[D4]
REID'S METHOD
Contributed by D H Reid

1. Introduction

The method used by D H Reid is essentially non-parametric in nature. A central feature is the use of an empirical estimate of the distribution of claims by size and delay-time, based on one chosen year of origin, the "base year". This estimated distribution is assumed to underlie all the other years of business. The number of claims for each year is assumed to be known or estimated. Instead of estimating a rate of development for the other years of business, each is compared with the base year. For example, if the proportions of the total number of claims from year of origin 2, developed to the end of the first, second, third etc year of their run-off are calculated, then the corresponding points of time for the same proportions in the base year are labelled r_{21}, r_{22}, r_{23}, etc.

The way in which corresponding points of time in the run-offs, and the corresponding sizes of claims are developed and used are described in the paper and shown graphically in Figure 5.

The complications arise mostly from the special treatment required for the end-periods, and for large claims.

Various complicated expressions are evaluated using numerical techniques which are typical of modern computer-based calculations; in principle, they do not affect the method, although they are part of the work of implementation.

1.1 This is a description of a reserving method first proposed by D H Reid (1978) and subsequently developed in a series of papers (Reference 1 to 3). It is a very powerful method of most relevance in direct business where data is available subdivided by claim size.

1.2 The following aspects of the claims reserving situation provide motivation for the particular approach taken:

1.2.1 In most reserving contexts for the claims arising from a particular period a correlation exists between their cost and the period of time elapsing between origin and settlement. Rates of settlement are at least partially within the control of claim officials and are not necessarily constant from year to year. Prima facie, adequate understanding of the developing experience of a claim portfolio upon which reserves can be constructed can thus be gained only by monitoring **both** development time and cost variables jointly. Thus both of these variables and in particular **cost** must be treated in any thorough reserving process.

1.2.2 The standard actuarial approach to graduation by means of a standard table would appear to provide a framework suitable for generalisation in this context. It turns out, however, that, because of the subjectivity involved in what constitutes a claim at the practical level — leading to varying proportions of "nil" claims — the relevant features of the process require careful specification.

1.2.3 The actual cost of claims is determined partly by factors extrinsic to the company, and partly by company policy. It is important that historic cost movements should be visible in a form which enables the effect of these sources to be viewed, before extrapolation to the future, and that this view should as far as possible be free of corruption through point 1.2.1 above.

1.2.4 It goes almost without saying that the general insurance market is highly competitive. Particularly in the context of a small market share there is a considerable premium on estimating claims experience levels as precisely as possible, and to this end maximum benefit from available data is needed. The objective should be to create claim estimation methods which make explicit use of data and use visible valuation bases and are thus directly under management's control. Present day computing power is such as to render computational labour transparent to the user.

1.2.5 Although the method is relatively complex, it is readily capable of being implemented on a PC.

1.2.6 The underlying model represents a deliberate simplification of the claim process (although more elaborate than any other currently in use). The extent to which it is refined in application will depend on the context — in most applications the model as proposed will be adequate. For some, for example where radically different types of payment are embraced within a claim, and where factors affecting each may be different, a version of the model which reflects this may be necessary (see e.g. Section 4 of Reference 2), or a "DP" solution may be adequate which subdivides the experience into two or more parts each of which can be valued separately. Nevertheless, it should be said that experience over 10 years has shown the basic model to be applicable in most cases without further elaboration.

1.2.7 Any valuation method — from case estimates onwards — involves the use (explicitly or implicitly) of a conceptual model, and it is critical to the success of the method that the degree of adequacy of the model should be visible — and where inadequate the model should be capable of modification, or, in the last resort, rejection.

1.2.8 Whilst the method has wide application — and can easily cope with, e.g., the effect of Excess of Loss reinsurance cover on claims experience, it is not readily applicable to "pathological" types of claim, such as certain long term industrial diseases — at least until considerably extended data bases are available.

1.2.9 It is important to mention that in the original paper much of the presentation was concerned with modelling the two way distribution of the base year in considerable detail. The author has now indicated that a relatively simple approach to the distribution based on linear interpolation will suffice for most practical purposes. This results in a much less labour-intensive approach than formerly.

2. The structure of the method is quite involved and the flowchart (fig. 1) may help clarify the interdependence of the various stages. The heart of the method is a bivariate distribution intended to express the relationship between ultimate cost and time of settlement of claims originating in one year.

