \sum_j v^j. (NC_{j-1} - PC_{j-1}) \\
& + \sum_j v^j. (WP_j - WP_{j-1}) - \sum_j v^j. (PC_j - PC_{j-1}) - \sum_j v^j. (NC_j - NC_{j-1}) \Big]
\end{aligned}
$$

Now v^t is constant if t is constant

$r.LRR.\sum_j (NC_{j-1} - PC_{j-1})$ is constant

$\sum_j (WP_j - WP_{j-1})$ is constant

$\sum_j (PC_j - PC_{j-1})$ is constant

$\sum_j (NC_J - NC_{J-1})$ is constant

Therefore: The change in the discounted total cash flow is proportional to the change in commission & brokerage.

The gradient of change is constant and equal to:

$$v^t\left[r.LRR.\sum_{All\,j} v^j(NC_{j-1} - PC_{j-1}) + \sum_{All\,j} v^j.(WP_j - WP_{j-1}) - \sum_{All\,j} v^j.(PC_j - PC_{j-1}) - \sum_{All\,j} v^j.(NC_j - NC_{j-1})\right]$$

Note: The gradient of change is geometrically affected by the time lag t of cash settlement.

APPENDIX 3

Providing the following are constant:

(i) Loss Ratio

(ii) Commission & Brokerage

(iii) Interest on Reserves Retained

(iv) Premium & Claim Patterns

(v) Loss Reserves Retained %

The change in the undiscounted & discounted cash flow is proportional to the change in premium reserves retained factor.

Proof:

(i) We are given the following:

UP1 = UP2

UC1 = UC2

TCB1 = TCB2

LRR1 = LRR2

Now:

Undiscounted Total Cash Flow Change

$$= UTCF_2 - UTCF_1$$

$$= (\cancel{UP_2} + INT_2 - \cancel{TCB_2} - \cancel{UC_2}) - (\cancel{UP_1} + INT_1 - \cancel{TCB_1} - \cancel{UC_1})$$

$$= INT_2 - INT_1$$

$$= r . PRR_2 . \sum_j (WP_{j-1} - WP_{j-5}) . UP_2$$

$$+ \cancel{r . LRR_2 . \sum_j (NC_{j-1} - PC_{j-1}) . UC_2}$$

$$- r . PRR_1 . \sum_j (WP_{j-1} - WP_{j-5}) . UP_1$$

$$- \cancel{r . LRR_1 . \sum_j (NC_{j-1} - PC_{j-1}) . UC_1}$$

$$= r . (PRR_2 - PRR_1) . \sum_{\text{All } j} (WP_{j-1} - WP_{j-5}) . UP$$

As r is constant

$$\sum_{\text{All } j} (WP_{j-1} - WP_{j-5}).UP \text{ is constant}$$

The change in undiscounted cash flow is proportional to the change in premium reserves retained factor. The gradient of change is constant and equal to

$$r.\sum_{\text{All } j} (WP_{j-1} - WP_{j-5}).UP$$

(ii) The change in the discounted (net present value) total cash flow is:

$$
\begin{aligned}
DTCF_2 - DTCF_1 = {} & \cancel{\sum_j v^{t+j}.UP_2.(WP_j - WP_{j-1})} - \sum_j v^{t+j}.PRR_2.(WP_j - WP_{j-4} - WP_{j-1} + WP_{j-5}).UP_2 \\
& + DINT_2 - \cancel{\sum_j TCB_2.v^{t+j}.(WP_j - WP_{j-1})} \\
& - \cancel{\sum_j v^{t+j}.UC_2.(PC_j - PC_{j-1})} - \cancel{\sum_j v^{t+j}.LRR_2.(NC_j - NC_{j-1}).UC_2} \\
& - \cancel{\sum_j v^{t+j}.UP_1.(WP_j - WP_{j-1})} + \sum_j v^{t+j}.PRR_1.(WP_j - WP_{j-4} - WP_{j-1} + WP_{j-5}).UP_1 \\
& - DINT_1 + \cancel{\sum_j TCB_1.v^{t+j}.(WP_j - WP_{j-1})} \\
& + \cancel{\sum_j v^{t+j}.UC_1.(PC_j - PC_{j-1})} + \cancel{\sum_j v^{t+j}.LRR_1.(NC_j - NC_{j-1}).UC_1} \\
= {} & -(PRR_2 - PRR_1).\sum_j v^{t+j}.(WP_j - WP_{j-1} + WP_{j-5} - WP_{j-4}).UP \\
& + DINT_2 - DINT_1 \\
= {} & -(PRR_2 - PRR_1).\sum_j v^{t+j}.(WP_j - WP_{j-1} + WP_{j-5} - WP_{j-4}).UP \\
& + r.(PRR_2 - PRR_1).\sum_j v^{t+j}.(WP_{j-1} - WP_{j-5}).UP \\
& + r.(LRR_2 - LRR_1).\sum_j v^{t+j}.(NC_{j-1} - PC_{j-1}).UC \\
= {} & (PRR_2 - PRR_1).\Big[r.\sum_j v^{t+j}.(WP_{j-1} - WP_{j-5}) \\
& \qquad - \sum_j v^{t+j}.(WP_j - WP_{j-1} + WP_{j-5} - WP_{j-4})\Big].UP
\end{aligned}
$$

Everything inside the square brackets is constant.

Therefore the changes in the discounted total cash flow is proportional to the change in premium reserves retained factor.

The gradient of change is constant and equal to:

$$
\Big[r.\sum_{\text{All } j} v^{t+j}.(WP_{j-1} - WP_{j-5}) - \sum_{\text{All } j} v^{t+j}.(WP_j - WP_{j-1} + WP_{j-5} - WP_{j-4})\Big].UP
$$