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On the Construction of a Combined Marriage and Mortality Table from Observations made as to the Rates of Marriage and Mortality among any body of Men; and on the Calculation of the Values of Annuities and Assurances that depend on the Contingency of Marriage as well as Death, and their application to determine the Rate of Premium for an Insurance against the Contingency of a Bachelor of a given age leaving Issue: illustrated by various Tables calculated from the experience of the British Peerage families. *By* T. B. SPRAGUE, M.A., *Manager of the Scottish Equitable Life Assurance Society.*

[Read before the Institute, 31 March 1879.]

IN valuing interests that involv the probabilitys ov marryge and ov leaving issue, we require to hav the means ov ansering such questions as the folloing among others. What is the probability that a bachelor ov a givn age wil (1) marry, or (2) die unmarryd, in an asignd year from the present time? or (3) be alive and stil unmarryd after the laps ov a givn number ov years? In ordinary life contingency calculations, the corresponding questions as to the probabilitys ov life and deth ar anserd by means ov the figurs containd in the familiar mortality tabl (formerly cald a *tabl ov mortality*, but by Dr. Farr a *life tabl*); and the great practical convenience ov that tabl sugests that, when we hav to deal with chances ov marryge as wel as ov deth, we shoud at the outset construct on analogos principls a combined marryge and mortality tabl.

It is my object in the present paper, not only to develop the theory ov the construction ov such a tabl from observations as to the marryge and deth rates among any population, but actually to construct a tabl from the experience ov the familys ov the British Peerage, and to sho how it may be aplyd to calculate the premiums for insurances agenst issue to bachelors; and I trust that the tabls I shal presently submit wil be found practically useful to actuarys.

In the preparation ov this paper I hav had to deal with the question ov the graduation ov mortality tabls, and in a note apended to the paper I hav described the grafic method which, after ful consideration, I decided to adopt. This or a similar method was employd by G. Davies,* and probably by others at home or abroad, but I cannot find that it has ever been clearly described by any riter. My subject also leads me to compare the rate ov mortality among bachelors with that among marryd men, and I shal hav ocasion to point out that the rate ov mortality among the men who ar selected at marryge, or (in other words) then taken out ov the general body ov bachelors, is, at the ages at which marryges ar most numeros, very similar to the rate ov mortality among lives recently selected for insurance.

If we hav a tabl ov the folloing form, it wil enabl us to trace completely the bachelors living at the yungest age in the tabl; for it shos how many ov them marry, and how many dy unmarryd, in each year ov life, and how many dy in each year ov life having previosly marryd.

Age.	AGREGAT COLUMS.						DECREMENT COLUMS.		
	Bachelors.			Marryd.			Decrement in a Year causd by		
	Ded.	Living.	Total.	Total.	Living.	Ded.	Bachelors Dying.	Bachelors Marrying.	Marryd Dying.
x	$b_x-(bl)_x$	$(bl)_x$	b_x	m_x	$(ml)_x$	$m_x-(ml)_x$	$(bd)_x$	$(bm)_x$	$(md)_x$

With regard to the heddings in the tabl, it is to be notist that thro'out this paper I uze the word "marryd" to denote a man who has been marryd, whether he is stil marryd or is a widoer (or divorst). I woud much prefer to uze som distinctiv and unambiguos word, but I can think ov non that wil anser the purpos. When it is necessary for any reason to keep in vew the distinction between men who ar marryd and those who hav becom

* Mr. S. Brown states (*J.I.A.* xi, 11) that G. Davies "obtained the rate of mortality for the average age in each rank, and then, as he says, 'laid down a curve, taking the age as the abscissa and the mortality per-cent as the corresponding ordinate, making the inflections thereof as regular as I could, so as to preserve the character of a continued curve.' This, it must be admitted, was a very rough way of deducing a table from original observations." I do not agree with Mr. Brown if he intends to imply that this graphic process is necessarily a very rough way of deducing a graduated table from the original facts.

widoers, this may be very satisfactorily don by calling the former "husbands".

In the ordinary mortality tabl we hav only two colums, l_x the number living at age x, and d_x the number dying between the ages x and $x+1$, the numbers in the latter colum being cald decrements. In a combined marryge and mortality tabl we require three decrement colums; for a bachelor may pas out ov observation as such, either by deth or by marryge, and we therefore require two colums shoing the numbers which pas out ov the clas in these two ways, and a third colum for the number ov marryd men dying. By way ov distinction from the decrement colums I propose to cal the others, which sho the total numbers ov bachelors and marryd men living or ded, "agregat" colums. It wil be convenient to hav distinctiv symbols to denote the figurs in each ov the colums ov the tabl; and I propose the folloing:—

b_x = the total number ov bachelors who hav either ataind the age x or dyd previosly;

$(bl)_x$ = the number ov bachelors living at the age x;

$(bd)_x$ = the number ov bachelors dying in the year ov age x to $x+1$.

The corresponding symbols for the marryd, ar m_x, $(ml)_x$, $(md)_x$. The analogy ov these symbols to the familiar l_x and d_x is obvios; and it wil be notist that b_x+m_x is constant. Lastly, I propose the symbol $(bm)_x$ to denote the number ov bachelors who marry in the year ov age x to $x+1$. No special symbols ar required for the numbers ov bachelors and marryd who are ded at a givn age, but these wil be suficiently represented by the differences $b_x-(bl)_x$ and $m_x-(ml)_x$.

If we introduce the element ov interest ov money, the figurs in the tabl wil enabl us to calculate for a bachelor ov any asumed age, x, the values ov such benefits as the folloing:—

(1) An anuity payabl so long as he shal continue alive and unmarryd; which we wil denote by $(ba)_x$.

(2) An asurance payabl on his deth unmarryd; $(b\mathrm{A})_x$.

(3) An endowment payabl on his marryge; $(bm\mathrm{E})_x$.

For a bachelor ov the first age in the tabl, say k, we shal be abl to calculate also the values ov the folloing benefits:—

(4) A postponed anuity to comence on his marryge and continue til his deth, $(bma)_k$;—for we kno the probability ov his being alive and marryd at any age.

(5) An asurance payabl on his deth after having marryd,

$(bmA)_k$; for we kno the probability ov his dying in any year ov age having previosly marryd.

We coud not calculate the values ov these latter benefits for bachelors ov any other age, such as $k+n$; for the numbers in the colums relating to marryd men, $(ml)_x$ and $(md)_x$, include those who marry before the age $k+n$, and they therefore do not giv us, or enabl us to find by any simple calculation, the probability ov a bachelor ov the age $k+n$ being alive and marryd after the laps ov a givn number ov years, or ov his dying marryd in a givn year ov life. The colums in the tabl therefore relating to the marryd ar litl more than matters ov curiosity. If the bachelors experienst the same rate ov mortality as the marryd, the abov benefit (4) coud be found by subtracting the benefit (1) from the ordinary anuity, and (5) coud be found by subtracting (2) from the ordinary asurance. But, as we shal see, the rate ov mortality among the bachelors is, under the age ov 45, much hevvier than among the marryd, and we therefore cannot proceed in this way except for ages not les than 45.

In practis the colums $b_x-(bl)_x$, b_x, m_x, $m_x-(ml)_x$, wil be ov no advantage to us, and I shal therefore adopt the folloing simpler form ov tabl.

Combined Marryge and Mortality Tabl.