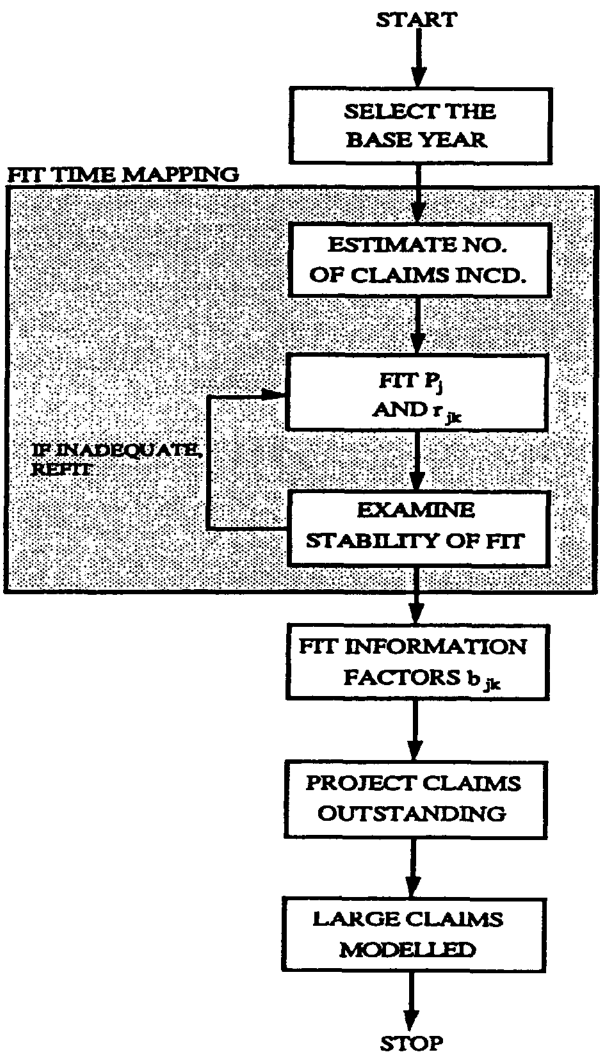

Figure 1

2.1 The variable x represents size of claim (aggregation of individual payments) The variable t represents time of development (time elapsing from the beginning of the origin year to the date of settlement).

Then M(x,t) represents the probability that a claim exceeds amount x and is settled at a time of development greater than t.

2.1.1 Origin date can refer either to date of originating event or date of notification to the company: in the second case estimation for known outstandings is produced; in the first some method of forecasting IBNR numbers is needed — then the method can produce total reserves (known cases and IBNR).

2.1.2 Consistency is required in the definition of a claim and its "time of settlement". The latter could for example relate to "time of first closure" — leaving liability in respect of reopenings to be determined separately. The important thing is consistency — other arrangements are possible.

2.2 The underlying method is as follows for a particular year of origin of claim:

- a proportion p of the claims are zero (i.e. settled at no cost);
- of the zero claims, $M^z(t)$ is the proportion whose time to settlement exceeds t;
- of the non-zero claims $M^{nz}(x,t)$ is the proportion whose cost exceeds x and time to settlement exceeds t.

2.3 The distributions $M^z(t)$, $M^{nz}(x,t)$ are empirically determined from the experience of a well (i.e. nearly completed) developed year of origin (the "base year"). For later years of origin this distribution is assumed to apply, although the model allows for different rates of settlement and for claim inflation by fitting mappings from the actual time and monetary amounts of later years to the operational time and monetary amounts of the base year. It also allows for varying proportions of zero claims.

2.4 The function $M^{nz}(x,t)$ is truncated so that large claims and claims settled at very late durations are treated differently in this analysis.

2.4.1 The definitions of cut-off points for large and late claims should be chosen to fit convenience.

2.4.2 Late claims are included in the last time period analysed (the "end group"). For most direct lines of business the end group should form a very small proportion of all the claims and no significant distortion need be introduced by this approach.

2.4.3 Large claims are modelled separately. The assumption here is that the number of large claims is binomially distributed and that the amount can be modelled by a Pareto curve. The most recent years will not (yet) supply much data for large claims and older years, including years earlier than the base year, can be used. The large claims modelled in this section may also include claims included in the main analysis so as to provide more data; obviously in any final calculations care should be taken to avoid double counting on any claim.

2.4.3.1 The proportion of large claims is settled from the base year data by judgement. This same proportion is then used to determine the actual large claims in each year of origin j.

2.4.3.2 Data from a number of years of origin are deflated back to the base year by a number of assumed large claims inflation factors. When a Pareto curve provides a good fit, this combination of inflation factor and curve is adopted. Projections of future sizes of large claims can then be carried out on a future large claims inflation factor, which may be based upon the fitting above.

2.4.3.3 The chosen proportion of large claims is used on the future probability of a large claim arising. This probability is applied to the total number of claims in each year of origin j to find the expected number of large claims for year j. This number is then combined with the size as found above.

2.5 A year which is well developed is chosen as the base year. Experience has shown that the actual choice of base year does not have a major effect on the final results, as the fitting of mappings between the base and later years will correct for any unusual effects. In any case of doubt, alternative base years can often be selected for comparison.

3. Fitting the time mappings

3.1 *"Operational Time"*

A critical feature of the model (necessary because of 1.2.1 above) is the manner in which allowance is made for varying rates of claim settlement on the claims arising in each origin year. This is achieved by the introduction of an "operational time" mapping in each origin year subsequent to the base year. The idea is that the proportions of claims settled at each point of development of a given origin year equate to those for the base year at the operational time value specific to that point of development.

The operational time scale for each origin year is determined entirely on the number of claims settled, and not the cost of claims.

Thus (refer to figure 2) r_{21} represents the value of operational time for origin year 2 corresponding to the stage of development (by number of claims and not amounts) which that origin year has reached at development time 1 year.

