Continuous Mortality Investigation

Working Paper 27

The "library" of Mortality Projections

July 2007

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1 Introduction

The CMI has in recent years incorporated projections of future mortality into its published mortality tables that have been extensively used by UK actuaries in pricing and valuing life insurance and pension scheme risks.

During its work on the “00” Series tables, the CMI undertook extensive research into mortality projections but came to the conclusion that it was unable to present a single view of the future, as had been attempted with preceding mortality tables. The final “00” Series tables adopted by the UK Actuarial Profession with effect from 1 September 2006 did not contain any projections. It soon became clear that the absence of projections left a gap that has caused much debate in recent months, both within the Profession and between the Profession and interested external stakeholders.

The CMI - and the Actuarial Profession as a whole - recognises the need to make the CMI’s recent work more accessible to actuaries. As a result, the CMI formed a Task Force which it hoped would:

- Gather users' perspectives on how they actually use mortality projections and associated measures of uncertainty;
- Interpret the CMI's recent research and recommend tools or education (e.g. a seminar or workshops) in order to make the research more accessible to actuaries;
- Propose terminology that will facilitate disclosure of mortality projections where this is required; and
- Seek to develop sets of projections which can be used as benchmarks.

The membership of the Task Force is: Gordon Sharp (Chair), Richard Humble, Angus Macdonald, George Russell, Andrew Walton, Richard Willets and Brian Wilson, with Dave Grimshaw as Secretary.

The Task Force hopes that this Working Paper, and the accompanying draft library of projections, is a major step towards achieving its objectives and invites feedback on the extent to which it meets actuaries’ needs.

We have included a number of specific questions on which we would appreciate feedback. These are indicated by shaded boxes. However comments are invited on all aspects of this Working Paper and the accompanying draft library of mortality projections. Any comments should be submitted via e-mail to projections@cmib.org.uk or in writing to: Dave Grimshaw, CMI, Cheapside House, 138 Cheapside, London, EC2V 6BW.

Comments should be received by **FRIDAY 17 AUGUST 2007** to be considered for the initial library which the CMI then intends to publish.

It remains the responsibility of any actuary or other person using a projection of future mortality to ensure that it is appropriate for the particular purpose to which it is put, regardless of whether the projection is contained within the library.

2 Background

Recent sets of mortality tables produced by the CMI and published by the Actuarial Profession have incorporated projections of future mortality for annuitants and pensioners. When the "92" Series tables were published, a single projection basis was incorporated, giving a single view of the future. These projections were principally based on analyses of past trends in the various CMI investigations and in the wider population.

The "92" Series projections were quickly found to understate the level of mortality improvements that were actually occurring in the subsequent CMI experience, as had tended to happen with previous projections. In addition, evidence had emerged of a "cohort effect", present in both population and CMI data. This effect appeared to show that a group of lives (born around the late 1920s / early 1930s) had experienced even more rapid improvements in mortality during the 1980s and 1990s than other generations. Given these people had recently reached (or would soon reach) retirement age, this had considerable significance for pension and annuity business, if actuaries had continued to use the latest published tables.

The CMI established the Mortality Projections Working Party (MPWP) to undertake research in this area and, in particular, to explore possible projection methodologies for use with the “00” Series tables. As a first step, in December 2002, the CMI published Working Paper 1, containing the "interim cohort projections". These reflected actual improvements in mortality to 1999 for this cohort and offered actuaries a choice of three projection bases, with no indication of which one actuaries should use (if any). They were not intended to carry any probabilistic interpretation; each simply offered an ad hoc adjustment to the original "92" Series projections that depended on how long a period of time this particular "cohort effect" was assumed to persist. The use of the term "interim" in the name was intended to reflect the ad hoc nature of these projections and that the CMI would undertake further work in this area.

Throughout its work, the MPWP has tried to involve the profession in the process, using a series of Working Papers (numbers 3, 11, 15, 20 and 25), seminars and the release of illustrative software accompanied by workshops.

The publication of Working Paper 25 on the Lee-Carter method completed the work of the Mortality Projections Working Party. The conclusions of the MPWP are set out in Working Paper 25. In brief, it found that both the Lee-Carter and P-spline methods (which had been considered in Working Paper 20) have relative benefits and drawbacks. Furthermore the projections each produces can appear reasonable, or not, depending on the choice of dataset and the age-range and period used, as well as how the model is parameterised. Neither can be regarded as the final answer in this area.

Working Paper 25 also briefly considered a modified version of the Lee-Carter model developed by Renshaw & Haberman. The MPWP concluded that the Lee-Carter Age-Period-Cohort (APC) model seems to produce better fits to UK data than the basic Lee-Carter model and preserves cohort effects, but that further testing is necessary before its suitability can be properly judged. No projections using this model have been included in the draft library and the CMI hopes that the Profession will encourage further research in this area.

3 The draft library of projections

Considerable work has been undertaken in the area of mortality projections, much of it of a highly technical nature. The CMI is publishing this Working Paper and the accompanying draft library of projections in the hope that it will result in a single reference source for much of this work for use by actuaries.

It also aims to establish a well-defined vocabulary for mortality projections; the need for this arises, for example, from:

- Scheme Funding discussions between employers and trustees, and
- Life offices' communications with rating agencies, analysts, shareholders and others.

The CMI hopes that each of the projections within the draft library is sufficiently well-defined that it can be uniquely identified. In addition within this document we seek to indicate where divergences from these projections need to be disclosed, for clarity, and in some cases suggest how this should be done.

It is very important to note that none of the projections is recommended for any particular situation and their inclusion in the library does not imply suitability.

Provision of the library will not take away the need for individual actuaries to use their judgement and make recommendations best suited to the firm or scheme.

A spreadsheet has been published alongside this Working Paper containing the draft library of projections. The projections in the draft library are summarised in the table in Appendix A. This section of the Working Paper seeks to explain how they can be used. More details on the derivation of the different projections are set out in subsequent sections of this paper.

The CMI does not intend that these projections should form part of the "00" Series mortality tables. Each of the projections contained within the draft library could – in theory – be used with any assumption of base mortality, i.e. projections are not uniquely associated with a particular base table as was the case with projections such as those contained within the "92" Series tables. It is, though, for the actuary to ensure the suitability of any particular projection in conjunction with the particular base table that is used as the starting point for a projection.

Each sheet within the spreadsheet contains a different projection (except the first page entitled "Notes"). The following points apply to all these projections:

- Each sheet contains a two-way table of cumulative mortality reduction factors, by age and calendar year.
- These cumulative reduction factors can be defined as:

  $RF(x,t) = q_{x,t} / q_{x,0}$

  where x is the age, t is the elapsed time from 1992.
- Thus each sheet starts from values of 100% in 1992 and subsequent columns show the cumulative reduction factor to the year in question.
- The improvements between 1992 and 2005 in each sheet are a mixture of projected values and actual values, as follows:
  - Where the Base Year of the projection equals 1992 (e.g. the Original "92" Series and Cohort projections) then all of the figures are projections.

  - For other projections where the Base Year is later than 1992 (e.g. P-spline projections using data to 2004) then the figures between 1992 and the Base Year are smoothed actual improvements, with the smoothing coming from the relevant model, except for the ONS Population Projections where we have included smoothed actual improvements using a P-spline age-cohort model for the period between 1992 and 2004 (this is discussed further in section 4).
- Actual smoothed improvements are indicated by shading within the draft library itself.
- In all cases, the projections in the draft library are shown to 2100, regardless of the length of the projection period used to derive the projection.

Naming convention
One of the aims of the library is to produce a standardised terminology for use between actuaries. The projections included in the library are not intended to include every projection that an actuary might consider it appropriate to use, nor does it seek to prescribe methods by which projections should be derived. However it is intended that if the naming convention is used, as a form of shorthand descriptor, then the projection should be used as set out in the library and in this document, or calculated in a consistent manner where indicated. Any departure from this should be specifically noted.

In an attempt to keep the proposed names brief, the names assigned to the P-spline and Lee-Carter projections intentionally do not include all aspects of the derivation of the projection. For example, the names of these projections do not currently state the age range that has been used; however it is intended that if projections are produced using a different age range to that indicated in the library, this would need to be specifically disclosed.

Q3.1 Do you agree that a defined naming convention is a desirable feature of the library? If not, please state why.

Q3.2 Do you agree with the naming convention adopted for the draft library? If not please state suggested changes, with reasons.

Age and year definitions
For each projection, "age" is defined as "age exact" as in base tables of mortality produced by the CMI. There is no precise definition of the calendar period to which CMI base tables relate. The "00" Series tables, for example, are based on data from calendar years 1999 to 2002. The actual point to which mortality rates graduated from this dataset apply depends on how data volumes are spread over the quadrennium and how experience varies over the quadrennium. However in order that the projections contained in the library can be used consistently, we have assumed that the mortality rates apply to lives attaining each particular age x at 30 June 2000.

A consistent approach should be taken with earlier CMI-produced tables, such as the "92" Series. If an actuary is using a base mortality assumption derived from other than a CMI table, they will need to have due regard to the definition of that table with regard to age and calendar year, but should always convert it to "age exact at 30 June" if it is then being projected using a projection from the library, or based on one from the library.

As an example, if one applies the medium cohort projection (sheet 4) to a base mortality assumption of 100% PNML00, then the generated mortality rates for a male aged 65 exact at 30/6/2000 would be:

Age	Year	Derivation	Rate
65	30/6/2000 – 30/6/2001	“00” Series tables based on age exact and assumed to relate to 30/6/2000, hence q_{65} at 30/6/2000 can be read from the table as q_{65} = 0.012853	0.012853
66	30/6/2001 – 30/6/2002	Base table value of q_{66} taken to be 0.014141; Improvement from 30/6/2000 to 30/6/2001 = 65.6255/68.4657 = 4.1484%; Adjusted value of q_{66} = 0.014141 * (1 – 0.041484)	0.013554
67	30/6/2002 – 30/6/2003	Base table value of q_{67} assumed to be 0.015689; Improvement from 30/6/2000 to 30/6/2002 = 62.2531/67.7614 = 8.1290%; Adjusted value of q_{67} = 0.015689 * (1 – 0.08129)	0.014414

(NB we have followed the CMI convention that mortality rates are rounded to 6 d.p.)

