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A CONSISTENT SYSTEM OF INVESTMENT AND BONUS DISTRIBUTION FOR A LIFE OFFICE

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INTRODUCTION

The object of this paper is to continue the discussion on the valuation and bonus problems of a life office that was recommenced in the recent papers by A. T. Haynes and R. J. Kirton (*T.F.A.* XXI, 141) and by F. M. Redington (*J.I.A.* LXXVIII, 286). Both of these papers were concerned, *inter alia*, with the principle of matching assets to liabilities or, in Redington's graphic phrase, of immunizing the life fund against the effects of a change in the general level of interest rates. Immunization of a life fund containing with-profits policies was shown by Redington to have far-reaching implications, and it was clear from the discussion on his paper that different opinions were held about the exact character of the immunization that might be most appropriate. For a non-profits fund total immunization may or may not be a suitable ideal with which to compare the practical position, and this would depend, in particular, upon the relative size of the capital and other free resources available and upon the level of the surrender and paid-up-policy options granted. But it is the purpose of this paper to argue that, for a with-profits fund, far from being a suitable ideal to be either aimed at or departed from, total immunization (including absolute matching) represents a dividing line between a zone of rational departure from another and more natural ideal and a zone in which a change in the level of interest rates would produce indefensible results.

In the first place, it seems to the present authors that the whole concept of matching assets to liabilities arises out of a mixture of two objectives and, in the case of with-profits business, a desideratum. The first of the two objectives of matching is to ensure that the requisite cash will always be available to meet the claims on the life fund as they arise, and the second is to ensure the parallel movement of the value of the assets and the value of the liabilities as the level of interest rates changes. The desideratum is a rather vague regard to the principles of equity in bonus distribution which, in fact, lie deep at the root of the whole subject. There is no need to adopt an altruistic attitude to equity. But at least it is important in a practical world so to act as to avoid being priced out of the market. This requirement not only governs the relations of premium rates between different ages and different periods of assurance but also influences the treatment for bonus purposes of policies issued at different times and, in particular, means that new business must not be required to lend financial support to existing business. It will be our purpose later in the paper to show that these considerations would require, in theory,

that sufficient attention should be paid to the notional linking of investments to individual policies at the time of acquisition as well as at the time of redemption. That is to say, matching liability-outgo and asset-proceeds is not enough for any 'natural' view of equity; there needs to be a matching of premium-income and investment. But more of this later.

The principle of matched assets and the consequences of failure to match are developed by Haynes and Kirton for a non-profits fund. It is only in a rather tentative way that they consider a with-profits fund, and there they make a notional division of the fund (or corresponding assets) into two parts. The first part is to cover the accrued liability calculated on a gross premium basis without allowance for future bonuses. The second part, or balance of the assets, is regarded by them as the sum of the current surplus and the additional reserve out of which current and future bonuses are to come. Incidentally, the principles and methods by which this second part would be divided into its two components were not discussed. This is, of course, the crucial valuation problem in practice for the common case of a with-profits fund whose solvency is not in question. Having split the assets into two parts in this way they regard matching as a principle applicable to the first part and free investment for profit as appropriate to the second part.

Redington, on the other hand, takes with-profits business in his stride and focuses attention more on the effects of changes of interest level on valuations of liabilities and assets. Matching of assets to liabilities is the by-product of an adjusting process rather than an end in itself. Attention is focused on valuation rather than on the need for cash to be available at the right time to pay the claims. The link between redemption of securities and the maturity of particular policies is broken as well as the link at the time of the investment of the policyholder's premiums. Redington would, we think, agree that his treatment of bonuses would be more suitable for guaranteed bonuses, and indeed he specifically points out the consequence of immunization that bonuses earned on new business become tied to the level of interest rates prevailing at the time of entry. While he acknowledges that, in practice, offices would depart from this position, he does not accept the conclusion that these consequences show the unsuitability of the principle of total immunization for a with-profits fund.

It may be that for a non-profits business which is not backed by adequate capital resources the principle of total immunization would be essential; but if surrender value or paid-up policy rights approaching the full theoretical level have been granted the office is in an unavoidable dilemma—should it immunize against the option to continue the policies or should it immunize against the option to discontinue the policies? The fact is that the justification for a wholly non-profits business has hitherto been the practical certainty of a sufficient rate of interest. But the post-war drive for ultra-cheap money brought to many actuarial minds the possibility of the realization of the Keynesian idea of a vanishing interest rate. The necessity either for premiums to be fixed on a low-interest basis or for a sufficiency of capital resources is obvious in the case of a new portfolio of non-profits business, but, as Haynes and Kirton and Redington show, the process of matching or immunization could in due course reduce the part to be played by capital resources in protecting the fund from fluctuations in interest rates at the price of restricting the office's freedom of investment. On the other hand, the future bonus loadings of with-profits policies provide a substantial buffer. This buffer

diminishes as the duration of the policies grows longer and as bonuses vest, and ultimately—particularly with a closed fund—some approach to matching seems to be essential.

With a progressive business of new and existing policies, not too heavily weighted with non-profits policies, it is apparent that the buffer provided by future bonus loadings, free reserves and capital, if any, greatly reduces the need for a rapid approach to a matched position. To what extent the with-profits premium rates should include a margin for bonuses, and what would be a proper balance between the rate of interest assumed in the calculation of the premium rates for non-profits business and the capital resources of the office, represent nice problems for the exercise of actuarial judgment in the light of the prevailing conditions. The smaller the buffers the more the need to escape from a speculative position. On the other hand, the greater the buffers the smaller the speculative element becomes from the point of view of solvency taken alone. In these circumstances the opportunity—and indeed the duty—exists for the office to operate its investments to the best financial advantage that is open to it. This position is, however, conditioned by the requirements of equity in the sense to which reference has already been made. It is with the effects of this condition, which arise out of what we regard as a *natural* approach to with-profits business, that this paper is mainly concerned.

INFLUENCE OF POLICYHOLDERS' EXPECTATIONS

When a policyholder effects a with-profits policy he expects his bonuses to reflect the general experience of the office. Lower expenses, lower mortality and increased interest earnings should all mean better bonuses for him. The reasonable policyholder would also expect his bonuses to suffer if experience in any of these factors should deteriorate. These thoughts are perhaps only vaguely present in his mind. On the other hand, there are some possibilities which he would definitely not expect to happen, nor acquiesce in if they did. For example, if the level of interest rates improved significantly shortly after entry any reasonable policyholder would expect to derive a fair share of benefit from these improved conditions, at least so far as the investment of his own premiums contributed to the improved interest earnings of the office. He would expect his office so to arrange its affairs that this expectation was in due course fulfilled. He would not expect his bonuses to be fixed on the basis of the investment conditions prevailing at the time of entry and to be impervious to subsequent changes in actual experience.

We can also be sure that where a policy is nearing maturity the policyholder would be expecting the proceeds of his policy to become more and more definite. He would not expect his office to have his money invested in such a way as to be risking substantial depreciation, or if it had arranged its investments in such a way he would not expect to have to bear the brunt of it even to the extent of a cut in his final bonus. Nor would he, if he is a reasonable man, expect to reap any benefit from any appreciation that might have come from the office's speculating with his maturing policy-proceeds.

It is our suggestion that these expectations of a reasonable policyholder are fundamental not only to a proper policy of bonus distribution but also to a properly planned arrangement of an office's investment portfolio. These two objectives must be absolutely consistent. Be that as it may, there is a practical point of prime importance to the management of the office which reinforces

these considerations in the case of policies of short duration, or even of longer duration if the office's scale of paid-up policies approaches the full theoretical values. Unless a recent entrant could depend on obtaining his fair share of the benefit of improved investment conditions, he would be foolish if he did not lapse his policy or convert it into a paid-up policy and effect a new one for the remainder of the term, in order to secure the benefit of the improved investment conditions. This option to lapse and re-enter would naturally be most valuable when there had been a sharp rise in the level of interest rates, and for long-term policies on which only a few premiums had been paid so that the loss on lapse would be at a minimum. This option is not the least of the evils which the logic of a policy of total immunization creates.

The possibility of lapse and re-entry has long been a factor in the development of industrial assurance for various reasons, and its control has largely been effected through the system of profit distribution. In ordinary life assurance the possibility has existed in a small way from time to time whenever non-profits rates have been reduced for new business, but premium-scale adjustments have usually been effected in small steps. This may not always be so, particularly in connexion with pension schemes, but control can, to some extent at least, be maintained through the contractual conditions of surrender. Redington emphasizes the financial dangers of surrender options but does not distinguish, as we feel should be done, between with-profits and without-profits policies or between a cash-surrender option and a paid-up-policy option.

THE BASIC PRINCIPLE

With the foregoing discussion as a background we may now propound and examine a fundamental principle by which a life office could arrange mutually consistent policies of investment and bonus distribution which would, it is suggested, adequately combine the office's financial and business needs with the participating policyholders' reasonable expectations. This principle is in two parts as follows:

(1) New business should not be required to support the benefits of existing business.

(2) Future premiums on existing policies should secure at least as favourable benefits as new business.

In order to give precision to this principle, it is necessary to have a rule by which future benefits under a policy are divided into two parts: (i) that part secured by past premium-payments and (ii) that part secured by future premium-payments. The obvious division is to associate part (i) with the paid-up policy and part (ii) with the balance of the benefits. For this purpose, however, the office paid-up policy is inappropriate because it would normally allow for a discontinuance penalty, an option charge or a direct contribution to the profit of the office. Some definition of a theoretical paid-up policy is required. Let us write $\pi_{x\overline{n}|}$ for the original scale of with-profits premiums, stripped of expense loadings, that was in use when the policy was effected t years ago, and let $\pi_{x+t:\overline{n-t}|}$ be the corresponding with-profits premium, on the same scale, for the present age of the life assured and for the remaining term of the policy. Then, ignoring irrelevant refinements about health and selection, the future premium-payments can be associated with the benefits under a new policy for a sum assured of $S\pi_{x\overline{n}|}/\pi_{x+t:\overline{n-t}|}$, where S is the original sum assured.

The paid-up policy or the part of the sum assured associated with the past premium-payments would be the balance of the sum assured, i.e.

$$S(1-\pi_{x\overline{n}|}/\pi_{x+t:\overline{n-t}|}).$$

Consistent with this arrangement would be the assumption that the existing bonuses would attach in full to the paid-up policy, which would also participate in future bonuses. Thus the part of the sum assured associated with future premium-payments would attract bonuses just like a new policy for the same amount, assuming that there has been no change in the premium scale. It is, of course, assumed that $\pi_{x+t:\overline{n-t}|}$ is available for all values of t from $t=0$ to $t=n-1$.

Such a division of benefits for a reversionary bonus system implies that the level office premium would comprise two varying parts. The part relating to the basic sum assured would generally increase with duration and that for the bonuses would generally decrease. This contrasts with the more usual view that the premium is divided into two level parts, a 'non-profits' premium and a bonus loading. It also contrasts with the suggestion made by Perks in 1933 (*J.I.A.* LXIV, 264) that the 'bonus reserve' in a net premium valuation could be included in the basic reserve by valuing a reducing net premium instead of by reducing the valuation rate of interest.

The notional division of the policy into two parts by the paid-up-policy principle can be seen more clearly by slightly re-arranging the expression for the valuation liability. A bonus reserve form is used to bring out the consistent treatment of the bonuses and it is assumed that the premium, experience and valuation bases are all the same. Ignoring expenses and allowing for a simple reversionary bonus rate of b_1, the valuation liability at duration t, i.e.

$$V(t)=(1+tb_1)\,A_{x+t:\overline{n-t}|}+b_1(IA)_{x+t:\overline{n-t}|}-\pi_{x\overline{n}|}\,\ddot{a}_{x+t:\overline{n-t}|},$$

can be split into two parts as follows

$$\left\{\left[\left(1-\frac{\pi_{x\overline{n}|}}{\pi_{x+t:\overline{n-t}|}}\right)+tb_1\right]A_{x+t:\overline{n-t}|}+b_1\left(1-\frac{\pi_{x\overline{n}|}}{\pi_{x+t:\overline{n-t}|}}\right)(IA)_{x+t:\overline{n-t}|}\right\}$$

$$+\left\{\frac{\pi_{x\overline{n}|}}{\pi_{x+t:\overline{n-t}|}}\left[A_{x+t:\overline{n-t}|}+b_1(IA)_{x+t:\overline{n-t}|}\right]-\pi_{x\overline{n}|}\,\ddot{a}_{x+t:\overline{n-t}|}\right\}.$$

The first part is the value of the paid-up policy as defined earlier. The second part relates to the future premiums and the benefits provided by them. This part is, accordingly, equal to zero. If the rate of interest changes at duration t for the remaining duration of the policy, a bonus at the rate of, say, b_2 can be supported in future by $\pi_{x+t:\overline{n-t}|}$. At the new rate of interest, and using b_2 instead of b_1, the second part of the expression for the liability remains equal to zero. The significant point is that the first part will alter by the same amount as the value of the corresponding assets if these have been matched to the paid-up policies, so that the bonus rate on this part of the contract will remain at b_1.

The rule for dividing the assurance into its two parts applies notwithstanding an alteration in the premium-scale from π to π'. The bonus to be credited in future to the part of the assurance provided by the future premiums would, however, be at the rate that $\pi_{x+t:\overline{n-t}|}$—not $\pi'_{x+t:\overline{n-t}|}$—would support in the new conditions.

In practice, an office does not continually alter its with-profits scale of premiums as financial conditions vary. Instead, it lets the bonus rates take up

the effect of the variations of experience over the long run. This means that a uniform reversionary bonus system (for the moment we assume uniformity only for different ages and terms but not necessarily for different entry dates) may become somewhat ill-adjusted to a set of premium rates after one or more changes in experience. On the other hand, the degree of maladjustment may be considerably overshadowed by the contribution to bonuses from miscellaneous sources, about which more will be said later. This type of maladjustment is sometimes corrected in varying degrees by using different bonus rates for whole-life and endowment assurances, for different ranges of maturity ages or by an occasional adjustment of the premium scales for new business. Such changes in premium scales are more often adjustments between ages and between terms of endowment assurance than adjustments to a different level of premium rates. The maladjustment that arises from the use of the same rate of reversionary bonus, whether simple or compound, for policies effected at different dates in different financial conditions is part of the main subject of discussion in this paper.

CONSEQUENCES FOR INVESTMENT AND BONUS

If the new business and the future premiums on existing business were to be linked together and then, for investment and bonus purposes, separated from the future benefits provided by past premium-payments on existing business, the consequence would be that the existing fund should be treated as earmarked to secure the due payment of the theoretical paid-up policies, with existing and future bonuses thereon, when they mature. It does not follow that absolute matching of investments and liabilities (quite apart from future bonuses on the theoretical paid-up policies) is possible, because the notionally closed fund may still be increasing if relatively few of the policies are near maturity. But the whole investment policy may, not inappropriately, be directed to immunizing the fund of paid-up policies according to the rule that we have suggested.

It is legitimate to think in terms of investing the relevant part of each premium to provide the corresponding increment in the theoretical paid-up policy, the future bonuses thereon, and the current bonus on the full sum assured less the paid-up sum assured before payment of the current premium. Each year's increments would be appropriately invested so as to set up a revised matched or immunized state on a paid-up basis. The process would be continuous. Thus the situation would be that, associated with the increment of paid-up policy arising on payment of the tth premium, there would be a rate of bonus b_t. This would apply to the bonus in the current year on the sum assured less the amount of the paid-up policy before payment of the current premium and to all future bonuses on the current increment of the paid-up policy. b_t is the rate of level bonus that could be supported by $\pi_{x+t:\overline{n-t}|}$ if the new conditions remained unchanged until maturity of the policy. In theory a minor adjustment is required if the premium scale in the new conditions supports bonuses which are not uniform. This complication is deferred until later.

To clarify the matter, it will be assumed that a simple reversionary bonus system for all policies is in operation, that the scale of premium rates has not been altered since the date when the oldest policy in force was effected, and that the same scale applies to new business. For simplicity it will also be

assumed that the policies in force are all endowment assurances maturing at a single given age, that the only source of variation from the original assumptions on which the premiums were based is the rate of interest (or, better still, that the effects of any such other variation are treated as part of the miscellaneous surplus, the disposal of which may be excluded from considerations of equity or financial policy), and that paid-up matching or immunization is in full operation. It will be argued later that if departure from paid-up immunization is justified on a sufficient prospect of benefit to the fund, the resulting profit or loss is also best considered as a part of the miscellaneous profit or loss.

We may now compare the effects on the bonus rate of a change in the rate of interest (i) if the fund is immunized on a paid-up basis, (ii) in conditions of total immunization in the Redington sense and (iii) in conditions in which the fund is invested dead short, e.g. on deposit at call. It will be assumed that the rates of interest being earned before and after the change in general level are the same for all three investment situations.

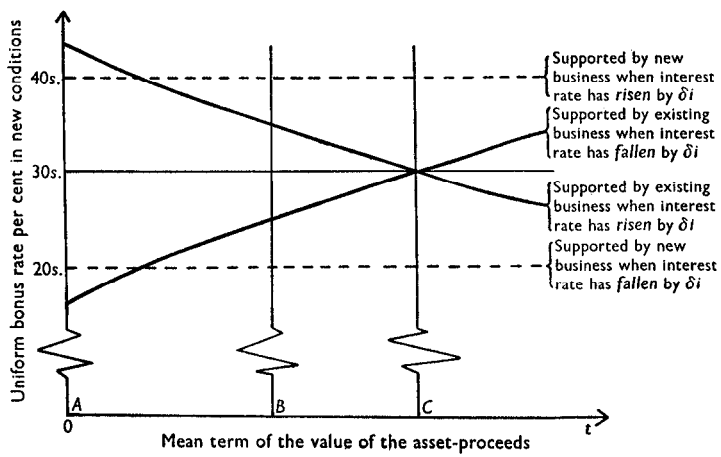


Fig. 1. Schematic representation of the effect on bonus rates of a uniform change in interest rates, according to different investment situations.

The *uniform* rate of future bonus that could be supported by the fund in the new conditions would not necessarily be appropriate for individual policies. This question will be considered in more detail later, but a general picture of the effects of different investment policies can best be seen by considering the uniform rate of bonus that the fund could support when conditions change suddenly. The different effects on the bonus rate are illustrated schematically in Fig. 1, which shows the changes in the *uniform* future bonus rates supportable by the new and existing business respectively before and after a change of δi in the rate of interest. The original bonus rate is assumed to be 30*s.*%. The effect of $\pm \delta i$ is assumed to be 10*s.*% for new business written after the change; this applies, of course, to all investment situations under discussion. The curved lines represent the uniform future bonus rate supportable by the existing business in the new conditions according to the mean term of the value of the asset-proceeds as defined by Redington.

The small effects of differences in the higher moments of the distributions of the value of the asset-proceeds and the value of the liability-outgo are ignored. In the figure ordinates are drawn at points corresponding to the mean terms of the value of the asset-proceeds when assets are invested (*A*) dead short, (*B*) to give paid-up immunization and (*C*) to give total immunization. The bonus curves for the existing business cut the ordinates at points corresponding to the uniform future bonuses that can be supported immediately after the change in conditions. For longer mean terms than are represented by the point (*C*) the result of an increase in the rate of interest would be to reduce the future bonuses on the existing business, and conversely for a decrease in the rate of interest.