Age.	Numbers Living.		Decrements caused by		
	Bachelors.	Marryd.	Bachelors Dying.	Bachelors Marrying.	Marryd Dying.
x	$(bl)_x$	$(ml)_x$	$(bd)_x$	$(bm)_x$	$(md)_x$
(1)	(2)	(3)	(4)	(5)	(6)

So far as I kno, ther ar very few sources from which we can obtain figurs to fil into our colums. Mr. Huie, in his work on the Valuation ov Widows' Funds, givs tabls relating to the Scoolmasters ov Scotland and the Ministers ov the Church ov Scotland, which sho the number ataining each age unmarryd, from 20 onwards, and the number who marry in the next year;—in other words, he gives the figurs that ar required for our colums $(bl)_x$ and $(bm)_x$. He asumes that the mortality among both bachelors and marryd agrees with that ov the Carlisle Tabl. Mr. Meikle, in his report on the Widows' Fund ov the Faculty ov Advocats as at 15 May 1877, givs a tabl from which the folloing is an extract.

(The bachelors ar asumed to be subject to the mortality ov the $H^{M(5)}$ Table.)

Tabl shoing the Number ov Bachelors Dying and the Number Marrying in each year.

Age.	Number entering upon each Year ov Life.	Number Dying [unmarryd].	Number Marrying.	Total Casualty.	Number remaining Alive and Unmarryd at end ov Year.
21	10,000	96	253	349	9,651
22	9,651	99	358	457	9,194
23	9,194	98	431	529	8,665
24	8,665	94	462	556	8,109
25	8,109	85	461	546	7,563
26	7,563	76	478	554	7,009
27	7,009	70	462	532	6,477
28	6,477	63	451	514	5,963
29	5,963	56	431	487	5,476
30	5,476	50	421	471	5,005

It is much to be regretted that neither Mr. Huie nor Mr. Meikle has givn the original statistics from which his figurs ar deduced.

In the Report ov Mes. Brown, Hardy, and Smith, on the Madras Military Fund (Laytons, 1863), ther ar many tabls givn, shoing the combined efect not only ov marryge and mortality but also in som cases ov withdrawal and retirement. These commonly contain colums corresponding to our $(bl)_x$, and $(bd)_x+(bm)_x$, but the component parts ov the latter ar not givn.

The annual reports ov the Registrar-General and the Census Reports, together furnish the means ov computing with considerabl accuracy the rate ov marryge among the general population; but the results so obtaind coud not, I think, be safely adopted in the calculations that actuarys hav to make for the purpos ov determining the proper rates ov premium for insurances agenst issue, or valuing reversionary interests in entaild estates, or estimating the financial position ov a Widos' Fund. The best statistics with which I am aquainted, on which such calculations can be based, ar containd in Mr. Day's paper *On the Statistics ov Marryges among the familys ov the Peerage* (*J.I.A.* x, 181); and from these I hav obtaind values ov the probability ov marryge, which hav enabld me to construct my Combined Marryge and Mortality Tabl. Before proceeding further, I cannot refrain from expressing the opinion that the great value ov the papers Mr. Day submitted to the Institute in former years, renders it a matter ov regret that he has ceast to contribute to its proceedings, and in particular that he has never completed (or, at al events, never publisht the results ov) his investigation referd to in his paper in the 12th volume ov the *Jurnl*, p. 185, *On the Statistics ov Second Marryges among the familys ov the Peerage.*

The folloing is an extract from the tabl on pages 186,7 in Mr. Day's abov-mentiond paper:—

TABL A.—*Peerage Familys* (*Bachelors*).

Age (x).	Completed the Age (x).	Dyd between the ages x and $x+1$ [Unmarryd].	Existing on 31 Dec. 1855, between the ages x and $x+1$ (Bachelors).	Marryd between the ages x and $x+1$.	Age (x).	Completed the Age (x).	Dyd between the ages x and $x+1$ [Unmarryd].	Existing on 31 Dec. 1855, between the ages x and $x+1$ (Bachelors).	Marryd between the ages x and $x+1$.
15	2,721	9	31	1	55	140	2	6	2
16	2,680	17	31		56	130	5	9	1
17	2,632	15	29	2	57	115	2	3	1
18	2,586	17	23	4	58	109	4	5	
19	2,542	14	36	18	59	100	3	2	2
20	2,474	20	46	25	60	93	1	2	
21	2,383	41	38	94	61	90	4	3	
22	2,210	25	29	105	62	83	4	2	
23	2,051	18	27	114	63	77	3	2	
24	1,892	17	29	120	64	72	4	2	
25	1,726	21	24	127	65	66	1	4	
26	1,554	23	30	111	66	61	3	2	
27	1,390	11	15	122	67	56	5	3	
28	1,242	14	24	99	68	48	2	4	
29	1,105	14	15	74	69	42	2	2	
30	1,002	18	19	73	70	38	2	3	
31	892	6	16	55	71	33	...	4	
32	815	5	14	58	72	29	3	2	
33	738	7	15	55	73	24	1		
34	661	5	14	48	74	23	2	2	
35	594	3	8	35	75	19	2	2	
36	548	10	10	33	76	15	...	1	
37	495	6	10	27	77	14	2		
38	452	5	10	22	78	12	...	1	
39	415	4	9	18	79	11	1		
40	384	6	7	23	80	10			
41	348	3	9	11	81	10	...	2	
42	325	4	10	15	82	8	1	1	
43	296	3	12	6	83	6	1		
44	275	2	8	8	84	5	1		
45	257	1	10	6	85	4	...	1	
46	240	2	5	6	86	3	...	2	
47	227	3	6	6	87	1			
48	212	2	10	2	88	1			
49	198	1	3	2	89	1			
50	192	2	8	1	90	1			
51	181	1	5	4	91	1			
52	171	...	6		92	1			
53	165	4	8	3	93	1		1	
54	150	5	4	1					
						42,974	445	736	1,540

From these figurs we lern that out of 2,721 bachelors who ataind the age ov 15, 9 dyd and 1 marryd before ataining the age ov 16, while 31 wer stil alive and unmarryd and under the age ov 16 at the date, 31 December 1855, at which the observation terminated. As is usual in investigations ov this kind, we asume that the 31 wer on the average under observation for six months each between the ages ov 15 and 16, so that what may be cald the *efectiv number under observation*, was 2,721 − 15·5 = 2,705·5. We proceed then upon the asumption that out ov 2,705·5 bachelors alive at the beginning ov the year 9 dyd and 1 marryd in the cours ov the year, leaving 2,695·5 alive and unmarryd at the age ov 16. This is equivalent to asuming that the 31 woud be found, if they coud be observd, to marry and dy until they ataind the age ov 16, at the same rate as the 2,690 who ar under observation for the whole ov the year ov age. If we adopt a radix ov 10,000 alive at the age ov 15, we conclude from Mr. Day's statistics that it is probabl that 33 ov these wil dy within a year and 4 marry, so that the total decrement wil be 37, leaving 9,963 alive and unmarryd at the age ov 16. The survivers out ov these at successiv ages being found by a similar process, we get the figurs containd in the folloing tabl B:—

Tabl B.—*Peerage Familys (Bachelors).—Shoing the Unajusted Numbers Marrying, Dying Unmarryd, and remaining Alive and Unmarryd at each Age, out ov* 10,000 *Bachelors Alive at* 15.