If mortality rates at age 65 are required as at 31 December 2000, for example, rather than at 30 June 2000 then it is necessary to incorporate an allowance for improvements during that half-year and the derivation of the rate at age 65 will become:

- “00” Series tables based on age exact and assumed to relate to 30/6/2000;
- Need to allow for improvements for half-a-year between 30/6/2000 and 31/12/2000;
- Improvement from 30/6/2000 to 30/6/2001 at age 65 = 65.6255/68.4657 = 4.1484%;
- Improvement from 30/6/2000 to 31/12/2000 assumed to be 1 - [(1 – 0.041484) ^ (184 / 365)] = 2.1132%;
- Hence q_{65} at 31/12/2000 can be estimated as q_{65} * (1-0.021132) = 0.125810.

Limiting Age

It has been the practice within recent CMI mortality tables to assume a limiting age of 120, i.e. that q_{120} = 1. There is very little data (within either the CMI or ONS datasets) to justify this practice explicitly, although the rarity to date of survivors beyond that age is perhaps justification in itself for base mortality assumptions.

This is a very convenient assumption, for practical purposes, and has been retained for all the projections within the draft library. However it is important to recognise that there is less justification for this assumption when future mortality improvements are taken into account, especially for example if considering a high-improvement scenario within a stress test. Actuaries should therefore consider whether it is appropriate to retain this assumption in their particular situation.

Differential smoking status

It is common practice to differentiate between smokers and non-smokers for certain assurances and similar practice is now being applied to annuity pricing. All of the projections within the draft library have been derived from data that is not differentiated by smoker status and actuaries will need to give additional consideration to whether modification is required for smoker-differentiated business. Similar considerations also apply in respect of substandard lives, especially if these constitute a significant part of the portfolio.

4 Previously-published tables of projections

The original "92" Series

Full details of the projections that were incorporated in the "92" Series tables are contained in section 6 of CMI Report No. 17.

In brief, the Committee sought to reflect recent trends in observed experience, with particular attention to the period 1975-1994. Despite differences between the various CMI investigations, it was decided to use a single projection. In particular this applied to females as well as males, even though no clear pattern could be discerned in recent female improvements.

The model adopted to allow for mortality improvement was essentially the same as that used for the "80" Series tables (see section 4.3 of CMI Report No. 10) whereby at each age the rate of mortality is assumed to decrease exponentially to a limiting value. For the "92" Series, the speed of convergence to the limit depended on age (in contrast to the "80" Series).

The model assumed that the long-term rate of mortality at each age will be a percentage of the rate in 1992, with the percentage equal to 13% at ages up to and including 60, 100% at ages 110 and over, and increasing linearly between.

In addition, the model assumed that a fraction of the total fall in the rate of mortality at each age will occur in the first 20 years. This fraction was set to 0.55 for ages up to and including 60, 0.29 at age 110, and reducing linearly between.

These values were chosen as a 'best fit' to male experience over 1975-1994, although the choice of age 110, above which there were no increases, was arbitrary.

The Interim Cohort Projections

Full details of these projections are contained in CMI Working Paper 1, published in 2002.

The "92" Series projections were quickly found to understate the level of mortality improvements that were actually occurring in the CMI experience and evidence had emerged of a "cohort effect", present in both population and CMI data. The CMI responded by publishing Working Paper 1, containing the "interim cohort projections" late in 2002.

Based on improvements in mortality to 1999, these tables offered an ad hoc adjustment to the original "92" Series projections. Key points in these adjustments are:

- The adjustment was in respect of one cohort only, born either side of 1926.
- This cohort was assumed to exhibit a faster rate of improvement than the original "92" Series projections for an arbitrary period – to 2010 for the "Short Cohort" projection, 2020 for the "Medium Cohort" projection and 2040 for the "Long Cohort" projection.
- The annual rates of improvement from 1993-1999 were based on smoothed actual rates of improvement during that period.
- From 2001, the improvement rates were assumed to reduce linearly to zero at the end of the cohort period.
- The rates of improvement were subject to minimum values of the improvements in the original "92" Series.

- Initially the cohort was taken to include years of birth between 1910 and 1942. After 2000, the 'width' of the cohort effect was reduced so that by the end of the cohort period it included only one year, which relates to lives born in 1926.

ONS 2004-based National Population Projections[1]

More details of these projections are contained in "National population projections 2004-based". This publication also contains useful background on recent trends in population mortality.

Key points underlying the approach to future improvements in mortality within the 2004-based population projections are:

- It is assumed that current rates of improvements converge by age and tend to a long-term "target" rate of improvement over the first 25 years of the projections (i.e. to 2029).
- For the principal projections, this long-term target is 1% p.a. applicable to m_x for all ages, for both genders and the different countries of the UK; broadly equivalent to the average annual rate of improvement over the whole of the 20th century.
- The transition from the assumed rates of mortality improvement by age and gender for the first year of the projection to the target rate is more rapid at first for males, and less rapid for females. (These transitions are illustrated in Table 7.2 of the "National population projections" paper and partially reproduced below. Note that for males, these figures apply only to England, Wales and Northern Ireland and that different transition rates are applicable to Scotland.)

Assumed percentage reduction in central death rates, m_x, for selected ages between selected consecutive calendar years in the projection period and the total reduction over 25 years

Age	2004-05	2011-12	2021-22	2028-29	Reduction over 25 years
Males (England, Wales and Northern Ireland)					
22	**3.31**	**2.38**	**1.36**	**1.00**	**38.7**
32	**1.86**	**1.52**	**1.14**	**1.00**	**28.8**
42	**1.48**	**1.28**	**1.08**	**1.00**	**25.9**
52	**0.80**	**0.75**	**0.93**	**1.00**	**16.0**
62	**1.87**	**2.19**	**0.93**	**1.00**	**28.5**
72	**5.01**	**2.31**	**1.32**	**1.00**	**41.3**
82	**3.22**	**2.86**	**1.35**	**1.00**	**41.2**
92	**1.47**	**2.25**	**1.49**	**1.00**	**33.7**
Females (UK)					
22	**2.47**	**2.15**	**1.62**	**1.00**	**37.5**
32	**0.58**	**0.67**	**0.82**	**1.00**	**17.3**
42	**1.97**	**1.76**	**1.41**	**1.00**	**32.6**
52	**1.42**	**0.83**	**0.91**	**1.00**	**19.7**
62	**1.30**	**1.81**	**0.91**	**1.00**	**25.5**
72	**4.37**	**2.07**	**1.44**	**1.00**	**39.5**
82	**2.01**	**2.61**	**1.58**	**1.00**	**40.6**
92	**0.30**	**1.56**	**1.87**	**1.00**	**30.1**

[1] Following the Government's acceptance of the recommendations of the Morris review, responsibility for the production of the official population projections for the UK and its constituent countries was transferred from the Government Actuary's Department (GAD) to the Office for National Statistics (ONS) with effect from 31 January 2006.

- Cohort effects are recognised in that the transitions for those born before 1960 (i.e. those shaded in the table above) have been projected by cohort, that is, diagonally downwards in the table.
- For generations born since 1960 (not shaded), there is little evidence of generation effects for these cohorts to date and the transitions in mortality rates have therefore been projected by calendar year, that is, horizontally in the table.
- The initial rates of mortality improvement by age and gender for 2004 have been estimated by analysing past data. The initial rates of improvement for ages 90 and over should be regarded as less 'robust' than those for younger ages because:
  - single year of age population estimates are not available for ages 90 and over so historical mortality rates at these oldest ages have to be estimated, and
  - the resulting estimated initial rates of improvement at ages 90 and over have been further adjusted to ensure that the future mortality rates produced from them look plausible compared to those for younger ages, and between males and females.
- "Variant" projections are also prepared, where the long-term target is 2% p.a. or 0% p.a. These are referred to as "High life expectancy" and "Low life expectancy" projections. As the "National population projections" paper states "These are intended as plausible alternative scenarios and not to represent upper or lower limits…" Adjustments were also made to the assumed rates of improvement in 2004-5 for these variants to reflect uncertainty about current rates of improvement.

The ONS 2004-based projections included in the draft library relate to:
- Males (England, Wales and Northern Ireland only) and Females (UK); and
- Principal, High life expectancy and Low life expectancy projections

These projections have not previously been published in age- and year-specific form and the CMI is grateful to the ONS for its permission to include these within the draft library.

In order that the ONS projections can be used with the "92" or "00" Series tables, if desired, we needed to include values up to the Base Year of 2004. As noted earlier, in section 3, we have included smoothed actual improvements using a P-spline age-cohort model in the draft library for the period 1992-2004.

Although this introduces subjectivity in the choice of a smoothing method, the task force felt this was preferable to using unsmoothed improvements during this period. Because unsmoothed improvements are age- and year-specific, they are relatively volatile and the application of the ONS projections preserved this volatility throughout the period of the projection. We felt this to be a very undesirable feature as annuity values at any particular age would not reflect the position at adjacent ages.

Note also that:
- The target rates used in the 2004-based projections after 2029 apply to improvements in m_x whereas we have expressed improvements in the draft library in the form of improvements in q_x. The improvements in the library will therefore be slightly lower than the target rates, with the difference increasing with age.
- As the ONS dataset of historical mortality made available to the CMI included age-specific mortality rates up to age 89 only, we have assumed that the average of the improvements at ages 87-89 apply to all ages from 90 to 119 inclusive for the period 1992-2004.

- The relevant sheets in the draft library contain (smoothed) actual improvements from 1992-2004 but the 2004-based projections were constructed before actual data to 2004 was available and a different smoothing model was used to analyse the data. This may give rise to discontinuities around 2004.

Q4.1 Are there any other previously-published tables of projections that should be included in the library for use in the UK? If so, please state which tables, with references, and explain why these may be useful.

Q4.2 Do you agree with the use of smoothed improvements for 1992-2004 being appended to the ONS projections in the library? Is the P-spline age-cohort model an acceptable choice of smoothing model?

5 Adjusted Cohort Projections

In the absence of any formal successor to the Interim Cohort Projections, some actuaries have modified these projections to make them more suitable for their use. This is entirely appropriate.

One consequence of the informal application of such modifications is that they are not necessarily undertaken in a consistent manner. The CMI has therefore included some variations that it understands are currently being used within the draft library to try to establish consistency of practice. As with other projections within the draft library, their inclusion should not be taken to infer that they are in any way recommended by the CMI.