EXAMPLES OF BONUSES

Examples have been worked out to show the effect of a change in the rate of interest on the future bonus rate in conditions of paid-up immunization. These bonus rates will be called 'natural' bonus rates. The premium rates have been calculated by the following formula using the A1924–29 ultimate table with 3% interest:

$$\pi_{x:\overline{60-x}|} = \frac{A_{x:\overline{60-x}|} + \cdot 015\,(IA)_{x:\overline{60-x}|}}{\ddot{a}_{x:\overline{60-x}|}}.$$

Thus allowance has been made for a simple reversionary bonus of 30*s.*% for each premium paid, and also for claims to be paid at the end of the year of death. Expense and other loadings have been ignored and attention has been confined to endowment assurances maturing at age 60. Specimen premiums are given in Table 1, together with some specimen theoretical paid-up policy values as previously defined (i.e. with full existing bonuses and future bonuses on the reduced sums assured); the corresponding values by the commonly-used proportionate rule are also shown for comparison.

Table 1. Specimen premium rates for with-profits endowment assurances, theoretical paid-up-policy values and corresponding proportionate values

Age at entry	Term of policy	Premium (%)	Paid-up-policy values at duration							
			5		10		20		30	
			T.	P.	T.	P.	T.	P.	T.	P.
20	40	2·294	·148	·125	·289	·250	·545	·500	·776	·750
30	30	3·226	·186	·167	·361	·333	·684	·667		
40	20	5·047	·258	·250	·506	·500				
50	10	10·224	·494	·500						

T. = theoretical; P. = proportionate.

Table 2 shows specimen 'natural' bonuses that could be supported in the future by existing policies when there has been a sudden rise in the rate of interest from 3 to $3\frac{1}{2}$% which is assumed to be permanent. As the investments held are assumed to comply with the requirements of paid-up immunization on the basis of the theoretical paid-up policies, the change in the rate of interest affects only the benefits earned by the future premium-payments. Accordingly, the bonus rates shown in Table 2 are in each case the weighted

averages of 30*s.*% on the paid-up policy and of the new rate on the balance of the sum assured, i.e. they are the uniform rates that could be supported on the full sum assured in the changed conditions.

Table 2. Specimen 'natural' reversionary bonus rates per cent for the remaining terms of policies of different durations when the interest rate changes from 3% to 3½%

Age at entry	Term of policy	Duration of policy at the date of change in the interest rate																					
		0			5			10			15			20			25			30			35		
		£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*
20	40	1	19	6	1	17	7	1	15	11	1	14	6	1	13	3	1	12	3	1	11	5	1	10	9
					1	*17*	*9*	*1*	*16*	*3*	*1*	*14*	*10*	*1*	*13*	*7*	*1*	*12*	*6*	*1*	*11*	*7*	*1*	*10*	*10*
30	30	1	18	4	1	16	3	1	14	7	1	13	2	1	12	0	1	11	0						
					1	*16*	*5*	*1*	*14*	*9*	*1*	*13*	*4*	*1*	*12*	*1*	*1*	*11*	*1*						
40	20	1	17	2	1	15	0	1	13	2	1	11	7												
					1	*15*	*0*	*1*	*13*	*2*	*1*	*11*	*7*												
50	10	1	16	4	1	13	3																		
					1	*13*	*2*																		

Note. The figures in italics are approximate 'natural' bonus rates calculated by using the proportionate paid-up policies.

The figures in Table 2 are perhaps a sufficient indication of the numerical effect of a change in interest rates in conditions of paid-up immunization. In particular, it will be observed that a permanent change of 10*s.*% in the interest rate gives rise to an average change of rather less than 5*s.*% in the future bonus rate for a block of business well spread over the various durations.

INVESTMENT IMPLICATIONS

An important implication of paid-up immunization is the shortness of the mean term of investment. This would not matter if it did not also imply a significant reduction in yield in the long run. Accordingly, an office might decide to invest in longer-term securities or in property and equities in order to improve the yield on its funds in the general interest of its participating policyholders. The extra yield from the longer term of its investments over what could be obtained from strictly following out the reasonable expectations of the policyholders may, not unreasonably, be regarded as a corporate benefit not identifiable with any particular group of policies. Similarly, the risk involved ought not to be related to any particular group of policies. This policy could seemingly be pursued without serious difficulty provided that the mean term of the investments were not allowed to go beyond the point of total immunization. To allow the mean term to pass beyond that point would be a speculation involving risks and consequences which need not be pursued here.

If short-term rates were not generally at variance with long-term rates and market conditions were stable, an office could, not inappropriately, regard every £100 of its fund, whenever received into the fund, as earning in each year the same rate of interest as every other £100. This, indeed, would be a suitable foundation for the orthodox contribution method of distributing surplus, but in practice such an assumption would be consistent only with

dead-short investment. During this century there have been fluctuations in medium-term and long-term interest rates from about $2\frac{1}{2}\%$ to over 5%. In such conditions paid-up immunization provides a suitable norm or safety point for an investment policy from which experienced judgment may in particular circumstances suggest a departure in one direction or the other. But there can be no doubt that any attempt to beat the market would be a dangerous practice for a life office. Moreover, in practice, interest rates are not uniform; the *differential* between short-term and long-term rates fluctuates widely. To depart from the norm with the view to changing direction at a later stage when the market has (if the speculation has come off) moved in the office's favour would be nothing more than a speculation, which would carry with it the obvious danger of being 'caught on the wrong leg'—hardly a suitable risk for a life office to take. In practice, of course, the stock market is such that only a relatively small office could indulge effectively in speculations of this kind.

It is suggested that for with-profits funds the only justification for departing from a position of paid-up immunization would be to secure a higher yield on the funds. It then becomes a nice problem calling for the highest judgment whether the expected improvement in the yield would be an adequate compensation for the speculative position that would be reached with respect to the norm. Adapting the famous saying of Elderton's about good new business and valuations, it may well be asked which should be abandoned when the immunization principle and a good investment conflict.

It is interesting to see the extent by which the mean term of investments under paid-up immunization falls short of the mean term for total immunization. For this purpose, the future progress of some simple types of fund comprising only with-profits policies is examined. All complications are ignored: the mortality experience is assumed to coincide both with the premium basis and with the valuation basis; there are no expenses, no income

Table 3. Future progress of Funds I and II, assuming no new business and (A) future premiums are paid when due, (B) no further premiums are paid, the policies being all converted into paid-up policies on the theoretical basis

Quinquennium ending 1 January	Net decrease in fund			
	I(A)	I(B)	II(A)	II(B)
1955	432	2,598	*−1,805*	1,394
1960	759	2,234	*−1,122*	1,605
1965	1,296	1,981	97	1,963
1970	1,881	1,612	1,793	2,130
1975	2,503	1,113	4,062	1,909
1980	3,129	462	6,975	999
Total	10,000	10,000	10,000	10,000
Mean term in years of the values of the liability-outgo	14·5	8·7	21·5	11·2

Note. The current premiums and current maturity claims due at the end of the quinquennium are assumed to have been paid. The first period strictly relates to 4 years only, though it includes 5 years' premiums.

tax and no miscellaneous sources of surplus. All policies remain in force until they become claims by death or by maturity.

On 1 January each year a 30-year endowment assurance policy for unit sum assured is issued to each of a number of lives aged exactly 30 years. The annual premium is on the same basis as before and is similarly loaded for a simple reversionary bonus of 30*s.*% for each year's premium paid.

Fund I, hitherto stationary, has been recruited by 575·48 entrants a year up to and including 1 January 1950, after which there have been no further entrants.

Fund II, hitherto increasing, has been recruited by 344·06 entrants on 1 January 1921 increasing at the rate of 5% each year to 1416·21 on 1 January 1950, after which there have been no further entrants. This rate of increase of new business is rather high, but, on the other hand, no allowance has been made for withdrawals: the constitution of the fund on 31 December 1950 might therefore be reasonably representative of a practical situation.

The valuation liability of each fund on 31 December 1950 on the premium basis, assuming a liability for future bonuses at the rate of 30*s.*% per annum, is 10,000. The net decrease in each fund during successive quinquennia is set out in Table 3 on the two different assumptions

(A) that premiums continue to be paid on all existing policies as they fall due;

(B) that no further premiums are paid.

Under (B), each policy is assumed to be converted to a paid-up policy from 1 January 1951 on the original premium basis and receives future bonuses at 30*s.*% on the reduced sum assured. Thus the paid-up sum assured of a policy of t years' duration on 31 December 1950 is $(1-\pi_{30:\overline{30}|}/\pi_{30+t:\overline{30-t}|})$ and the existing declared bonuses amount to $\cdot 015t$.

The future progress of the funds is plotted in Fig. 2. Ordinates are drawn to show the corresponding mean terms of the values of the liability-outgo.

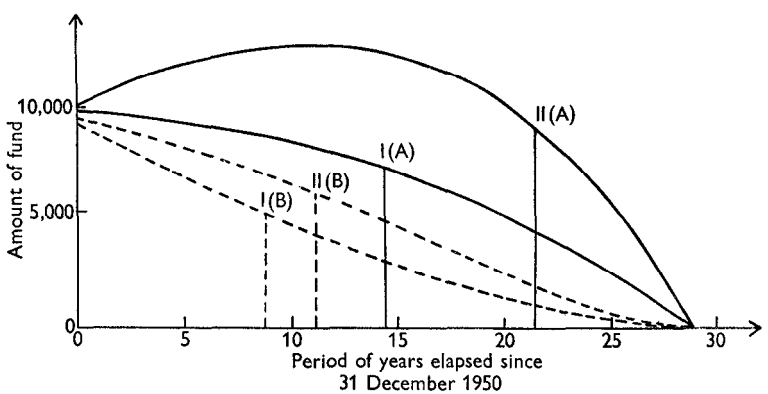


Fig. 2. Future trends of different closed funds showing the mean terms of the present values of liability-outgo. Fund I(A): Stationary up to 1950, premiums continuing on existing policies. Fund I(B): Stationary up to 1950, no further premiums payable. Fund II(A): Increasing up to 1950, premiums continuing on existing policies. Fund II(B): Increasing up to 1950, no further premiums payable.

Suppose now that each fund is invested in 3% bonds at par, the redemption dates being chosen in such a way as to immunize the fund against a change in the rate of interest. Redington has shown that there is an infinite variety of asset-patterns which will satisfy this requirement. In order to pursue the numerical illustration only one such pattern has been selected for each of the funds, but it has to be remembered that there are in fact several other practical distributions.

Where the fund decreases throughout, the distribution of the redemption dates has been chosen to coincide with the net decreases in the fund in Table 3. This provides an 'absolute match'. For II(A), where the fund is humped, the selection has been made which gives rise to the least number of changes in the portfolio as the fund works itself out.

The mean terms of the values of the asset-proceeds are, of course, the same as those of the values of the liability-outgo shown at the foot of Table 3. The arithmetic means of the redemption dates are: Fund I(A), 19·4 years; Fund I(B), 10·8 years; Fund II(B), 14·2 years. The distribution of the redemption dates for the asset pattern selected for the humped Fund II(A) is given in Table 4.

Table 4. Distribution of assets for the humped Fund, II(A)

Quinquennium ending 1 January	Bonds redeemable during quinquennium
Up to 1970	0
1975	1,233
1980	6,975
Irredeemable	1,792
Total	10,000

The effect of making the policies paid-up on 31 December 1950 would be to reduce the mean term of the values of the liability-outgo by 40–50%. The effect on the mean term of the liability-outgo itself is rather more marked.

VALUATION

So long as the experience of a fund coincides with the premium basis the valuation basis presents no difficulty. The valuation should release surplus in such amounts as to permit the declarations of bonuses exactly as provided for in the premiums. The obvious solution is a bonus reserve valuation on the premium basis, including as an asset the value of the future bonus loadings and as a liability the value of future bonuses of the amounts allowed for in the premiums. This applies whatever the form of the bonus. In the particular case of a uniform simple or compound reversionary bonus closely similar results can, of course, be obtained by omitting the future bonuses and bonus loadings and providing the 'bonus reserve' by an appropriate reduction in the valuation rate of interest or by some other method.

It is when the experience and premium bases diverge and when the unknown future experience cannot be expected to be the same as the past experience that the valuation problem becomes difficult. This is, of course, the normal practical situation which is further complicated by the incidence of a varying

experience for policies of different class and duration and by the changes in premium scales that may have taken place from time to time.

Our present concern is to consider a situation in which the experience and premium bases for a group of policies participating on the uniform simple reversionary bonus system have been in accord so far and have now diverged by reason of a sudden permanent rise in the rate of interest. It is assumed for the time being that investments have been made according to the principle of paid-up immunization. Now it is obvious that, on the interest basis applicable before the rise, the value of the paid-up policies and of the future bonuses thereon is identical with the usual bonus reserve value and also approximates to the corresponding net premium value. All these valuations fit the value of the assets. Immediately after the change in the rate of interest the value of the paid-up policies (and of the immunized future bonuses thereon) at the new rate of interest will be exactly equal to the new value of the assets. If these paid-up policies and assets were followed through the subsequent years in the new conditions, the surplus would be released in the right amounts to provide the corresponding bonuses whichever of the following two methods were used: (1) valuing both the liabilities and the assets at the old rate of interest or (2) valuing both the liabilities and the assets at the new rate of interest. In method (1) the assets are, of course, valued at more than the market values, whereas in method (2) the process would imply the gradual amortization of the amount written off the value of the assets at the time when the interest rate rose.

For the balance of the sums assured, and future bonuses thereon, and the corresponding future premiums on the existing policies, the valuation problem would be exactly the same as for the new business. The scale of premiums would in the new conditions support a higher rate of bonus, but this would not be uniform—it would vary by class of assurance, by age at entry (present age for existing policies) and by term of assurance (remaining term for existing policies). A bonus reserve valuation at the new rate of interest, and allowing in each case for the appropriate bonus rate, would obviously meet the situation, but this would be a complicated procedure. However, the variation in the new rates of bonus would not be large, as is shown by the figures for duration 0 in Table 2, and an average figure could no doubt be selected which would produce sufficiently accurate results in practice, particularly if the premiums for new business were adjusted in due course to this average bonus rate.

When the existing business is all off the books, the appropriate bonus reserve basis would obviously incorporate the new rate of interest and the new average rate of bonus. Meanwhile, this basis would apply to the future premiums, and corresponding benefits, on the existing policies. But, for the paid-up benefits, the valuation could be either at the old rate of interest with the old bonus rate or at the new rate of interest, again with the old bonus rate, provided that the assets were valued correspondingly. The practical problem is to determine a single basis for all the business in force at each valuation date from the date of change until the only business on the books is business written after the change in the interest rate. Clearly, the new rate of interest (with assets correspondingly valued) could be used throughout, and the problem then reduces to fixing the appropriate rate of future bonus (or rates of bonus if it is decided to differentiate between sections of the business) for each valuation.

The average rate of interest being earned on the funds at the valuation date (coupled with assets at cost prices*) might be a suitable basis. Such a basis would gradually merge into the new conditions when all the business on the books has been effected after the rise in the rate of interest and when, because of paid-up immunization, all the assets held are assets bought after the change. It would have the further merit that it does not treat the change in the level of interest rates as being necessarily permanent but gradually adjusts the valuation basis to the new conditions as they actually influence the interest earnings on the fund.

Whatever rate of interest is used the level of future bonuses to be assumed in the valuation presents a difficult problem. In general, after a change in the level of interest rates, the appropriate level of future bonuses to be assumed in a bonus reserve valuation and the current rate of bonus to be declared are not necessarily the same. The valuation rate of interest and the rate of future bonuses to be assumed are, of course, intimately linked, and there is reason to think that, if the current bonus is to be equivalent to the natural bonuses, equality between the current and future bonus rates can be obtained only when the valuation is based on a rate of interest somewhere between the rate consistent with the market value of the assets and the rate being earned on the fund with assets taken at cost prices, that is, on a kind of average future rate of interest on the fund assuming that the current market rate remains unchanged. This is a matter for investigation and will be illustrated numerically with various other possibilities.

Turning to the net premium method, the first question is 'what change in the valuation rate of interest is necessary to attain a net premium valuation basis suitable for the new business (and the benefits associated with the future premiums on the existing business), having regard to the increase in the rate of interest and the corresponding increase in the average rate of bonus that the new business (and future premiums on the existing business) will support?' Because of the higher bonus rate to be provided for, the necessary increase in the valuation rate of interest is much less than the increase in the investment rate of interest. The increase is, of course, affected by whether the existing bonuses are to be valued at the same rate of interest as is used for the policy values or at the investment rate.

For the existing business, immediately after the change in investment conditions the old valuation rate of interest would be suitable, assuming that the assets then held are not written down to their value at the new rate of interest. Apart from the automatic adjustment through the gradual increase in the amount of the existing bonuses, the effect on the valuation reserves of passing from the original to the ultimate basis would be small and would be spread over a long period.

If, for balance-sheet purposes or other reasons, it is thought to be necessary to value the assets at the new interest rate, the problem of valuing the liabilities becomes much more complicated. Clearly, an immediate weakening of the valuation basis would be necessary, followed by a gradual strengthening to meet the position when the new rates of bonus, as well as the new rate of interest, become fully operative.

* Throughout the paper the expression 'cost prices' is used to mean purchase prices subject to amortization of premiums and discounts compared with redemption values.

The foregoing discussion will now be illustrated by some numerical examples. For this purpose, consider a fund containing only 30-year endowment assurances effected at age 30. The same premium basis is assumed as in the previous illustrations. A further simplification is introduced by assuming that $l_{30}\,10^{-3}$ persons have effected policies for unit sums assured on 1 January of every fifth year from and including 1921. It is assumed that the experience up to 31 December 1950 coincides with the premium basis and that there are no lapses. It is also assumed that annual valuations and distributions are made and, unless stated to the contrary, that all previous bonuses have been allotted at natural rates. (This simplification enables the funds to be calculated independently of previous valuations.) The numerical examples will illustrate methods of valuation on and after 31 December 1950 assuming, unless stated to the contrary, that the fund is immunized at all times on the paid-up basis and assuming that the interest rate changed from 3 to $3\frac{1}{2}\%$ on 31 December 1950.

BONUS RESERVE VALUATIONS

For a bonus reserve valuation on 31 December 1950, the assets may be valued at either 3 or $3\frac{1}{2}\%$. The former requires a valuation of the policies in two parts: the paid-up benefits, including future bonuses thereon at 30*s.*% per annum, would be valued at 3%, and the balance of the sums assured, with bonuses thereon according to the scale shown in Table 2 for duration 0 corresponding to the attained age and unexpired term, and the future premiums, would be valued at $3\frac{1}{2}\%$. The results may be summarized as follows:

Valuation balance-sheet as at 31 December 1950

Liabilities:		*Assets:*	
Value at 3% of the paid-up sums assured, existing bonuses at 30*s*% per annum, and future bonuses on the paid-up sums assured (at 30*s.*)	37,282	Value at 3% of the investments	37,850
Value at $3\frac{1}{2}\%$ of the balance of the sums assured and corresponding future bonuses thereon less the value at $3\frac{1}{2}\%$ of the future premiums	0		
Value of net liability	37,282		
Surplus	568		
	37,850		37,850

The cost at 3% of a simple reversionary bonus of 30*s.*% on the sums assured on policies in force on 31 December 1950 exactly equals the surplus of 568. (The cost of interim bonuses is excluded from the surplus in this and in subsequent examples.)