More generally r_{jk} is the operational time at which the base year has reached the same stage of development as origin year j at its kth year of development.

3.2 *Total number of claims by origin year*

If the origin year is defined as year of notification, there is no difficulty in ascertaining the number of claims. If however the origin year is defined as year of event, some means is required at this stage for estimating the eventual number of IBNR claims. Usually a simple procedure based on reporting patterns will suffice.

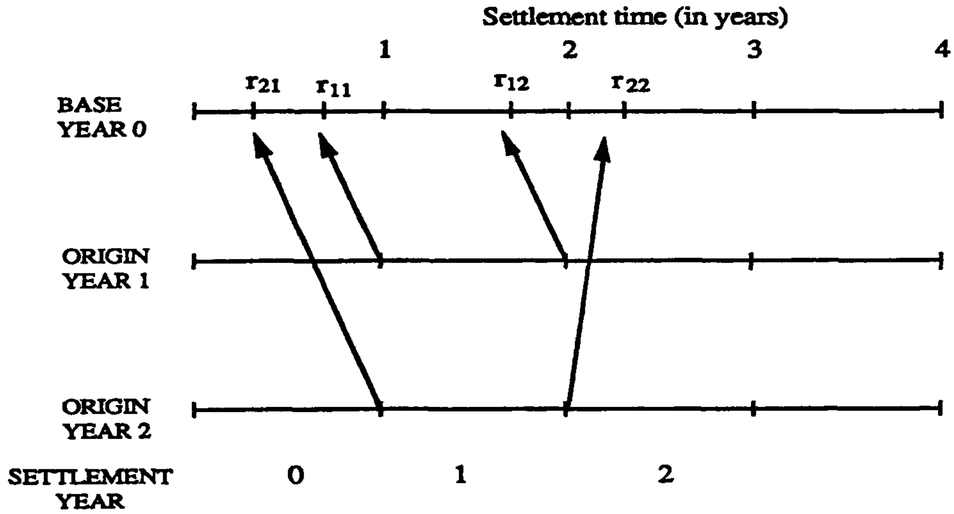

Figure 2

3.3 *Fitting r_{jk} and p_j*

The data will then provide us with:

$Q^{nz}{}_{jk}$ = the number of origin year j claims settled at positive cost in calendar time (j+k, j+k+1).

$Q^{z}{}_{jk}$ = the number of origin year j claims settled at zero cost in calendar time (j+k, j+k+1).

Q_j = the number of origin year j claims still unsettled at calendar time s (where s is the final calendar year of development currently available).

Diagrammatically:

non-zero	Q^{nz}_{j1}	Q^{nz}_{j2}	...	Q^{nz}_{js-1}	Q_j
origin year j					
zero	Q^{z}_{j1}	Q^{z}_{j2}	...	Q^{z}_{js-1}	
settlement time t = 0	1	2..	s−j−1	s−j	

Figure 3

We would expect these relative ratios to be:

$(1-p_j)$* $[M^{nz}(0,0)-M^{nz}(0,r_{j1})]$	$(1-p_j)$* $[M^{nz}(0,r_{j1})-M^{nz}(0,r_{j2})]$	...		$p_j M^z(r_{j\ s-j})$
p_j* $[M^z(0)-M^z(r_{j1})]$	p_j* $[M^z(r_{j1})-M^z(r_{j2})]$	...		$+ (1-p_j)M^{nz}(r_{j\ s-j})$

Figure 4

We can construct a log-likelihood function for this fit:

$$L = \sum_{j=1}^{s-1} \left\{ \sum_{k=0}^{s-1-j} [N_{jk}^{z} + N_{jk}^{nz}] + N_j \right\}$$

where

$$N_{jk}^{z} = Q_{jk}^{z} \, ln \, \{M^{z}[r_{jk}] - M^{z}[r_{j\ k+1}]\} + Q_{jk}^{z} \, ln \, p_j$$

$$N_{jk}^{nz} = Q_{jk}^{nz} \, ln \, \{M^{nz}[r_{jk}, 0] - M^{nz}[r_{j\ k+1}, 0]\} + Q_{jk}^{nz} \, ln \, (1 - p_j)$$

$$N_j = Q_j \, ln \, \{p_j M^{z}[r_{j\ s-j}] + (1 - p_j) M^{nz}[r_{j\ s-j}, 0]\}$$

Estimates of r_{jk} and p_j are found by maximising the log-likelihood function. This is achieved by standard computational methods.

3.4 *Validation*

At this stage the actual and predicted results can be compared, and if the base year is inappropriate for the data this should become evident.

3.5 *Alternatives*

In the event that a significant diversion should be found at this stage between actual and predicted results, various alternative models are available — for example it may be appropriate in some contexts to sever completely the connection between zero and non-zero claims and proceed accordingly, allowing different r_{jk} for zero and non-zero claims.

4. Fitting the monetary mapping

The development of numbers of claims in each origin year has been compared to the base year, and it is now necessary to compare each origin year to the base year on the basis of the cost of claims. This will produce a set of inflation factors.