Applying a minimum value

This modification seeks to apply a minimum improvement rate at all ages and calendar years to the mortality improvements in the Interim Cohort Projections. In their end-2005 and end-2006 FSA Returns a number of UK insurance companies adopted such an approach, using a variety of different minimum values.

Within the draft library we have included a single illustrative projection to an otherwise unadjusted cohort projection – based on applying a 1.00% minimum improvement rate to the q_x from the Medium Cohort projection. This should not be taken to imply that 1% is a recommended minimum. Other minima can be used, denoted by changing the value in the name of the projection, but should be calculated in a consistent manner to the example unless specifically noted otherwise.

Imposing a minimum value is relatively straightforward at most ages. From the cumulative improvement rates for the original projection, derive the annual rate of improvement for each age and calendar year. Any rates below the required minimum are replaced with the minimum value and the cumulative improvement rates are then re-calculated.

However the imposition of a minimum value to the cohort projections could be done in a variety of ways at older ages, although the overall financial impact of the different approaches is unlikely to be material. This arises because the original “92” Series projections (and, in most cases, the interim cohort projections) assume no improvements above age 110. Hence this assumption could be retained, even if the minimum improvement is applied elsewhere. If this is not done, then consideration of the limiting age is required. In many cases the underlying tables (and certainly those published recently by the CMI) use a limiting age of 120, as noted in section 3. Applying improvements to q_{120} will extend the table beyond that age and this may cause systems issues.

For the purposes of the illustrative projection in the draft library, the CMI has assumed that the minimum value does apply above age 110 but that the limiting age of 120 is retained. If users state that they are applying a different minimum value to a cohort projection, they should either do so in a consistent manner or explicitly state the approach they have adopted.

Using a percentage of the cohort projections
This modification uses a percentage of the mortality improvements in the Interim Cohort Projections.

Within the draft library we have included a single illustrative projection – based on using 90% of the Medium Cohort projection. This should not be taken to imply that 90% is a recommended adjustment. Other figures can be used, to adjust the relevant cohort projection up or down, but should be applied in a consistent manner to the example and can be denoted by changing the value in the name of the projection.

For the purposes of the illustrative projection in the draft library, the CMI has assumed that the approach to applying the percentage is as follows. From the original projection, derive the annual rate of improvement for each age and calendar year. Apply the required percentage and the cumulative improvement rates are then re-calculated.

Note that this approach applies the relevant percentage to all of the improvement rates within the projection, not just those rates that were uplifted by the Interim Cohort Projections from the original "92" Series projections.

Unlike the imposition of a minimum value to the cohort projections (see preceding section), the application of a percentage does not give rise to particular issues at older ages, as applying a percentage maintains the assumptions of no improvements above age 110 and the limiting age of 120.

Blending two cohort projections
This modification uses a mixture of the mortality improvements in two of the Interim Cohort Projections.

Within the draft library we have included a single illustrative projection – based on using an average of the Medium Cohort projection and the Long Cohort projection. Other mixtures can be used but should be applied in a consistent manner to the example and can be denoted by changing the name of the projection.

For the purposes of the illustrative projection in the draft library, the CMI has assumed that this modification is applied by deriving the annual rate of improvement for each age and calendar year for each of the original projections, averaging these and then re-calculating the cumulative improvement rates.

Note that this approach (like the application of a percentage) does not give rise to particular issues at older ages.

Blending two cohort projections and applying a minimum value
For the avoidance of doubt, the draft library includes an example of a minimum value (1.5% p.a.) applied to an average of the Medium Cohort projection and the Long Cohort projection. This has been calculated assuming that the blending of the projections is undertaken BEFORE the minimum is applied. Any divergence from this practice should be specifically disclosed.

We have included one example of each variation in the draft library to illustrate their application:

Q5.1 ***Do you see benefit in including additional examples of each variation? If so, please state what examples and explain why these are needed.***

Q5.2 ***Do any additional examples need to be included in the library, or within the CMI Tables Program?***

Q5.3 ***Are there other variations to the cohort projections that are currently being used that might be suitable for inclusion within the library? Please provide full details.***

Q5.4 ***Do you disagree with the proposed method for applying any of these variations? If so, please explain your alternative approach with reasons, if possible.***

Q5.5 ***In particular, should a minimum value be applied to*** m_x ***for consistency with ONS projections, rather than to*** q_x?

6 P-spline projections

More details of the Penalised Spline (or P-spline) projection methodology are contained in Working Paper 15 and Working Paper 20:

- Working Paper 15 sets out the CMI Mortality Projections Working Party's work towards developing stochastic methodologies. Section 2.3 gives a brief description of the P-spline model.
- Working Paper 20 provides practical advice on using the P-spline model, gives examples based on the P-spline methodology and discusses various features of the model.

Both papers contain further useful references.

Key points to note regarding the P-spline model are summarised below:

- The P-spline model is an example of a non-parametric smoothing model. It is a local model that fits cubic splines to the data, and was used to model the CMI Assured Lives dataset in CMI Working Paper 1 that introduced the Interim Cohort Projections.
- A 2-dimensional model can be fitted to mortality data using either the age and calendar year (age-period) dimensions or the age and year of birth (age-cohort) dimensions.
- Coefficients of the model are selected using a maximum likelihood approach subject to a penalty being imposed. The penalty acts to ensure that there is an appropriate balance between the level of smoothness and goodness of fit.
- The use of the penalty also enables the model to be used to generate projections, extrapolating recent trends in the data.
- P-spline age-period and age-cohort models are both able to identify cohort effects, if they exist, in the region of the data. However, the age-period model will only project the stronger cohort effects into the future. Examples of cohort features in projections using the age-period and age-cohort models are shown in Appendix E of Working Paper 20.
- The P-spline model generates standard deviations which can be used to generate percentiles to reflect parameter uncertainty. This is considered further in section 9.

P-spline projections included in the draft library

A number of applications of the P-spline model are included in the draft library. These illustrate the impact of using:

- Age-period and age-cohort versions of the model;
- CMI and ONS datasets for males. For females only the ONS dataset has been used;
- Data to 2003, 2004 and 2005, thus illustrating the impact of adding an additional year's data.

All of the projections have been generated using the CMI's illustrative software and in all cases the 50^{th} percentile projection has been included in the draft library. This can be considered as a best estimate from the model.

Further details of the method and parameters used to generate the projections are contained in Appendix B.

Q6.1 The naming convention does not fully determine the projection for the P-spline model. Is it sufficient? If not, what other features of the fit do you think need to be included within the name?

Q6.2 Are there other variations on P-spline projections that should be included in the library? If so, please state which projections and explain why these are needed.

7 Lee-Carter projections

More details of the Lee-Carter projection methodology are contained in Working Paper 15 and Working Paper 25:

- Working Paper 15 sets out the CMI Mortality Projections Working Party's work towards developing stochastic methodologies. Section 2.2 gives a brief description of the Lee-Carter model.
- Working Paper 25 provides practical advice on using the Lee-Carter model, gives examples based on the Lee-Carter methodology and discusses various features of the model.

Both papers contain further useful references.

Key points to note regarding the Lee-Carter model are summarised below:

- The Lee-Carter model is a bilinear model in age (x) and time (t) of the following form:
  $$\log \mu(x, t) = a(x) + b(x)\, k(t) + z(x, t)$$
- The force of mortality, $\mu(x, t)$, in the region of the data is derived by fitting the model to the mortality data and obtaining estimates of the parameters. The components of the model describe:
  - the average level of mortality over time for a particular age, $a(x)$;
  - the overall change in mortality over time, $k(t)$;
  - the pattern of deviations by age from the overall level of changes in mortality, $b(x)$; and
  - the random errors (stochastic innovations), $z(x, t)$.
- The parameters are selected to fit the model to the data using a maximum likelihood approach. To achieve a unique choice of parameters, some constraints on the parameters are required. These are usually $\sum_x b(x) = 1$ and $\sum_t k(t) = 0$.
- Projected $\mu(x, t)$ are obtained by projecting $k(t)$ forward. If this is done by fitting a time-series model, such as an ARIMA (Auto-Regressive Integrated Moving Average) process, then stochastic projections are generated.
- If the stochastic error is excluded, then a unique central projection of the average projected $\mu(x, t)$ is generated. This is the method that has been used to generate the projections in the draft library.
- Allowing for the stochastic error will generate sample paths for the projected $\mu(x, t)$. These are random unless the generation is controlled, by using a non-random seed. As the number of scenarios increases the mean of the projected mortality rates will tend towards the central projection.
- Generating $\mu(x, t)$ in this way has no regard for parameter risk. This can be introduced using a technique known as parametric bootstrapping (see Appendix C for a brief description) and generating a number of synthetic datasets. Each synthetic dataset is used as a basis for a simulation of $\mu(x, t)$.
- The Lee-Carter model does not smooth the volatility in mortality rates across calendar years to the same extent as the P-Spline model. This may make it more difficult to identify features in the region of the data and the structure of the model means that cohort features are not projected into the future.

Lee-Carter projections included in the draft library

A number of applications of the Lee-Carter model are included in the draft library. As for the P-spline projections, these illustrate the impact of using:

- CMI and ONS datasets for males. For females, only the ONS dataset has been used;
- Data to 2003, 2004 and 2005, and thus illustrate the impact of adding an additional year's data.

All of the projections have been generated using the CMI's illustrative software and in all cases the central projection has been included in the draft library. This can be considered as a best estimate from the model and is generated without any allowance for uncertainty. This is considered further, along with illustrations of allowance for some of the uncertainty inherent in any projection of future mortality, in section 9.

Further details of the method by which the projections included in the draft library have been generated is summarised in Appendix C.

Q7.1 The naming convention does not fully determine the projection for the Lee-Carter model. Is it sufficient? If not what other features of the fit do you think need to be included within the name?

Q7.2 Are there other variations on Lee-Carter projections that should be included in the library? If so, please state which projections and explain why these are needed.

8 Illustrating the choice of projection

Where an actuary needs, or chooses, to disclose the projection used they may find it useful to refer to the shorthand names adopted within the draft library. Where the projection from the draft library has been modified in any way, this should also be disclosed.