If, on the other hand, the assets are valued at $3\frac{1}{2}\%$ the policies must be valued at $3\frac{1}{2}\%$ with future bonuses at the natural rates depending upon the durations of the policies (see Table 2). The bonus reserve valuation can then

be made in conventional form and the results may be summarized as follows:

Valuation balance-sheet as at 31 December 1950

Liabilities:		*Assets:*	
Value at $3\frac{1}{2}\%$ of sums assured and existing bonuses	45,950	Value at $3\frac{1}{2}\%$ of the investments	36,524
Value at $3\frac{1}{2}\%$ of future bonuses at natural rates	6,114		
	52,064		
less value at $3\frac{1}{2}\%$ of future premiums	16,081		
Value of net liability	35,983		
Surplus	541		
	36,524		36,524

The cost at $3\frac{1}{2}\%$ of a simple reversionary bonus of 30*s.*% exactly equals the surplus of 541.

It is interesting to notice that to bring out the same surplus to enable the appropriate bonus of 30*s.*% to be declared by means of a bonus reserve valuation at $3\frac{1}{2}\%$ with a uniform rate of future bonus, the rate of future bonus required is 34*s.*% If the declared rate and the future rate included in the valuation are to be the same, the rate works out at 33*s.* 7*d.*%.

If the assets are valued at cost prices and a valuation is made at the rate of interest being earned on the fund, i.e. at 3%, the appropriate rate of future bonus would be 30*s.*% throughout. The results would then be the same as in the first example above, except that the value of the future premiums and the value of the corresponding benefits would equate at a higher figure. This would be the position also for a 3% bonus reserve valuation of a corresponding fund for which the assets had been invested to provide total immunization, i.e. so that future bonuses were entirely immunized at 30*s.*%. The corresponding $3\frac{1}{2}\%$ valuation would show the same results as in the second example above, except that the value of the future bonuses would be lower by an amount which would exactly balance the further reduction in the value of the assets. This is because the future bonuses would be entirely at the 30*s.* rate, and the 3% bonds constituting the assets would have a correspondingly longer mean term.

At subsequent annual valuations it is obvious that the mixed basis as in the first example, with assets valued at cost prices, and the $3\frac{1}{2}\%$ basis as in the second example, with assets valued at $3\frac{1}{2}\%$, would both produce surpluses exactly sufficient to provide the natural bonuses. This is a necessary consequence of paid-up immunization. The make-up of the figures would, however, be different, and there would also be slight differences between the equivalent uniform rates of bonus that the surpluses on the two bases would support. It will be sufficient and arithmetically simpler, however, to illustrate the position at the subsequent valuations by using the $3\frac{1}{2}\%$ basis. The annual valuation results at 31 December 1955 are summarized in the statement below. The rate of interest is assumed to have remained at $3\frac{1}{2}\%$ since 31 December 1950, and a further $l_{30}\,10^{-3}$ policies are assumed to have been effected on the same terms on 1 January 1951.

Valuation balance-sheet as at 31 December 1955

Liabilities:		*Assets:*	
Value at $3\frac{1}{2}\%$ of sums assured and existing bonuses	46,219	Value at $3\frac{1}{2}\%$ of the investments	37,141
Value at $3\frac{1}{2}\%$ of future bonuses at natural rates	6,395		
	52,614		
less value at $3\frac{1}{2}\%$ of future premiums	16,081		
Value of net liability	36,533		
Surplus	608		
	37,141		37,141

The cost at $3\frac{1}{2}\%$ of simple reversionary bonuses at natural rates exactly equals the surplus of 608. These natural rates are given in the figures in Table 2 for age at entry 30. The corresponding uniform rate of bonus that could be declared out of the surplus of 608 is 33*s.* 9*d.*%. On the other hand, the reserve for future bonuses could be reproduced by allowing for future bonuses at the uniform rate of 35*s.* 6*d.*% instead of at the natural rates. If it had been assumed that new business entered every year, instead of every fifth year, the uniform percentage rates of current and future bonus would have been 34*s.* 2*d.* and 35*s.* 9*d.* respectively. On the assumption of new business increasing at the rate of 5% each year, the uniform rates of bonus are 35*s.* 2*d.* and 36*s.* 5*d.* respectively. These uniform rates are, of course, weighted averages, in the one case using $S \times A$ and in the other using $S \times (IA)$ as the weights.

The results of successive valuations are shown in Table 5. It is assumed that the rate of interest remains at $3\frac{1}{2}\%$ and that a further $l_{30}\,10^{-3}$ policies are effected every five years.

The equivalent uniform bonuses given in the last three columns of Table 5 show the effect of following the practice of using uniform bonuses instead of the natural rates required on the principles developed in this paper. By declaring uniform bonuses some advantage is given to policies of long duration at the expense of policies recently issued, but the range of the difference steadily narrows as time goes on. In practice, with a progressive business subject to lapses, the recently entered policies and the new business are likely to be much more heavily weighted than in the model used for our illustrations. Moreover, the discrepancy between the earned and the declared bonuses becomes relatively insignificant if considered in relation to total bonuses on any particular policy. But the most important solvent of the discrepancy in practice is the contribution to the bonus rate from miscellaneous sources. None of these, except perhaps to some extent the profit from mortality and expense loadings, seems to be attributable in any particular degree to individual policies or groups of policies entitled to share in profits, and the total expressed as a bonus rate, even after allowing for any arbitrary contribution to the office's free resources, is usually considerable and well able to absorb the discrepancy between declared uniform and earned natural rates of bonus when viewed over the whole duration of the policies.

Table 5 also shows that, in the given conditions, if natural bonuses are to be declared, the uniform rate of future bonus to be valued must be somewhat

Table 5. Results of bonus reserve valuations of a fund containing 30-year endowment assurances when the rate of interest changes from 3 to $3\frac{1}{2}\%$ at the commencement of the period

No. of years since the change in the interest rate	Valuation liability at $3\frac{1}{2}\%$ allowing for future bonuses at natural rates	Surplus for the year	Natural rate of bonus that could be declared for the year on policies at the valuation date of duration						Equivalent uniform rate of bonus % that the surplus for the year would provide	Equivalent uniform rate of future bonus % that the reserve for future natural bonuses would support	Uniform rate of bonus when the future and current rates are the same
			5 years	10 years	15 years	20 years	25 years	30 years			
			£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.
0	35,983	541	1 10 0	1 10 0	1 10 0	1 10 0	1 10 0	1 10 0	1 10 0	1 14 0	1 13 7
5	36,533	608	1 18 4	1 16 3	1 14 7	1 13 2	1 12 0	1 11 0	1 13 9	1 15 6	1 15 4
10	37,199	630	1 18 4	1 18 4	1 16 3	1 14 7	1 13 2	1 12 0	1 15 0	1 16 9	1 16 7
15	37,899	651	1 18 4	1 18 4	1 18 4	1 16 3	1 14 7	1 13 2	1 16 1	1 17 7	1 17 6
20	38,574	669	1 18 4	1 18 4	1 18 4	1 18 4	1 16 3	1 14 7	1 17 1	1 18 1	1 18 0
25	39,141	683	1 18 4	1 18 4	1 18 4	1 18 4	1 18 4	1 16 3	1 17 10	1 18 4	1 18 3
30 and later	39,493	691	1 18 4	1 18 4	1 18 4	1 18 4	1 18 4	1 18 4	1 18 4	1 18 4	1 18 4

higher than the uniform equivalent of the current bonuses, until the policies effected before the change in the interest rate have all matured. At each valuation the equivalent uniform rate of future bonus lies between the uniform rate that could be declared as the current bonus and the rate that could be supported by a new policy at the entry age. This accords with common sense. If the original premium scale is maintained when interest rates rise, the existing business cannot support so high a rate of bonus as will the new business. There must follow a period of rising bonuses until the level supported by the new business is reached.

When uniform current and future bonus rates are equated the current distribution is too generous. But such a course could be justified if the contribution to bonuses from miscellaneous sources overshadowed the small discrepancy between the uniform equivalents of the current and future natural rates. The converse holds of course for a fall in the rate of interest. The conclusion is that to equate uniform current and future bonus rates tends to amplify the fluctuations in bonuses caused by variations in the rate of interest. This is the consequence of using market values and the corresponding interest rate for the valuation.

It is interesting to compare the position under total immunization. In these conditions all policies effected before the change in the interest rate would 'earn' a bonus rate of 30*s.*% throughout, and all policies effected after the change would earn 38*s.* 4*d.*% throughout, assuming that the premium scale remains unaltered. Table 6 shows the equivalent uniform rates. The figures in the last column assume that all previous bonuses have been declared at the 'earned' rates.

Table 6. Uniform bonus rates corresponding to those in Table 5 but assuming total immunization instead of paid-up immunization

No. of years since the change in the interest rate	Equivalent uniform rate of bonus % that the surplus for the year would provide	Equivalent uniform rate of future bonus % that the reserve for future bonuses (at 30*s.* or 38*s.* 4*d.* as the case may be) would support	Uniform rate of bonus when the future and current rates are the same
	£ *s.* *d.*	£ *s.* *d.*	£ *s.* *d.*
0	1 10 0	1 10 0	1 10 0
5	1 11 0	1 12 3	1 12 1
10	1 12 1	1 14 4	1 14 1
15	1 13 5	1 16 1	1 15 10
20	1 14 10	1 17 6	1 17 3
25	1 16 6	1 18 4	1 18 2
30 and later	1 18 4	1 18 4	1 18 4

As would be expected the rise in bonus rates is much slower under total immunization than under paid-up immunization, and the relationship between the current and future uniform rates is different.

Returning to paid-up immunization we may now illustrate the operation of a bonus reserve valuation at the interest rate being earned on the fund with assets taken at cost prices. Such a valuation expresses an agnostic attitude to the rate of interest to be earned on future investments and is to be regarded

as an approximation to the mixed basis in which the paid-up policies at the date of the increase in the rate of interest are valued at the old interest rate and the balance of the contracts at the new interest rate. This mixed basis is equivalent to a valuation using natural future bonuses and a rising interest rate* corresponding to the rate earned on the fund in successive years. To use the current earned rate throughout means departing from the accepted principles of the bonus reserve method by excluding some of the future interest on the present assets; consequently a reduced rate of future bonus compared with natural rates must be used. For the numerical illustrations at successive valuations Table 7 shows, as the valuation rate of interest, the rate

Table 7. Results of bonus reserve valuations corresponding to those in Table 5, assuming paid-up immunization and using the interest rate being earned on the fund with assets at cost prices

No. of years since the change in the interest rate	Valuation rate of interest (%)	Uniform rate of bonus equivalent, on the valuation basis, to natural rates for the current bonus	Uniform rate of future bonus necessary to provide a current surplus just sufficient to provide		Uniform future and current rates of bonus when the future rate is such as would be supported from entry by the premiums on the valuation basis	
			natural bonuses or the equivalent uniform bonuses	bonuses at the same rate as is assumed for the future	Future bonus rate	Current bonus rate
(1)	(2)	(3)	(4)	(5)	(6)	(7)
		£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.
0	3·000	1 10 0	1 10 0	1 10 0	1 10 0	1 10 0
5	3·117	1 13 9	1 11 4	1 11 6	1 11 11	1 7 3
10	3·227	1 15 0	1 12 10	1 13 0	1 13 9	1 5 7
15	3·326	1 16 1	1 14 6	1 14 8	1 15 5	1 7 1
20	3·408	1 17 1	1 16 2	1 16 3	1 16 9	1 10 9
25	3·467	1 17 10	1 17 6	1 17 6	1 17 9	1 15 2
30 and later	3·500	1 18 4	1 18 4	1 18 4	1 18 4	1 18 4

being earned on the fund at the valuation date with assets at cost prices. Three separate sets of results are shown: the future uniform bonus rate which will produce just sufficient surplus to support current bonuses at natural rates, the uniform rate of current and future bonuses when these are equated, and the rates of future and current bonuses when the future bonuses are at the rate that the premiums would support from entry at the valuation rate of interest. As in previous illustrations it is assumed at each valuation that previous bonuses have all been declared at natural rates.

If natural bonuses are to be declared, the uniform rate of future bonus to be valued must be somewhat lower than the uniform equivalent of the current bonuses. This contrasts with the reverse position when market values and the corresponding rate of interest are used (Table 5). When the reserve for future

* The rate earned on the assets existing at the valuation date rises because those at the old interest rate tend to mature before those invested at the new rate. Incidentally, a similar trend in the average rate of interest applies in practice because of the non-uniformity of market rates and the process of redemptions.

bonuses is at the rate which the premium would support from entry at the valuation rate of interest the current bonus is substantially depressed. The bonus rates which the premium scale would support at the attained ages and for the unexpired term might produce a more acceptable current bonus rate, but it is apparent that for annual valuations the current bonus is extremely sensitive to small changes in the assumed future bonus rate. When uniform current and future bonus rates are equated the rise in the bonus rates from the original level to the ultimate new level is slightly delayed, but we have already mentioned that, in practice, the incidence of the contribution to bonuses from miscellaneous sources is likely to overshadow a small discrepancy of this nature.

Compared with the position when market values and the corresponding rate of interest are used (Table 5), the use of cost prices and the earned rate tends to reduce the fluctuations in the bonuses caused by variations in the investment rate of interest.

For total immunization and assuming that the premium scale remains unchanged, the rate of interest earned on the fund with assets at cost prices rises more slowly, and so also do the corresponding bonuses. Table 8 gives figures corresponding to those in Table 7 in conditions of total immunization, previous bonuses being assumed to be at the rates of 30*s.* and 38*s.* 4*d.* for business effected before and after the change respectively.

The numerical illustrations have assumed that the premium, experience and valuation bases all coincide, and that a change in interest rates is at once reflected in a corresponding change in bonus rates. Expenses have also been omitted from consideration. It has been assumed that any discrepancy between actual expenses and expense loadings and the disturbance due to initial expenses have been absorbed in miscellaneous profit. In these conditions some form of bonus reserve valuation exactly fits the situation. In fact, the valuation becomes equivalent to a net premium valuation for a guaranteed bonus policy. This result inevitably follows from the assumption that the future experience is known. In practice, the situation is much more obscure. Apart from the problems of fixing the appropriate rates of interest and future bonuses to be used having regard to the method of valuing the assets, there are the difficult questions of the reserve for future expenses and the mortality basis. All these problems were illuminated by Elderton's important principle that the basis for a bonus reserve valuation should be one upon which the office premiums can be justified.

NET PREMIUM VALUATIONS

Corresponding numerical illustrations are now given for the net premium method. In the idealized conditions assumed, the net premium process is, of course, nothing more than an approximation to the results of one or other of the bonus reserve processes.

As the simple reversionary bonus system is being used in the numerical work it is necessary first to decide whether the existing bonuses should be valued at the same rate of interest as the policy values or at the rate of interest earned on the fund. The circumstances that have been assumed involve mixed investment rates, and this would complicate the arithmetic considerably if the earned rate were to be used. Accordingly, it has been decided to follow the customary practice of using the same rate of interest for the existing bonuses as for the policy values.

Table 8. Results of bonus reserve valuations corresponding to those in Table 7 but assuming total immunization instead of paid-up immunization

No. of years since the change in the interest rate	Valuation rate of interest (%)	Uniform rate of bonus equivalent, on the valuation basis, to 'earned' rates of current bonus (i.e. 30*s.* for business effected before, and 38*s.* 4*d.* for business effected after, the change in conditions)	Uniform rate of future bonus necessary to provide a current surplus just sufficient to provide		Uniform future and current rates of bonus when the future rate is such as would be supported from entry by the premiums on the valuation basis	
			30*s.* for business effected before, and 38*s.* 4*d.* for business effected after, the change in conditions, or the equivalent uniform bonuses	bonuses at the same rate as is assumed for the future	Future bonus rate	Current bonus rate
		£ *s.* *d.*	£ *s.* *d.*	£ *s.* *d.*	£ *s.* *d.*	£ *s.* *d.*
0	3·000	1 10 0	1 10 0	1 10 0	1 10 0	1 10 0
5	3·021	1 11 0	1 10 1	1 10 2	1 10 4	1 8 0
10	3·065	1 12 2	1 10 4	1 10 6	1 11 1	1 4 4
15	3·135	1 13 6	1 11 0	1 11 3	1 12 3	1 0 10
20	3·232	1 14 11	1 12 5	1 12 7	1 13 10	1 0 1
25	3·355	1 16 6	1 14 9	1 14 11	1 15 11	1 4 11
30 and later	3·500	1 18 4	1 18 4	1 18 4	1 18 4	1 18 4

While the interest rate remains at 3% the appropriate rate for the net premium valuation for the particular model with bonuses at 30*s.*% and assets at cost prices works out at 2·473%. If bonuses were valued at 3% the appropriate rate for the policy values would be 2·148%.

The results of a net premium valuation at 2·473% may be summarized as follows:

Valuation balance-sheet as at 31 December 1950

Liabilities:		*Assets:*	
Value of sums assured and existing bonuses	50,543	Value at 3% of the investments	37,850
less value of future net premiums	13,293		
Value of net liability	37,250		
Surplus	600		
	37,850		37,850

The cost, at the valuation rate of interest, of a simple reversionary bonus of 30*s.*% on the policies in force on 31 December 1950 is exactly 600.

When the policies effected before the change in the interest rate have all matured, the appropriate interest rate for a net premium valuation corresponding to an earned interest rate of 3½%, a bonus rate of 38*s.* 4*d.*% and assets valued at cost prices, works out at 2·820%, i.e. ·347% more than before the change.* If existing bonuses were valued at 3½%, the appropriate interest rate for the policy values would be 2·312%.

If instead of valuing the assets at cost prices on 31 December 1950 they were valued at 3½%, it would be necessary to increase the rate of interest for the net premium valuation to 3·159% in order to release the amount of surplus required to provide a bonus of 30*s.*%, valued at 3·159%.

Table 9 shows the position at successive valuation dates if surpluses are to be released each year of the exact amounts required to declare bonuses at the natural rates earned by the policies.

Table 9. Rates of interest to be used in net premium valuations of a fund containing 30-year endowment assurances in order to release surplus just sufficient to provide bonuses at natural rates, when the investment rate of interest changes from 3 to 3½% at the commencement of the period

No. of years since the change in the interest rate	Yield on the fund if the assets are valued at cost prices	Valuation rate of interest if the assets are valued at	
		cost prices	3½%
	(%)	(%)	(%)
0	3·000	2·473	3·159
5	3·117	2·603	3·023
10	3·227	2·700	2·926
15	3·326	2·767	2·865
20	3·408	2·805	2·832
25	3·467	2·820	2·820
30 and later	3·500	2·820	2·820

* The corresponding increase for compound bonuses would be about one-half of this figure, assuming a similar bonus loading.

The common practice of offices using the net premium method is to follow what Redington has called a passive valuation policy. That is to say, they keep the valuation basis unchanged, or make only gradual changes over a long period, and they value the assets at cost prices. It is true that capital profits are often used to write down certain book values, and investments reserves are built up out of surplus allocations. But, in principle, the assets are not valued at current market prices. This applies whether the prices rise or fall. It is only when, despite writings down and the existence of investment reserves, the market prices have fallen so much that a suitable market value certificate cannot be appended to the balance-sheet without making special provision, that the passive valuation policy would normally be abandoned, and it is legitimate to question whether indeed this should be done, even in such circumstances, if the gap in asset values is due solely to a change in the level of interest rates and is not due to intrinsic loss of investment income and of security of capital.