4.1 b_{jk} represents the factor by which the cost of claims originating in year j and settled in development year k exceeded that of those base year claims which were settled between r_{jk} and $r_{j\,k+1}$, i.e. the equivalent time in the base year.

4.2 Again it is possible to fit b_{jk} for all the j,k available using a log-likelihood method.

To bring size of claim into the analysis we group by size into bands (x_i, x_{i+1}). If we then let Q_{ijk} represent those claims included in Q^{nz}_{jk} falling between x_i and x_{i+1}, where the x_i are dividing points between the bands, we can express the distribution of Q_{ijk} as:

$$M^*_{ijk} - M^*_{i+1jk}$$

where

$$M^*_{ijk} = M^{nz}\left\{\frac{x_i}{b_{jk}}, r_{jk}\right\} - M^{nz}\left\{\frac{x_i}{b_{jk}}, r_{j\,k+1}\right\}$$

The log-likelihood function is proportional to:

$$\sum Q_{ijk} \log (M^*_{ijk} - M^*_{i+1jk})$$

and the values of b_{jk} which maximise this can be found by numerical techniques.

4.3 Again, at this stage we can examine the b_{jk} to determine whether they accord with intuitive understanding. We can also compare the fitted and actual distributions of claims by size. In the event that what appears to be a poor fit is obtained three possibilities need to be distinguished.

- The sample of claims involved may be so small that appreciable random fluctuation is anticipated.

- The choice of bands of claim size on which the fit has been carried out may be inappropriate relative to the weight of the distribution concerned.

- It may be that a substantive change in the underlying distribution has taken place which does not permit of representation by the underlying surface mapped by the successive transformations involved.

In the first of these cases typically recourse would be had to comparative values from earlier years, or alternatively the period concerned would be grouped with one or more surrounding periods in order to provide a more statistically viable sample.

In the second case, an alternative choice of fitting points can be examined, and it would be the intention to develop an "expert" system to provide this facility automatically.

In the third case a decision must be made as to whether the feature concerned is one which can be expected to persist in future and should therefore be carried forward or may safely be disregarded from the point of view of future projections.

4.4 It is now possible to examine the effect of inflation on past settlements. Before doing so, however, it is necessary to allow for changes in rate of settlement, which may distort the observed values of b_{jk}, (see section 1.2.1). This is done by deriving adjusted inflation factors B_{jk}, which relate to fixed periods of operational time rather than calendar time.

i.e. whereas b_{jk} relates to the period between **settlement** times k and k+1, corresponding to operational times r_{jk} and $r_{j\,k+1}$, B_{jk} relates to the period between **operational** times k and k+1.

Comparisons of B_{jk} for successive origin years should then be free of the distorting effect of changes in rates of settlement and should reflect the "true" effect of inflation.

Note — for notational convenience the fixed periods of operational time to which the B_{jk} relate are termed "groups".

4.4.1 The B_{jk} are obtained as weighted averages of those b_{jk} which lie wholly or partly between operational time k and k+1. The weights used are the contributions to the mean non-zero claim cost from the component parts of the base year distribution.

This derivation assumes that each b_{jk} applies uniformly over the period to which it relates. Other assumptions, and other methods of combining the component b_{jk} could be used. It would also be possible to calculate the B_{jk} directly, using b_{jk} derived from them to fit the data.

4.4.2 Since the B_{jk} correspond to periods of operational time it is desirable to be able to relate them to calendar times so that secular changes in claim cost can be properly measured. This is done via R_{jk}, which is defined as the time to settlement in origin year j of the beginning of group k; i.e. B_{jk} relates to the period of settlement R_{jk} to R_{jk+1}. The R_{jk} are obtained from the r_{jk} by linear interpolation, though other means for obtaining them could be used. Figure 5 demonstrates the relationship between the b_{jk} and B_{jk} and between the r_{jk} and R_{jk}.

4.4.3 Because groups and calendar years of development rarely correspond exactly, the situation often arises, for a particular origin year, where past b_{jk} provide information for only part of a group, the remaining part being outstanding. Such outstanding parts of groups are known as fringe groups. Usually each origin year will have one fringe group (never more than one).

The treatment of these groups has to be considered carefully. If only a small part of the group is outstanding then it may be appropriate to apply the B_{jk} obtained from the settled part to the outstanding part. Conversely, if only a small part is settled then it would be more appropriate to use preceding origin years' B_{jk} for the same group, along with projected inflation, as a guide to the B_{jk} for the outstanding part.

