EXAMINATION

September 2007

Certificate in Practical Financial Economics

EXAMINERS' REPORT

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

M A Stocker
Chairman of the Board of Examiners

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Comments

Comments on individual questions are given in the solutions that follow.

1 *This should have been an easy question but a number of candidates didn't attempt it. Despite the careful wording of part (i) of the question, nobody showed that Malcolm's "a" was 120 by rearranging the equation: everybody that attempted the question substituted a=120 into both sides of the equation and showed that it balanced. Was it really that difficult?*

(i) If Malcolm is indifferent between two strategies, the expected utility of the two strategies will be equal

So $U(43156) = \{U(1000) + U(100000)\}/2$

$\ln(a + 43.156) = (\ln(a + 1) + \ln(a + 100)\}/2$

multiply both sides by 2 and take exponentials to get

$(a + 43.156)^2 = (a + 1)(a + 100)$

which simplifies down to $14.688a = 1762.44$, so $a = 120$ as required

(ii) Expected utility if Malcolm takes a random box is

$\{U(1) + U(50) + U(2000) + U(100000) + U(250000)\}/5$
$= \{\ln(120.001) + \ln(120.05) + \ln(122) + \ln(220) + \ln(370)\}/5$
$= \ln(120.001*120.05*122*220*370) / 5$
$= 5.13731$

This corresponds to a certainty equivalent of x where

$5.13731 = \ln(120 + x/1000)$
so $x = 50{,}258$

So £50,258 is the upper bound for an acceptable offer

(iii) Expected utility from a random box is now

$(U(2000) + U(250000))/2$
$= (\ln(122) + \ln(370))/2$
$= \ln(122*370)/2$
$= 5.35876$

This corresponds to certainty equivalent x where

$5.35876 = \ln(120 + x/1000)$
so $x = 92{,}462$ and minimum acceptable offer is £92,462

(iv) Utility of the £500,000 certainty equivalent is ln(620)

If there is probability p of getting the answer right, expected utility from answering the question is

$pU(1000000) + (1 - p)\, U(32000)$
$= \ln(152) + p(\ln(1120) - \ln(152))$

So for answering the question to maximise utility, need

$\ln(152) + p(\ln(1120) - \ln(152)) > \ln(620)$

so

$p > (\ln(620) - \ln(152)) / (\ln(1120) - \ln(152)) = 0.704$

i.e. need to be 70.4% sure of the answer before gambling

So:
(a) would take the £500k with no lifelines ($p = 0.25$)
(b) would go with Penny's answer ($p = 0.8$)
(c) would not risk the 50/50 ($p = 0.5$)

Correct answer was C, but no marks for that. Apologies to the real life Malcolm and Penny who tell me that they both knew the answer was C.

2 *This question was generally well answered. Most candidates recommended that the boss's change not be implemented.*

(i) Advantages:

- Small projects may be much more **numerous**. As significant time is required to evaluate each project having relaxed criteria for small expenditures will mean **less time is spent** overall on evaluation
- Small projects generally have a much **lower financial impact, therefore cause less risk** to the organisation
- **Less management time** will be required, therefore lower hurdle rate may be justified

Disadvantages:

- This could encourage people to **split large projects into smaller projects** for easier acceptance

- **Smaller projects may actually have just as large financial impact** in some companies — thus a more relaxed view might not be reasonable given the financial impact

- Determining the higher hurdle rate to be used may be very subjective and **reject perfectly good projects**

(ii) The actual recommendation is not important — the answer depends on the individual aspects of the company concerned and the types of projects.

The conclusion should use the pros and cons to support.

3 *Another well answered question.*

Hire purchase key features:

- Goods are hired with regular rental payments being made.

- Legal ownership changes at the end of the rental period when the final payment is made.

- Failure to make payments means the seller can take the goods back.

Credit sale key features:

- Goods are bought but with an agreement to pay for them over a set period of time.

- Legal ownership changes at outset.

- The seller cannot reclaim the goods even if the buyer defaults — all he can do is sue through the courts.