Table 9 shows that the process of writing down the assets requires a violent increase in the valuation rate of interest followed by a gradual strengthening until the new equilibrium is reached. On the other hand, the maintenance of the passive valuation policy, despite the gap in asset values, would require a gradual weakening of the valuation basis until the new equilibrium is reached if natural bonuses are to be declared.

Although the numerical illustrations have been confined to the circumstances of an increase in the level of interest rates it is obvious that with appropriate changes of sign and *mutatis mutandis* the conclusions to be drawn from them apply equally to a reduction in the level of interest rates. It will be remembered that during the cheap-money period offices using the net premium method gradually strengthened their valuation bases by reducing the rate of interest and retaining their assets at cost prices. Although this strengthening appeared to be made at the expense of current surplus the figures in Table 9 imply that some strengthening was necessary to siphon off what was not really earned surplus at all, if natural bonus rates are taken as the criterion. The instinctive process of passing gradually from the old level to the new level of interest rates was thoroughly justified.

In the conditions of Table 9 the office might well decide to pursue a passive policy and refrain from weakening its valuation basis with the object of gradually reaching an exceptionally strong position in the new conditions. Such a course would mean that declared bonuses would be somewhat lower than earned bonuses for a time followed by higher bonuses later. For a regular flow of 30-year endowment assurances in the conditions used for our illustrations the total strengthening over the 30 years following the change in the interest rate from 3 to $3\frac{1}{2}\%$ would be about 2% of the liability. The declared bonuses would rise slowly from the 30*s.* rate but would gather momentum as the interest on past strengthenings caught up with current strengthenings. The declared bonus would pass over the uniform equivalent of the natural bonuses about 20 years after the change and would settle down at about 1*s.* 3*d.* above the earned rate of 38*s.* 4*d.* after 30 years. There would then be no further strengthening and the interest on the additional reserve would regularly fall into surplus.

Some automatic weakening or strengthening of the fund may also take place after a change in level of interest rates if the appropriate changes in the investment portfolio are not made so that paid-up immunization, or the same

degree of departure from paid-up immunization, will be maintained. This is because any such failure would have the effect of increasing current investment income, and hence of increasing current surplus, at the expense of investment income and surplus in later years or *vice versa*, according to the direction of the departure from the previous investment position. References to paid-up and to total immunization in the illustrations assume, of course, that all the necessary steps to maintain the investment position are being carried through.

SUCCESSIVE CHANGES IN INTEREST RATES

The paper has so far been concerned with the effects of a sudden long-term change in the level of interest rates but, in practice, changes in interest levels are neither sudden nor known in advance to be for a long term. A change of level which gradually develops over a short period of a few months or a year or two, possibly with temporary set-backs, is not greatly different from a sudden change of level. It is, of course, assumed that paid-up immunization is in operation or at least that the effect of any departure therefrom, such as a speculative attempt to take advantage of the changing market, contributes to miscellaneous profits or losses in the long run. Similarly, a sudden rise followed by an early fall can be regarded as a single change of levels by the amount of the net difference in the two changes.

The effect of a gradual trend from one level of interest rates to another, with or without temporary fluctuations, requires consideration. This kind of development is equivalent to a succession of sudden changes and seems to require that the contribution of each premium-payment to the growing paid-up policy should be immunized according to the investment conditions operating at the time of the investment of the relevant part of the premium. This suggestion calls for some extension of the principles hitherto examined. The appropriate idea is to associate with each premium-payment the corresponding increment in the paid-up policy. The investment rate of interest for this increment would govern the bonus rate applicable for one year to the whole balance of the sum assured after deducting the accrued paid-up sum assured and before including the increment for the current premium. The same rate would apply to the future bonuses on the current increment in the paid-up sum assured. This would leave the balance of the sum assured and future bonuses thereon to be met by the future premiums whatever the rates of interest may be when these premiums are paid, and there is available the additional buffer against a catastrophic fall in the interest rate represented by the immunized future bonuses on the paid-up policy which, of course, are not yet vested additions to the policy.

The extension of the theory to deal with a succession of changes in the rate of interest can be developed symbolically in the same way as for a single change. Considering an endowment assurance policy effected at age x for a term of n years, let π_0 and π_r represent the with-profits premiums, loaded for a simple reversionary bonus, at ages x and $(x+r)$ for terms n and $(n-r)$ years respectively in the conditions ruling at the time of entry, and let ${}^t b_r$ represent the level rate of bonus for the remaining duration of the policy that can be supported by π_r at the investment rate of interest ruling at time t. In general, ${}^t b_r$ will be constant for $t = 0$, but for other values of t it will vary somewhat with r.

Now the valuation liability at duration 1 can be written in the same form as before, as follows:

$$V(1)=\left\{\left[\left(1-\frac{\pi_0}{\pi_1}\right)+{}^0b_0\right]A_{x+1:\overline{n-1}|}+{}^0b_0\left(1-\frac{\pi_0}{\pi_1}\right)(IA)_{x+1:\overline{n-1}|}\right\}$$
$$+\left\{\frac{\pi_0}{\pi_1}[A_{x+1:\overline{n-1}|}+{}^1b_1(IA)_{x+1:\overline{n-1}|}]-\pi_0\,\ddot{a}_{x+1:\overline{n-1}|}\right\}.$$

The interpretation of the two parts of this expression in conjunction with the principle of paid-up immunization would be on the same lines as before, since only one change in investment conditions is in question. For duration 2 the valuation liability would be as follows:

$$V(2)=\left\{\left[\left(1-\frac{\pi_0}{\pi_1}\right)+B_0\right]A_{x+2:\overline{n-2}|}+\left(\frac{\pi_0}{\pi_0}{}^0b_0-\frac{\pi_0}{\pi_1}{}^0b_1\right)(IA)_{x+2:\overline{n-2}|}\right\}$$
$$+\left\{\left[\left(\frac{\pi_0}{\pi_1}-\frac{\pi_0}{\pi_2}\right)+B_1\right]\mathrm{A}_{x+2:\overline{n-2}|}+\left(\frac{\pi_0}{\pi_1}{}^1b_1-\frac{\pi_0}{\pi_2}{}^1b_2\right)(IA)_{x+2:\overline{n-2}|}\right\}$$
$$+\left\{\frac{\pi_0}{\pi_2}[A_{x+2:\overline{n-2}|}+{}^2b_2(IA)_{x+2:\overline{n-2}|}]-\pi_0\,\ddot{a}_{x+2:\overline{n-2}|}\right\}.$$

If the interest rate is constant, all the b's become equal and the expression for $V(2)$ contracts to the usual expression for the policy value of a guaranteed bonus policy. The coefficient of the increasing assurance in the first part of the expression for $V(2)$ reduces to ${}^0b_0(1-\pi_0/\pi_1)$ since ${}^0b_0={}^0b_1$ by definition, but the way in which it is written shows the symmetry of the whole expression.

The interpretation of this analysis of $V(2)$ into three parts follows the same lines as for a single change in the rate of interest except that there are now two changes and three rates of interest, i.e. i_0 and i_1 associated with the investment of the relevant parts of the first two premiums and i_2 the rate ruling at the valuation date two years after entry. If the bonus rates supported by the premium scale remained uniform when the rate of interest changed from i_0 to i_1 the second part of the expression for $V(2)$ would reduce to the value of the second year's increment in the paid-up policy together with future bonuses thereon at the rate of ${}^1b$. The expression given, however, takes care of the adjustment which the non-uniformity of ${}^1b_r$ requires.

The first part of $V(2)$ represents the first year's contribution to the paid-up policy, the first year's declared bonus, that part of the second year's declared bonus that is secured by the first premium, and the future bonuses secured by the first premium. B_0 is thus the first year's bonus plus part of the second year's bonus. The assets corresponding to the first part of $V(2)$ are assumed to be invested according to the paid-up immunization principle. Accordingly, the value of this part of the benefits and the value of the corresponding assets will equate at both i_0 and i_2, i.e. at the rates applicable respectively to cost prices and market value of the assets.

Similarly, the second part of $V(2)$ represents the second year's increment of the paid-up policy, that part of the second year's declared bonus that is secured by the second premium and the future bonuses secured by the second premium. The immunization principle applies also to the corresponding assets. Again, the value of this part of the benefits and the value of the corresponding assets equate at both i_1 and i_2.

The third part of $V(2)$ represents the future premiums and the benefits that they will support in current conditions and so, by definition of ${}^{2}b_2$, this third part, valued at i_2, is precisely zero and there are, of course, no corresponding assets.

The symbolic expression for the valuation liability can be generalized as follows:

$$V(r)=\sum_{t=0}^{r-1}\left[\left(\frac{\pi_0}{\pi_t}-\frac{\pi_0}{\pi_{t+1}}+B_t\right)A^{i_t \text{ or } i_r}_{x+r:\overline{n-r}|}+\left(\frac{\pi_0}{\pi_t}{}^{t}b_t-\frac{\pi_0}{\pi_{t+1}}{}^{t}b_{t+1}\right)(IA)^{i_t \text{ or } i_r}_{x+r:\overline{n-r}|}\right]$$

$$+\left[\frac{\pi_0}{\pi_r}[A^{i_r}_{x+r:\overline{n-r}|}+{}^{r}b_r(IA)^{i_r}_{x+r:\overline{n-r}|}]-\pi_0\,\ddot{a}^{i_r}_{x+r:\overline{n-r}|}\right]$$

B_t is, of course, the total amount of bonus already declared up to duration r that has been secured by the tth premium. If the valuation is made at the varying rates i_t the appropriate value of the assets is their cost price values, with amortization. If the valuation is made at i_r the assets would be valued at i_r and their value would be the market values provided that the current interest structure is uniform. The scope of this restriction is very much reduced if the valuations are made at the varying rates i_t.

It will be seen from the analysis of $V(r)$ that the natural bonus rate that can be declared in respect of the $(r+1)$th premium is

$$\sum_{t=0}^{r-1}\left(\frac{\pi_0}{\pi_t}{}^{t}b_t-\frac{\pi_0}{\pi_{t+1}}{}^{t}b_{t+1}\right)+\frac{\pi_0}{\pi_r}{}^{r}b_r$$

$$=\sum_{t=0}^{r-1}\left[{}^{t}b_t\left(\frac{\pi_0}{\pi_t}-\frac{\pi_0}{\pi_{t+1}}\right)+\left({}^{t}b_t-{}^{t}b_{t+1}\right)\frac{\pi_0}{\pi_{t+1}}\right]+\frac{\pi_0}{\pi_r}{}^{r}b_r,$$

which is independent of the valuation basis, as it should be.

The first term in the summation represents the sum of the immunized bonuses on the successive increments in the paid-up policies. ${}^{r}b_r\pi_0/\pi_r$ represents the level bonus that π_0 will support on the balance of the sums assured for the remainder of the term in the conditions ruling at time r. The second term in the summation provides the small correction required in theory by the fact that, except for the first year, ${}^{t}b_r$ (t constant) varies with r; the bonus rates supported by the original premium scale are precisely uniform only at the rate of interest ruling at entry. If this small adjustment is omitted and the proportionate rule is used for the paid-up policies an approximation to the amount of the $(r+1)$th declared bonus is given by

$$\frac{1}{n}\left[\sum_{t=0}^{r-1}{}^{t}b_t+(n-r)\,{}^{r}b_r\right].$$

The total bonuses at maturity of the policy on this approximate basis amount to

$$[(2n-1)\,{}^{0}b_0+(2n-3)\,{}^{1}b_1+\ldots+{}^{n-1}b_{n-1}]/n.$$

The foregoing discussion, apart from the proportionate paid-up policy approximation, applies also to whole-life policies and limited payment policies, with the appropriate changes in the notation. Single-premium policies represent a limiting case in which bonuses would be immunized from entry.

A similar analysis, although more complicated in detail, can be applied to compound reversionary bonuses and is given in the appendix. Subject to certain qualifications closely similar rules apply to the calculation of the natural compound bonuses at each valuation.

LOGICAL CONSEQUENCES FOR VALUATION

The logical conclusion from the analysis of $V(r)$ is that, in conditions of paid-up immunization, the future rate of interest, and, indeed, the rate ruling at the valuation date, can be regarded as irrelevant for purposes of valuation. The future premiums and corresponding benefits are treated as new business and therefore cancel each other. The valuation would naturally take the form of valuing the increments of the paid-up policies separately, including existing bonuses and an explicit or implicit reserve for future bonuses on the paid-up policies, and comparing the result with the corresponding value of the assets. The resulting surplus would be just sufficient to permit the declaration of natural bonuses provided that their value is taken consistently, i.e. at the relevant interest rates used in the valuation itself. It is when the valuation formula is put into the conventional form for the Board of Trade Returns that consideration has to be given to the valuation assumption about the investment of the future premiums. Even if this were not so, the use of a single rate of interest would usually be favoured in practice in place of the amalgam of interest rates implicit in the structure of immunized increments to the paid-up policies. Similarly, the use of a uniform rate of future bonus would also be a convenience. To use the current rate of interest i_r consistent with the market values of the assets would require the assessment of a future bonus rate consistent with the amalgam of natural bonus rates, including ${}^{r}b_{r}$, the rate appropriate to the balance of the sums assured at i_r. The figures in the last few columns of Table 5 indicate that, in general, the uniform rate of future bonus would be different from the uniform rate suitable for the current declaration and accordingly would be difficult to fix. We have also seen that to equate the uniform rates of current and future bonus could have the effect of amplifying the swing of bonus declarations as interest rates fluctuate. In any event, the procedure implies changes in basis from valuation to valuation, including the writing up and down of the value of the assets. These features are, perhaps, more troublesome for annual valuations than for quinquennial valuations, particularly the discrepancy between the uniform rates of future bonus and current bonus, because a small error in the rate of future bonus would have a much greater effect on the rate of bonus declared for one year than for a longer period.

On the other hand, the rate of interest being earned on the fund at the valuation date is the weighted average at that point of time of the amalgam of interest rates implicit in the system and is therefore consistent with the total of the asset values at amortized cost prices. Similarly, the uniform rate of bonus corresponding to the immunized bonuses on the paid-up policies is a suitable rate for the current bonus declaration thereon. The earned interest rate and this average bonus rate on the paid-up policies would tend to move in sympathy as paid-up policies mature and corresponding investments are redeemed. In effect, the use of these two rates would ignore small parallel future changes in each of them. Finally, there is reason to think that this average bonus rate approximates to the average bonus rate which the premium scale would support at the rate of interest being earned on the fund.

To express the foregoing conclusions symbolically we may first note that, to a close degree of approximation, the bonus rate supported by the premium scale is linearly related to the rate of interest employed, i.e. b_t and i_t are so related. Now, writing ΔW_t for the paid-up increments at time t and summing over all these increments, the average rate of bonus on the increments of the paid-up policies is the weighted average $\Sigma b_t \Delta W_t / \Sigma \Delta W_t$. Similarly, the rate of interest earned on the fund is $\Sigma i_t \Delta W_t F^{it} / \Sigma \Delta W_t F^{it}$, where F^{it} is the composite valuation factor for the corresponding paid-up increment, including the value of the associated declared and future bonuses. The factors F^{it} do not greatly disturb the weighted average, so that the average rate of bonus on the paid-up policies is closely similar to the rate supported by the premium scale at the earned rate of interest. In fact, in the conditions of Table 7 the two rates are almost the same at all durations since the change. The equivalent uniform bonus rates on the paid-up policies for the current declaration and for the future bonuses differ somewhat from the average rate of bonus on the paid-up policies, one in one direction and the other in the opposite direction, because of the inclusion of the respective weights A and (IA) with progressions in opposite directions. There remains the question of the appropriate current rate of bonus b_{r-1} for the balance of the sums assured $\Sigma(1 - W)$. If i_{r-1} exceeds the earned rate, b_{r-1} exceeds the average bonus rate on the paid-up policies. If i_{r-1} is less than the earned rate the opposite is the position.

These considerations suggest that a valuation in orthodox bonus reserve form at the earned rate of interest with the rate of future bonus fixed to produce a surplus just sufficient to permit a current declaration at the same rate would be a suitable arrangement. It would take broad account of the actual facts of the development of the fund, and it would not incorporate any estimate of the unknown future. It has the effect of slightly delaying the changes in declared bonuses compared with natural bonuses as interest rates rise or fall. It thus introduces a desirable element of gradualism, and it irons out the swings in the natural bonuses that result from fluctuating rates of interest.

For net premium valuations, a passive policy involving a gradual adjustment of the valuation rate of interest according to changes in the earned rate hardly seems capable of improvement. The whole objective is to allow the developments of the future to unfold themselves and gradually to affect the interests of the policyholders.

This discussion of the effects of a trend in interest rates also suggests the practical line in the case of a sudden long-term change in the level of interest rates. Such a change cannot be known to be a long-term effect for many years after the change, and in practice there can be little doubt that only a gradual acceptance of its relative permanence would be appropriate.

DEPARTURES FROM PAID-UP IMMUNIZATION

The figures of Table 8 suggest that in conditions of total immunization similar conclusions apply regarding the valuation procedures that would suitably reflect the consequences of that system. The inference is that similar conclusions would also apply for any intermediate position between paid-up and total immunization, so that in this wide range the market values of the assets are strictly irrelevant for a sound development of a with-profits fund. This is, of course, widely recognized as long as market values do not fall below book

values. There remains the question of the appropriate action to be taken on a change in the level of interest rates taking place if the investment position is shorter than for paid-up immunization or longer than for total immunization.

It is no part of the purpose of the present discussion to consider the treatment of intrinsic losses on investment account or the practical question of asset values for the public presentation of the accounts of a life office and the steps necessary to enable the balance-sheet certificates under the 1909 and 1946 Acts to be given. These problems may arise even though an office's investment position is within the immunization range. From the present point of view, however, it is because of the possibility of the investment position being outside this range that the question of the asset values may be important.

We need not linger over the unusual case of an ultra-short position. Here the capital position would be intact. The loss of future interest would be the risk, and this applies whether the current short-term rate is high or low. For with-profits business, however, this risk should be covered by the bonus loadings in the premiums. That this is usually so can easily be seen from the fact that the average premiums of twenty commission-paying offices for with-profits endowment assurances at age 40 are about 10% greater than the pure premiums on the A1924–29 table with interest at ½% per annum. The developments of this paper are, of course, concerned with full-profits policies. We have not considered the corresponding problems for premiums loaded for small bonuses.

Even when the investment position is ultra-long and there has been depreciation it does not necessarily follow that the use of market values coupled with a corresponding valuation of the liabilities is a suitable procedure. The difficulty is in the assumption, implicit in the use of market values and the consequential treatment of the liabilities, that where sales of investments have to take place in due course as a result of the failure to match, each of these transactions will take place on the basis of the yield implicit in the *current* market value of the particular investment. This assumption is made even if the sale is only a notional one to the fund itself in respect of future premiums on existing or new business. It involves the capitalization, on one particular basis, of an assumed loss that may never occur. In practice with a continuing business if this 'loss' materializes it takes the form of a loss of investment income between the date of the notional sale and the date of redemption of the investment in question. A loss arises only if a better yield could have been obtained by a reinvestment at the date of the notional sale, assuming, of course, that the income of the investment has not been affected in the meantime. If the position is such that there is good reason to expect a significant loss of income as a result of investing over-long, an objective estimate may need to be made with due regard to the nature of current market conditions. There then arises the difficult question of what action should be taken on the basis of the estimate: which policyholders are to bear the burden? The view has already been expressed that any profit or loss that sooner or later accrues to a fund which is invested over-long is a contribution to miscellaneous surplus. Any decision to apportion such an estimate between current and future bonuses must call for the highest judgment about the validity of the estimate. It is, perhaps, a virtue of the use of a bonus reserve valuation with the assets at market values that a deliberate decision on this matter has to be taken, whereas the use of a bonus reserve valuation at the earned interest rate or a net premium valuation with assets at cost prices may lead to an automatic

apportionment unless a separate examination is made and an appropriate modification of the valuation and surplus is carried through. In practice, of course, the 'loss' may be covered by an open investment reserve fund, or by the hidden margin which is built up by profits on sales and by a policy of not amortizing stocks bought at a discount.