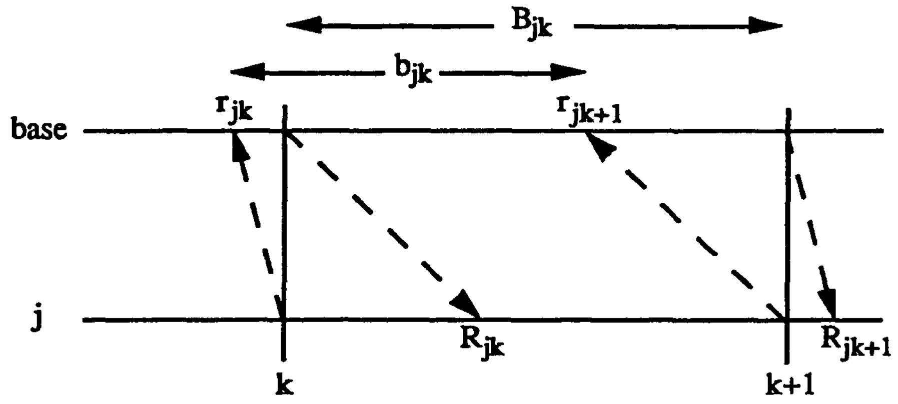

Figure 5

5. Estimating Reserves

5.1 The process of estimating reserves arises as a natural consequence of the underlying conceptual framework — a bivariate claims distribution (by size and time) is modified to reflect the observed experience of individual origin years. The required modifications are produced by the B_{jk}.

In the reserving context, what is required is that part of the modified bivariate distribution which is outstanding at the time of inspection, i.e. that part which lies after $r_{j(s-j+1)}$.

5.2 Estimation of reserves thus boils down to estimation of the outstanding B_{jk}. This may normally be done by assuming a constant rate of future claims inflation (f, say) and applying the formula $B_{j+1\ k}=(1+f)B_{jk}$ for outstanding groups but other adjustments may also be made to allow for, e.g., anticipated changes in rate of settlement or inflation rates which vary by origin year or settlement year.

5.2.1 The expected settlement amount for each outstanding group is then calculated as the product of

(i) B_{jk}
(ii) mean non-zero claim cost for the group
(iii) expected number of non-zero claims in the group.

To obtain (iii), the expected number of non-zero claims in each group, the estimated total number of outstanding non-zero claims (as obtained from the fitting procedure described in 3.3) is spread over the group pro rata to the proportion of all non-zero claims attributable to each group.

It is then a simple matter to accumulate the total settlement amount outstanding.

5.2.2 It should be borne in mind that, as mentioned in 2.1, the claim costs reserved represent aggregations of individual payments, i.e. the ultimate cost of outstanding claims. To obtain the outstanding monetary amounts any payments made on account on these claims should be deducted from the reserve calculated as described above.

5.2.3 Fringe groups, described in 4.4.3, require special consideration. The appropriate B_{jk} should be obtained as discussed in 4.4.3 and the mean non-zero claim cost and expected number of non-zero claims are derived from the distribution for only the outstanding part of the group.

5.2.4 Large claims also require special consideration. When assigning the outstanding non-zero claims to groups, the proportion of large claims may be reduced in line with the number of such claims already settled for each origin year, in order to allow for the time development of these claims. The appropriate B_{jk} may be obtained from an assumed constant inflation rate, though it should be borne in mind that for these claims origin year is the important determining factor for cost level.

6. Miscellaneous

6.1 The advantages of the method cover two main headings:

(i) *Analysis*

By removing the effect changes in the rate of settlement have on observed claim cost the method allows a proper analysis of the underlying movements in claim cost and, unlike other standard methods (e.g. chain ladder, separation method) is free from the distorting influence of such changes. Inspection of the fits, both by size and time, may also indicate whether observed changes can be attributed to secular movement or relate to underlying changes in the nature of the business, in which case appropriate steps may be taken to amend any assumptions for the future.

(ii) *Control*

By suitable adjustment of the parameters affecting the reserves (e.g. rate of inflation, individual B_{jk}, proportion of large claims) senior management can ensure that the reserves reflect their judgements as to general economic conditions and the nature of the business (possibly as indicated by the method's own analysis). The flexibility of the method allows such adjustments to be precise and specific.

6.2 The method can also be extended to experience rating of larger commercial risk, though a number of constraints may be required because of greater variability, e.g. by reference to an extended model using B_{jk} from a larger portfolio. This is of particular significance to those contexts where rating is based on "burning cost". (For a fuller discussion see Reference 3.)

6.3 As pointed out in the original paper, the method gives rise to the possibility of estimating confidence intervals for outstanding claims. Beyond this it becomes a practical possibility to examine the "strength" of reserves in terms of the trade off of variability against mean cost at a given reserve level.

7. Example

Example Accounting Date: 31.12.87

MODEL (Section 2)

The Base Year for the model is 1982 with truncation points of 6 for operational time t and £80,000 for claim size x (see paragraph 2.3–2.5).

The model for Nil claims, $M^z(t)$, is:
(x 10^{-5})

t:	0.00	0.5	1	2	3	4	5	6
$M^z(t)$:	100,000	84,835	37,299	1,288	46	13	3	1

The model for Non-nil claims, $M^{nz}(x,t)$, is:
(x 10^{-5})