Leasing key features:

- A lease is an agreement whereby the owner of the lease (the lessor) agrees to make the asset available to a user (the lessee) in return for periodic payments

- Legal ownership does not change hands with leases

- Operating leases are short-term cancellable

- where the lessor carries most of the risk associated with owning the assets

- Whereas financial leases are long term non-cancellable where the lessee takes on most of the risk

4 *This was a question with a wide range of marks. Candidates that could not answer part (i) were always going to struggle.*

(i) $Ep = x_A E_A + x_B E_B$
$Vp = x_A^2 V_A + x_B^2 V_B + 2 x_A x_B C_{AB}$

(ii) $Vp = x_A^2 V_A + x_B^2 V_B + 2 x_A x_B C_{AB}$
but note $x_A = 1 - x_B$

so we have:

$Vp = x_A^2 V_A + (1 - x_A)^2 V_B + 2 x_A (1 - x_A) C_{AB}$
$Vp = x_A^2 V_A + (1 + x_A^2 - 2 x_A) V_B + 2 x_A (1 - x_A) C_{AB}$
$Vp = x_A^2 V_A + V_B + x_A^2 V_B - 2 x_A V_B + 2 x_A C_{AB} - 2 x_A^2 C_{AB}$
$Vp = x_A^2 (V_A + V_B - 2C_{AB}) + 2 x_A (C_{AB} - V_B) + V_B$

Now two possible methodologies are:

1. Differentiating

Now differentiate Vp with respect to x_A

$dVp/dx_A = 2x_A V_A + 2x_A V_B - 2V_B + 2C_{AB} - 4x_A C_{AB}$

For Vp to be minimised first differential needs to be zero so solve:

$2x_A V_A + 2x_A V_B - 2V_B + 2C_{AB} - 4x_A C_{AB} = 0$
$x_A (2V_A + 2V_B - 4C_{AB}) - 2V_B + 2C_{AB} = 0$
$x_A (V_A + V_B - 2C_{AB}) - V_B + C_{AB} = 0$
$x_A (V_A + V_B - 2C_{AB}) = V_B - C_{AB}$
$x_A = (V_B - C_{AB}) / (V_A - 2C_{AB} + V_B)$

But note this is only a minimum if $d^2Vp/dx_A^2 = 2V_A + 2 V_B - 4C_{AB} > 0$

2. Completing the square

Can rearrange in the form:

$$Vp = (V_A + V_B - 2C_{AB})\{x_A + (C_{AB} - V_B)/(V_A + V_B - 2C_{AB})\}^2 + V_B - (C_{AB} - V_B)^2/(V_A + V_B - 2C_{AB})$$

This is clearly minimised by setting the squared term to zero….

(so $x_A = (V_B - C_{AB}) / (V_A - 2C_{AB} + V_B)$ as required)
….provided the squared term has a positive coefficient

i.e. provided $V_A + V_B - 2C_{AB} > 0$

Whichever methodology is followed, need to show that
$V_A + V_B - 2C_{AB} > 0$

Say return on A is R_B and return on B is R_B

Then $V_A + V_B - 2C_{AB}$

$= ER_A^2 - (ER_A)^2 + ER_B^2 - (ER_B)^2 - 2E(R_AR_B) + 2ER_AER_B$
$= E(R_A^2 + R_B^2 - 2R_AR_B) - ER_A^2 - ER_B^2 + 2ER_AER_B$
$= E(R_A - R_B)^2 - (E(R_A - R_B))^2$
$= \text{Var}(R_A - R_B) > 0$

5 *This question was inspired by an idea from a consultant that sounded very much like the CAPMAPs in the question. He stressed that the beauty of CAPMAPs is that the CAPMAP for the group is equal to the sum of the CAPMAPs for the individual business units. Their downside is examined later in the question.*

Although the maths in the model solution looks pretty daunting, it is the conclusions that are important and that the marks were weighted towards. Students of CAPM should be familiar with how the beta for a portfolio can be derived from the betas of the individual investments.