Age.	Remain.	Dy Unmarryd.	Marry.	Exit.	Age.	Remain.	Dy Unmarryd.	Marry.	Exit.
x	$(bl)_x$	$(bd)_x$	$(bm)_x$	$(bd)_x+(bm)_x$	x	$(bl)_x$	$(bd)_x$	$(bm)_x$	$(bd)_x+(bm)_x$
15	10,000	33	4	37	38	2,412	27	119	146
16	9,963	64	0	64	39	2,266	22	99	121
17	9,899	57	8	65	40	2,145	34	130	164
18	9,834	65	15	80	41	1,981	17	63	80
19	9,754	54	69	123	42	1,901	24	89	113
20	9,631	79	98	177	43	1,788	18	37	55
21	9,454	164	376	540	44	1,733	13	51	64
22	8,914	102	426	528	45	1,669	7	40	47
23	8,386	74	469	543	46	1,622	14	41	55
24	7,843	71	501	572	47	1,567	21	42	63
25	7,271	89	539	628	48	1,504	14	14	28
26	6,643	99	479	578	49	1,476	8	15	23
27	6,065	48	535	583	50	1,453	15	8	23
28	5,482	62	441	503	51	1,430	8	32	40
29	4,979	64	336	400	52	1,390	0	0	0
30	4,579	83	337	420	53	1,390	35	26	61
31	4,159	28	259	287	54	1,329	45	9	54
32	3,872	24	278	302	55	1,275	19	19	38
33	3,570	34	269	303	56	1,237	49	10	59
34	3,267	25	240	265	57	1,178	21	10	31
35	3,002	15	178	193	58	1,147	43	0	43
36	2,809	52	171	223	59	1,104	33	22	55
37	2,586	32	142	174	60	1,049			

These figurs, it will be observd, ar wholly unajusted, and therefore exhibit very considerabl irregularities, which render them, as they stand, altogether unsuitabl for practical use. In order to graduate them we must, ov cours, deal separatly with the probabilitys ov marryge and deth. Take, for example, the age 25: the tabl informs us that out ov 7,271 bachelors who atain the age ov 25, 539 marry and 89 dy unmarryd within a year, or before ataining the age ov 26. Hence the probability ov a bachelor ov 25 marrying within a year is $539 \div 7{,}271$, or $\cdot 0741$, and the probability ov a bachelor ov 25 dying unmarryd within a year is $89 \div 7{,}271$, or $\cdot 0122$. We get, ov cours, the same result from Mr. Day's original figurs: 1,726 atain the age ov 25; and deducting 12 (half the "existing"), we get 1,714 as the efectiv number under observation, ov whom 127 marry and 21 dy unmarryd before ataining 26: then $127 \div 1{,}714 = \cdot 0741$, and $21 \div 1{,}714 = \cdot 0122$, as before. Mr. Day calculates the value ov a different function: he deducts from the 1,726 not only 12, half the existing, but also 10·5, half the number dying between ages 25 and 26; then he cals $1{,}726 - 12 - 10\cdot 5 = 1{,}703\cdot 5$, the "number who miht contract marryge" between 25 and 26, and gets $127 \div 1{,}703\cdot 5 = \cdot 07455$ as the "annual marryge rate" at the age 25. This ratio may be more fully described as the annual marryge rate *among the bachelors who do not dy in the year.* In the same way, if from the 1,726 we deduct 12, half the existing, and 63·5, half the number marrying between the ages 25 and 26, we get $1{,}726 - 12 - 63\cdot 5 = 1{,}650\cdot 5$ as the efectiv number exposed to the risk ov deth; and then, $21 \div 1{,}650\cdot 5 = \cdot 01272$, is the annual deth rate *among the bachelors who do not marry in the year.*

We may employ in our calculations either ov the abov described ratios relating to the marryges; and ther is also a third, which is in som respects preferabl to both ov them, namely, the force ov marryge, which is analogos to the familiar force ov mortality. In order to find the aproximat value ov the force ov marryge at the midl ov any year ov age, we hav to divide the number marrying in the year by the number under observation in the midl ov the year. We hav, therefore, in getting our divisor, to subtract not only half the dying in the year but also half the marrying in the year; and it wil be observd that the divisor is the same in calculating the force ov mortality in the midl ov the year. Thus, the force ov marryge at the age $25\frac{1}{2}$ is $127 \div (1{,}726 - 12 - 10\cdot 5 - 63\cdot 5) = 127 \div 1{,}640 = \cdot 07744$; and the force ov mortality at the same age is 21 ÷ 1,640 = ·01280. In conection with this subject, it may be useful to quote the remark of the late Mr. S. Brown (*J.I.A.* xi, 15). "It is convenient to hav the percentage rates ov mortality, marryge, &c., computed upon 100 living in the midl ov the year ov age, by deducting from the living under observation half ov al who enterd or left from any caus, and ascertaining the proportion ov deths, &c., thereto. The number left woud then be common to al, when several events, such as mortality, withdrawal, retirement, or marryge, hav to be compared together. The usual plan is to compare them with 100 entering upon a year ov life, which requires a different initial number in each clas ov facts." It wil be seen that the percentages calculated in the manner which Mr. Brown adopts, ar 100 times the forces ov mortality, marryge, &c., at the midl ov the year ov age under consideration.

Whichever ov the ratios abov described is uzed by an author, he shoud describe it clearly, so that ther may be no dout on the point in the minds ov his readers, and he shoud carefully avoid anything that is likely to confuse it with either ov the other ratios. This is not always suficiently atended to. Mr. Day, having added together the numbers in his tabl in quinquennial groops, gets what he cals "annual marryge rate, quinquennial groops"; and then on page 190 ov his paper he cals this same quantity the probability ov marrying in a year, which seems to me an inaccurat description ov it. Mr. Brown also, in his paper abov referd to, compares Mr. Day's figurs with his oen, without noticing that they ar calculated on a different principl, and that one or other ov the two sets ov figurs o't to be ajusted before they are properly comparabl. It is to be notist that, in consequence ov the very complete way in which Mr. Day has givn his original facts, and the very clear way in which he has stated his conclusions from them, we hav somwhat unusual facilitys for verifying his calculations and determining his exact meaning. It is much to be wisht that al future riters on similar subjects wil follo his exampl in this respect.

As it conduces to clearnes ov ideas to hav a separat symbol for each distinct ratio, I propose the folloing :—

$(bmq)_x$ = the probability ov marrying within a year,

$(bdq)_x$ = the probability ov dying unmarryd within a year;

$(bmr)_x$ = the annual marryge rate among the bachelors who do not dy in the year;

$(bdr)_x$ = the annual deth rate among the bachelors who do not marry in the year;

$(bm\mu)_{x+\frac{1}{2}}$ = the force ov marryge in the midl ov the year ov age x to $x+1$;

$(bd\mu)_{x+\frac{1}{2}}$ = the force ov mortality at the same age.

For brevity, I wil denote these varios quantitys by q_1, q_2, r_1, r_2, μ_1, μ_2. Then we hav the formulas

$$q_1=(bmq)_x=(bm)_x \div (bl)_x\,,$$
$$q_2=(bdq)_x=(bd)_x \div (bl)_x\,;$$
$$r_1=(bmr)_x=(bm)_x \div \{(bl)_x-\tfrac{1}{2}(bd)_x\}\,,$$
$$r_2=(bdr)_x=(bd)_x \div \{(bl)_x-\tfrac{1}{2}(bm)_x\}\,;$$
$$\mu_1=(bm\mu)_{x+\frac{1}{2}}=(bm)_x \div \{(bl)_x-\tfrac{1}{2}(bm)_x-\tfrac{1}{2}(bd)_x\}\,,$$
$$\mu_2=(bd\mu)_{x+\frac{1}{2}}=(bd)_x \div \{(bl)_x-\tfrac{1}{2}(bm)_x-\tfrac{1}{2}(bd)_x\}\,.$$

With reference to the notation it is to be observd that q_x ordinarily denotes the probability that a person ov the age x wil dy within a year, and also the annual rate ov mortality to which persons ov the age x ar subject, these quantitys being identical when we hav to deal only with the operation ov mortality. But, when we hav to deal with the combined operation ov marryge and mortality, they influence each other in such a way that the probabilitys ov marryge and deth ar no longer identical with the marryge and deth rates. We hav therefore to introduce a fresh symbol, and on the whole it seemd to me more convenient to adapt the q symbol to the former quantity rather than the latter.