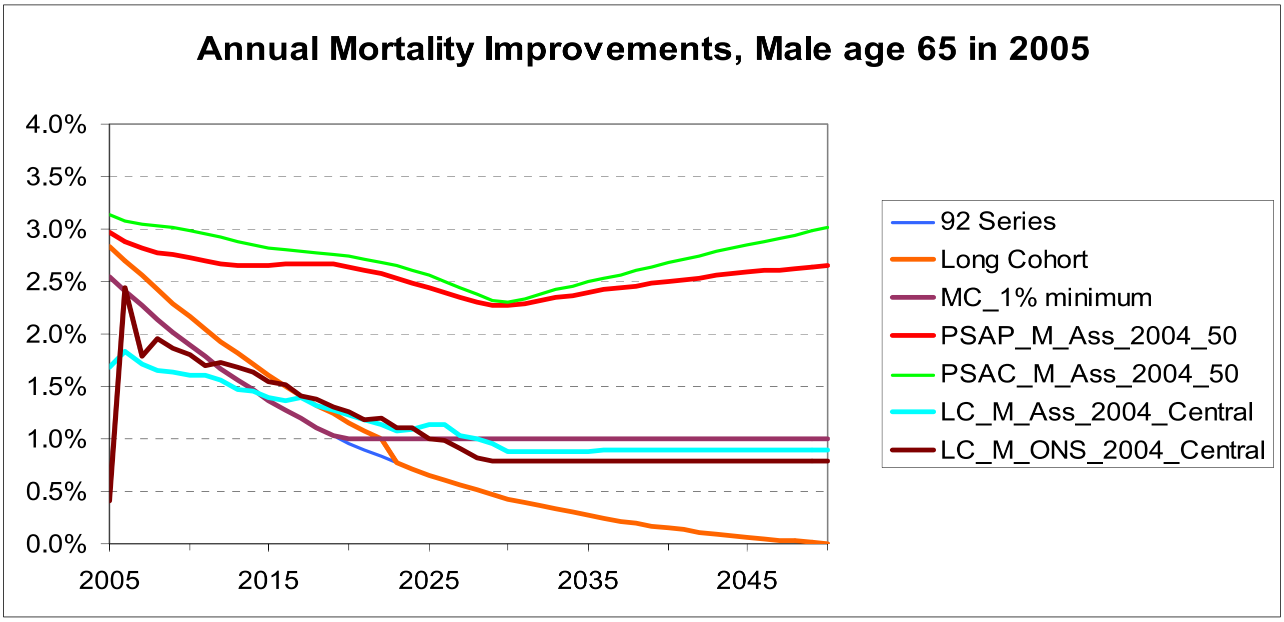
Whilst this may be appropriate for disclosing the projection to other actuaries, it will convey little to most users of actuarial advice and alternative approaches are required. The CMI believes that it would be beneficial if there was some discussion of alternative approaches and their advantages and disadvantages. Possible approaches include:

a) Heat Maps

These will now be familiar to many actuaries and are particularly useful to illustrate surface features, such as cohort effects. However they may be considered too complex for many situations, as it can take time to explain their many facets, diverting attention from the mortality features. Two heat maps are contained in Appendix D.

b) Mortality improvement graphs

Much simpler pictorial representations may be preferred. The graph below illustrates the future improvements in mortality implied by a sample of projections from the draft library. It shows the improvements for a male age 65 in 2005 throughout the remainder of his life, compared to the mortality rates of a male one year older (i.e. in year 2025 it shows the improvement from $q_{85,2024}$ to $q_{85,2025}$):



c) Expectations of Life

These have the benefits that they are easily understood by non-actuaries and that they succinctly summarise mortality rates across a range of ages. They are also commonly used.

In most situations, actuaries will only be concerned with mortality during the mid- and late-years of life and hence expectation of life at, say, age 65 will be more appropriate than quoting the expectation of life at birth. Life companies are required to disclose the mortality bases used in calculating statutory reserves in FSA Returns and in order to avoid any ambiguity, are also required to provide figures for the expectations of life at ages 65 and 75 for annuities and pensions in payment, and contingent expectations of life at 65 for lives currently aged 45 and 55.

However there are also disadvantages from our perspective in that figures for expectation of life:

- encapsulate both the projection and the base assumption; and
- differences at very old ages may be given undue weight compared to their financial significance.

d) Annuity Values

Using annuity values is a comparable approach but the effect of discounting eliminates the second disadvantage of expectation of life, noted above.

We do not see the need to use an interest rate as a disadvantage, although it would of course be appropriate to reflect any indexation of benefit in the interest rate chosen, if the benefit amount is not modelled explicitly. Where differences in mortality bases for a number of companies or schemes are being considered it would be appropriate for a common interest rate to be adopted.

As with expectations of life, annuity values do not distinguish whether any difference arises from the base mortality assumptions or the projection, as we would ideally like. Within this paper we have overcome this issue by using a common starting assumption across all the projections (other than differentiating between males and females).

Q8.1 We would welcome views on which method(s) of illustrating differences in projections are found to be most useful.

Q8.2 Are there alternative suggestions for illustrating different projections?

Illustrative figures

The tables below set out illustrative annuity due values over a range of ages for the year of use 2005. Values for expectation of life at various ages are also shown for 2005 and for age 65 in 2015 and 2025. These have all been calculated using 100% of PCMA00 or PCFA00 in 2000, for males and females respectively, an interest rate of 5% (for the annuity values) and the relevant projection from the draft library. (Note that the PCMA00 and PCFA00 base tables only provide values of q_x for ages 50 and above. For the younger ages we used the extensions to younger ages provided in Working Paper 26.)

In each case, a two-way table of q_x was produced by applying improvement factors from the draft library. The values of q_x have been rounded to 6 decimal places, as is normal practice in the CMI Tables Program (STP). Note that since the "00" Series tables apply to mortality rates in 2000, the improvements from 2000-2005 are a mixture of projections and (smoothed) actual improvements, as noted earlier in section 3. For comparison purposes, values are also shown using just the base mortality and interest (and no projection).

Q8.3 Does the range of ages and values within these tables provide a useful means of comparing the various projections? If not please state how you would suggest amending them.

Males	Annuity values at 5% p.a. for a life aged *x* exact on 1 July 2005						Expectation of life for a life aged 65 exact on 1 July		Expectation of life for a life aged *x* exact on 1 July 2005			
Projection	$_{20\|}\ddot{a}_{45}$	$_{10\|}\ddot{a}_{55}$	$\ddot{a}_{60}$	$\ddot{a}_{65}$	$\ddot{a}_{70}$	$\ddot{a}_{80}$	**2025**	**2015**	e_{60}	e_{65}	e_{70}	e_{80}
No projection *	3.944	6.796	13.441	11.944	10.245	6.762	18.401	18.401	22.523	18.401	14.504	8.118
“92” Series	4.538	7.462	14.107	12.542	10.754	7.054	21.315	20.732	24.593	19.976	15.645	8.610
Short Cohort	4.624	7.613	14.310	12.815	11.098	7.303	21.966	21.407	25.263	20.680	16.354	8.981
Medium Cohort	4.698	7.738	14.474	13.033	11.389	7.530	22.645	22.088	25.921	21.360	17.065	9.368
Long Cohort	4.860	8.012	14.832	13.503	11.919	7.862	24.373	23.794	27.561	23.035	18.611	10.033
Medium Cohort_1% minimum	4.781	7.822	14.556	13.109	11.456	7.581	23.754	22.778	26.422	21.733	17.333	9.503
90%_Medium Cohort	4.630	7.650	14.376	12.929	11.278	7.454	22.244	21.731	25.586	21.066	16.810	9.242
Average(MC_LC)	4.778	7.873	14.650	13.264	11.648	7.692	23.481	22.912	26.712	22.167	17.808	9.689
Average(MC_LC)_1.5% minimum	4.950	8.038	14.797	13.378	11.727	7.767	25.783	24.262	27.626	22.760	18.158	9.891
ONS_2004_Males_Principal	4.704	7.886	14.656	13.280	11.558	7.565	24.099	23.254	26.871	22.263	17.641	9.545
ONS_2004_Males_HLE	4.992	8.187	14.943	13.536	11.772	7.683	27.176	25.223	28.303	23.294	18.333	9.795
ONS_2004_Males_LLE	4.435	7.610	14.393	13.044	11.361	7.456	21.656	21.645	25.681	21.392	17.045	9.318
PSAP_Male_Ass_2003_50	5.103	8.150	14.822	13.236	11.372	7.435	28.429	25.340	28.015	22.536	17.449	9.371
PSAP_Male_Ass_2004_50	5.076	8.113	14.783	13.199	11.340	7.418	28.037	25.083	27.820	22.395	17.354	9.337
PSAP_Male_Ass_2005_50	5.139	8.187	14.859	13.274	11.414	7.487	28.875	25.645	28.245	22.720	17.601	9.477
PSAP_Male_ONS_2003_50	4.124	7.103	13.818	12.355	10.602	6.888	19.024	19.223	23.483	19.274	15.143	8.291
PSAP_Male_ONS_2004_50	5.767	9.019	15.786	14.318	12.549	8.441	38.211	33.105	34.254	27.790	21.757	11.735
PSAP_Male_ONS_2005_50	5.757	9.040	15.822	14.352	12.555	8.265	38.345	33.255	34.446	27.912	21.742	11.332
PSAC_Male_Ass_2003_50	5.183	8.191	14.838	13.229	11.345	7.407	29.113	25.506	28.038	22.479	17.360	9.315
PSAC_Male_Ass_2004_50	5.168	8.171	14.817	13.209	11.329	7.400	28.920	25.381	27.943	22.411	17.317	9.302
PSAC_Male_Ass_2005_50	5.207	8.218	14.866	13.259	11.380	7.449	29.457	25.750	28.227	22.631	17.487	9.401
PSAC_Male_ONS_2003_50	5.590	8.602	15.241	13.664	11.768	7.501	37.122	30.098	31.073	24.647	18.796	9.567
PSAC_Male_ONS_2004_50	5.874	9.060	15.760	14.212	12.332	7.944	41.004	33.937	34.362	27.360	20.930	10.584

PSAC_Male_ONS_2005_50	5.936	9.128	15.816	14.262	12.364	7.939	42.029	34.622	34.819	27.661	21.093	10.591
LC_Male_Ass_2003_Central	4.585	7.510	14.161	12.614	10.841	7.158	22.486	21.417	25.037	20.349	15.956	8.829
LC_Male_Ass_2004_Central	4.597	7.527	14.180	12.635	10.861	7.173	22.534	21.467	25.093	20.397	15.997	8.852
LC_Male_Ass_2005_Central	4.656	7.606	14.271	12.728	10.955	7.254	22.905	21.784	25.406	20.666	16.222	8.996
LC_Male_ONS_2003_Central	4.578	7.511	14.164	12.612	10.829	7.124	22.236	21.259	24.939	20.262	15.873	8.752
LC_Male_ONS_2004_Central	4.646	7.605	14.271	12.722	10.937	7.206	22.658	21.622	25.295	20.565	16.122	8.894
LC_Male_ONS_2005_Central	4.683	7.658	14.333	12.787	11.000	7.253	22.875	21.817	25.491	20.735	16.261	8.971

* Note that the ‘No Projection’ values are not true “Annuity Values in 2005” or “Expectation of life in 2005/2015/2025” figures as they are calculated using a mortality assumption that relates to 2000 and make no allowance for improvement between 2000 and 2005/2015/2025.

