The Actuarial Profession
making financial sense of the future

CMI – Mortality projections
Issues and questions

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Longevity risks

- Non-diversifiable
- No traded markets in longevity risk, so price not directly observable
- Not easily hedged, though can be offset
- Price for risk is calculated by purchasers (insurance companies)

The current position

- Similarities with 1950s when interest rates very low and below rates used in pricing bases
- Precipitated move from non-profit to with-profit
- Issuers of long-term guarantees based on future longevity in similar position, but now have methods for measure of systemic risk
- Working Party believes a measure of uncertainty should be provided with projections of future mortality rates ….
- …. but users responsible for approach taken in their own circumstances

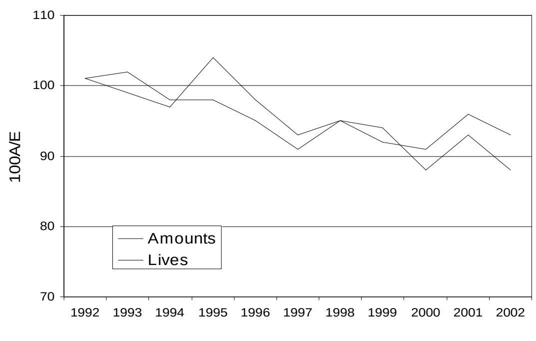
Agenda

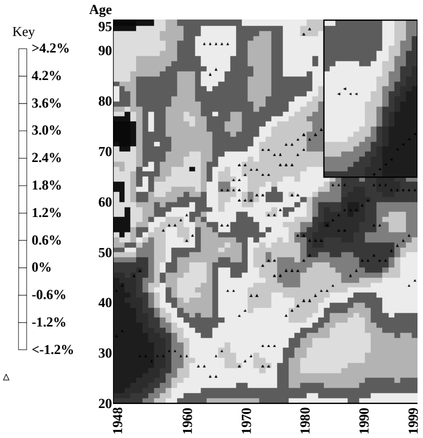
- Reasons for new projections
- Considerations affecting those projections
- Modelling q(x) and what you can do as a result

Reasons for new projections

- Experience for 1999 generally lighter than that projected for 1999 under “92” tables, repeating past history of projections in mortality improvement being too low

Life Office Pensioners 100A/E using the “92” Series projected mortality rates : Males





Past mortality improvements of pensioners and male assured lives

Reasons for new projections

- Experience for 1999 generally lighter than that projected for 1999 under "92" tables, repeating past history of projections in mortality improvement being too low
- Advances in methodologies for projecting mortality
- Need to give some measure of uncertainty

Projection methodologies

- Process-based
- Explanatory-based
- Extrapolative

Process-based methodologies

- Model mortality rates from bio-medical perspective
- Processes causing death need to be understood
- Mathematical models need to be developed
- Not really practical at present….
- ...but could become more relevant in future

Explanatory-based methodologies

- Explanatory links need to be understood
- Underlying economic or environmental factors need to be modelled…
- … not just for short term but for 50+ years
- May provide partial attempts for projecting minimum/maximum improvements (e.g. links with patterns of smoking)

Extrapolative methodologies

- Project historical trends into the future
- Include some subjective element
- Simple extrapolation only reliable to extent that conditions leading to changes in past mortality have similar impact in the future
- Can be invalidated by medical advances or emergence of new diseases

Extrapolative models

- Trend projection – relationship between mortality at different ages often ignored
- Parametric methods – e.g. fitting parameterised curves to past data and projecting trends in parameters forward
- Targeting approach – interpolating between current mortality rates and targets assumed to hold at a given future date

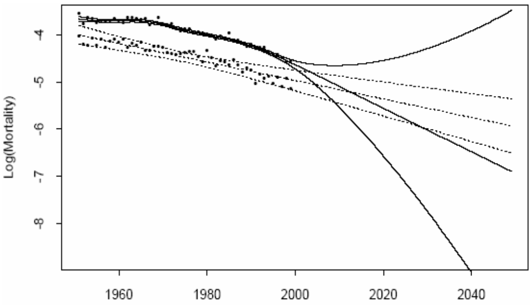
Sources of uncertainty

- Model uncertainty
- Parameter uncertainty
- Stochastic uncertainty
- Measurement error
- Heterogeneity
- Past experience may not be good guide
  (e.g. change in business mix)