I also propose the symbol $(bp)_x$ for the probability that a bachelor ov the age x will be alive and stil unmarryd at the end ov a year, so that

$$(bp)_x=(bl)_{x+1} \div (bl)_x\,.$$

From the manner in which our combined marryge and mortality tabl is formd, we see that

$$(bl)_x=(bm)_x+(bd)_x+(bl)_{x+1}\,.$$

Hence $$(bmq)_x+(bdq)_x+(bp)_x=1\,.$$

This equation is analogos to the familiar one, $q_x+p_x=1$, and expresses that it is certain that a bachelor ov the age x, wil either marry within a year, or dy unmarryd within the year, or be alive and unmarryd at the end ov the year.

In practis the problem we hav to deal with offen is: Givn a ratio relating to the marryges and another relating to the deths, to find the number of bachelors remaining alive and unmarryd at the end ov any year out ov a givn number who comence the year. For brevity, put l insted ov $(bl)_x$, and ${}^1l$ insted ov $(bl)_{x+1}$.

Then, if we hav q_1 and q_2, the marryges in the year ar q_1l, and the deths q_2l; and therefore

$${}^1l=l\,(1-q_1-q_2).$$

It is very rarely, however, that q_2, the probability ov dying unmarryd in a year, wil be givn. We shal more commonly have q_1, the probability ov marrying within a year, and r_2, the annual deth rate to which the bachelors ar subject. In this case the number ov marryges is, as before, q_1l. Asuming these to be distributed uniformly over the year, the bachelors who marry in the year ar together exposed to the risk ov deth for $\frac{1}{2}q_1l$ years. Therefore the efectiv number ov bachelors exposed to the risk ov deth for a year, is $l-\frac{1}{2}q_1l=l(1-\frac{1}{2}q_1)$; and the deths among these ar $l(1-\frac{1}{2}q_1)r_2$. Therefore

$${}^1l=l(1-q_1-r_2+\tfrac{1}{2}q_1r_2).$$

Next supose that r_1 and r_2 ar givn, and let y be the number who marry and z the number who dy in the year. Then the efectiv number ov bachelors exposed to the risk ov marryge is $l-\frac{1}{2}z$, and the number ov marryges among these wil be $(l-\frac{1}{2}z)r_1$. But the number ov marryges being by supposition y, we hav the equation

$$(l-\tfrac{1}{2}z)r_1=y.$$

Similarly, the efectiv number ov bachelors exposed to the risk ov deth during the year, is $l-\frac{1}{2}y$; and the number ov deths consequently $(l-\frac{1}{2}y)r_2$, which givs us the second equation,

$$(l-\tfrac{1}{2}y)r_2=z.$$

Solving these two equations, we get

$$y=\frac{lr_1(1-\frac{1}{2}r_2)}{1-\frac{1}{4}r_1r_2},\qquad z=\frac{lr_2(1-\frac{1}{2}r_1)}{1-\frac{1}{4}r_1r_2}.$$

Therefore

$${}^1l=l\left(1-\frac{r_1+r_2-r_1r_2}{1-\frac{1}{4}r_1r_2}\right)=l\,\frac{(1-r_1)(1-r_2)-\frac{1}{4}r_1r_2}{1-\frac{1}{4}r_1r_2}.$$

Lastly, supose that we kno μ_1 and μ_2; and let y and z, as before, denote the numbers ov marryges and deths. Then asuming, as usual, the deths and marryges to be uniformly distributed over the year, the number ov bachelors remaining alive and unmarryd in the midl ov the year wil be $l-\frac{1}{2}(y+z)$. Hence the force ov marryge in the midl ov the year wil be $y \div \{l-\frac{1}{2}(y+z)\}$; but this is givn $=\mu_1$, so that we get the equation

$$y=\mu_1\left(l-\frac{y+z}{2}\right).$$

Similarly we get the equation

$$z=\mu_2\left(l-\frac{y+z}{2}\right),$$

and solving these equations we get

$$y=l\mu_1 \div \left(1+\frac{\mu_1+\mu_2}{2}\right), \quad z=l\mu_2 \div \left(1+\frac{\mu_1+\mu_2}{2}\right),$$

and consequently

$$'l=l.\frac{1-\frac{1}{2}(\mu_1+\mu_2)}{1+\frac{1}{2}(\mu_1+\mu_2)}.$$

Ther is a fixt relation between r_1 and μ_1; so that, if we kno one, we can calculate the other without having any information as to the deths, or without knoing the value ov q_2 or r_2 or μ_2. But when q_1 is givn, we cannot find the value ov r_1 or μ_1 without knoing the deth rate. This is very simply proovd as follos:—

We hav $$r_1=\frac{(bm)_x}{(bl)_x-\frac{1}{2}(bd)_x};$$

and since $$q_1=\frac{(bm)_x}{(bl)_x}, \quad \text{and } q_2=\frac{(bd)_x}{(bl)_x},$$

we get $$r_1=\frac{q_1}{1-\frac{1}{2}q_2}.$$

Similarly, $$\mu_1=\frac{q_1}{1-\frac{1}{2}(q_1+q_2)}.$$

Hence $$\frac{q_1}{r_1}=1-\tfrac{1}{2}q_2, \quad \frac{q_1}{\mu_1}=1-\tfrac{1}{2}(q_1+q_2),$$

and therefore $$\frac{q_1}{r_1}-\frac{q_1}{\mu_1}=\tfrac{1}{2}q_1,$$

or $$\frac{1}{r_1}-\frac{1}{\mu_1}=\tfrac{1}{2}.$$

Hence $$\mu_1=\frac{2r_1}{2-r_1}, \quad \text{and } r_1=\frac{2\mu_1}{2+\mu_1}.$$

It wil be notist that these ar the wel knoen aproximat equations conecting the probability ov dying in a year with the force ov mortality in the midl ov the year; and it miht perhaps hav been foreseen that this woud be the case. The preceding demonstration shos us that r_1 and μ_1 depend only on the marryge rate, but q_1 depends also on the deth rate. This apears also from the analogy ov the quantitys with those relating to the deths; for it is evident without any demonstration that r_2 and μ_2 depend only on the deth rate, but q_2 depends also on the marryge rate.

It is to be observd that our formulas are only aproximat and not exact, in consequence ov our having asumed that the deths and marryges ar distributed uniformly over the year. We shoud get more exact, and in som respects simpler formulas, by employing the differential calculus as Mr. Makeham has don in som analogos problems.