Females	Annuity values at 5% p.a. for a life aged *x* exact on 1 July 2005						Expectation of life for a life aged 65 exact on 1 July		Expectation of life for a life aged *x* exact on 1 July 2005			
Projection	$_{20\|}\ddot{a}_{45}$	$_{10\|}\ddot{a}_{55}$	$\ddot{a}_{60}$	$\ddot{a}_{65}$	$\ddot{a}_{70}$	$\ddot{a}_{80}$	**2025**	**2015**	e_{60}	e_{65}	e_{70}	e_{80}
No projection *	4.466	7.565	14.359	12.903	11.240	7.716	20.853	20.853	25.264	20.853	16.677	9.675
“92” Series	4.958	8.129	14.936	13.444	11.717	8.007	23.571	23.062	27.232	22.397	17.827	10.195
Short Cohort	5.032	8.259	15.110	13.676	12.011	8.224	24.142	23.658	27.835	23.027	18.461	10.529
Medium Cohort	5.101	8.376	15.263	13.879	12.279	8.441	24.785	24.310	28.477	23.687	19.145	10.910
Long Cohort	5.263	8.648	15.616	14.339	12.809	8.798	26.567	26.083	30.211	25.445	20.790	11.662
Medium Cohort_1% minimum	5.198	8.480	15.366	13.977	12.369	8.514	26.119	25.183	29.142	24.195	19.520	11.109
90% Medium Cohort	5.045	8.303	15.179	13.787	12.180	8.370	24.424	23.985	28.170	23.413	18.905	10.787
Average(MC_LC)	5.181	8.511	15.438	14.107	12.540	8.616	25.654	25.174	29.321	24.542	19.943	11.275
Average(MC_LC)_1.5% minimum	5.366	8.698	15.610	14.248	12.646	8.719	28.247	26.771	30.462	25.317	20.431	11.561
ONS_2004_Females_Principal	5.104	8.442	15.348	14.003	12.300	8.254	25.982	25.166	29.115	24.269	19.367	10.671
ONS_2004_Females_HLE	5.329	8.669	15.559	14.188	12.451	8.338	28.581	26.765	30.263	25.068	19.885	10.853
ONS_2004_Females_LLE	4.894	8.234	15.153	13.832	12.158	8.174	23.876	23.835	28.142	23.578	18.909	10.502
PSAP_Female_ONS_2003_50	4.183	7.255	14.054	12.594	10.895	7.299	17.972	18.818	23.740	19.590	15.582	8.856
PSAP_Female_ONS_2004_50	5.506	8.831	15.700	14.253	12.500	8.534	31.655	28.700	31.646	25.902	20.453	11.375
PSAP_Female_ONS_2005_50	5.453	8.771	15.651	14.229	12.511	8.559	31.011	28.308	31.383	25.785	20.450	11.416
PSAC_Female_ONS_2003_50	5.457	8.750	15.591	14.104	12.295	8.219	31.497	28.238	31.060	25.237	19.723	10.700
PSAC_Female_ONS_2004_50	5.812	9.242	16.141	14.690	12.891	8.682	36.723	32.494	34.621	28.153	21.988	11.785
PSAC_Female_ONS_2005_50	5.840	9.269	16.165	14.711	12.918	8.684	37.184	32.783	34.829	28.303	22.105	11.804
LC_Female_ONS_2003_Central	4.973	8.150	14.964	13.497	11.792	8.097	24.850	23.824	27.710	22.801	18.165	10.406
LC_Female_ONS_2004_Central	5.031	8.228	15.053	13.594	11.896	8.191	25.267	24.185	28.054	23.106	18.429	10.579
LC_Female_ONS_2005_Central	5.059	8.265	15.097	13.642	11.946	8.236	25.437	24.340	28.208	23.243	18.549	10.658

* Note that the ‘No Projection’ values are not true “Annuity Values in 2005” or “Expectation of life in 2005/2015/2025” figures as they are calculated using a mortality assumption that relates to 2000 and make no allowance for improvement between 2000 and 2005/2015/2025.

9 Illustrating Uncertainty

Any projection of future mortality is uncertain. Section 4 of CMI Working Paper 3 highlights a number of different sources of uncertainty:

- Model uncertainty, which arises because the "correct" underlying model is not known;
- Parameter uncertainty, which arises because for any particular model, the parameters are estimated from a finite set of data;
- Stochastic uncertainty. This reflects the random variations which would occur in future, even if the model and the parameters were known;
- Measurement error. The raw data that underlies any projection is unlikely to be entirely accurate, for example due to late reported deaths;
- Heterogeneity exists if there are subsets within the data with different experience, for example with regard to socio-economic profile. Uncertainty is introduced into our projections if we parameterise a model ignoring these differences; and
- Past experience not being a good guide to the future arising, for example from a change in business mix.

The projections considered in sections 4 and 5 of this paper do not illustrate any of the types of uncertainty noted above, unless they are compared to other projections, in which case they illustrate a degree of model risk.

P-spline and Lee-Carter models do both produce measures of uncertainty but these are not directly comparable to each other. The use of a structured model, such as Lee-Carter, gives greater weight to the choice of model than a non-parametric model, such as P-spline, which seeks to derive its shape from the data. The upshot is that there appears to be less uncertainty associated with a Lee-Carter projection than a P-spline projection, but the CMI does not believe that we can necessarily have more confidence in the former.

The approaches to generating the measures of uncertainty are detailed in Appendix B for P-splines and Appendix C for Lee-Carter.

The nature of the measures generated by these models is also very different. P-spline projections generate standard errors which allow confidence intervals around the mortality rates to be calculated directly, but these relate to parameter uncertainty only.

The Lee-Carter model generates sample paths, which may be considered advantageous if one wishes to incorporate these with economic scenarios in a combined model. These sample paths reflect both parameter uncertainty and stochastic uncertainty and can also be used to generate percentiles but, as explained in Appendix C, this can be done in different ways:

- The mortality rates at each age could be ranked to generate the required confidence interval but these rates would arise from different sample paths.
- Assumptions can be made as to base mortality and interest rates to calculate an annuity value for each sample path, which can then be ranked to generate confidence intervals. This approach produces much narrower confidence intervals than ranking mortality rates. This approach was adopted in Working Paper 25, except that the 50th percentile values were based on the mean annuity value, not the ranking.

A further difference between the measures of uncertainty generated by the models arises for joint life annuities. Directly-calculated percentiles, such as those from the P-spline models, carry an implicit assumption that the mortality improvements for the two lives are dependent; whereas sample paths implicitly assume them to be independent.

Q9.1 ***We have not included any P-spline projections other than 50th percentiles in the draft library. Do you see benefit in including examples based on other percentiles? If so, please state what examples and explain why these are needed.***

Q9.2 ***We have only included central Lee-Carter projections in the draft library. Do you see benefit in illustrating uncertainty from the Lee-Carter model within the library?***

Q9.3 ***If so, should this be done by:***

- ***Including percentiles based on ranking mortality rates at each age, or***
- ***Including percentiles based on ranking annuity rates, derived using a stated set of assumptions regarding base mortality and interest rates, at each age?***
- ***Making available CDs containing sets of (say) 1,000 simulations for actuaries to manipulate themselves?***

10 Recent trends in mortality

This section does not aim to provide a comprehensive overview of trends in mortality. Recommended reading for more background includes Willets et al (2004) and Willets (2004). There is also much useful material located in presentations, for example those to the Profession's "Mortality & longevity" seminars in June 2006 and April/May 2007.

Q10.1 Would actuaries find it helpful to establish a "Recommended Reading" list on the Profession's website? Suggestions for inclusion would be welcomed.

Q10.2 What else should the Profession or the CMI consider doing to help actuaries further enhance their understanding in this field?

An indication of recent trends can be obtained from recent CMI results and also from the draft library itself, for the datasets used for projections (i.e. CMI Assured Lives for males and ONS for males and females). The draft library includes the output from fitting the P-spline and Lee-Carter models to these datasets since 1992, during which period the improvement rates are smoothed actual rates, rather than projections. Recent trends in ONS mortality are also apparent from the ONS projections in the draft library, where the figures from 1992 to the start year of the projection (2004) are smoothed using a P-spline age-cohort model.