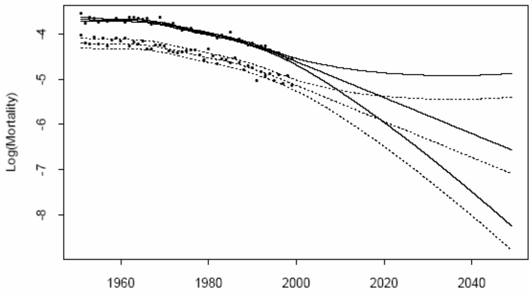
Quantifying uncertainty

- Estimates of parameter uncertainty can be made for regression and time series models, after model has been chosen
- For model uncertainty, can try different models and assess sensitivity of results, but ….
- …. no easy method for providing probabilistic statements on model risk
- A further question is what past data should be used

Fitted and projected model of larger (top) and smaller (bottom) mortality experience. P-spline model with separate smoothing parameters. 95% c.i.s shown.



Fitted and projected model $\log \mu_{65}(t) = a + \log \mu_{60}(t)$ of larger (top) and smaller (bottom) mortality experience. P-spline model with smoothing parameter chosen to favour goodness-of-fit. 95% c.i.s shown.



Projections - conclusions so far

- Will use extrapolative parametric(?) methods
  - E.g. adjusted Lee-Carter and/or P-splines
  - Fitting difficult, over dispersion (shocks)
- Stochastic model(s) will be provided
- COD analyses may be used to “explain” results
- Model uncertainty ignored, problem too big
- Parameter uncertainty, reflected in ci’s
- Data risk, use the largest data sets

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Modelling q_x stochastically

An example

- Consider a £10,000 pa annuity
- Male age 60, PMA92(B=1944)mc, 0%
- … traditional value = £261k
- 50% chance this is too big or too small – 100% chance that it is wrong
- … but used to reserve, calc transfer values etc.

Another way …

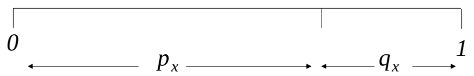
- What size fund will give me 99% certainty that the annuity can be paid?
- … easy calc for one life
- For age 60 just find y such that $\frac{l_y}{l_{60}} = 0.01$
- y = 103.8!
- Fund = (103.8 − 60) × £10k = £438k
- Note that y = 87.5 for 50% and, from the last slide, that a_{60} @ 0% = 26.1
- i.e. (87.5 – 60) ≈ 26.1 - *Modes and medians*

So comparison is

- Pay £261k for the annuity and get 0% chance of insolvency with a 0% chance of surplus
- Or put £261k in fund => 50% chance of insolvency and a 50% chance of surplus
- Or put £438k in fund => 1% chance of insolvency and a 99% chance of surplus
- Call the difference “Risk Capital” = £177k or 68% of the annuity cost.

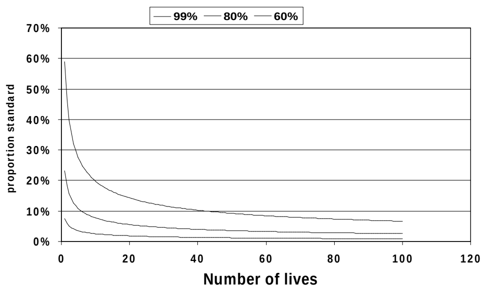
More lives?

- Need a different approach
- … one is stochastic.

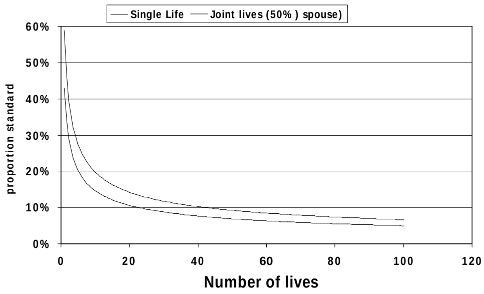


- Run this 1,000 times and order the results
- look for 50% (500th) and 99% (990th) percentiles
- Risk capital for 99th percentile is the difference

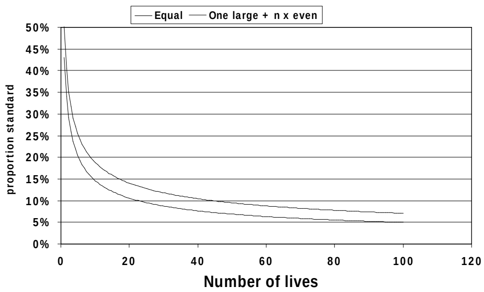
Risk capital by no. of lives



Joint life v single life @ 99%



Comparing RC for uneven sized pensions (Joint lives)