In illustration ov the abov theory, supose that 10,000 bachelors ov the age 21, among whom the probability ov marrying in a year is ·0253, ar subject to the $H^{M(5)}$ rate ov mortality ($q_{21}=\cdot 009657$). Then 253 wil marry in the year; and, asuming their marryges to be uniformly distributed over the year, they wil be under observation as bachelors for only six months each on the average: the efectiv number ov bachelors at risk for a year wil therefore be 10,000−126·5=9,873·5, and the deths among these wil be 95; (for 9,873·5 × ·009657=95·349). Uzing the symbols abov explaind, if r_2 be the annual deth rate among the bachelors, the number, $(bd)_x$, dying unmarryd during the year ov age x to $x+1$, wil be equal to $r_2\{(bl)_x-\frac{1}{2}(bm)_x\}$. It wil be found that Mr. Meikle, in his tabl from which an extract is givn on p. 410, calculates the number ov deths by the formula $r_2(bl)_x$. On the other hand, Mr. Huie calculates the deths by the formula $r_2\{(bl)_x-(bm)_x\}$: see p. 33 of his book. The latter therefore makes the deths too few, and the former too many.

The questions here considerd as to the combination ov the probabilitys ov marryge and deth ar, to a great extent, the same as those considerd by Mr. Samot in his paper (p. 288) *On the probabilitys which ocur in the question ov Invalidity.* He puts p_x for the probability ov living for a year, and i_x for the probability ov becoming incapabl in a year: hence, l_x being the number living at the beginning of the year, the number who becom incapabl in the year is $l_x i_x$. These wil on the average be each under observation for half-a-year after becoming incapabl; and asuming that they ar subject to the same deth rate $q_x (=1-p_x)$ as the helthy, the number of them who dy in the year is $\frac{1}{2}l_x i_x(1-p_x)$; and the probability ov first becoming incapabl and then dying in the year is therefore $\frac{1}{2}i_x(1-p_x)$. The efectiv number who are under observation during the year as capabl is $l_x-\frac{1}{2}l_x i_x$, and the number therefore who dy while capabl is $l_x(1-\frac{1}{2}i_x)(1-p_x)$; and the probability ov dying while capabl in the course of the year is $(1-\frac{1}{2}i_x)(1-p_x)$. The number who becom incapabl but do not dy, is $l_x i_x-\frac{1}{2}l_x i_x(1-p_x)=\frac{1}{2}l_x i_x(1+p_x)$; and the probability ov being alive but incapabl at the end of the year is therefore $\frac{1}{2}i_x(1+p_x)$. Lastly, the number who do not becom incapabl and do not dy in the year, is

$$l_x(1-i_x)-l_x(1-\tfrac{1}{2}i_x)(1-p_x)=l_x(p_x-\tfrac{1}{2}i_x-\tfrac{1}{2}i_x p_x),$$

and the probability ov neither dying nor becoming incapabl, that is to say, ov being alive and capabl at the end ov the year, is $p_x-\frac{1}{2}i_x(1+p_x)$. These formulas ar identical with Mr. Samot's; and the reasoning by which I hav obtaind them, altho very much shorter than his, seems quite as satisfactory. It wil be observd that he asumes that the incapabl are subject to the same rate ov mortality as the capabl, which is very unlikely to be the case. If we supose the incapabl to be subject to a different deth rate q', the number becoming incapabl in the year is unalterd, namely, $l_x i_x$; but the number ov these who dy in the year is now $\frac{1}{2}l_x i_x q'$, and the probability ov becoming incapabl and then dying in the year is $\frac{1}{2}i_x q'$ or $\frac{1}{2}i_x(1-p')$, supose, and the probability ov being alive but incapabl at the end ov the year is $\frac{1}{2}i_x(1+p')$. The other probabilitys, ov being alive and capabl at the end ov the year, and ov dying while capabl in the year, remain unalterd; and these ar entirely analogos to the probabilitys we hav had to deal with, namely, those ov being alive and unmarryd at the end ov the year, and ov dying while unmarryd in the year.

Altho Mr. Samot's formulas ar quite corect, I thought it not unlikely that the authors whom he criticizes, Heym and Wiegand, had not really faln into the error ov principl he atributes to them; but Mr. Samot has kindly sent me extracts from their papers, which hav satisfyd me that his description ov their reasoning is corect.

The function which I hav delt with, is the probability ov marrying in a year. This has facilitated the comparison ov my results with those ov Huie and Meikle, but subsequent consideration has led me to believ that it woud probably be better to make use ov one ov the other ratios, namely, either the marryge rate among bachelors who do not dy, or the force ov marryge. These ratios ar, as we hav seen, independent ov the mortality, whereas the probability ov marrying in a year depends partly on the rate ov mortality. It is quite conceivabl that ther may be two bodys ov men, such that the force ov marryge is the same in both, while one body is subject to a much hevvier deth rate. In this case, the probability ov marrying in a year wil not be the same in the two bodys; and it follos that, in comparing results deduced from different observations, the use ov the probabilitys ov marrying may lead to inaccurat conclusions that woud be avoided by employing either ov the other ratios.

The ajusted probabilitys which I hav deduced from Mr. Day's statistics, by the proces described in the note apended to this paper, ar givn in colum (7) ov the folloing Tabl C:—

TABL C.—*Probability ov Marrying in a Year,* $(bmq)_x$.

Age.	HUIE.		MEIKLE.			SPRAGUE.	Age.
	Scool-masters.	Clergymen.	Clergymen.	Advocats.	Peerage Familys.	Peerage Familys.	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
15	...	...	...	...	...	·0001	15
6	...	...	...	...	...	·0004	6
7	...	...	...	...	...	·0009	7
8	...	...	...	...	...	·0020	8
9	...	...	...	...	...	·0050	9
20	·01625	·00125	...	·01792	·01024	·0100	20
1	·01904	·00242	·05123	·02530	·03199	·0300	1
2	·02701	·00407	·08379	·03709	·04403	·0500	2
3	·04275	·01132	·11492	·04691	·05612	·0600	3
4	·04880	·01567	·13676	·05333	·06500	·0700	4
5	·05270	·02435	·15634	·05680	·07251	·0760	5
6	·05772	·03849	·14831	·06322	·07676	·0775	6
7	·06420	·04891	·14531	·06586	·07786	·0773	7
8	·07277	·05521	·13751	·06955	·07743	·0770	8
9	·08312	·06328	·13356	·07219	·07595	·0760	9
30	·08472	·06625	·12735	·07682	·07350	·0750	30
1	·07624	·07006	·12326	·08053	·07213	·0735	1
2	·07694	·07171	·11379	·08053	·07147	·0715	2
3	·08092	·07288	·10738	·08158	·06891	·0692	3
4	·08068	·07379	·09784	·08308	·06741	·0660	4
5	·08270	·07736	·09337	·08255	·06447	·0623	5
6	·08358	·07800	·08117	·07826	·06041	·0585	6
7	·08322	·07551	·07948	·07923	·05620	·0547	7
8	·07714	·07792	·07473	·07507	·05383	·0509	8
9	·07370	·08019	·07001	·06800	·04875	·0472	9

TABL C—(*continued*).