There are three considerations that need to be borne in mind in an examination of such a situation.

First, there is the fact that the current yield basis is always exceptional because stock exchange values are always marginal. It is, therefore, a questionable basis for estimating future sale prices.

Secondly, the assumption of a uniform rate of interest is unsound for the purpose of estimating the present value of deferred temporary income. An office may have an $(n+m)$-year stock instead of an n-year stock which the principles of matching might require. We are not considering for the moment the possibility that the former stock may have been bought just because its yield was sufficiently greater than the yield on the shorter-term stock. The point we wish to make is that when the stock comes to be 'sold' it will be only an m-year stock, a shorter-dated security. To use the rate of interest currently obtainable on an $(n+m)$-year stock to estimate the price n years' hence of an m-year stock would be indefensible. Even on the current yield-structure it might well be that the value based on i_{n+m} would show a loss, while the value based on i_m would show a profit. This would be the case if $i_{n+m} > i > i_m$, where i is the yield at the original purchase price. These conditions are not at all unlikely in practice.

Thirdly, any sales or notional sales that must take place in the future will be spread over a long period of time during which the rate of interest will, no doubt, fluctuate over a considerable range. There is no reason to assume that the current level of interest rates represents in any sense a mean value.

The conclusion is that for a with-profits fund no hard and fast principles can be laid down to cover the circumstances of ultra-long or ultra-short investment positions. Market values and corresponding valuations of liabilities can be positively misleading. All that can be done is to rely, as must always be done ultimately, on the judgment of the actuary, in the light of all the circumstances, including in particular the bonus policy which it is intended to pursue and the treatment in the office's investment ledgers of capital profits and losses on sales and redemptions.

For a with-profits fund there can rarely be any question about the solvency of the fund without obvious pointers to the unsatisfactory position. But owing to the possibility of intrinsic losses on investment account as well as from a maladjusted mean term of the investments some reference to market values may be essential. Might it not completely meet the situation if it were necessary merely to state whether the asset values were covered by the market values, and if they were not so covered to require the actuary to certify the amount of the deficit and whether it was covered by the reserve for future profits, implicit or explicit, in the valuation of the liabilities? This would leave to the actuary the responsibility of controlling the rate of release of surplus for distribution without the encumbrance of irrelevant requirements concerning the assets with their equally irrelevant curtailment of the freedom of the actuary in deciding upon his valuation basis. It would require some modification of the 1946 Act certificates, in which the life fund is assumed to be the liability.

CONCLUSION

The paper has been concerned solely with the problems of changing investment conditions and their effects on a with-profits life-assurance business. The theory of paid-up immunization does not, of course, extend to the other changing factors in the business, i.e. taxation, mortality, expenses, withdrawals, etc. Nevertheless, our conclusions go a long way to support much of what has been traditional practice in the operation of the British system of reversionary bonuses. Indeed, much of the literature on the subject of valuations and surplus before the publication of Suttie's papers seems to have taken for granted some such principle as paid-up immunization which we suggest is fundamental to the subject. Moreover, it provides a suitable standard for considering financial policy and broad questions of equity in practice in relation to individual policies.

Our indebtedness to Redington for the immunization principle is obvious if only because it was from this principle that the present paper has germinated. But the theory based on paid-up policies does not necessarily involve the immunization principle, because if securities in the form of saving certificates for all periods were available it would always be possible to secure an absolute matching for paid-up policies. What is sufficient to the theory is that the rates of interest at which the interest on the existing securities is reinvested should be fixed at the time of purchase.

Some aspects of the present paper were foreshadowed by the remarks of both of the present writers in the discussion on Redington's paper, and also by Ogborn's remarks in the same discussion. His remarks in the discussion on Suttie's papers (*J.I.A.* LXXII, 220 and LXXIII, 60) are also relevant. We have already mentioned that the paper by Haynes and Kirton is largely concerned with non-profits business. It remains to mention that a paper by J. L. Anderson entitled 'Notes on the effects of changes in rates of interest on the bonus-earning power of an office paying a uniform compound reversionary bonus' (*T.F.A.* XVII, 137) contains much in common with the present paper. His remarks in the discussion on Suttie's paper should also be mentioned. His instinctive thinking runs parallel to our own in many ways.

Our grateful thanks are due to G. E. Wallas who has helped with the arithmetic and to C. M. O'Brien who has checked all the calculations. Finally, we are indebted to M. E. Ogborn for his helpful suggestions and kindly interest in the paper throughout its preparation.

APPENDIX

COMPOUND REVERSIONARY BONUSES

A consistent treatment of simple reversionary bonuses in conditions of paid-up immunization has been explained in detail in the foregoing pages, and it remains to consider what modifications to the theory are necessary if distributions of surplus are made in the form of compound reversionary bonuses.

It is necessary to consider, as before, a succession of changes in the rate of interest, and the appropriate solution can obviously be obtained by again examining the valuation liabilities of a particular policy at successive durations. The results are, however, quite independent of valuation factors, and a certain amount of repetition is avoided if the full valuation formula is eliminated.

Consider an endowment assurance policy effected at age x for a term of n years. Let π_0 and π_r represent the with-profits premiums, loaded for an annual compound reversionary bonus, at ages x and $(x+r)$ for terms n and $(n-r)$ respectively in the conditions ruling at the time of entry, and let ${}^t b_r$ represent the level rate of bonus for the remaining duration of the policy $(n-r)$ that could be supported by π_r at the investment rate of interest ruling in the tth year from entry.

As before, consider the position at successive durations immediately before a premium is paid, and assume that the interim bonuses paid on death claims are at the same rate as that declared on policies in force at the end of the year.

Duration 0

The rate of bonus supported by a new policy for 1, at $t=0$, age x, is ${}^0 b_0$. The future bonuses to be reserved for at valuation are:

(i) in respect of premiums already paid, nil,

(ii) in respect of the first premium, ${}^0 b_0$, and

(iii) in respect of subsequent premiums, ${}^0 b_0$ on $\frac{\pi_0}{\pi_0}$, of which $\left\{ {}^0 b_0 \text{ compounding on } (1+{}^0 b_0) \textit{ minus } {}^0 b_1 \text{ compounding on } \frac{\pi_0}{\pi_1} \right\}$ is immunized on payment of the first premium and the balance, ${}^0 b_1 \left(\text{compounding on } \frac{\pi_0}{\pi_1} \right)$, being the bonus supported by subsequent premiums, is *not* immunized.

Duration 1

The total bonus declared for the past year is ${}^0 b_0$.

The rate of bonus supported by a new policy for $\frac{\pi_0}{\pi_1}$ at $t=1$, age $x+1$, is ${}^1 b_1$. This rate will be equal to ${}^0 b_1$ only if conditions have remained unchanged.

The future bonuses to be reserved for at valuation are:

(i) in respect of premiums already paid, ${}^0 b_0$ on $(1+{}^0 b_0)$, $-{}^0 b_1$ on $\frac{\pi_0}{\pi_1}$, immunized by the payment of the first premium,

(ii) in respect of the current premium, ${}^1 b_1 \frac{\pi_0}{\pi_1}$, and

(iii) in respect of subsequent premiums, ${}^1b_1$ on $\frac{\pi_0}{\pi_1}$, of which $\left\{{}^1b_1 \text{ compounding on } (1+{}^1b_1)\frac{\pi_0}{\pi_1} \textit{ minus } {}^1b_2 \text{ compounding on } \frac{\pi_0}{\pi_2}\right\}$ is immunized on payment of the current premium.

Duration 2

The sum assured and existing bonuses may be written:

$$(1+{}^0b_0)-\frac{\pi_0}{\pi_1},$$

$$+\frac{\pi_0}{\pi_1}.$$

The total bonus declared for the past year is

$$\text{(i)}\quad {}^0b_0(1+{}^0b_0)-{}^0b_1\frac{\pi^0}{\pi_1},$$

$$+\quad \text{(ii)}\quad {}^1b_1\frac{\pi_0}{\pi_1}.$$

The rate of bonus supported by a new policy for $\frac{\pi_0}{\pi_2}$ at $t=2$, age $x+2$, is ${}^2b_2$.

The future bonuses to be reserved for at valuation are:

(i) in respect of premiums already paid,

$${}^0b_0 \text{ on } (1+{}^0b_0)^2,\quad -{}^0b_1 \text{ on } (1+{}^0b_1)\frac{\pi_0}{\pi_1},$$

$$+{}^1b_1 \text{ on } (1+{}^1b_1)\frac{\pi_0}{\pi_1},\quad -{}^1b_2 \text{ on } \frac{\pi_0}{\pi_2},$$

immunized by the payment of past premiums,

(ii) in respect of the current premium, ${}^2b_2\frac{\pi_0}{\pi_2}$, and

(iii) in respect of subsequent premiums, ${}^2b_2$ on $\frac{\pi_0}{\pi_2}$ of which $\left\{{}^2b_2 \text{ compounding on } (1+{}^2b_2)\frac{\pi_0}{\pi_2} \textit{ minus } {}^2b_3 \text{ compounding on } \frac{\pi_0}{\pi_3}\right\}$ is immunized on payment of the current premium.

Duration 3

The sum assured and existing bonuses may be written:

$$(1+{}^0b_0)^2-(1+{}^0b_1)\frac{\pi_0}{\pi_1},$$

$$+(1+{}^1b_1)\frac{\pi_0}{\pi_1}-\frac{\pi_0}{\pi_2},$$

$$+\frac{\pi_0}{\pi_2}.$$

The total bonus declared for the past year is

$$\text{(i)}\ {}^{0}b_0(1+{}^{0}b_0)^2-{}^{0}b_1(1+{}^{0}b_1)\frac{\pi_0}{\pi_1}$$

$$+{}^{1}b_1(1+{}^{1}b_1)\frac{\pi_0}{\pi_1}-{}^{1}b_2\frac{\pi_0}{\pi_2},$$

$$+\quad\text{(ii)}\qquad {}^{2}b_2\frac{\pi_0}{\pi_2}.$$

The rate of bonus supported by a new policy for $\frac{\pi_0}{\pi_3}$ at $t=3$, age $x+3$, is ${}^{3}b_3$.

The future bonuses to be reserved for at valuation are:

(i) in respect of premiums already paid,

$${}^{0}b_0 \text{ on } (1+{}^{0}b_0)^3,\quad -{}^{0}b_1 \text{ on } (1+{}^{0}b_1)^2\frac{\pi_0}{\pi_1},$$

$$+{}^{1}b_1 \text{ on } (1+{}^{1}b_1)^2\frac{\pi_0}{\pi_1},\quad -{}^{1}b_2 \text{ on } (1+{}^{1}b_2)\frac{\pi_0}{\pi_2},$$

$$+{}^{2}b_2 \text{ on } (1+{}^{2}b_2)\frac{\pi_0}{\pi_2},\quad -{}^{2}b_3 \text{ on } \frac{\pi_0}{\pi_3},$$

immunized by payment of past premiums,

(ii) in respect of the current premium, ${}^{3}b_3\frac{\pi_0}{\pi_3}$, and

(iii) in respect of subsequent premiums, ${}^{3}b_3$ on $\frac{\pi_0}{\pi_3}$, of which $\left\{{}^{3}b_3 \text{ compounding on } (1+{}^{3}b_3)\frac{\pi_0}{\pi_3} \textit{ minus } {}^{3}b_4 \text{ compounding on } \frac{\pi_0}{\pi_4}\right\}$ is immunized on payment of the current premium.

Duration $r+1$

Generalizing, the following results are obtained.

The sum assured and existing bonuses amount to

$$S(r)=\sum_{t=0}^{r-1}\left[(1+{}^{t}b_t)^{r-t}\frac{\pi_0}{\pi_t}-(1+{}^{t}b_{t+1})^{r-t-1}\frac{\pi_0}{\pi_{t+1}}\right]+\frac{\pi_0}{\pi_r}.$$

The total bonus declared for the past year is

$$B(r)=\text{(i)}\sum_{t=0}^{r-1}\left[{}^{t}b_t(1+{}^{t}b_t)^{r-t}\frac{\pi_0}{\pi_t}-{}^{t}b_{t+1}(1+{}^{t}b_{t+1})^{r-t-1}\frac{\pi_0}{\pi_{t+1}}\right],$$

$$+\quad\text{(ii)}\quad {}^{r}b_r\frac{\pi_0}{\pi_r}.$$

The rate of bonus supported by a new policy for $\frac{\pi_0}{\pi_{r+1}}$ at $t=r+1$, age $x+r+1$, is ${}^{r+1}b_{r+1}$.

The future bonuses to be reserved for at valuation are:

(i) in respect of premiums already paid,

$$\sum_{t=0}^{r}\left[{}^{t}b_t \text{ on } (1+{}^{t}b_t)^{r-t+1}\frac{\pi_0}{\pi_t},\quad -{}^{t}b_{t+1} \text{ on } (1+{}^{t}b_{t+1})^{r-t}\frac{\pi_0}{\pi_{t+1}}\right]$$

immunized by payment of past premiums,

(ii) in respect of the current premium, ${}^{r+1}b_{r+1}\dfrac{\pi_0}{\pi_{r+1}}$, and

(iii) in respect of subsequent premiums, ${}^{r+1}b_{r+1}$ on $\dfrac{\pi_0}{\pi_{r+1}}$, of which $\left\{{}^{r+1}b_{r+1}\right.$ compounding on $(1+{}^{r+1}b_{r+1})\dfrac{\pi_0}{\pi_{r+1}}$ *minus* ${}^{r+1}b_{r+2}$ compounding on $\left.\dfrac{\pi_0}{\pi_{r+2}}\right\}$ is immunized on payment of the current premium.

The rate of bonus that may be declared for the past year, at duration $r+1$, is of course the ratio of $B(r)$ to $S(r)$.

Now $B(r)$ may be expressed alternatively as

$$\sum_{t=0}^{r-1}\left[{}^{t}b_t(1+{}^{t}b_t)^{r-t-1}\left\{\frac{\pi_0}{\pi_t}(1+{}^{t}b_t)-\frac{\pi_0}{\pi_{t+1}}\right\}\right.$$
$$\left.+\{{}^{t}b_t(1+{}^{t}b_t)^{r-t-1}-{}^{t}b_{t+1}(1+{}^{t}b_{t+1})^{r-t-1}\}\frac{\pi_0}{\pi_{t+1}}\right]+\frac{\pi_0}{\pi_r}{}^{r}b_r.$$

The second term within the summation provides the small correction required in theory because, in the conditions ruling at t, the original premium scale does not support a uniform rate of bonus for all ages at entry.

If this small adjustment is omitted

$$B(r)\fallingdotseq\sum_{t=0}^{r-1}\left[{}^{t}b_t(1+{}^{t}b_t)^{r-t-1}\left\{(1+{}^{t}b_t)\frac{\pi_0}{\pi_t}-\frac{\pi_0}{\pi_{t+1}}\right\}\right]+\frac{\pi_0}{\pi_r}{}^{r}b_r.$$

Similarly, the total sum assured and existing bonuses

$$S(r)\fallingdotseq\sum_{t=0}^{r-1}\left[(1+{}^{t}b_t)^{r-t}\frac{\pi_0}{\pi_t}-(1+{}^{t}b_t)^{r-t-1}\frac{\pi_0}{\pi_{t+1}}\right]+\frac{\pi_0}{\pi_r},$$

and the sum assured and total bonuses at maturity

$$S(n)\fallingdotseq\sum_{t=0}^{n-1}\left[(1+{}^{t}b_t)^{n-t}\frac{\pi_0}{\pi_t}-(1+{}^{t}b_t)^{n-t-1}\frac{\pi_0}{\pi_{t+1}}\right].$$

Comparing the expressions $B(r)$ and $S(r)$ we see that the rate of bonus to be declared in respect of the $(r+1)$th premium is simply a weighted average of ${}^{t}b_t$ for all values of t from $t=0$ to $t=r$ inclusive.

The weighted average will be very little affected if approximations are introduced into the weights by substituting therein:

(i) an average value of ${}^{t}b_t$ throughout, say b, and

(ii) the expression P_0/P_t for $(1+b)^{-t}\dfrac{\pi_0}{\pi_t}$, where P_0 and P_t are taken as the 'non-profit' premiums obtained from π_0 and π_t by deducting the bonus loading for a compound rate of bonus b.

Dividing both $B(r)$ and $S(r)$ by $(1+b)^r$ the weights then become

$$\left[\frac{P_0}{P_t}-\frac{P_0}{P_{t+1}}\right] \text{ for } {}^tb_t \quad \text{and} \quad \frac{P_0}{P_r} \text{ for } {}^rb_r.$$

The approximate weights are therefore the paid-up policy increments of the 'equivalent' non-profits assurance. The effect of approximation (i), however, is to reduce somewhat the relative weights that should, in theory, be given to the higher values of ${}^tb_t$ and correspondingly to increase the weights for the smaller values.

In considering the foregoing analysis and the application of the other principles of the paper to compound reversionary bonuses it is helpful to bear in mind that for a given premium the bonus content of the policy is the same whether the system of declaration is simple or compound. Assuming paid-up immunization, the bonus earnings secured by the investments in respect of each premium are similar. The only difference is that at each duration the division of the total bonuses already paid for and immunized, between declared and undeclared bonuses, is different for the two systems; the compound system holds up more of the bonus already immunized for later declaration. Comparing the two systems, there is a difference in the incidence of bonuses between early death claims and late death and maturity claims. The valuation problems of the two systems are identical in principle; the greater complication for the compound system arises solely out of the different rate at which bonuses are declared and the different form that they take.

ABSTRACT OF THE DISCUSSION

Mr Wilfred Perks, in introducing the paper, said that it was based on the simple common-sense idea that with-profits premiums would normally be invested in the conditions ruling at the time of their payment, and that no attempt would be made to invest them in advance or to use other policyholders' money to give the same effect.

He would like also to emphasize that the mean term of investment would not be so short as might be construed from the figures on page 25. The mean terms shown there were, of course, the unexpired periods for securities bought on the average a good many years earlier.

The authors' premises themselves were not new; they went back a long way in actuarial history, and so did the recognition of the need for consistency. J. A. Higham (*J.I.A.* XXII, 44) had said in support of a particular valuation process that

> It places the policyholders surviving a distribution on equal terms with new entrants for future bonus. It makes the surplus dependent on the facts; and leaves no room for ingenuity in discovering a pure premium which will yield the surplus desired. It is intelligible to the assured. He can understand us when we say we hold in respect of his policy the same sum which he must put into the chest to obtain a second policy precisely like the first.

The need for consistency was purely self-evident as the primary condition of all satisfactory reasoning.

