

EXAMINATIONS

6 April 2001 (am)

Certificate in Derivatives: Mathematics and Basic Principles

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only but notes may be made. You then have three hours to complete the paper.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*
4. *Mark allocations are shown in brackets.*
5. *Attempt all 8 questions, beginning your answer to each question on a separate sheet.*

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

In addition to this paper you should have available actuarial tables, derivatives formula sheet and an electronic calculator.

- 1** (i) Compare and contrast the features of a strangle and a straddle under the headings of construction, maximum loss, maximum profit and the relationship between cost and maximum profit. [4]
- (ii) An equity fund manager holds a large position in a particular stock. She believes that there will be a large movement in the price of the stock in the next three months but is uncertain as to the direction of that movement.

Describe briefly how strangles and straddles could be used to capitalise on her belief. [4]

[Total 8]

- 2** An Australian mining company wishes to enter into a US dollar foreign currency forward contract.

- (i) Derive a formula to determine the forward exchange rate, defining all symbols used. [7]
- (ii) Discuss how the relationship between the forward rate and the spot rate varies with the relative levels of US and Australian interest rates and the term to maturity of the forward contract. [3]

[Total 10]

- 3** (i) (a) Define what is meant by the hedge ratio in respect of hedges involving futures contracts.
- (b) The objective of a particular trader is to minimise the variance of the change in the value of her hedged position (cash position plus futures position).

Derive a formula for the minimum variance hedge ratio in terms of the co-efficient of correlation between ΔS and ΔF , the standard deviation of ΔS and the standard deviation of ΔF .

ΔS is the change in the spot price, S , during the life of the hedge.
 ΔF is the change in the futures price, F , during the life of the hedge. [3]

- (ii) You work for a gold mining company. The company is concerned about the price at which it will be able to sell its gold production in the last quarter of its financial year. A colleague at work has suggested that the company can eliminate risk by using futures. Discuss this assertion. [9]

[Total 12]

- 4 Table 1 shows the portfolio of over-the-counter options on the US dollar held by an investment bank.

Table 1

Type of Option	Option Delta	Option Gamma	Option Vega	Size of Position
Call	0.5	2.2	1.8	−1,500
Call	0.8	0.6	0.2	−750
Put	−0.4	1.3	0.7	−3,000
Call	0.7	1.8	1.4	−750

- (i) Calculate the delta, gamma and vega of the investment bank's portfolio. [4]

- (ii) An exchange offers a traded option on the US dollar with the following parameters:

Delta: 0.6
Gamma: 1.5
Vega: 0.8

Calculate the position in the traded option and in dollars needed to make the investment bank's portfolio both delta and gamma neutral. [3]

- (iii) Calculate the position in the traded option and in dollars needed to make the investment bank's portfolio both vega and delta neutral. [3]
[Total 10]

- 5 (i) Distinguish between the credit risk faced by:

- (a) the writer and the purchaser of an over-the-counter option; and
(b) the two counterparties to an over-the-counter forward contract [4]

- (ii) Discuss how the credit risks in (b) could be reduced by:

- (a) reducing the probability of default; and
(b) reducing the underlying exposure [5]

- (iii) A pension fund has entered into an agreement with an investment bank whereby the pension fund will transfer ownership of a zero coupon bond to the investment bank in return for cash. In three days time the investment bank will return the zero-coupon bond to the pension fund in return for the initial cash payment plus interest at the repo rate.

Discuss the credit risks faced by the investment bank and suggest ways in which they could be reduced. [5]

[Total 14]

- 6** (i) Explain the meaning of the following terms in the context of futures trading:
- (a) settlement price
 - (b) open interest; and
 - (c) volume of trading [3]
- (ii) (a) Describe briefly the use that an investment bank, that trades in a futures market, might make of trends in the items of information in (a), (b) and (c) in part (i).
- (b) Suggest one reason why the volume of trading for a day might be significantly greater than the change in open interest over the trading day. [8]
- (iii) A futures market has only three participants A, B and C. On the morning of the first day of trading in a particular contract, A enters a long futures contract with B as counterparty and in the afternoon of the same day A closes out the contract again with B as counterparty. There are no other trades in the contract on the exchange on that day.
- (a) State the open interest at the end of the day and the volume for the day.
- (b) State the open interest at the end of the day and the volume for the day if A closed out the contract with C instead of B. [2]
- (iv) A trader takes a long position in 10 gold futures contracts when the futures price is \$300 an ounce. When trading in the contract ceases, the final futures price is \$250 an ounce.

Calculate the amount of cash that the trader hands over on the delivery date in return for taking delivery of the gold.

You may assume that each contract covers 100 ounces of gold. [3]
[Total 16]

7 Let S be the price of an asset with

$$dS = \mu S dt + \sigma S dZ$$

where Z is a Brownian motion process.

Let f be the price of a European call option contingent on S .

(i) State the assumptions needed to derive the Black-Scholes formula for valuing f . [4]

(ii) (a) Write down an expression for the stochastic derivative df of f .

(b) By considering a portfolio consisting of:

–1 units of the derivative, and

$\frac{\partial f}{\partial S}$ units of the stock S

Show that

$$\frac{\partial f}{\partial t} + rS \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf \quad [6]$$

(iii) Describe briefly what equation (ii)(b) above tells us about the risk preferences used to value f . [4]

[Total 14]

8 Let \mathbf{S} be a process representing the set of possible stock values over time such that the stock values S_t at time t follow the branches of a binomial tree. Let \mathcal{P} be a set of real numbers between zero and one which measures the probability of upward jumps at each node on the tree. Let the filtration \mathcal{F}_t represent the history of the stock process until time t along the binomial tree with \mathcal{F}_0 being the root node of the tree.

(i) Define and explain what it means for the process \mathbf{S} to be a martingale process with respect to measure \mathcal{P} and filtration \mathcal{F} . [4]

(ii) Define what is meant by a previsible process ϕ with respect to the filtration \mathcal{F} and explain what this means relative to the stock price S_t at time t . [2]

(iii) Explain how the Binomial Representation Theorem can be used for valuing a European call option and for constructing a portfolio of stock and cash which matches the derivative in value. You may ignore the changing value of money over time (or equivalently assume that the relevant risk free interest rates are zero). [10]

[Total 16]