Age.	HUIE.		MEIKLE.			SPRAGUE.	Age.
	Scool-masters.	Clergymen.	Clergymen.	Advocats.	Peerage Familys.	Peerage Familys.	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
40	·06347	·07967	·06077	·06138	·04546	·0435	40
1	·05956	·06805	·05927	·05446	·04102	·0399	1
2	·06024	·05834	·05100	·04571	·03664	·0363	2
3	·05473	·04879	·04291	·03700	·03220	·0328	3
4	·03999	·04596	·03591	·03254	·02918	·0294	4
5	·02829	·04391	·03453	·02911	·02531	·0261	5
6	·02480	·04026	·02827	·02685	·02194	·0232	6
7	·02262	·03923	·02384	·02465	·01842	·0208	7
8	·01947	·03996	·02095	·02251	·01555	·0188	8
9	·01784	·04159	·01834	·02043	·01379	·0171	9
50	·01724	·04168	·01573	·01841	·01100	·0158	50
1	·01657	·03987	·01340	·01643	·00821	·0146	1
2	·01585	·03437	·01143	·01447	·00573	·0134	2
3	·01420	·03102	·01070	·01257	·00356	·0122	3
4	·01287	·02643	·00819	·01067	·00176	·0111	4
5	·01235	·02380	·00688	·00881	·00015	·0100	5
6	·01226	·02216	·00575	·00695	...	·0090	6
7	·01168	·02172	·00478	·00489	...	·0081	7
8	·01106	·01881	·00395	·00223	...	·0073	8
9	·01094	·01612	·00324	·00037	...	·0065	9
60	·00922	·01528	·00263	...	...	·0057	60
1	·00680	·01495	...	...	...	·0051	1
2	·00591	·01341	...	...	...	·0046	2
3	·00493	·01228	...	...	...	·0040	3
4	·00386	·01034	...	...	...	·0035	4
5	·00337	·00884	...	...	...	·0030	5
6	·00212	·00716	...	...	...	·0026	6
7	...	·00602	...	...	...	·0023	7
8	...	·00555	...	...	...	·0021	8
9	...	·00418	...	...	...	·0019	9
70	...	...	...	...	...	·0018	70
1	...	...	...	...	...	·0016	1
2	...	...	...	...	...	·0015	2
3	...	...	...	...	...	·0013	3
4	...	...	...	...	...	·0012	4
5	...	...	...	...	...	·0010	5
6	...	...	...	...	...	·0009	6
7	...	...	...	...	...	·0007	7
8	...	...	...	...	...	·0006	8
9	...	...	...	...	...	·0004	9
80	...	...	...	...	...	·0003	80
1	...	...	...	...	...	·0001	1

It wil be observd that Mr. Day's statistics do not enabl us to determin the marryge rate at ages abov 60; for altho they sho 912·5 years ov life abov that age, ther is not a singl marryge. If, however, our tabl is to be ov practical servis, it becoms necesary to aproximate to the marryge rate at hiher ages. I hav don this by means ov the figurs givn in the folloing tabl, which is extracted from Mr. Day's paper* (see vol. x, p. 190).

Age.	Probability ov Marrying in a Year.			
	Bachelors.		Widoers.	
	General Population.	Peerage Familys.	General Population.	Peerage Familys.
15–	·00464	·00192	..	...
20–	·11209	·04213	·30766	·17858
25–	·12209	·07700	·35791	·14910
30–	·07851	·07137	·28627	·12485
35–	·04558	·05473	·20313	·10460
40–	·02798	·03947	·14075	·09954
45–	·01448	·01977	·08858	·07715
50–	·00705	·01074	·05711	·05907
55–	·00349	·01046	·03201	·04246
60–	·00152	...	·01745	·03268
65–	·00146	..	·00862	·02173
70–	·00031	...	·00316	·01884
75–	·00059	. .	·00100	·00825
80–	·00000	...	·00067	·00702

On examining these figurs carefully we see that the marryge rate among the bachelors ov the peerage familys bears relations, more or les regular, to the marryge rates among the other classes ov men to which the tabl relates. This wil be more evident from Section 1 ov Tabl D (page 424), which shos the ratio ov the unajusted marryge rate among the peerage bachelors to the other

* The folloing tabl, calculated by Mr. A. H. Morgan, ov the *Scottish Equitable* Life Assurance Society, relates to marryges ov bachelors in Scotland, and may be compared with Mr. Day's figurs (*J.I.A.* viii, 130 and 133). The Scotch Census Report ov 1871 givs (vol. ii, p. 137) the number ov bachelors alive at each quinquennium ov age on 3rd April 1871; but, for the purpos ov strict comparison with the marryges in the year 1871, we require the number alive in the midl ov the year. The Registrar-General for England (40th Annual Report, p. 107) estimates that the total male population ov Scotland increast from 1,603,143 on 3rd April to 1,607,276 on 30th June, being in the ratio ov 1 to 1·002578; and it was asumed that the number ov bachelors at each quinquennium ov age increast in the same proportion. Mr. Day dos not seem to hav tho't it worth while to make this corection for increas ov population. The marryges ov bachelors, as stated in col. 3, ar givn in the 17th Detaild Annual Report ov the Scotch Registrar-General, p. 25.

In one respect the Scotch statistics apear greatly preferabl to the English, for in 10 cases only out of 21,050 wer the ages ov bachelors who marryd not stated, whereas in the English statistics used by Mr. Day, the number ov bachelors whos ages wer not stated was 84,088 out ov 132,643. Comparing the two tables it wil be seen that under the age ov 25 the marryge rate is considerably loer in Scotland than in England, from 25 to 30 it is practically the same, and abov 30 it is uniformly hiher than in England. If therefore the figurs ar to be rates. We see that the ratios in colum (2), with a singl exception, increas with the age; and that the ratios in colum (4) first increas as the age increases, and then decreas rapidly. We also see that the regularity ov the progression is interrupted when we reach the ages 55—, in consequence ov the probability ov marryge among the peerage bachelors at those ages, ·01046, being very nearly the same as at the ages 50—, whereas, if it had folloed the same law as at the previos ages, it woud hav been considerably les. It is clear that in ajusting the marryge probability we must reduce the probability shoen by the observations at the ages 55—; but, considering the purposes our tabl is to serv, it is important that the probabilitys, as a whole, shoud not be too lo, but shoud er, if at al, on the side ov exces. I therefore, while reducing the probability shoen by the facts at the ages 55—, and taking ·01 as the ajusted probability at the age ov 55, increas the probability at the ages 50—, and also asign a magnitude to the probability at

trusted, marryges, as a rule, take place at later ages in Scotland than in England. It is to be notist, however, that Mr. Day asumes that the 84,088, whos ages ar not stated, may be distributed over the varios ages in the same proportion as the 48,555 whos ages ar stated. If this is not the case, but the former ar on the average older than the latter, this may perhaps be suficient by itself to explain the difference between his figurs and these.

Ages.	Number of Bachelors		Force ov Marryge at the midl age ov the quinquennium.	Midl Age.
	Living in Scotland on 30th June 1871.	Who married in Scotland in 1871.		
(1)	(2)	(3)	(4)	(5)
15–	167,185	681	·004073	$17\frac{1}{2}$
20–	116,816	9,152	·078345	$22\frac{1}{2}$
25–	56,632	6,947	·122669	$27\frac{1}{2}$
30–	28,670	2,515	·087722	$32\frac{1}{2}$
35–	16,520	974	·058959	$37\frac{1}{2}$
40–	13,113	444	·033860	$42\frac{1}{2}$
45–	9,055	186	·020541	$47\frac{1}{2}$
50–	7,901	80	·010125	$52\frac{1}{2}$
55–	5,718	36	·006296	$57\frac{1}{2}$
60–	5,286	17	·003216	$62\frac{1}{2}$
65–	3,481	6	·001724	$67\frac{1}{2}$
70 & upwards	4,961	2	·000403	..
Total .	435,338	21,040	...	
Not stated	. . . . .	10	...	..
...	...	21,050	...	..

ages 60 onwards, which fully compensates for the reduction at ages 55—.

TABL D.