The authors had deliberately freed themselves from the restriction of assuming that the market value of the assets was an essential ingredient of valuation for bonus distribution, and they did not pursue the question of valuation for solvency. They did not say that there was no problem of investment losses, but they believed that market values did not provide the one and only answer. They had also rejected the idea that maintenance of bonus rates was a primary objective. Maintenance of profit sources was important, as was implicit in their premises, and this included leaving future premiums to be invested in the conditions ruling when they were received. About twenty years before Britt had made that distinction (*J.I.A.* LXIV, 303); but, of course, the distinction went back a long way, to the arguments in the early volumes of the *Journal*. The reinsurance method, the net premium method and the original bonus reserve method had all been aimed at preserving the profit sources rather than at the maintenance of bonus rates. All those methods had been developed, moreover, at a time when mortgages comprised a large section of the assets and the question of market values did not seriously confuse the issue. The need to realize the assets had not then arisen and it did not now arise.

In conclusion he said that it had been a great joy to him personally to work with Mr Bayley and to play some small part in the production of the paper.

Mr J. A. S. Lamb, in opening the discussion, said that the effect of appreciation or depreciation in assets due to a change in the general level of interest rates had been considered in a number of papers since the war. To appreciate the background of the subject, reference should be made to the papers by Suttie (*J.I.A.* LXXII, 203 and LXXIII, 37) and by Anderson (*T.F.A.* XVII, 137).

The Faculty paper by Haynes and Kirton (1952) developed the theory outlined by Kirton in 1933 during the discussion on Penman's paper, *A review of investment principles and practice* (*J.I.A.* LXIV, 387). Although the Faculty paper

dealt principally with the question of absolute matching of investments and liabilities, attention had been drawn to the fact that, assuming a uniform change in the general level of interest rates, it was possible to immunize a fund by a suitable combination of investments of varying date.

Redington (*J.I.A.* LXXVIII, 286) had broken away from tradition and developed the idea of immunization as opposed to matching. For the first time he demonstrated mathematically the conditions necessary if the relationship between the value of the assets and the value of the liabilities was to remain undisturbed on a change in the general level of interest rates. For a with-profits fund, however, the logic of his theory demanded that, when such a change had taken place, there should be a cleavage between the rate of bonus on new business and the rate of bonus on existing business. Alternatively, the rates of premium charged on new business should be amended.

The theory of the paper under discussion had been developed on the basis of the expectations of the reasonable policyholder—an actuarial fiction bearing a strong resemblance to that legal fiction, the reasonable man, in that he rarely, if ever, existed. That approach to the problem was eminently logical, but its weakness lay in the fact that it was all too easy for the protagonist of a particular system to persuade himself that his system was reasonable. Indeed, from a study of the literature of what was a controversial subject it would appear that equity, like beauty, lay chiefly in the eye of the beholder.

On page 16 the authors had set out the expectations of that paragon, the reasonable policyholder. He agreed that the conditions laid down were reasonable; but he was not convinced that other conditions could not be found, equally reasonable, which would justify some other system. For example, the authors said that

> where a policy is nearing maturity the policyholder would be expecting the proceeds of his policy to become more and more definite.

That appeared to mean that the bonus declared should steady as the policy approached maturity, until the final bonus was virtually guaranteed. The system devised in the paper carried the idea into effect by immunizing future bonuses on the increment to the paid-up sum assured secured by each premium. Would not a policyholder be equally reasonable, however, if he expected the whole of his future bonuses to reflect the future experience of the office? Such an approach could be used to justify a system in which the whole of the sum assured and declared bonuses would be immunized, future bonuses being left out of account. To immunize future bonuses did, however, ensure that the bonus 'buffer' was not required to meet a deficiency in interest earnings at a time when it was also required to cover other contingencies.

It was helpful, in considering the value of an immunization theory, to keep well in mind the fundamental purpose of an investment policy—to obtain the maximum profit consistent with capital security. An office ought not to risk a capital loss which might impair its ability to meet its liabilities, but it also needed to earn a good yield on its investments if it were to survive competition. An immunization theory would indicate a mean term of investment and point to the consequences of going 'long' or 'short' of that mean term. So it safeguarded the capital position at the expense of restricting freedom of investment. However, freedom from restriction on the investment policy was essential if a maximum profit was to be earned, and he suggested, therefore, that it was possible to over-immunize. Indeed, since the total immunization theory

immunized the current and future bonuses, which were not guaranteed, as well as the sum assured and existing bonuses, which were guaranteed, that theory might well, for a with-profits fund, be considered over-immunization. He would support the suggestion of the authors that total immunization was more suitable for guaranteed bonuses.

The authors contended that it was more reasonable to immunize that part of a policy and its related bonuses (both existing and future) supported by past premiums. It followed that, although the whole of the sum assured was guaranteed, only part of it would be immunized, but part of the future bonuses was immunized also, although they were not guaranteed. That lack of relation between what was guaranteed and what was immunized set a limit to the conditions under which the paid-up immunization theory was applicable. No one would dispute that the sum assured under a policy was sacrosanct. It followed, therefore, that the authors' theory would be applicable only provided the bonus loading was sufficient to absorb any foreseeable fluctuation in interest earnings. If the premium scale reflected the anticipated mortality experience, and if expense loadings were adequate, a possible criterion was that the with-profits rates should be at least equal to the corresponding non-profits rates calculated at the minimum rate of interest which could reasonably be expected to be earned on future investments. It was, he thought, unlikely that opinion would be unanimous on what that minimum rate of interest should be.

Given those conditions, he felt that the authors' theory had much to commend it. It was simple in its conception, if not in its mathematics. It had an advantage over total immunization in that it produced a better equation of bonus earning power between existing business and future new business. Moreover, immunization of new business was not dependent on the presence of a large fund of existing business.

Redington had said (*J.I.A.* LXXVIII, 294)

> It may well be concluded that immunization theory should not dictate investment policies, although it is enlightening about the consequences and points to a norm.

That remark seemed to be equally applicable to the policy of bonus distribution. Paid-up immunization in theory demanded, after a change in interest rates, a bonus declaration at rates varying with duration, class of policy, age, term and so on. Such a distribution would be quite impracticable and some compromise must be made, either an equivalent uniform bonus being declared, or a rate varying according to a relatively few broad groupings of the business. Few offices would willingly abandon a form of bonus distribution which had been accepted by their policyholders for years. The authors' theory, therefore, shared with Redington's the disadvantage that some departure from a consistent bonus policy was in practice inevitable. It was, in fact, impossible in a changing world to combine consistency of investment and bonus policy with a uniform rate of bonus, and that difficulty must be faced. Happily, except in the case of a violent fluctuation, the smoothing out of the bonus rate could be effected by the application of miscellaneous surplus, not identifiable with any particular body of policyholders.

Under the heading 'Investment Implications' the authors pointed out that paid-up immunization implied a short mean term of investment, approximately one-half, in fact, of that demanded by total immunization. They suggested that if an office, in search of an improved yield, went 'long' of the paid-up immunization position, the extra yield obtained and the risks involved should be treated

on a corporate basis. That was expedient, but he did not see it as a natural outcome of the expectations of the reasonable policyholder. It might be felt that different policyholders contributed in widely varying degree to any profit and took widely differing shares of any risk. The justification of the policy suggested appeared to him to be the insurance principle of the pooling of risks and profits.

It was necessary to keep in mind, however, that the position of an office must be looked at as a whole, not as a series of separate funds each with separate assets and related to a specific section of the office's business. A short mean term for with-profits business might well prove attractive for investment purposes as an offset to the long mean term demanded by an expanding portfolio of group-pension and deferred-annuity business.

Application of the paid-up immunization theory resulted in the bonus rate being relatively sensitive to fluctuations in interest earnings. That was a natural outcome of the postulates laid down. In practice, however, an actuary would be reluctant to allow his bonus rates to swing as the theory demanded, and, particularly in the case of an office making annual declarations, some departure from theory on that account would be inevitable. The treatment of bonuses advocated by Redington did, it was true, introduce a greater degree of gradualism, but the speaker did not feel that that in itself was sufficient to justify the application of Redington's theory to a with-profits fund. He did not see how gradualism could be introduced into the paid-up-immunization model except by making it more complicated. The model had as it stood the great merit of simplicity and, in so far as gradualism was required, it seemed to him that the proper place for its introduction was in the choice of valuation method adopted.

In spite of the necessity for some practical divergence from the logical consequences of the strict theory, he felt that the authors had made out a strong case for the applicability of paid-up immunization to a with-profits fund, always provided that the bonus loadings were sufficient to justify its use. The obvious approach to the problem of immunization of the with-profits fund was on the basis suggested by Haynes and Kirton, whereby sum assured and existing bonuses were immunized, assets representing the value of future bonuses being left free. It would have been interesting if the authors had had space to give fully the reasons for their rejection of that system. Would not some sacrifice of consistency be justified if it would lead to a theory of more universal application? As had been seen, the consistency of the paid-up immunization theory was likely to become blurred in practice.

The authors concluded their paper by considering the situation arising when an office had invested 'short' of the paid-up immunization position or 'long' of the total immunization position. In the latter case departure from the paid-up model had been substantial indeed, resulting in the position that a rise in interest rates, which the paid-up model assumed would produce a rise in bonuses, would, in fact, lead to a fall. As the authors said, if interest rates rose when investment was ultra-long it might be necessary to make an objective estimate of the future loss of investment income. Unless what Redington described as the 'estate' was sufficient to cover that loss, the bonus prospects of new business might well be affected, since most actuaries would be reluctant to throw the whole of the loss, which might never materialize, on current bonus. That would in its turn tend to render the bonuses for new assurances non-competitive. It seemed that an office wishing to go 'long' of the total immunization position should build up adequate reserves.

Mr T. R. Suttie, in a written contribution, suggested that it was, perhaps, unfortunate that the authors had used the word 'consistent' in the title of their paper since their arguments led, on the one hand, to a mean term of investments which they admitted to be shorter than was likely to be used in practice and, on the other, to bonuses varying by original term and duration, which they promptly translated into a uniform bonus.

The authors did not consider in any detail the comparison between the bonuses resulting from investments corresponding to paid-up immunization and those from investments corresponding to total immunization. If interest rates were to fluctuate around the level ruling when the paper was written, the investments corresponding to total immunization would produce higher bonuses because of the higher yield which was normally obtainable on longer term investments. If interest rates were to fall permanently to a lower level, the investments corresponding to total immunization would have the double advantage of the higher basic yield and of the fact that the existing business would be immunized against the result of the reduction in yield. If interest rates were to rise to a high level permanently, the higher basic yield under the total immunization investments would operate to offset the advantage under paid-up immunization of subsequent premiums being invested at a higher rate.

The only case in which the paid-up immunization investments would have an advantage would therefore be a permanent rise in the rate of interest by an amount greater than the margin between the rates of interest obtainable on investments for the mean term of the paid-up immunization and the mean term of total immunization. On balance there was a strong probability that the investments corresponding to total immunization would give better results, and in some circumstances the differential might be considerable.

He felt that convincing arguments were necessary to justify a theory which led to results which could not be applied in practice.

He referred also to the expectation of the 'reasonable policyholder' and pointed out that the normal method of selling with-profits policies concentrated the policyholder's attention on the rate of bonus being allowed at the date his policy was effected. Because of that emphasis at the time of sale, it seemed equally possible to argue that the policyholder expected the company to do everything possible to ensure that the actual bonuses declared on his policy corresponded with his expectations when the policy was taken out. That argument led directly to investments corresponding as closely as possible to total immunization.

As a subsidiary argument, the authors had raised the danger of lapse and re-entry. For a purely ordinary office, lapse and re-entry had never, he believed, been a problem when changed conditions made it necessary to introduce new without-profits premium rates. The advantage, if any, of lapse and re-entry would be more obvious to the holder of a without-profits policy than to the holder of a with-profits policy. That was particularly true if any paid-up policies allowed did not participate in future profits.

He agreed with the authors' basic principle that new business should not be required to support the benefits of existing business. Under the method of bonus distribution which they recommended for practical use, bonuses would in fact be at a rate lower than that which could be supported by the new business alone. The authors might reply that that was more than compensated by miscellaneous profits. In an expanding office it was by no means certain that profits from the 'estate' would in fact exceed current contributions to building up the 'estate'.

Much play was made with the difficulty of repeated alterations in with-profits premium rates. To satisfy reasonable equity such alterations did not have to be more frequent or drastic than those in the without-profits premium rates, which had never been a serious problem. He believed that a truly consistent method of bonus distribution could best be arrived at by combining total immunization with alterations in the with-profits premium rates whenever circumstances were such that the actuary decided that an alteration was necessary in his without-profits premium rates.

Mr F. M. Redington welcomed the paper because there were several aspects of his own paper which he had felt could profitably have been expanded, and the authors had dealt with one of them. His own paper had been concerned primarily with the technique of immunization, of 'how' to immunize rather than 'what'. When he came to with-profits business he had had to make an assumption about bonus policy and chose what the authors called total immunization, but his paper contained no word of advocacy of that system as a principle; indeed, he had pointed out many limitations and qualifications, and had gone so far, in talking of the effect on the individual policyholder, as to say that the logical consequences were entirely foreign to accepted practice and tradition. None the less he had not chosen it idly; he had thought that it tallied roughly with practice and with what would be done, and he still thought so, in broad terms.

The authors had sought a consistent system, and he sympathized with them there. The fact that there was in practice no such consistent system was unsatisfactory. Theoretically there were a number of consistent systems—indeed, he supposed an infinite number—but they were all to some extent incompatible with current practices and traditions, and the reason was, as Mr Perks had said, that there was a fundamental incompatibility under changing conditions between uniform bonuses and a fixed scale of premiums.

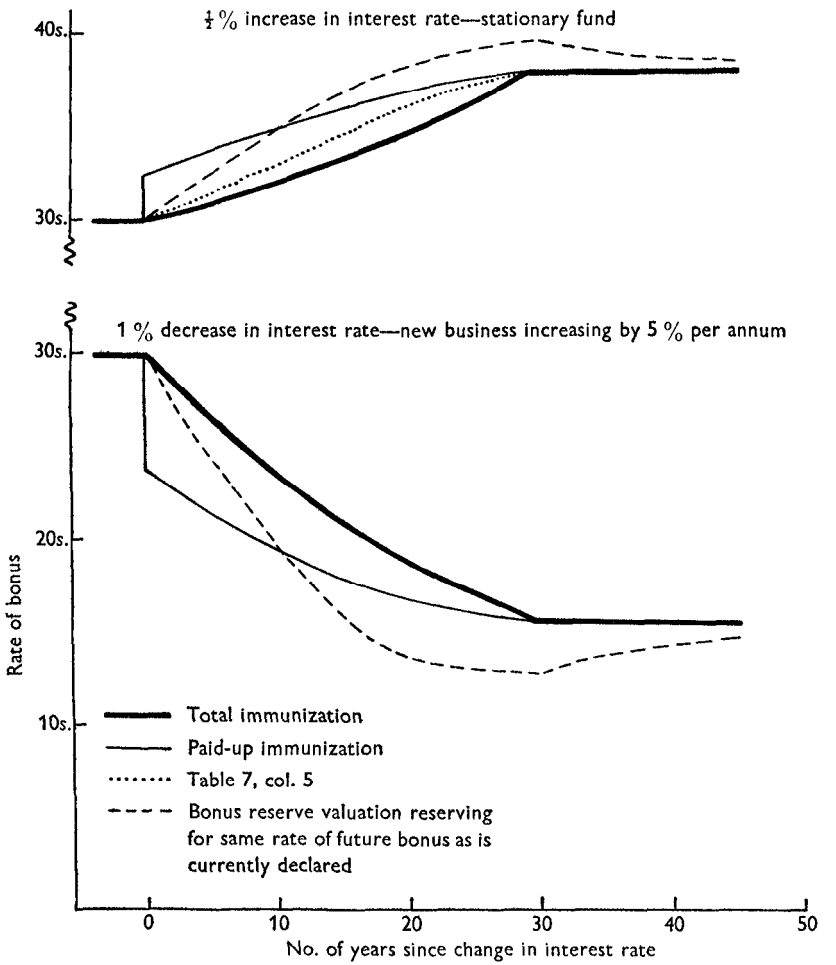
The authors' system, as set out in Table 2, was such a consistent system. If that table were elaborated to give the results under varying conditions, one would get a set of bonuses which was impracticable under conditions then current, but that was no criticism of their system, because it was equally true of any other, and in many ways less true of that table than of, for example, total immunization.

So far so good, but the authors had to go on, as was always necessary, to make practical modifications. They went on to uniformity and gradualism. Here he had found considerable difficulty in understanding parts of their paper and had also found difficulty in making himself understood by other people, so he had prepared two graphs.

The top graph was based on a $\frac{1}{2}\%$ increase in the rate of interest, but the broad picture was the same if it was 1% or $\frac{1}{4}\%$. Under total immunization the existing business supported a 30*s.* bonus and the new business 38*s.* 4*d.* at the new rate of interest. The mean effect of the combined business was as shown. The graph of paid-up immunization showed an immediate jump to 32*s.* 5*d.* The authors had naturally taken quinquennial values for their bonus figures, but that obscured the fact of the immediate jump. The paid-up policies earned 30*s.* and the 'unpaid policies' earned 38*s.* 4*d.* It might seem surprising that the mean of the two was as low as 32*s.* 5*d.*, but that was due to the effects of compound interest. Even if there were no new business at all, the paid-up immunization took the form of a sagging curve rising to 38*s.* 4*d.*; the effect of the new business,

all earning 38*s.* 4*d.* was to give the curve shown. The top graph applied to a stationary fund, with no withdrawals.

The picture would be rather different for a typical modern fund of expanding business with withdrawals. The total immunization curve instead of being convex would be concave, and the immediate jump under paid-up immuniza-



tion would be bigger. The greater area under the paid-up immunization curve represented the profit from investing shorter. If there were a drop in the rate of interest, there would be a corresponding loss.

The authors went on from 'uniformity' to advocate 'gradualism'. At that point he found himself perplexed in general principle, because it seemed to him that gradualism was not a practical modification of their theory but in direct conflict with it. Their basic principle was that the bonus should be responsive,

and immediately responsive, to changes in the rate of interest on the grounds that that was what the policyholder expected. Gradualism, however, meant the reverse of that. It meant that bonuses must not be immediately responsive. He had expected a clash of principle, and he thought that there was one.

He had looked in the paper for the rate of bonus that the authors would actually recommend, and the figures that he found were in Table 7, column 5. Those were the figures on the bonus reserve method of valuation which the authors recommended as a suitable arrangement. The rates from that table were shown on the top graph and there were two striking things about them. They were much closer to total immunization than to paid-up immunization; and they were obviously not the equivalent of the paid-up immunization. By altering the valuation basis it was not possible to alter the total area.

He had then realized—and he thought that many people had been under the same misconception as himself—that the figures in Table 7 were based, for understandable reasons of simplicity as was made clear in the paper, on the assumption that natural bonuses were in fact declared. But that obscured the true consequences of their method of valuation.

Although there were no figures to indicate what the authors finally intended, their bonus reserve method was quite explicit. He had worked out approximate figures of the resulting bonuses and they were shown on the graph.

He could find little to defend in the suggested method of valuation, but the main difficulty lay in gradualism. The 'natural' bonuses changed so rapidly that it was not possible to be very gradual; gradualness at the beginning had to be paid for later. To defer the shock well into the next generation, and even in perpetuity, as it were, would be hard to explain to the policyholder.