t: x	0	0.5	1	2	3	4	5	6
£0	100,000	71,556	28,296	2,278	598	243	112	70
£25	95,126	68,026	26,889	2,147	568	234	106	68
£100	56,228	43,280	19,881	1,852	513	213	99	65
£200	35,885	28,612	14,386	1,552	454	201	97	63
£500	15,727	13,189	7,577	1,136	384	180	93	59
£1,000	7,336	6,529	4,158	859	334	160	84	53
£1,500	4,512	4,115	2,798	728	304	150	80	4
£2,000	3,030	2,792	1,989	615	275	139	76	49
£3,000	1,529	1,425	1,126	462	234	122	68	44
£4,000	918	876	745	376	211	112	68	44
£5,000	595	570	511	310	184	104	65	42
£6,500	414	407	372	255	160	99	59	40
£8,000	285	279	258	190	133	87	55	40
£10,000	203	203	186	148	112	76	49	38
£15,000	104	104	97	85	74	51	34	28
£20,000	68	68	66	63	57	47	34	28
£25,000	55	55	55	51	49	40	28	25
£30,000	40	40	40	38	36	27	21	19
£40,000	28	28	28	28	27	21	15	13
£50,000	17	17	17	17	17	15	11	9
£65,000	17	17	17	17	17	15	11	9
£80,000	13	13	13	13	13	13	9	8
£100,000	6	6	6	6	6	6	4	2

The above matrices are based on 23,679 Nil claims and 52,643 Non-nil claims forming the actual data for the Base Year as seen at 31/12/87. The values of $M^z(t)$ and $M^{nz}(x,t)$ at intermediate values of x and t are derived from linear interpolation of ln(M) against t and/or ln(x). For this purpose, the device is used of re-assigning the lowest value of x to 1, in order to avoid singularities when taking logs.

For large claims (see paragraph 2.4.3) the model consists of a truncated pareto distribution over the range £80,000 to £420,000 with parameter 1.29. The proportion of large claims is 0.00012. This is slightly different from the proportion shown in the bivariate model above because it is derived from the inspection of a number of years' data as described in paragraph 2.4.3, as are also the truncated limit £420,000 and the pareto parameter 1.29.

FITTING THE "TIME" MAPPINGS (Section 3)

The actual number of claims settled and outstanding for later years is:

ORIGIN YEAR		YEAR OF DEVELOPMENT 0	1	2	3	4	OUTSTANDING
1983	Nil	15,204	7,609	289	13	6	}
	Non-nil	39,052	14,006	964	192	76	} 92
1984	Nil	16,381	9,569	457	35		}
	Non-nil	43,971	16,561	1,299	280		} 217
1985	Nil	18,132	11,025	786			}
	Non-nil	51,167	18,919	1,712			} 725
1986	Nil	18,511	12,885				}
	Non-nil	58,257	21,218				} 4,104
1987	Nil	19,080					}
	Non-nil	62,522					} 46,531

Fitting the p_j and r_{jk} as described in paragraph 3.3 gives the following parameter values:

ORIGIN YEAR	p_j	r_{j1}	r_{j2}	r_{j3}	r_{j4}	r_{j5}
1983	0.7015	1.014	1.982	2.918	3.720	4.486
1984	0.7019	0.981	1.900	2.743	3.569	
1985	0.7073	0.974	1.826	2.638		
1986	0.7195	0.958	1.780			
1987	0.7361	0.906				

which generate fitted data as follows:

ORIGIN YEAR		YEAR OF DEVELOPMENT 0	1	2	3	4	OUTSTANDING
1983	Nil	14,902	7,917	302	10	3	2
	Non-nil	39,515	13,558	931	193	79	91
1984	Nil	16,282	9,702	449	23		6
	Non-nil	44,051	16,430	1,302	302		223
1985	Nil	18,314	10,980	647			46
	Non-nil	50,959	18,964	1,856			704
1986	Nil	19,358	12,016				871
	Non-nil	57,423	22,027				3,280
1987	Nil	19,091					14,717
	Non-nil	62,548					31,777

FITTING THE MONETARY MAPPING (Section 4)

The actual numbers of claims settled in size bands are shown in detail in the Appendix. The numbers shown represent the numbers of claims settled at a cost greater than the corresponding value of x (claim size). In this way, the number of claims settled greater than £0 can be seen to correspond to the number of non-nil claims settled in fitting the time mapping above. Numbers of claims settled within size bands, Q_{ijk}, can easily be obtained by differencing.

Fitting b_{jk} as described in paragraph 4.2 gives the following results:

ORIGIN YEAR	b_{j0}	b_{j1}	b_{j2}	b_{j3}	b_{j4}
1983	1.138	1.091	1.235	1.103	1.113
1984	1.216	1.190	1.216	1.406	
1985	1.327	1.355	1.458		
1986	1.372	1.471			
1987	1.416				

which generate the modelled numbers of claims by size shown in the Appendix.

As described in paragraph 4.4, the fitted b_{jk} are transformed to B_{jk}, which are free of the distorting effect of changes in settlement rates evidence from the r_{jk}. The resultant B_{jk}, using the method described in paragraph 4.4.1, are:

ORIGIN YEAR	B_{j0}	B_{j1}	B_{j2}	B_{j3}	B_{j4}
1983	1.138	1.093	1.228	1.105	1.113
1984	1.216	1.191	1.251	1.406	
1985	1.328	1.363	1.458		
1986	1.376	1.471			
1987	1.416				

Note that each of the B_{jk} in the bottom diagonal corresponds to a fringe group, as discussed in paragraph 4.4.3. The effect of the special considerations applied to fringe groups will be apparent in the section on estimating reserves.