Considering the period from 1992-2005, the following table shows the average annual rates of actual (unsmoothed) improvement in ONS mortality rates in quinquennial age bands:

	Males			Females		
	1992-1997	**1997-2001**	**2001-2005**	**1992-1997**	**1997-2001**	**2001-2005**
20-24	-1.8%	2.9%	4.8%	0.6%	1.4%	2.3%
25-29	-1.3%	2.1%	3.5%	0.4%	0.5%	0.4%
30-34	-0.3%	-0.7%	3.5%	0.6%	0.4%	1.5%
35-39	2.3%	-0.8%	1.2%	-0.4%	2.6%	2.4%
40-44	0.4%	1.7%	0.9%	0.9%	1.4%	1.2%
45-49	0.8%	0.2%	1.6%	0.4%	1.0%	1.4%
50-54	2.4%	1.3%	1.3%	1.5%	0.9%	1.4%
55-59	2.1%	2.7%	2.6%	1.8%	1.9%	1.8%
60-64	3.1%	2.7%	2.4%	2.5%	2.2%	2.7%
65-69	2.8%	3.8%	3.1%	2.1%	3.5%	1.9%
70-74	1.6%	3.6%	3.4%	0.6%	3.0%	3.1%
75-79	2.0%	1.9%	3.3%	1.3%	1.4%	2.1%
80-84	1.1%	3.0%	1.5%	0.4%	2.3%	0.8%
85-89	0.4%	1.5%	2.8%	0.1%	1.2%	1.7%

For males it can be seen that:

- Mortality deteriorated at the younger ages during the 1990s, but has subsequently shown significant improvements.
- At older ages, rates of improvement have been relatively stable at around 3% p.a..
- The peak rates of improvement relating to the cohort born around 1931 are apparent at age band 60-64 in 1992-1997, 65-69 in 1997-2001 and 70-74 in 2001-2005.

For females:

- Rates of improvement are generally lower than those for males, but similar patterns exist.
- In particular, these figures suggest the cohort effect exists for females too.

Within the most recent period included in the table above, the rates of improvement have fluctuated between years, as illustrated below:

	Males				Females			
	2001-2	**2002-3**	**2003-4**	**2004-5**	**2001-2**	**2002-3**	**2003-4**	**2004-5**
20-24	3.5%	3.5%	9.3%	6.1%	9.0%	-4.6%	3.0%	1.8%
25-29	-2.0%	6.0%	6.9%	4.4%	1.0%	-5.4%	0.4%	5.3%
30-34	5.0%	2.7%	1.1%	6.7%	-5.9%	3.4%	5.3%	3.0%
35-39	2.0%	-0.6%	1.5%	2.0%	1.1%	1.6%	-1.4%	8.6%
40-44	-1.8%	-0.3%	4.3%	1.5%	2.4%	-2.5%	5.9%	-1.1%
45-49	-0.7%	3.5%	5.1%	-1.1%	-2.2%	2.6%	3.7%	1.6%
50-54	1.1%	-1.6%	2.1%	3.8%	2.3%	-0.7%	5.2%	-1.0%
55-59	3.6%	1.0%	6.4%	0.2%	3.5%	0.5%	4.2%	-0.8%
60-64	1.4%	0.9%	6.0%	1.8%	2.1%	2.3%	4.6%	2.7%
65-69	2.8%	2.1%	4.4%	4.4%	2.1%	0.8%	4.3%	0.9%
70-74	2.9%	3.5%	5.8%	2.7%	2.1%	2.1%	5.8%	3.4%
75-79	2.0%	3.1%	5.6%	4.1%	0.0%	0.7%	6.7%	1.5%
80-84	1.0%	-0.2%	3.0%	2.3%	0.6%	-2.1%	4.4%	0.3%
85-89	0.6%	-0.3%	9.0%	2.9%	-1.8%	-2.9%	8.5%	2.7%

In particular it can be seen that for most age bands the rates of improvement between 2002 and 2003 were lower than for this period as a whole, but these were followed by particularly high rates of improvement between 2003 and 2004.

Whilst such year-to-year fluctuations may be of little significance to actuaries considering long-term pension liabilities, they can have a major impact on projections generated using some of the methodologies included in the draft library. In particular, the P-spline method is most sensitive to improvements in the most recent years and calendar year fluctuations such as those illustrated above will particularly influence age-period projections. This is especially true when the ONS dataset is used, as the P-spline model will fit much more closely to this than to a dataset with lower data volumes, such as the CMI assured lives dataset. The impact of this can be seen from the expectation of life figures in section 8 in both the volatility using P-spline projections as 2004 ONS data is added and in the reducing figures into the future, which indicates that some of the projections (e.g. PSAP_Male_ONS_2003_50) imply worsening mortality in future years.

These patterns of improvement are illustrated in heat maps in Appendix D, which show improvements smoothed using a P-spline age-cohort model on ONS data to 2005.

Year-to-year fluctuations in CMI insured experience tend to be more volatile because of the lower data volumes, and hence in the table below we consider improvements rates across the entire period, from the average of 1992-4 to the average of 2003-5. The corresponding ONS figures are included for comparison:

	Average annual rate of improvement in mortality from 1992-4 to 2003-5				
	Males			**Females**	
	ONS E&W	**CMI Assured Lives**	**CMI Life Office Pensioners**	**ONS E&W**	**CMI Life Office Pensioners**
20-24	1.6%	-1.0%		1.1%	
25-29	1.2%	-0.6%		0.4%	
30-34	0.8%	2.0%		0.7%	
35-39	0.8%	2.2%		1.0%	
40-44	0.9%	2.3%		1.2%	
45-49	0.7%	1.9%		0.8%	
50-54	1.5%	2.0%		1.2%	
55-59	2.1%	2.3%		1.6%	
60-64	2.3%	2.5%		2.1%	
65-69	2.7%	2.4%	1.5%	2.2%	2.2%
70-74	2.5%	2.0%	2.6%	2.0%	3.4%
75-79	2.2%	1.7%	2.4%	1.5%	0.5%
80-84	1.8%	1.6%	2.1%	1.2%	1.2%
85-89	1.4%	1.1%	1.7%	0.7%	0.1%

Improvement rates for male assured lives appear higher than for the population at younger ages and lower at older ages however there is little obvious difference between insured pensioners and the general population over this period.

11 Future Updates

The CMI is not committing to any specified review dates for the library. However it will be appropriate to supplement the library from time-to-time:

- To incorporate subsequent years' data, e.g. projections using data to 2006;
- To incorporate experience from a new dataset, e.g. from the CMI SAPS investigation, when there is sufficient data;
- To incorporate new "intuitive" projections, in the light of likely or actual medical advances; or
- If future work on projection methodologies indicates that a new methodology is worthy of inclusion, e.g. the Lee-Carter Age-Period-Cohort model.

In addition to the Lee-Carter APC model, other methodologies are regularly being developed. For example see the LifeMetrics paper which comments on the relative merits of a number of methodologies. The CMI is keen to contribute to further research within the Profession into methodologies but does not anticipate leading such research.

The CMI intends to draw up criteria that can be used to govern the inclusion of projections within the library. Draft criteria are that new projections must be:

- A worthwhile addition to what is already contained in the library;
- Publicly available;
- Clearly described and documented; and
- Independently Peer Reviewed.

It may of course be appropriate to revise these criteria over time.

The process by which the CMI supplements the library may depend on the extent and impact of the new projections. For example:

- A minor change, such as adding projections based on subsequent data, may be incorporated without prior consultation;
- In contrast incorporating new projections generated from a "new" methodology is likely to only be done after consultation, perhaps by means of a Working Paper.

Whilst adding an additional year's data may be considered a routine update, comparison of the figures in section 8 shows that it can have a substantial impact on Lee-Carter and, especially, P-spline projections. Actuaries making use of projections based on the latest year's data should not do so without due care, given the volatility of some projections to new data

Note that as none of the projections in the library is "recommended", there is unlikely to be a corresponding need to "withdraw" projections.

Q11.1 Do you have any comments on the draft criteria for including new projections within the library, including suggestions for additional criteria?

Q11.2 Do you have any other comments on the process by which future versions of the library should be managed?

Q11.3 Do you have any views on what other projection methodologies the Profession should seek to research and how such research is best organised?

References

CMI Report No. 10: Graduation of the 1979-82 Mortality Experience - the "80" Series (May 1990)

CMI Report No. 17: Graduation of the 1991-94 Mortality Experience - the "92" Series Standard Tables (June 1999)

CMI Working Paper 1: An interim basis for adjusting the "92" Series mortality projections for cohort effects (December 2002)

CMI Working Paper 3: Projecting future mortality: A discussion paper (March 2004)

CMI Working Paper 11: Responses to Working Paper 3 entitled *Projecting future mortality: A discussion paper* and further commentary thereon (January 2005)

CMI Working Paper 15: Projecting Future Mortality: Towards a proposal for a stochastic methodology (July 2005)

CMI Working Paper 20: Stochastic Projection Methodologies: Further progress and P-Spline model features, example results and implication (April 2006)

CMI Working Paper 21: The Graduation of the CMI 1999-2002 Mortality Experience: Final “00” Series Mortality Tables – Assured Lives (August 2006)

CMI Working Paper 22: The Graduation of the CMI 1999-2002 Mortality Experience: Final “00” Series Mortality Tables – Annuitants and Pensioners (August 2006)

CMI Working Paper 25: Stochastic projection methodologies: Lee-Carter model features, example results and implications (April 2007)

CMI Working Paper 26: Extensions to Younger Ages of the "00" Series Pensioner Tables of Mortality (April 2007)

Pensions Institute Discussion Paper: PI-0701 – A quantitative comparison of stochastic mortality models using data from England & Wales and the United States – Andrew J.G. Cairns, David Blake, Kevin Dowd, Guy D. Coughlan, David Epstein, Alen Ong and Igor Balevich (March 2007). lifeMetrics available via http://www.jpmorgan.com/pages/jpmorgan/investbk/solutions/lifemetrics

National population projections 2004-based (GAD, Series PP2 No 25, 2006) available from http://www.statistics.gov.uk/StatBase/Product.asp?vlnk=4611&Pos=&ColRank=1&Rank=256 or http://www.gad.gov.uk/Publications/Demography_and_statistics.htm

Renshaw A. E. and Haberman S. (2006) A cohort-based extension to the Lee-Carter model for mortality reduction factors. Insurance: Mathematics and Economics 38 (3), 556-570.