He was perhaps being unfair in emphasizing the valuation so much, because it was not a basic part of the paper. The bonus reserve method was full of traps, as he had tried to show in his own paper, and the authors might not have intended quite that method of valuation. Although they gave no figures under a net premium valuation it seemed clear from their description that their recommended net premium valuation gave quite different results from their bonus reserve valuation; it responded quickly and smoothly and finished at the end of 30 years as it should, but it was not gradual.

There would be general agreement on the authors' first principle, that future new business should not be involved. The second principle, that future premiums on existing policies should secure the same benefits as new business, was a different matter. He sympathized with the authors' approach, because he remembered his own sympathy with Suttie's first paper, which had been founded on the same general principle. Suttie had developed it quite differently, but the principle was the same as the authors', and the result was a short investment policy. But it was a dangerous guide. Future premiums on existing business were not the same thing as new business. They were renewal premiums, and could not be made by any statement of principle into new premiums. Secondly, whatever the policyholders might expect—and it was doubtful whether they expected just what the authors said—he doubted whether the authors' second principle was basic. That principle did not apply to non-profit or to low-bonus policies, and it would still leave the danger of insolvency.

The paper, indeed, only became possible because of an accident of history, namely, that the bonus loadings were high enough to remove the danger of insolvency. He thought it was doubtful whether that accident of history should be elevated into the basic principle of with-profits business.

If the authors' second principle was not right for non-profits business and not right for low-bonus policies, when did it become right? Was it when the bonus was 20*s*. or 25*s*.? If it did become right, was it suddenly at some point? His own feeling was that the real principles of with-profits business were to steer clear of trouble, to avoid insolvency, and, subject to that, to maximize profit. On non-profits business it was necessary to be very careful and to keep to a narrow path, but on with-profits business the path was wider, and the bigger the bonus loading the wider the path. He thought that it was quite right to be flexible and, within the wide path, to invest short or long according to investment conditions.

The path was sufficiently wide to embrace the authors' system, but there were three corollaries. One was that the bonus loadings must be kept large enough to embrace the system, another was that the policyholder should know and be warned that there might be big drops in his bonus if the rate of interest fell, and a third, he supposed, was that all companies should follow broadly the same pattern.

That the authors' system was possible was shown by the fact that in South Africa, Australia and Canada the lack of long investments made it necessary to invest short, though whether or not the offices liked it he did not know and they certainly would not like it if there were a heavy fall in the rate of interest.

Even if the authors' basic theory were dismissed, there was still the problem which they had rightly tried to attack, that of finding a consistent system. Tentatively, he would like to put forward an idea: a consistent system could be had by altering with-profits premiums. He had been interested to find that Suttie had referred to the same idea. He was far from advocating it and the thought was only seeping into his mind, but, by investing long and altering with-profits premiums in changing conditions, equity could be had and much of the speculation could be removed from with-profits policies. After all, their business was insurance and minimizing risk.

Mr A. T. Haynes said that he had approached the paper with the greatest possible interest. He had come away rather puzzled, and with a growing conviction that the authors were really dealing with a different problem from that which he and Kirton had attempted to investigate earlier in the year. He was also a little frightened that the two problems might become confused.

The problem which he and Kirton had tried to analyse was fundamentally that of financial security, and for simplicity their analysis was related mainly to non-profits business. It seemed to him that the authors were concerned with a different problem, namely, bonus policy in a with-profits fund. They had stated that he and Kirton 'only in a rather tentative way' had considered a with-profits fund. What in fact he and Kirton had done in a rather tentative way was to consider a mixed fund of with- and without-profits business, which, after all, was the type of fund which was normally met in practice. The authors did not appear even to have approached the mixed fund in their consideration of their problem.

He wondered whether there were any funds which were wholly with-profits. It was necessary to take into account in this context the presence of annuity business, immediate and deferred, most of which was non-profits. If there were such a fund, consisting entirely of with-profits business, he would agree that financial security was a very minor matter. Obviously if there were received nothing but premiums of rather more than £5 per annum to pay £100 *plus* profits at the end of 20 years or on previous death, the result could not be far

wrong if the £5 were put each year in the bank. The bonuses would probably be small, but the financial security would be intact. In other words, the problem for such a fund must be that of being a little more adventurous in investment policy, in order to provide attractive bonuses.

The moment, however, that the proportion of non-profits business in any life fund began to grow, the problem of security grew with it. That problem was not only to maintain the capital intact at the right future date but to maintain the interest yield on the fund as a whole (i.e. on the investment of future premiums as well as on present reserves) for the right future term. That, to his mind, inevitably required a policy of full immunization.

It might be asked why, in a mixed fund, with-profits business should not be immunized on a paid-up basis. His answer to that question would be that in a mixed fund the future bonus loadings were required as a reserve for the business as a whole, and not only for the with-profits business—a reserve to meet all departures from expectation, options which might be exercised against the office, voluntary departure from what he and Kirton had called the standard asset distribution, as well as all the hazards of mortality and every other contingency which might require such a reserve. If those bonus loadings were hypothecated in the first place as a reserve against the with-profits business itself, they might not be free for the more general purpose for which they were required.

In other words, he felt that in a mixed fund security must come first. Security, not bonus policy, should dictate the standard and the method of immunization. After all, a child was immunized against diphtheria to keep it secure from diphtheria, not to make its cheeks more rosy nor to give it any benefits which it would not otherwise have had. Bonus policy must certainly be allowed to come into the picture in considering the extent to which there should be voluntary departure from a matched asset position. That was, to his mind, the stage at which bonus policy was a very proper consideration.

If the fact that security must come first were accepted, he would argue that security was complete only when all future premiums could be seen invested to meet all contractual liabilities; i.e. when a full immunization test had been satisfied. His conclusion was therefore that most funds, of the typically mixed type, should be invested long for safety, and not, as had been suggested, merely for profit.

Mr K. J. Britt remarked that all the recent papers of the kind under discussion had been most interesting, but the authors of all of them agreed that it would be justifiable at times to depart from the rigid conception which they found it necessary to make to achieve perfect security. When interest rates were very low, he thought that going very short would be justified, because the risk that interest rates would then go still lower could not be great. When interest rates were very high, it seemed that the least risk would be found by going very long and so keeping those high yields for as long as possible.

In spite of what Mr Haynes had just said, there were some funds which to the extent of nearly 90% were really with-profits funds. There were some offices which made investments chiefly in high-class redeemable securities and amortized the values by redemption dates, and need not, therefore, write up or down because they would never lose, presumably, in interest or capital. In such a case, bonus rates would gradually rise as interest rates rose, or when interest rates were falling the bonuses would also fall slowly. He would have thought that that was a most desirable state of affairs. Why should the policy-

holder be subject to violent fluctuations in bonus, merely because market values fluctuated? He reminded his audience that the Joint Stock Banks had recently departed from precedent in the treatment of short-term redeemable investments.

He had a great deal of sympathy with the partial or paid-up immunization system at a time of very low interest rates. If the paper had been written in 1946 he thought that the offices might well have secured great advantage from it. He thought that the authors would agree, however, that at the time of speaking offices would be justified in departing widely from it and going long, of course with their eyes fully open to the risks, which he thought were slight unless interest rates rose to a very high level indeed.

Personally, he preferred the passive system of valuation, with gradually rising or falling rates of bonus. Of course, an office which had been able to accumulate big reserves, even if it did believe that it must cover fluctuations in market values, would be in much the same position as an office which did not write up or down, except that with its bigger reserves it would naturally be able to declare bigger bonuses.

Mr C. M. O'Brien, dealing with the investment implications of the paper, said that, as Mr Haynes had pointed out, most modern life funds were a combination of participating and non-participating business, and the latter included a fast-growing volume of deferred annuity business. In the paper, the authors dealt specifically and exclusively with the technique of the participating fund. He thought that they would agree that, if paid-up immunization was a yardstick for participating assurance, total immunization was the yardstick for non-participating assurance, so that in practice both ideas would be combined having regard to the composition of the business between the two classes.

The 'mean term' was an expression which was used in the paper and had been defined by Mr Redington. It was important not to be misled by that phrase. When considering the assets, it was the mean term of interest income and redemption money, and was different in nature from, and essentially less than, the average period to redemption.

The authors gave in Table 4 an asset distribution which matched Fund II(A) in Fig. 2. As Redington had pointed out in his paper, there was an infinity of patterns which would match that fund on total immunization, and it was interesting to look at one other pattern. If it were assumed that 15% of the fund was dead short, e.g. in open mortgages, then one pattern would be 15% dead short, 50% redeemable in the period 1975–80, and no less than 35% irredeemable. He thought that it was not unreasonable to consider that an office might well have about 15% of its funds in mortgages or comparable short investments.

He had referred to the mean term, and he thought that it was revealing to realize that, on a 3% basis, the maximum term of an irredeemable investment was $33\frac{1}{3}$ years, while an existing deferred annuity, assuming that the annuity was taken, might well have a mean term of no less than 50 years. He thought that, bearing in mind the composition of business in most offices including deferred annuity business, and having regard to the significance of the words 'mean term', many actuaries would be surprised if they looked at the relative length of their assets and their liabilities. He thought that they would be very likely to find that they were already invested shorter than the norm implied by total immunization for the non-participating business and paid-up immunization for the participating business.

Mr L. Brown reminded members that the investment of life funds was essentially a practical business. He felt that if the authors had had amongst their responsibilities the actual determination of investment policy, and particularly the responsibility for carrying out that policy, their paper would have benefited in its presentation by giving clearer recommendations and more explicit results. Moreover, they would have had so many experiences of events proving their views to have been wrong that the opinions which they expressed might have been a little less dogmatic. Again, if they had had the responsibility of battling with leading accountants, and had had to make the effort to reconcile actuarial theory with accountancy views, they would not, perhaps, have been quite so strong in their expressions, such as their statement that the current rate of interest was irrelevant to the presentation of accounts.

The argument on which the paper was based, namely, the expectation of a policyholder when he took out a new policy, had already been criticized. He thought that that expectation could be expressed in other ways. He would agree that a policyholder would probably, if asked, expect to benefit by improvements in the experience, but the speaker was not at all sure that he would expect to suffer from any worsening of the experience, particularly if he had been first told that it was within the power of the actuary to protect him, at any rate to some extent, on that account. He also disagreed with the conception because he thought that most policyholders would think that a fall in the rate of interest was something which, by reason of the appreciation of investments, would greatly benefit their bonus prospects.

He found the basic principles of the paper a little confusing. To put them into his own words, it appeared that new policyholders must do as well in the future as old policyholders, and old policyholders must do at least as well as new policyholders. It reminded him of a political dissertation which he once heard, to the effect that when the forthcoming revolution took place everybody would be as well off as everybody else, and perhaps a bit better.

Nevertheless, if his remarks were critical he welcomed the paper warmly. As he looked at the problems, from a financial or investment point of view, it seemed to him that the logical thing to do when making investments was to ask what was the purpose of them. If he were a banker, and had to invest against money which might be withdrawn at any time, he would want his investments to be realizable. If he were dealing with a charity fund, where the only consideration was expenditure of income and where it would never be necessary to call the capital in, he wanted the highest income that he could get and the longest term—perpetuity, if possible. He thought that the basic life assurance policy was a non-profits policy, and, in so far as it was possible to cover the investment risk involved when a non-profits policy was issued, it seemed to him that that was the logical thing to do. The with-profits policy was, to his mind, a frill on the basic non-profits policy, though perhaps a rather large frill.

In modern times, as a result, as Mr Redington said, of an accident of history, with high bonuses and a substantial loading charged in the premium to cover and produce those bonuses in future, he was inclined to think that they ought to look on with-profits business as guaranteed bonus policy business, without the guarantee. In other words, a bonus was charged for, and it was not good enough to throw the whole of that bonus into reserve against catastrophe, if part of it could be secured.

He felt that in dealing with the investment of a life fund the right attitude was to vary the investment programme in accordance with the long-term views

which might be held from time to time. He was sure that there was no definitely right answer at any time.

Mr M. E. Ogborn expressed his disagreement with Mr Brown, who regarded participating business as a frill whereas he himself regarded it as basic to life assurance. He did not think that it was any accident of history that participating business with substantial bonus loadings came to be the staple business of life assurance; he thought that it was the logic, not an accident, of history.

In putting security first, actuaries should not look to policies of so-called immunization to give them security. Where to look for security was in the size of the premiums which were charged, not in what was done with the premiums afterwards. It was because security lay in the size of the premiums charged, that there was in fact participating business at all.

To deal with a side issue first, he thought that too much emphasis was apt to be laid on the security which could be obtained from a policy of immunization; that emphasis came from the word 'immunization' itself, which gave a feeling of security which was not there. Immunization was an attempt to neutralize the effects of fluctuations in market values; when it was put in that way it would be seen at once that there was a severe limitation on what could be done, since all securities did not respond to changes in market levels in the same way. The price of a security did not in fact depend only on its date and on the rate of interest; there were many other considerations to bring into account. While, therefore, 'immunization' could provide some lead as to distribution by date, it could not give security because a particular distribution by date had been selected.

It had seemed to him as the discussion proceeded, and in listening to the other discussions which had taken place, that there were widely divergent views about the nature of participating business. To take two extremes, one view was that by choosing suitably long investments and neutralizing or 'immunizing' the fund against changes in market price the rate of bonus would be fixed according to the conditions ruling at the time of entry. At the other extreme, there was the view that consideration need be given only to the part of the contract which had been guaranteed and that the whole of the reserve for future bonuses, the bonus fund, could be taken as being a free fund for investment for profit. That meant, of course, that for many years after the issue of a with-profits contract there was nothing to neutralize or immunize, since the reserve on the with-profits contract, excluding the value of future bonuses, was negative in the early years; for many years after writing a block of new business the office would be free, by that theory, to invest as it liked. Those two views were, he thought, at opposite extremes, and it was perhaps unusual to find Mr Perks, one of the authors, providing a happy mean between extremes. Perhaps that was because he was running in double harness.

To return to what he had been saying earlier, the office's real security lay in charging premiums that were high enough, and that was in fact how the bonus system had come about. He did not mean to say that the first actuaries sat down and deliberately 'doubled the answer'; when they came to make their calculations the future conditions had been obscure and they had felt, particularly since the calculations had been made in the first place for a mutual fund, that they had to make a fair estimate—by 'fair' he meant one which they thought would not go wrong—of the mortality, and of the rate of interest, which they assumed to be 3 %. It was an historical fact that their estimates showed considerable profits, and when those profits had been earned they were distributed to the policy-

holders as bonuses. The origin of participating business was as simple as that.

Were not actuaries still in the same position? Could conditions be foreseen sufficiently for business to be transacted only on a non-participating basis, or, if they could not be foreseen accurately, was there any investment policy which would enable business to be transacted solely on a non-participating basis? Personally, he did not believe that there was. He believed that for the security and safety of life assurance business the bulk of the business should be participating. That was a matter to which those who were concerned with a fair proportion of deferred annuity business on guaranteed terms had to give close attention.

Another aspect of the matter was to see, in reading past history, how the ideas about the distribution of surplus had been linked with the particular investment policies that were possible or desirable at the time. Investment policy was not something which was wholly within the office's own control; to some extent the office was dependent on the types of investment that were available. That was a theme which it would be interesting to illustrate from the past.

In reading discussions on the contribution method of distribution it was important to notice how closely the method was originally linked with the conception that the proper investments for a life fund were mortgages. In the early days, his own office invested in Government stock, but when rates of interest were low and prices were high, in the 1820's, the office endeavoured to increase its proportion of mortgages; arising out of the experience in those years, it came to be the accepted view that mortgages were the only suitable investment for life office funds, which would be generally remembered from Bailey's Canons. Such an investment policy had in fact been widely carried out in practice; Murray (*T.F.A.* XVI, 247) showed that ten leading life offices had over half their funds in mortgages in 1871, while the proportion in his own office went up to two-thirds, or even a little more. Those were staggering proportions by modern standards.

The extensive investment in mortgages was linked with funds which were wholly, or almost wholly, participating. With such a fund it was possible to consider each policyholder as having a share of the total funds in a way which the Americans had called 'the asset-share', a typically American conception of life assurance. That conception, however, was not serviceable for a fund which invested in securities where the price of the security fluctuated with market prices. It was one of the merits of the paper under discussion that the authors introduced their readers to another conception of assurance which would replace the 'asset-share' conception, namely, one under which each premium purchased a part of the assurance, so that the interests of the existing policyholders were the parts of their assurances which they had purchased to date.

Mr G. E. Wallas said that the authors referred to the mean term of the liabilities. It was practicable to estimate this quantity accurately, using only the valuation summaries and what was, in effect, an application of the n-ages method. He had had access to the model funds used in the paper, and had obtained, with little labour, results which were remarkably accurate. It was hoped to submit a short note to the *Journal* setting out the method in detail.

It seemed to him that Redington's definition of mean term had the same general superiority over the arithmetic mean term of redemption dates as, in statistics, the mean had over the median and the mode. In the first place it became possible to cope easily with irredeemable investments. Secondly, which

was more important, it could be manipulated arithmetically, both in changing the point of time from which the term was to be measured, and also in obtaining a combined mean term for several blocks of liabilities or assets from their separate mean terms.

Mr N. Benz thought that paid-up immunization represented the sacrifice of the practical benefits of immunization, without any compensation of practical value.

He asked the authors what they thought the views of the 'reasonable policyholder' would be on the subject of surrender values and fully-paid values, and as to whether such values should carry guarantees, or not.

On page 17 there was a reference to the distinction between surrender options and fully-paid options. He would have thought that that distinction depended very largely on the past practice of the particular office concerned. Most offices guaranteed proportionate fully-paid values, and some guaranteed surrender values as a matter of routine. Perhaps also of practical importance was the implied guarantee which followed the quotation of a surrender value to which no formal guarantee attached. It could be extremely embarrassing to have to go back on one already quoted. The authors might have mentioned in that context policy loans as representing a special type of option, which might be exercised against the office as often as there were major downward changes in interest rates.

On page 23 the authors said that 'there can be no doubt that any attempt to beat the market would be a dangerous practice for a life office'. It was difficult to reconcile that statement with what they said in the first complete paragraph on page 16, but perhaps he had missed something.

He would have found Tables 5–8 much easier to understand if there had been more detailed references to the effect of a fall in interest rates. He thought that those tables showed clearly the difficulties of interpreting the results of bonus reserve and net premium valuations, and he had the feeling that the authors took the view that it was easier to go wrong with a bonus reserve valuation than with a net premium valuation.

Studying the paper he had more than once found himself thinking that for a life office there was no substitute for strength, however that strength was expressed. It might be regrettable, but actuaries could not by their own efforts make bonuses, or even assurance funds, absolutely sure.

In conclusion, he would make a plea for some tidying up of the terminology associated with the subject. Some confusion was caused by the use of 'total', 'absolute', 'full' and other adjectives which had been applied to immunization.

Mr J. L. Anderson, in closing the discussion, said he felt that his primary task was to focus attention on the basic principles of the subject. As applied to non-profit business the issue was essentially a simple one. For any fund it was possible to ascertain the mean maturity date of the assets which, for a given rate of interest, would ensure that fluctuations in the rate of interest could not render the fund insolvent, unless the office weighted the scales against itself by giving too many options. That problem had been examined by Haynes and Kirton in their paper to the Faculty. The paper under discussion dealt with the corresponding problem for a with-profits fund, a problem already examined, though not exhaustively, by Redington.