Ages. (1)	Ratio ov Probability ov Marryge among the Peerage Bachelors to that among the — Bachelors ov the General Population. (2)	Widoers ov the General Population. (3)	Widoers ov the Peerage Familys. (4)
Section (1).—Unajusted Probabilitys.			
15–	·414	...	...
20–	·376	·137	·236
25–	·631	·215	·516
30–	·909	·249	·571
35–	1·201	·269	·523
40–	1·411	·280	·397
45–	1·365	·223	·256
50–	1·523	·188	·182
55–	2·997	·327	·247
Section (2).—Probabilitys (relating to the Peerage Bachelors) ajusted by Mr. Sprague.			
15–	·362	..	...
20–	·393	·143	·246
25–	·629	·214	·515
30–	·905	·248	·569
35–	1·201	·269	·523
40–	1·300	·258	·365
45–	1·464	·240	·275
50–	1·903	·235	·228
55–	2·344	·256	·193
60–	3·013	·262	·140
65–	1·630	·276	·110
70–	4·774	·468	·079
75–	1·220	·720	·087

The second section ov Tabl D shos the relations which my ajusted probabilitys for the Peerage Bachelors bear to the unajusted probabilitys for the three other classes ov lives; and the general progression ov the figurs apears to me fully to justify the values ov the probabilitys I hav asigned at the hiher ages. The next tabl (E) compares the number ov actual marryges with the number expected acording to my ajustment, and shos how very closely on the whole I hav folloed the original facts.

TABL E.—*Peerage Bachelors.—Comparison ov the actual marryges with the expected acording to Mr. Sprague's ajustment.*

Ages.	Efectiv Number who miht Marry.	Marryges.	
		Actual.	Expected.
15–19	13,086	25	21
20–24	10,925·5	458	459
25–29	6,963	533	534
30–34	4,069	289	291
35–39	2,480·5	135	137
40–44	1,605	63	59
45–49	1,117	22	24
50–54	843·5	9	11·42
55–59	581·5	6	4·84
60–64	409·5	0	1·89
65–69	265·5	0	·66
70–74	141·5	0	·22
75–	118·5	0	·06
TOTAL	42,606·0	1,540	1,544·09

I hav givn in Tabl C, alongside ov my oen ajusted figurs, the probabilitys ov marryge acording to Mr. Huie and Mr. Meikle. Mr. Huie givs in his Tabl 1 the number ov scoolmasters ataining each age unmarryd and the number who marry in the next year, and from these ar calculated the ratios in colum (2). Similarly the ratios in colum (3) ar obtaind from Mr. Huie's Tabl 2, which relates to clergymen ov the Church ov Scotland. It dos not apear that any method ov graduation has been employd by Mr. Huie. The figurs in colums (4), (5), (6), ar taken from the tabl givn by Mr. Meikle on page 8 ov his *Report on the Widos' Fund ov the Faculty ov Advocats,* "shoing the probabilitys ov bachelors marrying during each year ov life", for "advocats", "clergymen ov Church ov Scotland", and "peers". Mr. Meikle dos not state where his figurs relating to the peers ar obtaind from, but I think we shal not be rong in asuming that they wer got by som process ov graduation from the figurs in Mr. Day's paper in the 10th volume ov the *Jurnl ov the Institute,* and I hav acordingly alterd the hedding from peers to "Peerage familys". Mr. Meikle's peerage probabilitys on the whole agree very wel both with Mr. Day's figurs and with my ajustment ov them, but it apears to me that for ages abov 45 Mr. Meikle has made the probability ov marryge sensibly too smal. The original facts as to the marryges of the advocats not being givn, I hav no remarks to make upon Mr. Meikle's probabilitys in colum (5), except that the graduation, altho probably quite satisfactory for his purposes, woud admit ov improovment, as the figurs do not proceed with so much regularity as coud be desired.

Passing now to the consideration ov the probabilitys ov marryge among the clergymen, it is to be notist that, altho Mr. Huie's and Mr. Meikle's probabilitys relate to the same clas ov lives, namely, clergymen ov the Church ov Scotland, their results are very widely different. Acording to Mr. Meikle's tabl, 25 is the age at which the probability ov marryge for a clergyman is greatest, and the probability at that age is more than dubl the probability ov marryge for either advocats or peerage familys, in which the maximum probability ocurs at the ages 34 and 27 respectivly. Acording to Mr. Huie, on the contrary, the probability ov marryge ov clergymen is greatest at about the age ov 38; under that age, the probability ov marryge is considerably les than among the advocats; and under the age ov about 30, les also than among the peerage familys. At more advanst ages, Mr. Huie makes the probability ov marryge among the clergymen very considerably greater than it is among either advocats or peerage familys. Comparing the two sets ov results, Mr. Huie's apears to be in itself much the les probable law ov marryge; and an examination ov the description he has givn of his proces ov finding the probabilitys, has led me to form the opinion that his figurs ar not trustworthy. On page 31 ov his book he givs an acount ov the materials from which he calculated his probabilitys: he says that he was abl to ascertain the age ataind ov the existing members ov the fund (ministers marryd or unmarryd); also the ages at deth ov the members who had dyd in the previos 18 years. He also knew the ages at first marryge ov the existing members. As regards the age at marryge ov the deceast members ov the 18 years, if they marryd after joining the fund, the date ov marryge was knoen; and the age at deth being also knoen, the age at marryge coud be ascertaind. He did not ascertain the age at marryge ov those deceast members who marryd before joining the fund, but the member's age at admission being knoen, it was asumed that his age at marryge was the average age at marryge ov the other members who marryd before the age at which he was admitted. Having these facts, and "folloing the same method as is ordinarily pursued in framing a mortality tabl", he found "how many members livd *thro'* each year ov age in a condition of bachelorhood". He then asumed that the marryges which wer found to hav taken place in each year ov age wer distributed at equal intervals thro'out the year, and that consequently the number ov bachelors living thro' the year was a mean between those under observation at the beginning ov the year, and those who remaind bachelors at the end. He thus obtaind a tabl ov percentages for ages 20 to 69, from which the folloing is extracted:—

Age.	Number Marrying out ov 100 living through the Year.
20	·1234
21	·2423
22	·4068
23	1·1385
24	1·5746
25	2·4417
26	3·8654
27	4·9102
28	5·2383
29	6·7985

Full as this description apears to be, I hav not been abl to understand exactly from it how the original facts wer treated. In particular, I do not see why Mr. Huie shoud wish to kno the average age at marryge ov those deceast members who wer marryd when they joind the fund. They wer marryd when they came under observation, and they shoud therefore hav been altogether discarded in an enquiry as to the rate ov marryge ov bachelors. Those existing members also shoud hav been discarded who wer marryd before admission to the fund; but it dos not apear that this was don. Agen, I do not gather from the description how those members who dyd unmarryd were treated. If each ov them was suposed to hav livd thro' half ov the year in which he dyd, Mr. Huie's percentage wil be symbolically exprest by $100(bm)_x:\{(bl)_x-\frac{1}{2}(bd)_x-\frac{1}{2}(bm)_x\}$. Mr· Huie proceeds: "As the figurs in the abov tabl represent the number who marry out ov 100 bachelors living thro' each year, it was necesary, before constructing a tabl ov marryges, to find the proportion which marryd out ov 100 beginning the year." He dos this by multiplying the percentages in his tabl by the ratio (deduced from the Carlisle Tabl) $\frac{l_x+l_{x+1}}{2l_x}=1-\frac{1}{2}\frac{d_x}{l_x}=1-\frac{1}{2}q_x$. In this step he has omitted to take into acount a fact which is very clearly bro't out by his description, namely, that his tabl givs the number marrying out ov 100 living thro' the year *in a state of bachelorhood;* and it follos that his process dos not giv the corect value ov the quantity he is seeking,—the number marryd out ov 100 beginning the year. On the whole, I consider Mr. Meikle's results ar much more likely to be corect than Mr. Huie's, and I hav no confidence in the figurs givn by the latter either as to the scoolmasters or the ministers.