Sessional meeting paper: Two dimensional mortality data: patterns and projections – John Ellam, Jennifer Hubbard, J L C Lu, Stephen Makin, Keith Miller, Stephen Richards. (March 2007)

Sessional Meeting Paper: The cohort effect: insights and explanations – R C Willets (March 2004)

Sessional Meeting Paper: Longevity in the 21st century – R C Willets; A P Gallop; P A Leandro; J L C Lu; A S Macdonald; K A Miller; S J Richards; N Robjohns; J P Ryan: H R Waters (April 2004)

Appendix A: Full list of projections in the draft library

The full list of projections included in the draft version of the initial library is shown below:

Projection	Sheet in spreadsheet	Base Year
Previously-published Projections		
Original "92" Series	2	1992
Short Cohort	3	1992
Medium Cohort	4	1992
Long Cohort	5	1992
ONS_2004_Males_Principal	6	2004
ONS_2004_Males_High life expectancy	7	2004
ONS_2004_Males_Low life expectancy	8	2004
ONS_2004_Females_Principal	9	2004
ONS_2004_Females_High life expectancy	10	2004
ONS_2004_Females_Low life expectancy	11	2004
Adjusted Cohort Projections		
Medium Cohort_1% minimum	12	1992
90% of Medium Cohort	13	1992
Average (Medium Cohort_Long Cohort)	14	1992
Average (Medium Cohort_Long Cohort)_1.5% minimum	15	1992
P-spline Projections		
PSAP_Male_Ass_2003_50	16	2003
PSAP_Male_Ass_2004_50	17	2004
PSAP_Male_Ass_2005_50	18	2005
PSAP_Male_ONS_2003_50	19	2003
PSAP_Male_ONS_2004_50	20	2004
PSAP_Male_ONS_2005_50	21	2005
PSAP_Female_ONS_2003_50	22	2003
PSAP_Female_ONS_2004_50	23	2004
PSAP_Female_ONS_2005_50	24	2005
PSAC_Male_Ass_2003_50	25	2003
PSAC_Male_Ass_2004_50	26	2004
PSAC_Male_Ass_2005_50	27	2005
PSAC_Male_ONS_2003_50	28	2003
PSAC_Male_ONS _2004_50	29	2004
PSAC_Male_ONS _2004_50	30	2005
PSAC_Female_ONS_2003_50	31	2003
PSAC_Female_ONS_2004_50	32	2004
PSAC_Female_ONS_2005_50	33	2005
Lee-Carter Projections		
LC_Male_Ass_2003_Central	34	2003
LC_Male_Ass_2004_Central	35	2004
LC_Male_Ass_2005_Central	36	2005
LC_Male_ONS_2003_Central	37	2003

LC_Male_ONS_2004_Central	38	2004
LC_Male_ONS_2005_Central	39	2005
LC_Female_ONS_2003_Central	40	2003
LC_Female_ONS_2004_Central	41	2004
LC_Female_ONS_2005_Central	42	2005

Appendix B: Generating the P-spline projections in the draft library

Choice of dataset

- The P-spline model requires age-specific data for successive years; a minimum of 20 years was suggested in Working Paper 20. Additionally, for the age-ranges fitted, a large amount of data is required in each year of observation.
- The only UK datasets, available to the CMI, that satisfy these criteria are the ONS England and Wales population (males and females) dataset and the CMI Assured Lives (males) dataset. These were the datasets used to illustrate the P-spline methodology in Working Paper 20.
- Datasets may be subject to retrospective adjustment. Ordinarily the projections in the library use the original dataset. For example, the CMI dataset for the projections based on data to 2003 used in Working Paper 20 was based on data collected to 2003. The CMI Assured Lives dataset has subsequently been amended reflecting revisions to the 1947-2003 data that arose during the processing of 2004 data but the projections in the draft library using CMI data to 2003 all use the original 1947-2003 dataset. If projections are undertaken using a more recent dataset with the last year's/years' data removed, this should be specifically disclosed.
- Note that whilst the CMI will be aware of such changes in its own datasets, it may not necessarily always have access to the first available ONS dataset.

Method of generating P-spline projections

- The P-spline model fits forces of mortality (i.e. μ_x) to the data. The age definition of the exposure and deaths for each of the datasets and the age (x) to which the fitted μ_x apply is as follows:

Dataset	**Age Definition**	**μ_x Estimate**
ONS	Age last birthday	$\mu_{x+½}$
CMI Assured Lives	Age nearest	μ_x

- Mean values of $\mu_{x,t}$ are produced for each age x and year t within the fitted region of the dataset and in the region of the projection.
- The $\mu_{x,t}$ can be used to estimate the values of the $q_{x,t}$ and from these the calendar year improvements can be determined for each age.
- For ages above 90 for the CMI Assured Lives data and above 89 for the ONS data the improvements are assumed to equal the improvements at ages 90 and 89, respectively, whilst $q(120)$ is assumed to equal 1.
- The draft library provides projected improvements to 2100. These have been derived from mean values of $\mu_{x,t}$ using the following approach:
    - For the CMI Assured Lives data, values for $q_{x,t}$ were estimated as:
    $$q_{x,t} = 1\text{- } \exp \{\text{- } ½ \,(\mu_{x,t} + \mu_{x+1,t})\ \}$$
    - For the ONS data, values for $q_{x,t}$ were estimated as:
    $$q_{x,t} = 1\text{- } \exp \{\text{- } \mu_{x+½,t}\ \}$$
    - The cumulative improvement for a particular year t has been calculated as $q_{x,t}\ / q_{x,0}$, where $q_{x,0}$ is the mortality rate for 1992.
- The parameters used to generate the projections are shown below.
- The positioning of knots has followed the convention outlined in Sections 7.9-7.10 of Working Paper 20. This explains that the knots have been positioned at both corners of the leading edge of the data. In practice, this means that:

- For the age-period model, knots are positioned at the highest age in the age dimension and in the final year of the dataset in the period dimension. The data is curtailed at younger ages, if necessary, so that a knot is also positioned at the lowest age.
- For the age-cohort model, knots are positioned at the highest age in the age dimension and, in the cohort dimension, on the cohort consistent with this age in the last year of the dataset. The data is again curtailed at younger ages, if necessary, so that a knot is also positioned at the lowest age.

Calculating percentiles for P-Spline projections

- The P-Spline model produces mean values for log $\mu_{x,t}$ and corresponding standard deviations for the log $\mu_{x,t}$, $\hat{s}_{x,t}$.
- A set of $\mu_{x,t}$ relating to a particular percentile can be calculated by applying the standard normal variable (Z), for the percentile in question, to the standard deviations and using this to adjust the mean $\mu_{x,t}$. This process is summarised by the following equation:

$$\mu_{x,t} = \exp\{\log(\mu_{x,t}) + Z \times \hat{s}_{x,t}\}$$

- These may be used to illustrate some of the uncertainty inherent in any projection of future mortality.

Parameters used to generate the projections

We have used cubic B-splines and a penalty order of 2 for all our fits. In all cases the models have produced projections for 100 years (Note that the models produced projections for 100 years, e.g. to 2103 for 2003 base year projections, but the projected improvements included in the draft library are only provided up until 2100. Changing the length of the projection period may alter the fit produced.)

Age-Cohort model

For datasets fitted using the age-cohort model the following parameters were used:

	Assured Lives Males	**ONS Males**	**ONS Females**
Calendar Year range	1947-2003/4/5	1961-2003/4/5	1961-2003/4/5
Age range	21-90	21-89	24-89
Knot spacing: - age dimension - cohort dimension	 Every 3 years Every 3 years	 Every 4 years Every 4 years	 Every 5 years Every 5 years
Fixed knot positions: - age dimension - cohort dimension	 90 Last year of data less 90	 89 Last year of data less 89	 89 Last year of data less 89
Minimum for penalty: - age dimension - cohort dimension	 0.0001 0.0001	 0.0001 0.0001	 0.0001 0.0001
Starting point for penalty: - age dimension - cohort dimension	 100 100	 100 100	 100 100

Age-period model

For datasets fitted using the age-period model the following parameters were used:

	Assured Lives Males	**ONS Males**		**ONS Females**
Calendar Year range	1947-2003/4/5	1961-2003	1961-2004/5	1961-2003/4/5
Age range	22-90	23-89	24-89	23-89
Knot spacing: - age dimension - cohort dimension	 Every 4 years Every 4 years	 Every 6 years Every 6 years	 Every 5 years Every 5 years	 Every 6 years Every 6 years
Fixed knot positions: - age dimension - cohort dimension	 90 Last year of data	 89 Last year of data		 89 Last year of data
Minimum for penalty: - age dimension - cohort dimension	 0.0001 0.0001	 0.0001 0.0001		 0.0001 0.0001
Starting point for penalty: - age dimension - cohort dimension	 100 100	 100 100		 100 100

For the projection using male ONS data to 2004 generated using the age-period model it was not possible to use the same parameters as those used for the projections with data to 2003. A fit was obtained by altering the knot spacing (to every 5 years) but other ways of achieving this may be possible. The same parameterisation was used for projections using data to 2005.

Appendix C: Generating the Lee-Carter projections in the draft library

Choice of dataset

- The data requirements for the Lee-Carter model are similar to those for the P-Spline model (described in Appendix B). However, the minimum number of successive calendar years covered by the data can be adjusted depending on the width of the age range being fitted. If a narrower age range is used then fewer than 20 calendar years of data are required.
- The same datasets have been used to illustrate the Lee-Carter methodology in Working Paper 25 and to generate the projections in the draft library as were used for the P-Spline projections.
- As noted in Appendix B for the P-spline projections, datasets may be subject to retrospective adjustment. Ordinarily the projections in the library use the original dataset. For example, the CMI dataset for the projections in the draft library and in Working Paper 25 does not reflect revisions to the 1947-2003 data that arose during the processing of 2004 data. If projections are undertaken using a more recent dataset with the last year's/years' data removed, this should be specifically disclosed.
- Note that whilst the CMI will be aware of such changes in its own datasets, it may not necessarily always have access to the first available ONS dataset.

Method of generating Lee-Carter projections

- The Lee-Carter model fits forces of mortality (i.e. μ_x) to the data. The ages included in the datasets are specified below.
- Values of $\mu_{x,t}$ are produced for each age x and year t within the fitted region of the dataset and in the region of the projection.
- The $\mu_{x,t}$ can be used to estimate the values of the $q_{x,t}$ and from these the calendar year improvements can be determined for each age.
- For ages above 90 for the CMI Assured Lives data and above 89 for the ONS data the improvements are assumed to equal the improvements at ages 90 and 89, respectively and $q(120)$ is assumed to equal 1.
- The draft library provides central projected improvements to 2100. These have been derived from mean values of $\mu_{x,t}$.
- In addition to the central projections, it is possible to calculate projected improvements for particular percentiles, i.e. 97.5th percentile (see section 9 for a brief explanation).