Candidates found this to be a difficult question – some candidates more than others.

(i) Comparing the profits of the business units to each other takes no account of the risks that the different BUs have taken.

BUs could generate extra profits by taking excessive risks if these risks are not recognised within the process for comparing performance.

So profits are adjusted to allow for the risks undertaken (capital requirements being risk sensitive), to get everybody onto a level playing field.

Also, group will want all EAPs to be positive — otherwise profits are not covering cost of the capital used to back them.

[other valid points could be made]

(ii) (a) Simple algebra shows that the EAPs will add up to group EAP if and only if capital requirements for individual business units add up to group capital requirements.

Proof: start from

$EAP_i = Profit_i – (Economic\ Capital_i * Cost\ of\ Capital\ \%)$

Sum over business units to get

$\Sigma EAP_i = \Sigma Profit_i – \Sigma(Economic\ Capital_i * Cost\ of\ Capital\ \%)$
$= Total\ profit - (\Sigma Economic\ Capital_i) * Cost\ of\ Capital\ \%$

Compare to

$Group\ EAP = Total\ profit – Group\ EC * Cost\ of\ Capital\ \%$

To see that

ΣEAP_i = Group EAP if and only if
$\Sigma Economic\ Capital_i$ = Group EC

Capital is unlikely to add up in this way because of:

- the possibility of some lines of business acting as natural hedges to others, and
- diversification benefits, where the capital required to cover A and B will be less than the total of the separate capital requirements for A and B

(b) Allocating the differences over business units boils down to dividing group capital over business units. Possible approaches are:

- dividing up group capital requirements in proportion to capital requirements for individual business units
- dividing up group capital requirements in proportion to individual business units' marginal addition to group capital

(iii) According to CAPM, the expected return on an asset is related to the expected return on the "market portfolio" according to

$E_i = r + \beta_i.(E_m – r)$

where E_i is expected return on asset i, E_m is expected return on market portfolio and β_i. is the beta factor of security i, defined as $Cov(R_i,$

R_M)/Var(R_m) where R_i is actual return on security *i* and R_m is actual return on market portfolio.

ABC is using this by using ith security as equity shares in the ith business unit and the market portfolio as "equities".

The theory behind CAPM assumes that non-market risks are diversifiable: a shareholder requires no extra return on these risks as he can create a diversified portfolio that reduces all diversifiable risks to negligible levels.

By extending CAPM to how it ranks the performance of business units, ABC is effectively telling the business units that it requires compensation for taking market risks but not for taking non-market risks. Although ABC has too few business units to be able to diversify away all non-market risks, it is assuming that its shareholders are able to do this, and are therefore indifferent to whether ABC's business units are taking such risks.

(iv) As already mentioned, β_i is equal to Cov(R_i, R_M)/Var(R_m) where R_i is actual return on security *i* and R_m is actual return on market portfolio.

The betas could be derived by analysing past profits within the different business units and past equity returns and inputting them to the formula.

If data is scanty, ABC might assign some credibility to the historic betas of companies with similar operations to the business units.

(v) Individual business units' CAPMAPS are given by

$C_i = \text{Profit}_i - NA_i(r_f + \beta_i(r_m - r_f))$ where NA_i are net assets for business unit *i*

At group level,

$C = \text{Profit} - NA(r_f + \beta(r_m - r_f))$ where non-subscripted terms refer to the group.

Now, C_i, NA_i and Profit_i clearly sum up over business units to get to *C*, *NA* and Profit for the group.

But what about β?

$\beta = \text{Cov}(R, R_M)/\text{Var}(R_m)$
$= \text{Cov}(\text{Profit}/NA, R_M)/\text{Var}(R_m)$
$= \text{Cov}(\Sigma\text{Profit}_i/NA, R_M)/\text{Var}(R_m)$
$= \text{Cov}(\Sigma r_i NA_i/NA, R_M)/\text{Var}(R_m)$
$= \Sigma\, NA_i/NA\ \text{Cov}(r_i, R_M)/\text{Var}(R_m)$
$= \Sigma\, NA_i/NA\ \beta_i$

So group beta is average of those for the business units, weighted by net assets.