Trend v diversifiable risk

- So far, only dealt with diversifiable risks
- … trend risk is same for all lives, cannot be diversified
- Use a stochastically generated set of $q_{x,t}$ to examine one case (with many lives etc)
- Can then work on many sets of $q_{x,t}$ to look at trend risks
- Stochastic models aggregate these risks

Implications of stochastic mortality modelling

- Diversifiable and non-diversifiable risks and their impact on risk capital
- Use a very simple model to illustrate issues
- Risk capital requirements
- Sources of uncertainty having highest impact
- Practical issues with nested stochastic models

A simple model allowing for trends

$q_x(t)$ = probability of life aged x at start of year t dying in year t

Then $q_x(t) = q_x(t\text{-}1) * [1\text{-}Imp(t)]$

$Imp(t) = X(t) + Y_x(t)$

Where $X(t)$ is the trend and $Y_x(t)$ is variations by age

$X(t) = X(t-1) + \sigma_x Z(t)$

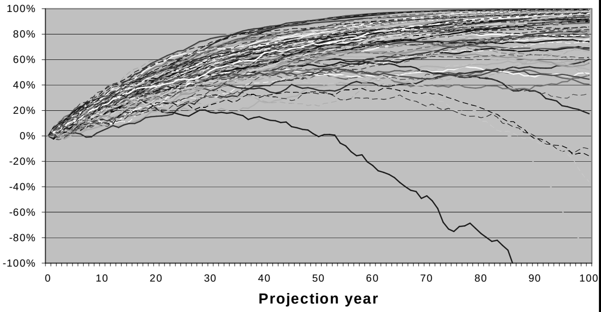
$Y_x(t) = \sigma_Y Z_x(t)$

$Z(t)$ is a random variable distributed as $N(0,1)$
and σ_x and σ_Y are the sds in X(t) and Y_x(t) respectively

Calibration of model

- $q_x(0)$ = PML92C1992
- $X(0)$ = 2.50%, *(the initial trend)*
- σ_X = 0.25%, *(the s.d. of the trend)*
- σ_Y = 2.00%, *(the s.d. of variation by age)*

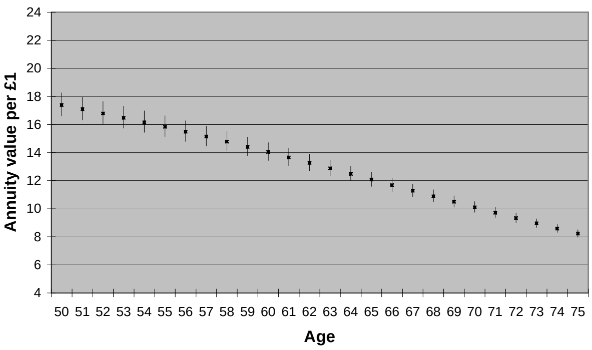
Cumulative mortality improvements = $(1 - q_{60}(t) / q_{60}(0))$ aged 60 at t = 0, 1,000 simulations



Start with non-diversifiable risk only

- Project mortality improvements
- Assume very large homogenous population
- Assume that annuity amounts are same for all annuitants

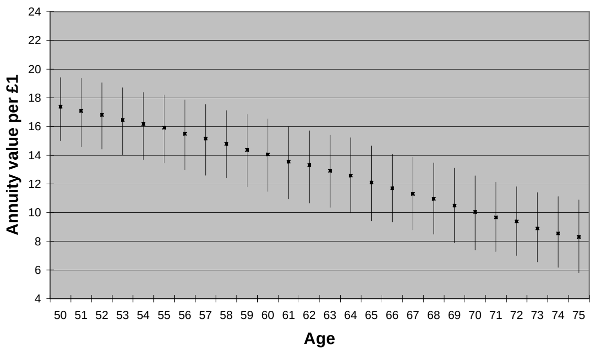
Homogeneous population, infinite number of lives, 95% CIs



Introduce diversifiable risks

- Start with risk arising from small population
- Add risk from non-homogenous population
- Add risk from different annuity amounts
- Issue - Nested models?
  - Can be avoided by assuming independence between diversifiable and non-diversifiable risks
  - Just do more un-nested simulations!