Parametric bootstrapping

The process of parametric bootstrapping generates each synthetic dataset using the following steps:

- Fit the Lee-Carter model to the data and calculate the $\mu(x, t)$.
- Use the $\mu(x, t)$ and the exposure data to determine the number of expected deaths, based on the Lee-Carter fit.
- Compare the actual deaths against the expected deaths to obtain deviance residuals for each age and year.
- For each age, randomly reallocate the deviance residuals across the years.
- Use the reassigned deviance residuals to simulate the number of deaths for each age and year.

- Re-fit the Lee-Carter model to the simulated deaths and the actual exposures and fit a time-series to the *k(t)* parameters.
- Use the fitted parameters to generate $\mu'(x, t)$ in the region of the dataset and the time-series to generate projected $\mu'(x, t)$. The $\mu'(x, t)$ form a simulation.

Calculating percentiles for Lee-Carter projections

- The percentiles for the Lee-Carter projections are determined from the scenarios generated.
- The $q_{x,t}$ can be calculated for each scenario. Percentiles could be generated by ordering the mortality rates from all the scenarios, for each age and year, and selecting those corresponding to a particular percentile. The volatility of the mortality rates projected using Lee-Carter means that confidence intervals around the mortality rates would be very wide.
- The approach used in Working Paper 25 was to assume a base table of $q_{x,0}$, reflecting actual experience in year zero [both "92" Series and "00" Series base tables were used] and an interest rate [4.5%] to calculate annuity values for each age and year, for each of 1,000 scenarios. The mean of these values is the figure shown in Working Paper 25 as the 50th percentile value.
- Values for other percentiles were generated by ordering the annuity values from all the scenarios for each age and selecting the value corresponding to that particular percentile.
- The resulting confidence intervals are much narrower than those around the projected mortality rates.

It is important to note that using the method adopted for Working Paper 25 necessitates assumptions regarding interest rates and base mortality and different assumptions could result in a different ranking of the scenarios, and hence different confidence intervals. Furthermore the ranking of the scenarios will differ according to the start age of the annuity.

For these reasons we have not included projected mortality rates, other than the central projection, within the draft library. Actuaries wishing to illustrate uncertainty by means of ranking scenarios using the Lee-Carter method will need to specify details of how these have been obtained if it is intended that another actuary should be able to reproduce them.

Parameters used to generate the projections

For all the Lee-Carter projections we have used an ARIMA(1,1,0) model to project the *k*(*t*) parameters.

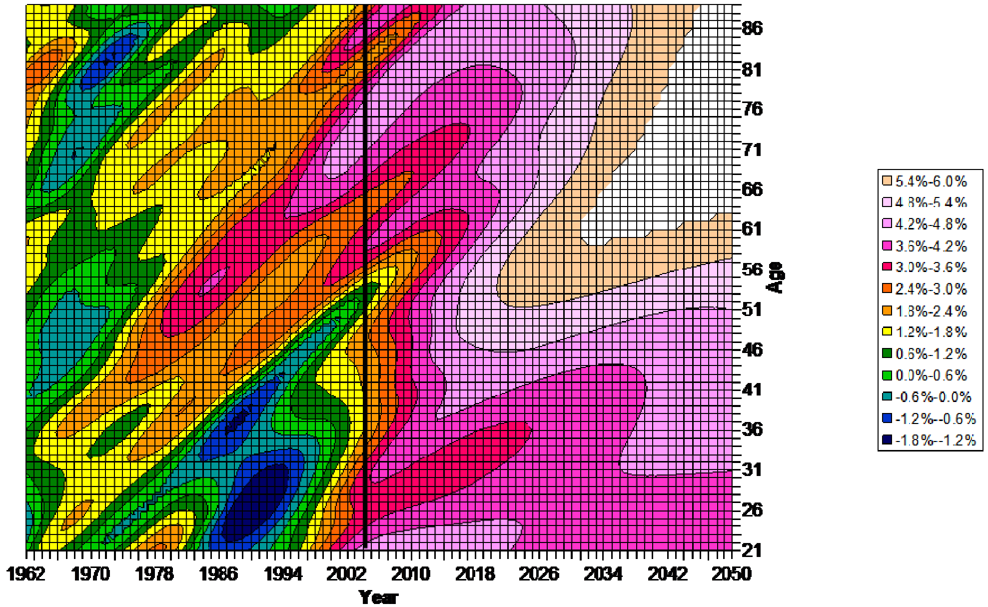
The following age ranges were used:

Assured lives, males	20-90
ONS, males	20-89
ONS, females	20-89

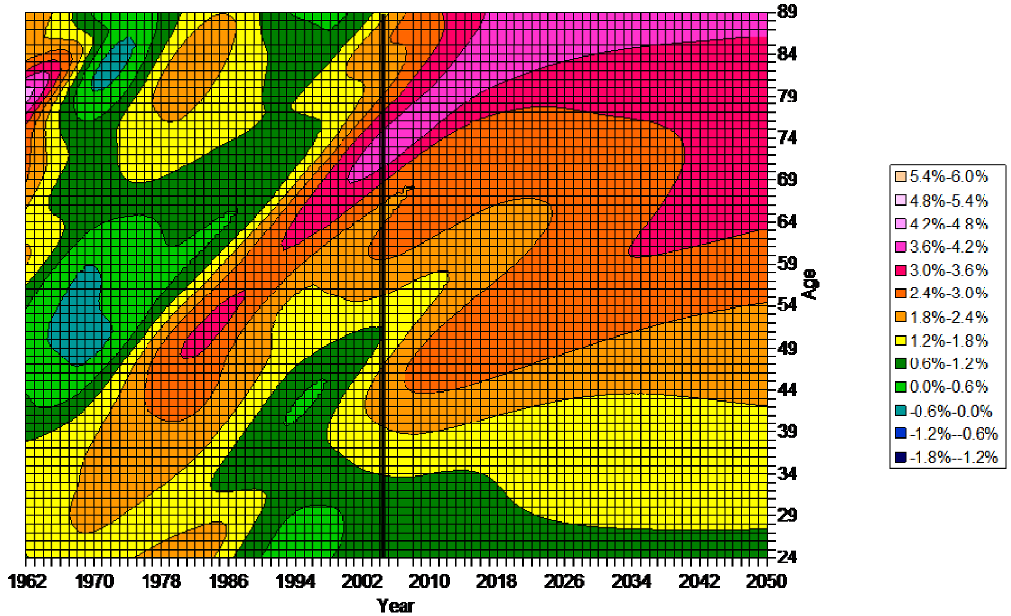
Appendix D: Recent patterns of improvement in mortality (see section 10).

Heat maps of projected improvements for ONS data from 1961-2005 using a P-spline age-cohort model.

Males ONS, ages 21-89 (PSAC_Males_ONS_2005_50)



Females ONS, ages 24-89 (PSAC_Females_ONS_2005_50)



Appendix E: Full list of specific questions for feedback

Feedback is invited on all aspects of this Working Paper and the accompanying draft library of mortality projections. We have also included a number of specific questions on which we would appreciate feedback. These are indicated by shaded boxes in the document but are repeated below for completeness:

Q3.1 Do you agree that a defined naming convention is a desirable feature of the library? If not, please state why.

Q3.2 Do you agree with the naming convention adopted for the draft library? If not please state suggested changes, with reasons.

Q4.1 Are there any other previously-published tables of projections that should be included in the library for use in the UK? If so, please state which tables, with references, and explain why these may be useful.

Q4.2 Do you agree with the use of smoothed improvements for 1992-2004 being appended to the ONS projections in the library? Is the P-spline age-cohort model an acceptable smoothing model?.

Q5.1 Do you see benefit in including additional examples of each variation? If so, please state what examples and explain why these are needed.

Q5.2 Do any additional examples need to be included in the library, or within the CMI Tables Program?

Q5.3 Are there other variations to the cohort projections that are currently being used that might be suitable for inclusion within the library? Please provide full details.

Q5.4 Do you disagree with the proposed method for applying any of these variations? If so, please explain your alternative approach with reasons, if possible.

Q5.5 In particular, should a minimum value be applied to m_x for consistency with ONS projections, rather than to q_x?

Q6.1 The naming convention does not fully determine the projection for the P-spline model. Is it sufficient? If not, what other features of the fit do you think need to be included within the name?

Q6.2 Are there other variations on P-spline projections that should be included in the library? If so, please state which projections and explain why these are needed.

Q7.1 The naming convention does not fully determine the projection for the Lee-Carter model. Is it sufficient? If not what other features of the fit do you think need to be included within the name?

Q7.2 Are there other variations on Lee-Carter projections that should be included in the library? If so, please state which projections and explain why these are needed.

Q8.1 We would welcome views on which method(s) of illustrating differences in projections are found to be the most useful.

Q8.2 Are there alternative suggestions for illustrating different projections?

Q8.3 Does the range of ages and values within these tables provide a useful means of comparing the various projections? If not please state how you would suggest amending them.

Q9.1 We have not included any P-spline projections other than 50th percentiles in the draft library. Do you see benefit in including examples based on other percentiles? If so, please state what examples and explain why these are needed.

Q9.2 We have only included central Lee-Carter projections in the draft library. Do you see benefit in illustrating uncertainty from the Lee-Carter model within the library?

Q9.3 If so, should this be done by:

- Including percentiles based on ranking mortality rates at each age, or
- Including percentiles based on ranking annuity rates, derived using a stated set of assumptions regarding base mortality and interest rates, at each age?
- Making available CDs containing sets of (say) 1,000 simulations for actuaries to manipulate themselves?

Q10.1 Would actuaries find it helpful to establish a "Recommended Reading" list on the Profession's website? Suggestions for inclusion would be welcomed.

Q10.2 What else should the Profession or the CMI consider doing to help actuaries further enhance their understanding in this field?

Q11.1 Do you have any comments on the draft criteria for including new projections within the library, including suggestions for additional criteria?

Q11.2 Do you have any other comments on the process by which future versions of the library should be managed?

Q11.3 Do you have any views on what other projection methodologies the Profession should seek to research and how such research is best organised?

Comments should be submitted via e-mail to projections@cmib.org.uk or in writing to:
Dave Grimshaw, CMI, Cheapside House, 138 Cheapside, London, EC2V 6BW.

Comments should be received by **FRIDAY 17 AUGUST 2007** to be considered for the initial library which the CMI then intends to publish.