Finally, sum of individual CAPMAPs is

ΣC_i

$= \Sigma\text{Profit}_i - \Sigma NA_i(r_f + \beta_i(r_m - r_f))$

$= \Sigma\text{Profit}_i - \Sigma NA_i(r_f + \beta_i(r_m - r_f))$

$= \Sigma\text{Profit}_i - \Sigma NA_i r_f \; - \; \Sigma NA_i\beta_i(r_m - r_f)$

$= \text{Profit} - NAr_f - NA\beta(r_m - r_f)$

$= C$

So group CAPMAP is sum of individual CAPMAPs.

(vi) Under the EAP approach, business units saw their profits adjusted downwards for any risks undertaken. Under CAPMAP, profits are only adjusted downwards to compensate for non-diversifiable/market risks.

So one might wonder whether business units will be tempted to start taking more non-diversifiable risks if this would result in more profits. For example, the life and non-life subsidiaries might reinsure less of their business if reinsurance premiums exceeded the corresponding expected claims.

Marks also available for other relevant points.

6 *Everybody seemed to make a decent stab of this question, with marks being relatively flat.*

(i)

- The Club should only invest in products which give a positive net present value
- …which should be calculated by discounting at the rate of return available on equivalently risky securities i.e. 50%
- £14m in 5 years time discounted at 50% gives £1,843,621 i.e. £1.84 per share

(ii) (a) The actuaries have 1m shares and have issued 1m new shares hence the value of their shares is £1,843,621. They originally invested £600,000 hence they have a £1,243,621 paper gain

(b)

- Ultimate responsibility for financial decisions is with the directors i.e. the actuaries
- The directors act on behalf of the ultimate owners — the shareholders — which includes Auger Close in this case
- This separation of ownership and management has the advantage that ownership can change without affecting management

- But there are disadvantages if the interests of the owners and managers diverge
- Such conflicts are referred to as principal — agent problems and give rise to agency costs
- These include the costs associated with monitoring actions and seeking to control them
- Information asymmetry may exist between different stakeholders so written agreements may be needed to specify key aspects of the relationship

(c)

- Auger Close will want to set terms so the actuaries have a strong incentive to work hard
- The actuaries are unlikely to get large salaries or secure contracts
- There are likely to be tight restrictions on cashing in for the actuaries
- Auger Close would perhaps have negotiated for preferred shares (which may be convertible in certain circumstances) rather than ordinary shares
- Auger Close may have demanded a seat on the Board

(iii) (a)

- This is a clear case of financial distress
- The face value of the outstanding bonds is £50m which exceeds the Company's total market value of £35m
- Hence if the debt matured today, the Company would default and the firm would be bankrupt

(b)

- The owners of the business will only recoup any value if the firm's value increases above £50m
- The owners may be tempted to go for high risk/high reward ventures
- This may be the case even if the projects have negative NPVs
- The owners can effectively gamble with the bondholders' money as they control the investment and operating strategy
- The owners may avoid positive NPV opportunities if fresh equity capital would be required
- Owners may try to take dividends out of the business
- Owners may cut corners in some operational areas e.g. maintenance, research and development etc
- Owners may be tempted to disguise the extent of the problems

(c)

- **Bankruptcy costs — direct**
  - Court, legal and admin fees
- **Bankruptcy costs — indirect**
  - Costs of attracting and retaining staff, customers, suppliers
  - Extra costs of management
- **Financial distress (with bankruptcy)**
  - Resolving conflicts of interests
  - Undertaking risky ventures
  - Failing to exploit opportunities
  - Over payment of dividends
  - Massaging of financial reports
  - Fire sale of assets

7 *Apart from question 1, this was the question with the widest range of marks. Parts (ii) and (vii) addressed common fallacies. Part (vi) is based on a real life problems for (with profit) life offices that need to calculate realistic balance sheets. It was pleasing to see that everybody used put-call parity to answer part (v).*

(i) There is a Black-Scholes formula for the prices of European call options on a non-income bearing security (or, equivalently, a total return index). The price is a function of the risk free rate and "volatility". Of these, only volatility is unknown.