The R_{jk}, discussed in paragraph 4.4.2, define the periods of calendar time covered by groups in the fitted data. Derived from the r_{jk}, they have the following values.

ORIGIN YEAR	R_{j0}	R_{j1}	R_{j2}	R_{j3}	R_{j4}
1983	0.986	2.019	3.103	4.366	5.000
1984	1.020	2.119	3.311	4.000	
1985	1.031	2.214	3.000		
1986	1.051	2.000			
1987	1.000				

ESTIMATING RESERVES (Section 5)

In this example the outstanding B_{jk} have been estimated by assuming a constant rate of 7.5% p.a. for future claims inflation but the following two cases of special treatment should be noted:

(a) *Fringe Groups (see paragraph 4.4.3)*

For 1985–87 the outstanding part of the fringe group has been assigned the same value of B_{jk} as was fitted to the settled part. For 1983–84, however, the settled part has been deemed to be too small to give a reliable indication of the B_{jk} for the whole group and the B_{jk} for the outstanding part has been projected at the assumed rate of inflation from the B_{jk} for the previous year.

(b) *End and Large Claims Groups (see paragraph 2.4)*

The B_{jk} for these groups are projected at the assumed rate of inflation from the Base Year, for which the B_{jk} are automatically equal to 1. A similar consideration also applies to Group 5. The resultant B_{jk} are:

	GROUP							
ORIGIN YEAR	0	1	2	3	4	5	END	LARGE CLAIMS
1983					1.075	1.075	1.075	1.075
1984				1.188	1.156	1.156	1.156	1.156
1985			1.458	1.277	1.242	1.242	1.242	1.242
1986		1.471	1.567	1.373	1.335	1.335	1.335	1.336
1987	1.416	1.581	1.685	1.476	1.436	1.436	1.436	1.436

The corresponding mean claim costs, derived from the model after application of the estimated outstanding B_{jk} given above, are:

	GROUP							
ORIGIN YEAR	0	1	2	3	4	5	END	LARGE CLAIMS
1983					6,229	7,257	15,768	165,539
1984				5,396	6,160	7,802	16,951	177,954
1985			2,653	5,107	6,622	8,387	18,222	191,300
1986		1,174	2,287	5,490	7,119	9,016	19,589	205,648
1987	448	796	2,459	5,902	7,653	9,692	21,058	221,071

Note the effect of the positive correlation between time to settlement and claim size on the outstanding fringe groups' mean claim costs, which are greater than those for the corresponding complete groups. The number of outstanding non-zero claims for each year, as estimated in "Fitting the Time Mapping" above, is assigned to individual outstanding groups pro rata to the corresponding probabilities from the model. This gives the following expected numbers of non-zero claims in each group:

	GROUP							
ORIGIN YEAR	0	1	2	3	4	5	END	LARGE CLAIMS
1983					28	21	34	7
1984				68	75	23	37	9
1985			260	248	89	28	44	10
1986		1,379	1,373	290	104	33	51	12
1987	5,091	24,562	1,585	335	120	38	59	14

Note that for 1983 the probability of being assigned to the Large Claims group has been reduced slightly because of the known settlement of one large claim. The resultant outstanding settlement amounts (product of numbers and averages) are, in £000:

ORIGIN YEAR	GROUP 0	1	2	3	4	5	END	LARGE CLAIMS
1983					177	155	530	1,171
1984				365	460	183	624	1,567
1985			691	1,269	590	234	799	2,007
1986		1,619	3,140	1,594	741	294	1,004	2,522
1987	2,281	19,560	3,898	1,979	920	365	1,246	3,130
Total	2,281	21,179	7,729	5,208	2,887	1,231	5,154	12,451

Aggregating these amounts for each origin year and subtracting payments made on account (see paragraph 5.2.2) gives the following table of claims reserves, in £000:

ORIGIN PERIOD	OUTSTANDING SETTLEMENT COSTS	AMOUNT PAID ON ACCOUNT	RESERVE
1983	2,033	789	1,244
1984	3,199	1,075	2,124
1985	5,590	1,525	4,065
1986	10,915	3,836	7,079
1987	33,380	10,435	22,945
Total	58,120	19,377	38,743

References

REID, D.H.:-

(1) 1978: Claim Reserves in General Insurance (with discussion), *Journal of the Institute of Actuaries*, Vol. 105 Part III pp 211–296, discussion p 297.
(2) 1980: Reserves for outstanding claims in non-life insurance. *Transactions of the International Congress of Actuaries*, Zurich and Lausanne, 2, pp 229–241.
(3) 1981: *Cahiers du CERO*, Vol. 23, 3–4.
(4) 1986: *Insurance: Mathematics and Economics* 5 pp 45–56.