Homogeneous population, variety of ages, 220 lives, 95% CIs

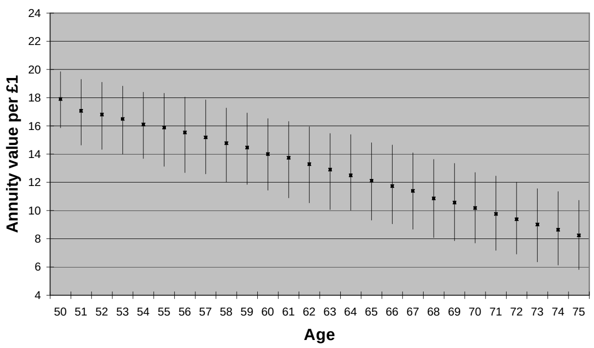


Introduce heterogeneity

- Assume 4 different sub-groups
- Average mortality of portfolio remains the same
- Assume that mortality improvements same for all the groups

	Group 1	Group 2	Group 3	Group 4
Proportion of portfolio by lives	10%	15%	35%	40%
Base mortality %PMA92C1992	67.5%	85.0%	110.0%	105.0%

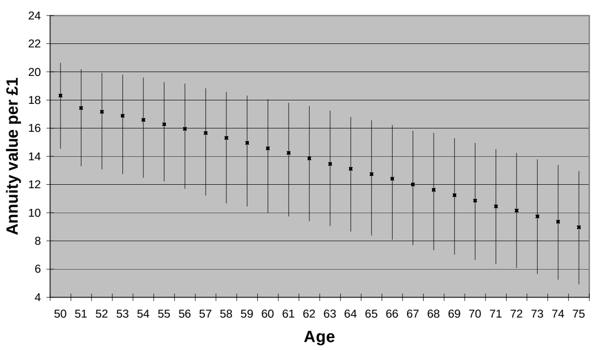
Heterogeneous population, same pension, 220 lives, 95% CIs



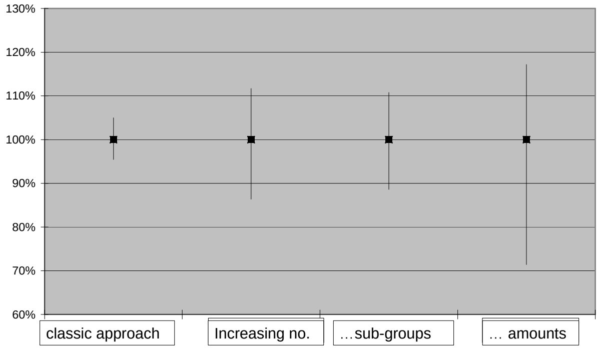
The sub-groups

	Group 1	Group 2	Group 3	Group 4
Proportion of portfolio by lives	10%	15%	35%	40%
Base mortality %PMA92C1992	67.5%	85.0%	110.0%	105.0%
Average annuity amount £	13,000	11,000	6,500	4,000
Proportion of portfolio by £	19%	24%	33%	23%

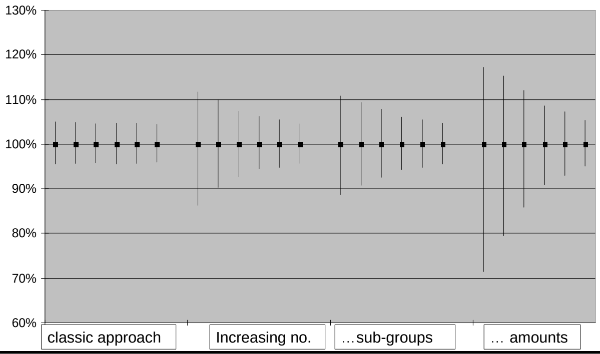
Heterogeneous population, varying pensions, 220 lives, 95% CIs



Risk Capital - 220 lives, varying diversifiable risks



Risk Capital – impact of varying diversifiable risks as number of lives increase (220 to 22,000)



Implications

- For smaller portfolios, the risk capital can be high
- Heterogeneity can be diversified away for large portfolios (perhaps)
- Need a portfolio of 20,000 plus lives to minimise costs of diversifiable risk – assuming that all sub-groups experience same improvements!
- Worthwhile for small to mid-size pension schemes to insure.

Questions

- Are these risks different for
  - Members – (Heterogeneous, cross subsidy, passed to scheme)
  - Company – (can merge schemes, insure, industry scheme)
  - Shareholders – Diversify risk across companies?
- Do you pay risk capital for?
  - Bulk buy-outs
  - Transfer values
  - Scheme mergers
- What about other risks?

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