Given the above, prices can be backsolved via Black-Scholes to get to implied volatilities. This has been done for a variety of European call option prices with a range of terms and exercise prices.

(ii) The derivation of the Black-Scholes formula is based on the assumption that volatility is independent of term and exercise price (or term and moneyness). So the fact that the matrix isn't flat would seem to cast doubt on Black-Scholes.

However B-S is a great tool for the market to use. It can convert between prices and volatilities, so that a market trader only needs to keep track of the volatility matrix rather than a huge number of prices.

(iii) (a) An exercise price of £1.25M corresponds to a TRIPE of 5,000*1.25/1 = 6,250

Term of option = 3 years

To get a suitable implied volatility, interpolate implied vols for 3-year options with exercise prices of 6,000 and 6,500 to get (15.0% + 16.3%) / 2= 15.65%

(b) Plug into B-S formula:

$d_1 = (\ln(1/1.25) + (0.05+0.5*0.1565*0.1565)*3)/(0.1565*3^{0.5})$
$= -0.1343$

$d_2 = -0.1343 - 0.1565*3^{0.5} = -0.4054$

$N(d_1) = 0.44658$

$N(d_2) = 0.34260$

$c = 1*0.44658 – 1.25*\exp(-3*0.05)*0.34260 = 0.077980$

i.e. call option worth £77,980 - £78k, say

(iv) By put-call parity, European put option prices are linked to European call option prices via $p = c + K\exp(-rt) – S$

$K\exp(-rt)$ and S are both independent of volatility

Provided that put and call option prices obey put-call parity, the volatility that sets c equal to the market price will be the same as the volatility that sets p equal to the market price.

So the volatility matrix for puts will be identical that for calls provided put-call parity holds.

That's **if** put-call parity holds. Matrices might not be **exactly** the same. *[This last point not necessary for full marks]*

(v) By put-call parity, $p = c + X\exp(-rt) – S$

So the option is worth
$77,980 + 1,250,000\exp(-0.05*3) – 1,000,000 = £153,865$
£154k, say

(vi) Policies in force will have outstanding terms throughout the range 0–10 years and a range of equivalent exercise prices (or moneynesses) so there's no individual cell in the volatility matrix where they belong.

One option is to calculate average exercise price (or moneyness) and average outstanding duration and to use this to pick a sensible volatility that applies to the "average policy". This won't necessarily get us to the correct number though.

Whatever volatility the model is calibrated to, policies in some cells will be overvalued and policies in some cells will be undervalued — fact.

So need to find some way to get comfort that undervaluations are more than made up for by overvaluations.

(vii) You could see this idea being used to eliminate any first order sensitivity (delta) to movements in the TRIPE.

However there is still the problem of second order sensitivity (gamma).

Lots of ways of trying to explain this, for example

This second order sensitivity will show up as losses following large market movements — the loss on the loss making derivative will be bigger than the profit on the profit making derivative, or

Put and call prices are always convex when plotted against the price of the underlying. At a given point, the proportions of puts and calls could be chosen so that the value of the combined portfolio is locally flat when plotted against the price of the underlying. The problem is that on either side of this, the value rises upwards, so that any market movement (in either direction) will result in the combined value of the options increasing.

8 *You know what? This was the best answered question on the paper, maybe as a result of being more directly relevant to candidates' day jobs. Take a bow everybody. Marks were awarded for drafting the note in a way that the board would understand. The majority of candidates recommended stochastic modelling ahead of stress testing, but either recommendation was fine if properly justified.*

(i)
To: Company board
From: Actuary
Date: 9 October 2006
Subject: Regulatory Capital Requirements: Recommended approach

I have been asked to consider the most appropriate method by which the company should determine the regulatory capital requirements.