APPENDIX — ACTUAL & MODELLED NUMBERS OF CLAIMS BY SIZE (ACTUAL DATA)

CLAIM SIZE	YEAR OF ORIGIN														
	1983					1984				1985			1986		1987
	YEAR OF DEVELOPMENT					YEAR OF DEVELOPMENT				YEAR OF DEVELOPMENT			YEAR OF DEVELOPMENT		YEAR OF DEVELOP-MENT
	0	1	2	3	4	0	1	2	3	0	1	2	0	1	0
£0	39,052	14,006	964	192	76	43,971	16,561	1,299	280	51,167	18,919	1,712	58,257	21,218	62,522
£25	37,285	13,407	920	181	69	42,140	15,859	1,236	270	49,496	18,284	1,632	56,728	20,627	61,101
£100	20,991	10,066	792	168	62	24,616	12,086	1,071	245	30,599	14,356	1,437	36,973	16,883	42,290
£200	12,506	7,201	658	141	54	14,593	8,841	909	217	18,295	10,732	1,193	21,837	12,710	25,081
£500	5,097	3,735	442	113	46	6,114	4,625	600	170	7,696	5,730	828	8,878	6,714	9,772
£1,000	2,138	1,948	326	96	42	28	211	326	119	64	3,039	581	3,891	3,660	4,312
£1,500	1,235	1,248	261	83	38	1,456	1,546	334	120	1,916	1,966	457	2,298	2,422	2,520
£2,000	802	840	212	74	36	920	1,286	273	105	1,264	1,373	371	1,544	1,697	1,655
£3,000	340	414	138	62	32	417	541	187	83	613	760	247	867	949	908
£4,000	177	236	96	42	28	211	326	119	64	316	388	164	527	577	522
£5,000	88	140	67	34	24	123	204	84	48	206	231	122	336	356	344
£6,500	41	79	42	26	20	68	97	56	41	122	118	70	188	192	183
£8,000	29	43	31	22	16	37	60	42	31	61	69	49	109	106	95
£10,000	14	19	22	18	13	21	34	22	17	33	35	34	57	61	42
£15,000	5	6	11	9	9	4	9	7	8	9	12	10	20	14	15
£20,000	2	1	7	3	5	0	5	6	3	3	3	7	8	4	4
£25,000	1	0	5	3	3	0	3	4	2	2	1	3	4	2	3
£30,000	1	0	5	2	3	0	2	3	0	1	1	3	3	2	2
£40,000	0	0	2	0	2	0	1	2	0	0	1	2	1	1	0
£50,000	0	0	1	0	2	0	1	1	0	0	0	1	1	1	0
£65,000	0	0	0	0	2	0	0	1	0	0	0	1	1	1	0
£80,000	0	0	0	0	1	0	0	1	0	0	0	1	0	1	0
£100.000	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
£150,000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
£200,000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

APPENDIX — ACTUAL & MODELLED NUMBERS OF CLAIMS BY SIZE (MODELLED DATA)

CLAIM SIZE	YEAR OF ORIGIN														
	1983					1984				1985			1986		1987
	YEAR OF DEVELOPMENT					YEAR OF DEVELOPMENT				YEAR OF DEVELOPMENT			YEAR OF DEVELOPMENT		YEAR OF DEVELOP-MENT
	0	1	2	3	4	0	1	2	3	0	1	2	0	1	0
£0	39,052	14,006	964	192	76	43,971	16,561	1,299	280	51,167	18,919	1,712	58,257	21,218	62,522
£25	37,239	13,336	910	181	73	41,972	15,793	1,228	265	48,906	18,080	1,625	55,712	20,304	59,823
£100	21,106	9,913	786	162	66	24,369	11,857	1,045	240	29,505	13,918	1,399	34,067	15,844	36,915
£200	13,014	7,228	666	138	59	15,260	8,786	871	212	18,962	10,642	1,195	22,088	12,336	24,071
£500	5,158	3,723	465	109	49	6,142	4,572	590	164	7,832	5,699	814	9,202	6,710	10,068
£1,000	2,103	1,948	329	91	42	2,540	2,408	405	133	3,321	3,067	558	3,934	3,653	4,316
£1,500	1,159	1,244	264	80	39	1,408	1,549	317	116	1,863	2009	433	2,214	2,417	2,427
£2,000	723	846	222	71	35	888	1,071	260	104	1,196	1,427	357	1,423	1,717	1,558
£3,000	306	425	155	58	29	391	550	175	84	555	766	250	675	962	753
£4,000	144	241	115	50	25	189	315	126	70	279	448	179	342	566	384
£5,000	72	140	88	42	22	96	192	95	61	146	283	135	183	361	208
£6,500	34	77	64	32	20	43	99	66	48	63	144	93	80	194	92
£8,000	20	47	51	24	17	25	63	51	39	36	91	69	45	115	50
£10,000	12	26	30	18	13	15	35	30	29	21	53	49	26	70	28
£15,000	5	8	12	11	9	7	12	11	16	9	18	19	11	24	12
£20,000	2	3	5	6	6	3	5	5	10	5	8	8	6	11	7
£25,000	1	2	3	4	5	1	2	3	6	2	3	5	3	5	3
£30,000	0	1	1	4	4	0	2	1	4	1	2	3	1	2	1
£40,000	0	0	1	3	2	0	1	1	4	0	1	1	0	2	0
£50,000	0	0	1	1	2	0	0	1	3	0	0	1	0	1	0
£65,000	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0
£80,000	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0
£100,000	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
£150,000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
£200.000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0