At the risk of emphasizing the obvious, he would repeat that, whereas with a non-profits fund there was, for any given rate of interest, a unique mean maturity date of the assets necessary to give protection against variations in

interest rates, for with-profits business the existence of the bonus loading in the premium gave scope for varying the mean term of the assets without involving the risk of insolvency. He did not think that difference had been brought out as clearly in the paper as it might have been, possibly because the authors had not been concerning themselves primarily with the question of solvency. It had, however, been brought out by several speakers in the discussion.

He found it helpful to pose the problem of the with-profits fund in the following way. For any given range of interest rates, it was possible to ascertain, for a particular with-profits fund, the maximum and minimum values of the mean term of the assets within which it was permissible to operate without running the risk of insolvency provided the rate of interest did not go outside the stated range. (He was dealing with the matter in its simplest form and taking no account of probable variations of the rate of interest according to the term of the investment.)

The maximum and minimum values of the mean term of the assets would depend, *inter alia*, on the age of the fund. Taking a single block of new with-profits endowment assurance business and tracing it through its history, at the outset it would be permissible to invest entirely either in irredeemable securities on the one hand or in cash on the other. As the fund grew older, however, the minimum limit would tend to rise and the maximum to fall. The movement of the limits would depend to some extent on the range of interest rates postulated, but, speaking broadly, he thought it would be true to say that, as the fund grew older, the minimum would rise off the floor before the maximum fell below the ceiling, so that for a period in the history of the fund it would be permissible to invest entirely in irredeemables but not entirely in cash. As the fund grew older still, the maximum and minimum would both tend to fall until eventually, as the maturity date approached, it would only be permissible to hold very short-dated securities. If the office guaranteed surrender and paid-up policy values, the limits would, of course, have to be narrowed, perhaps drastically.

From the solvency standpoint, therefore, when considering a with-profits fund, the appropriate action was not to ascertain any one mean asset term, such as was appropriate for a non-profits fund, but to ascertain the appropriate maximum and minimum mean terms for what seemed a suitable group of interest rates. (When the rate on $2\frac{1}{2}\%$ Consols was a little over 4%, the assumed range for the yield on this stock might be from $2\frac{1}{2}\%$ to 6%, with corresponding yields for other types of securities.) These maximum and minimum terms, when calculated, should provide the management with all the guidance it needed as to the spread of the maturity dates of its investments in order to protect the fund against insolvency.

Where, then, did the mean terms corresponding to full and paid-up immunization fit in? That would depend on the age of the fund, but, generally speaking, both those mean terms would lie between the maximum and minimum limits. He was therefore a little puzzled by the general statement near the foot of page 22 that to allow the mean term to pass beyond the point of total immunization would be a speculation. It seemed to him that in certain circumstances it would be possible to pass beyond that point without any serious risk of insolvency. Admittedly a fall in the rate of bonus following a rise in interest rates would take some explaining, but that was not a valid consideration when dealing with solvency. Perhaps the significance of the authors' statement to which he referred had not been made as clear as it might be.

So much for the question of solvency. Both Redington's paper and the

authors' dealt also with the question of equity between existing and future business, and different classes of business. On the occasion of his own paper to the Faculty (*T.F.A.* XVII, 137), he had emphasized that it was absurd to attempt to test the equity of a bonus system without first finding a solution to the problem of allocating between different classes of policy the interest earned on the investments of the fund and the profit or loss sustained by their appreciation or depreciation. For that purpose, he found it convenient to postulate a notional spread of investments which agreed fairly closely with the spread obtained from paid-up immunization. He had treated any profit or loss due to variations of the actual investments from those postulated as something which went into the pool and belonged to the policyholders as a whole. In that way he attempted to measure the relative bonus-earning powers of different classes of policy following a change in the rate of interest, but he did not draw any deductions about the merits of investment according to the principle of paid-up immunization.

The authors went further. They put forward arguments based on the policyholder's expectations; and it had been interesting to hear the different ideas of what the policyholder's expectations were likely to be. Policyholders had been referred to as 'reasonable' and as 'intelligent' and so on; in fact, Mr Brown's policyholder must have been very intelligent. He himself was of the authors' opinion on that point; he felt that if the policyholder's expectations were considered, paid-up immunization was a sensible compromise to ensure that policyholders did not get too much of a shock either from the unexpected immunity of bonus rates from fluctuations in the rate of interest on the one hand, or from too rapid variations in bonus rates on the other.

While he agreed that the expectations of policyholders were to be taken into account when framing investment policy, he thought that it would be a mistake to let them weigh too heavily. So long as the mean term of the assets was kept well within the maximum and minimum limits necessary to ensure solvency for a reasonable range of interest rates, it seemed to him that the management should be free to invest as long or as short as they thought proper with the aim of producing the best results for their with-profits policyholders. He would go further and say that it was the duty of the management of a pre-eminently with-profits fund to seek to benefit their with-profits policyholders by an active investment policy, and he saw no reason why they should not try to beat the market. He would emphasize that the position was quite different for a non-profits fund, where the scope for departing from a fully-immunized position must be strictly limited by the extent of the free reserves.

On valuation problems he had nothing to add to what had been said, but he would like to ask the authors one question: what bonus would they in fact declare if their fund were invested in securities on the principle of paid-up immunization and there were a rise in interest rates?

He would like to put on record that the principles and practice of immunization were by no means new, although it was only in recent years that the subject had received the attention which it deserved in the records of the Faculty and the Institute. He did not wish to detract in any way from the work which had been done on the subject in recent years, but it would be a pity if the rising generation were to gain the impression that offices had been unaware in the past of the need to match investments. In his own office the principles of matching had been understood, and the investment policy of the office designed to give effect to those principles, for many years before the war, and he imagined that other offices must have acted on similar lines.

Furthermore, that action, and the action of the offices after 1932 in gradually reducing bonus rates but at the same time strengthening their valuation bases, were not merely the result of 'instinctive' thinking; they were the result of logical reasoning. He thought that the older generation of actuaries should be given full credit for this, and it should not be suggested to their successors that they had been unaware of the principles involved and had reached the right answer more by instinct than by reason.

The President (Mr W. F. Gardner), in proposing a vote of thanks to the authors, said that the subjects which had been discussed were important; they were subjects which many, if not all, of those present would want to go away and ponder over for some considerable time. He would like to recall the first sentence in the Institute's *Year Book*, which defined broadly the function of the actuary in modern civilization as 'the application of the theory of probability to practical problems'. It was important, he thought, to notice that the theory came first and the practice second; and so in the matter under discussion he felt that it was important that they should all acquaint themselves first of all with the theoretical consequences on investments and on bonuses of both paid-up immunization and total immunization, examining the cases both when interest increased and when it decreased and not overlooking the important matter of options, which had been referred to by several speakers. He thought that they should absorb those matters of theory and that they should work out their own experiments in addition to those which the authors had shown them. The authors, he was sure, would be the first to feel that it was helpful that everyone should do his own arithmetic, learning at the same time from the arithmetic which they had put forward.

Since no one, he understood, would advocate declaring in practice the bonuses theoretically produced by any of the methods which had been discussed, they were in the end faced with the necessity of taking an arbitrary decision, and it was not surprising that there should be considerable differences of view. It was necessary to ponder the practical problems carefully, and particularly the circumstances in which they had to form a practical judgment—the circumstances of their own offices. Members would, he was sure, give due weight to the opinions which the authors had expressed and to those which others had expressed in the discussion.

Anyone who had had any experience of arbitration would feel some comfort in the reflection that the supporters of paid-up immunization advocated longer-term investment than their theory implied, while the supporters of total immunization advocated a shorter term than their theory would seem to suggest. That must give renewed heart to those actuaries who performed the important work of the investment of life funds.

It was rare but not unique for two successive Institute discussions to be devoted substantially to the same subject. He thought it would be agreed that that unusual sequence of events had been justified by the value of the paper which the authors had submitted and by the important discussion which it had provoked. It gave him great pleasure, therefore, to propose a vote of thanks to the authors for their useful and interesting paper.

Mr G. V. Bayley, in a preliminary reply, thanked the members for the reception which they had given to the paper.

In one approach to the investment policy of a participating fund, referred to by Mr Lamb, a part of the assets was earmarked to immunize or match

certain liabilities, while the other part of the assets was left 'free'. For example, under the authors' system the assets corresponding to the immunized future bonuses on the paid-up sums assured could be regarded as the free part. Each investment of the free assets had nevertheless a mean term. It might be more or less difficult to know what it was, but it was a sobering thought that the free assets as a whole had in fact a mean term. It seemed to the authors to be desirable, therefore, to link those assets to the corresponding liability maturity dates in order to provide a norm for investment purposes. Unless that norm were known it was not in fact possible to tell whether, with the free assets, a view was being taken or not, and, if so, which view: it was not known whether a rise in the rate of interest was a good thing or a bad thing for the free assets.

In any system in which a large part of the assets could be regarded as completely free, almost any practicable distribution of the total assets would appear to be satisfactory, because a block immunizing the fixed liabilities could be selected from the total portfolio, leaving the rest, whether short or long on the average, as the free part.

If part of the assets should have a mean term a and the other part a mean term b, the whole of the assets should have a mean term somewhere between a and b. The authors had concluded that a single norm or mean term for the assets was the more practical idea, although there was no objection in principle to thinking of the assets in two parts, with their respective mean terms for investment.

It had been suggested that by favouring gradualism when considering the valuation problem they were departing from the model or system which they had put forward. They had considered several methods of valuation to see how they conformed to the requirements of the model, and whether they would release surplus at the right rate consistent with it. They preferred one of the bonus reserve methods of valuation to the others because it caused a damping rather than a resonance of bonus fluctuations; but no departure in principle from the basic objectives had been intended, and when a policy became a claim by death or by maturity a comparison of the bonuses payable should be made not with the equivalent uniform bonuses shown in column 3 of Table 7 but with the natural bonuses which they would regard as the appropriate criterion. Over the whole duration of the policy, if investment policy had been consistent, the valuation method suggested would produce total bonuses closer to the criterion of natural bonuses than to the corresponding criterion of any other system known to them.

The authors would agree, of course, that the method of valuation referred to—equating current and future rates of bonus and using the earned rate of interest on the fund—was not suitable in all circumstances. It released surplus rather too early if there was a sustained fall in the rate of interest. The logical remedy, however, was not to devise a different investment policy, but to correct the surplus for the shortcomings of the valuation method. Miscellaneous sources of surplus might or might not be sufficient for that purpose.

The authors had suggested gradualism because if, for example, the rate of interest changed suddenly from 3% to $3\frac{1}{2}$% (or $2\frac{1}{2}$%) it would not be known in practice whether that change was a permanent one, and it seemed to them inexpedient to act as if it were. Gradualism associated with successive changes in the rate of interest seemed to them to be a satisfactory expedient in practice, but that was not to say that they thought that a consistent system of total immunization would be a more appropriate model just because it had a damping effect on bonus fluctuations.

Some speakers felt that a policyholder did not expect bonuses to be so sensitive to fluctuations in the rate of interest as was implied in paid-up immunization. The authors, on the other hand, felt that a reasonable policyholder would not expect his bonuses to become tied to the rate of interest operating at the date when he took out his policy. They would use the expedients of gradualism in valuation and miscellaneous sources of surplus to smooth out the bonuses in their system. Those who favoured total immunization would, if he had understood Mr Redington correctly, favour expedient departures from that model to overcome the objections to that system, so that it would be necessary to depart from both models in practice. Which system, therefore, should be chosen as a guide to practical affairs? He could only give the authors' reasons for choosing their own model. (He would emphasize, as Mr Haynes had done effectively, that the authors were dealing with a full with-profits fund.) First, comparing the two theoretical models as such, the system of paid-up immunization secured a smooth junction of bonuses between existing business and new business: total immunization did not. Secondly, comparing departures from either system in practice, they felt that the departures under their system were smaller than under the other, but he admitted that that was a matter of opinion, and it came in as subjective evidence. Thirdly, in practice, offices did keep their with-profits premiums more or less constant, and they allowed fluctuations in the rate of interest to work themselves out in the bonus rates. That idea was more easily accommodated in the authors' system than in a system of total immunization. There was, of course, a great deal more in it than that, and he could only sum up by saying that they had concluded that a system of paid-up immunization more closely represented sound practice.

The authors would agree that what they would regard as speculation others might well not so regard, and he supposed that it depended on the proportion of the fund that was being considered for that operation.

In conclusion, he would like to thank Mr Perks very much for allowing him to be associated in the paper. To put it briefly and inadequately, it was a signal honour to have a paper accepted by the Institute for discussion, and it had been a great pleasure to share that honour with Mr Perks.

The authors have written as follows in amplification of Mr Bayley's reply to the discussion:

The reasonable policyholder has been abused but, while not abandoning him, we may point out that he belongs to the preliminary discussion of features which an acceptable system must possess. His expectations set out on page 16 still appear to us the most reasonable that have been suggested. We are unconvinced that a policyholder could reasonably expect his bonuses to be tied to investment conditions at entry when that involves presenting him with a running option against the office. Nor are we able to agree that it is justifiable for him to expect the best of both worlds. Our basic principle has been misunderstood but it is not indispensable because other starting points can be used. For example, our system can be erected on the sole premise that the part of each premium not required to provide the year's life cover is deemed to be invested at the interest level ruling at the time of its receipt.

Pages 21–41 of our paper were intended to be a factual analysis of our system and of its consequences; we analyse and illustrate, but advocate very little at that stage. We certainly do not propose a valuation rule of thumb; we expressly

recognize that there is no substitute for the actuary's judgment. After discussing the exact valuation solutions of the situation behind Table 7 in which the rate of interest has changed to a permanently higher level, we proceed to approximate solutions designed to release surplus as required. An approximate solution is unavoidable if, as is usual in practice, a single rate of interest and, for bonus reserve valuations, a uniform rate of future bonus are to be used. Table 7 contains three bonus reserve approximations of which that illustrated in column 5 has certain virtues, e.g. the wave of small errors introduced is self-adjusting in the long run. The reason for these errors is stated immediately above the Table. Mr Redington emphasized the errors of this approximation but has overlooked, we feel, the essential conditions which we attached to its use, namely the existence of a contribution to bonuses from miscellaneous sources sufficient to overshadow the small discrepancies. This would be particularly important in the case of a large prolonged reduction in the rate of interest. A precautionary strengthening of the valuation basis would, of course, be considered in practice, as indeed we suggest for a net premium valuation in somewhat similar circumstances (page 37). Incidentally, the errors of this particular method of valuation for total immunization are shown in Table 8 and are comparable in size but have a different incidence. The results of net premium valuations are given in general terms on pages 36 and 37 and we do not follow Mr Redington's comment that it is not possible to be gradual there.

As was pointed out in the oral reply it is, we think, more important to study the total bonuses when a policy becomes a claim. The bonuses resulting from the valuation procedure of column 5 of Table 7 compare reasonably well with natural bonuses at maturity but the corresponding comparison for total immunization (Table 8) is not so satisfactory. Where there are successive changes in the rate of interest the same comparison for paid-up immunization is very satisfactory but the corresponding comparison for total immunization remains poor.

We agree that our principles require bonuses to be responsive to the rate of interest—this is in fact their virtue—but the immediateness of the response is a question of degree and a matter for control without violating the principles. The total at the date of claim is what matters. This thought suggests that if bonuses were declared only on the paid-up sum assured gradualism would be automatically introduced into the system. Such a system has a long record of practical application in the form of a bonus depending on the number of years' premiums paid. The ordinary uniform system can be regarded as a partial anticipation of the later immunized bonuses under such a system. Perhaps that is how it should be regarded.

It is only after considering the effects of successive changes in the rate of interest and when we consider the practical situation in which the future is entirely unknown that we conclude (page 42) that a particular form of bonus reserve valuation, which could be applied in practice, would be 'a suitable arrangement' and that a gradually changing net premium valuation would be 'hardly capable of improvement'—we mean, of course, that it would be exceptionally suitable! We do not recommend the former for the highly idealized situation of Table 7 without qualification. But that situation places a severe test upon the bonus results under our system and we claim that the method works surprisingly well in such extreme circumstances over as long a period as 30 years. Incidentally, the strengthening over the 30 years is closely similar to that of a passive net premium valuation process, described on page 37.

Mr Suttie and Mr Redington both put forward the suggestion that with-profits premium rates might be varied instead of the bonus rate when the interest rate changes. This would be consistent with the total immunization system and the objections to it still remain; there is the further one that the suggestion is quite contrary to traditional practice in this country.

We completely agree with Mr Ogborn's view about the nature of participating business and that security is to be found only by charging adequate premiums. This applies to non-profits as well as to with-profits business. As in general insurance, additional reserves need to be built up out of profits or margins in the premiums.

Mr Britt has pointed out that there are many almost wholly with-profits funds—industrial and ordinary—but we agree with Mr Haynes that mixed funds of with- and without-profits business present special problems. The case of a small proportion of non-profits business is not significantly different in principle from the complete with-profits fund. When the with-profits business is a small proportion of the whole, we have an unusual situation, although we agree that it is becoming more common.

The with-profits policyholders are not only joining together in mutual life assurance but they are participating in the profits and losses of non-profits life assurance and pension scheme business. The security needs of such a fund may be acute but it is not within our province to pursue the matter here except to agree with Mr Ogborn that total immunization of with-profits business is not the way to security.

To the extent that the margins in the premiums over what is required for the basic sum assured have been received and have not been distributed as bonus we agree with Mr Haynes that they are available as a buffer for the whole business. But the future margins are much less tangible because, if large lossses should materialize in any section of the business, these future margins might well evaporate as a result of surrender or lapse by policyholders unwilling to pay for bonuses that they have no prospect of receiving. But apart from this weakness there is the question of determining the amount and incidence of these margins. Under any system of total immunization they are dependent upon the investment conditions at the date of entry but their incidence over the durations of the policies has not been precisely defined by supporters of the system. Under our system the margins for bonus are dependent upon the conditions prevailing when each premium is paid. The size of this contingent buffer under the two systems depends (apart from the question of incidence under total immunization) upon whether the interest rate at entry is greater or less than the average rate over the duration of the policy. No advantage in principle can therefore be claimed for total immunization on this line of argument.

We do not accept Mr Benz's suggestion that the statements on pages 16 and 23 are inconsistent. To depart from the investment norm to obtain a higher yield is a different operation from attempting to 'beat the market'. We think that there are limits beyond which a life office ought not to go—perhaps it is all a matter of degree. On pages 15 and 16 we discuss the conditions in which an office, for reasons of solvency, ought to adhere to a matched position.

It has been suggested that our system conflicts with the desire to obtain a high yield on longer-dated investments. This is not so—as Mr O'Brien has explained. In any case, the interest-rate structure is continually altering: short-term rates have sometimes been greater than long-term rates and in 1952 the net redemption yield on medium-dated stocks has exceeded both.

The reason for the remark at the foot of page 22 which Mr Anderson referred to and which has nothing to do with solvency was our view that a prolonged fall in bonuses on a rise in interest rates would be something of a disaster to an office. We hope that there is nothing in our paper which suggests that 'matching' is a new idea. We are sorry if our phrase 'instinctive thinking' is unhappy. We intended it to mean that sound principles were so ingrained in him that he could not get his thinking wrong. When he appears to differ from us it is usually due to faulty phrasing on our part.