Two approaches are possible and both have been considered, stress & scenario testing and stochastic modelling.

Stress & scenario testing is the less sophisticated of the two options and involves consideration of the possible adverse events which may impact the company. Examples of stress & scenario tests might include:

- adverse investment performance
- adverse claims experience including catastrophes to the household book
- losses to the service company
- combinations of the above

The pros of this method are:

- relative simplicity

- ease and speed of application
- ease of understanding the results

The cons of this method are:
- difficulty in assessing what scenarios relate to the 99.5% level
- difficulty in assessing the combination of events in required scenarios
- might be viewed as too simplistic by external parties (regulators, investors, rating agencies)
- does not allow further detailed analysis to be carried out (e.g. capital allocation, assessment of different reinsurance strategies, assessment of different investment strategies, changes in the mix of business)

Stochastic modelling is the more sophisticated option commonly used by large insurers. The aim here is to build a financial model of the company which can then be used to determine all possible financial outcomes. Stochastic refers to the use of Monte Carlo simulation; i.e. using random drawings from selected distributions to mimic the financial outcomes of the company. These distributions would be used to represent the outcomes of the investments, the household and motor classes and potentially the service company.

Unlike stress & scenario testing where only a small number of situations are considered, stochastic modelling can be used to consider tens of thousands of outcomes and explicitly to determine the amount of capital required to protect to the 99.5% level.

The pros of this method are:
- ability to consider all outcomes
- ability to capture all sources of variability/risk
- ability to determine the capital at the required percentile level explicitly
- ability to carry out further analysis (see con above)

The cons of this method are:
- complexity
- cost and time required
- difficulty in explaining the method
- difficulty in explaining the results

Recommendation: I would recommend stress & scenario testing, at least initially, given that we are a small company and this is a new area.

[As stated above, there was no "correct" recommendation: either was acceptable if the candidate could provide justification]

(ii) *Definitions in italics not necessary, but explanations of ratings were expected to indicate some understanding of the definition.*

Insurance risk

The risk of loss arising from the inherent uncertainties about the occurrence, amount and timing of insurance liabilities and premiums

Medium/High

Insurance risk would account for a majority of the risk of the business. Household business can be catastrophe exposed and motor insurance can be exposed to large impacts from adverse court awards and very large claims (e.g. the land rover onto the train tracks). This would suggest a medium to high risk.

With market and credit risk, credit risk from corporate bonds could be included under either heading.

Market risk

Market risk refers to the risk that arises from fluctuations in values of or income from assets, in interest rates or in exchange rates.

Low/Medium

Investment is in bonds; however that still leaves interest rate risk. We are not told of the quality of the bonds or their currency.

Credit risk

Credit risk refers to the risk of loss if another party fails to perform its obligations or fails to perform them in a timely fashion. For syndicates, key counterparties include reinsurers, brokers, insureds, reinsureds, coverholders and investment counterparties

Unknown

There may be exposure from corporate bonds, through reinsurers or through brokers. However there is not enough information to make an assessment.

Operational risk

Operational risk refers to the risk of loss resulting from inadequate or failed internal processes, people and systems, or from external events.

Medium/Unknown

The service company could be included in operational or group risk – but not both. A general level of operational risk would be included.

Liquidity Risk

Liquidity risk refers to the risk that sufficient financial resources are not maintained to meet liabilities as they fall due.

Low/Unknown

There is not sufficient information; however investment in bonds (assuming `they are gilts or well traded) would usually lead to a low liquidity risk.

Group Risk

Group risk refers to the potential impact of risk events, of any nature, arising in or from membership of a corporate group.

Medium/Unknown

A general allowance for group risk might be required — however there is not sufficient information.

This was the last CPFE exam. Thank you to the Actuarial Profession's staff for their support work and to the team of examiners (listed in question 1) for setting and marking the exams.

END OF EXAMINERS' REPORT