Having got the ajusted probabilitys ov marryge, as set out in colum (7) ov Tabl C, it was next necesary to decide as to the proper rate ov mortality to uze in the calculations. If Mr. Day's figurs had been suficiently large, I miht hav graduated the probability ov a bachelor dying unmarryd in any year ov age; but the results so got woud not hav admitted ov direct comparison with the probabilitys givn by ordinary mortality tabls. I therefore preferd to calculate the annual deth rate to which the bachelors wer subject.

The rate for each age was found by the formula explained in an earlier part ov this paper, $(bd)_x : \{(bl)_x - \frac{1}{2}(bm)_x\}$; and having graduated the values by a grafic proces, I got the figurs in the folloing tabl:—

TABL F.—*Peerage Bachelors.—Ajusted Annual Deth Rate,* q_x.

Age x	q_x	Age x	q_x	Age x	q_x	Age x	q_x
15	·0040	26	·0128	37	·0109	48	·0136
16	·0045	27	·0126	38	·0110	49	·0144
17	·0052	28	·0122	39	·0111	50	·0152
18	·0064	29	·0117	40	·0112	51	·0160
19	·0082	30	·0113	41	·0113	52	·0170
20	·0096	31	·0110	42	·0115	53	·0182
21	·0108	32	·0109	43	·0117	54	·0198
22	·0117	33	·0109	44	·0119	55	·0216
23	·0123	34	·0108	45	·0122	56	·0225
24	·0128	35	·0108	46	·0126	57	·0243
25	·0130	36	·0108	47	·0130		

The numbers remaining under observation at hiher ages ar so few that no reliance can be placed on results deduced from them.

As our combined marryge and mortality tabl is to sho the deths among the marryd as wel as among the bachelors, it is necesary to kno the rate of mortality to which the former ar subject. It would obviosly be unsafe to asume that they ar both subject to the same rate ov mortality. Mr. Meikle remarks on this subject in his report abov mentiond, "It has been frequently observd that the mortality ov unmarryd males is considerably greater than among the marryd. In the Registrar-General's Reports for Scotland this has been frequently referd to, and Dr. Farr has cald atention to the fact in one ov the recent reports ov the Registrar-General for England and Wales." He concludes, from an examination ov his statistics, that the mortality among the unmarryd advocats agrees more nearly with that ov the $H^{M(5)}$ Tabl than with any other, and he says that the mortality among the marryd advocats coincides nearly with the H^M Tabl. I have been led by this remark to compare the mortality among the Peerage Bachelors with that ov the $H^{M(5)}$ Tabl; and I hav givn in colum (8) of Tabl G, the expected deths acording to that tabl, the actual deths being given in colum (10). We see from this that up to the age of 40 the rate ov mortality is greatly in excess of the $H^{M(5)}$ rate, but becoms lihter from 40 onwards. This result is not such as to encourage us to asume that the rate ov mortality among the marryd wil agree with that ov any standard tabl; and it is clear that our best cours wil be *to ascertain, if possibl,* the rate ov mortality among marryd men belonging to the same clas ov society as the bachelors, namely, the Peerage familys.

Mr. Day's paper in the 10th volume ov the *Jurnl,* which has hitherto supplyd us with statistics, givs only figurs relating to the marryges and deths ov bachelors and widoers, and givs no information as to the deths ov the marryd men generally; and I am not aware ov any colection ov statistics that relates exclusivly to marryd men. The valuabl paper, however, by Messrs. Bailey and Day in the 9th volume ov the *Jurnl* (p. 305) givs us ful and exact information as to the mortality among the Peerage males (including both bachelors and marryd); and a comparison of the figurs containd in the two papers wil enabl us to deduce indirectly the mortality among the marryd. As a first step it is desirabl to compare the results ov the two sets of statistics (which, it will be rememberd, do not relate to precisely the same set ov persons), and see how far they ar consistent with each other. This is don in the folloing Tabl G :—

Tabl G.

Ages.	Males.					Bachelors.					Ages.
	At Risk.	Expected Deths.		Actual Deths.	Rate ov Mortality.	At Risk.	Expected Deths.		Actual Deths.	Rate ov Mortality.	
		Berridge.	Sprague.				HM(5) Tabl.	Sprague.			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
15–19	10,965·0	79·3	74·0	72	·0066	13,073·5	61·3	73·5	72	·0055	15–19
20–24	10,433·5	110·6	115·7	115	·0110	10,696·5	105·5	121·1	121	·0113	20–24
25–29	9,877·5	96·6	97·8	98	·0099	6,696·5	66·8	83·9	83	·0124	25–29
30–34	9,354·0	81·1	80·3	80	·0086	3,924·5	36·1	43·2	41	·0105	30–34
35–39	8,797·5	78·6		78	·0089	2,413·0	25·6	26·3	28	·0116	35–39
40–44	8,065·5	86·7		88	·0109	1,573·5	18·1	18·1	18	·0114	40–44
45–49	7,215·0	96·6		99	·0137	1,106·0	16·0	14·5	9	·0081	45–49
50–54	6,325·5	103·7	101·9	102	·0161	839·0	15·6	14·4	12	·0143	50–54
55–59	5,323·5	107·7	106·8	101	·0190	578·5	14·5	...	16	·0277	55–59
60–64	4,406·5	119·7	122·9	125	·0284	409·5	...	...	16	·0391	60–64
65–69	3,392·5	142·0	150·3	156	·0460	265·5	...	...	13	·0490	65–69
70–74	2,338·5	168·3	...	160	·0684	141·5	...	...	8	·0565	70–74
75–79	1,377·0	148·3	...	144	·1046	69·0	...	...	5	·0725	75–79
80–84	668·0	100·6	...	105	·1571	37·5	...	...	3	·0800	80–84
85–89	174·5	...	..	50	·2866	8·5	...	...	0	...	85–89
90–94	14·5	...	...	6	·4138	3·5	...	...	0	...	90–94
Total	88,728·5	...	...	1,579	...	41,836·0	...	...	445	...	Total

It wil be seen that at the ages 15-19 ther is a remarkabl discrepancy between the two sets ov results, which I am quite unabl to explain. The years ov life among the bachelors ar many more than among the males, (see colums 2 and 7), but the number ov deths is identical (colums 5 and 9), so that the rate ov mortality among the bachelors, as shoen in colum (10), is considerably les than that (in colum 6) among the males. This is, ov cours, a wholly inadmissibl result; for, at the ages in question, the number ov the marryd is so very trifling as to produce no apreciabl efect upon the rate ov mortality, either in the way ov diminishing or increasing it. I therefore decided to discard the probabilitys derived from the experience ov bachelors at these ages, and to adopt those deduced from the observations on the males. Passing on now to the ages 20 to 44, the rate ov mortality among the bachelors is thro'out greater than among the males, the difference first increasing rapidly, and then gradually diminishing with advancing age. This indicates a stil greater difference between the rate ov mortality among the bachelors and that among the marryd, and is consistent with the results given by other observations. At the ages 45–54 the rate ov mortality among the bachelors is very much loer than that among the males; and at 55–64 very much hiher. These results do not seem intrinsically probabl, for I can see no reason why the rate ov mortality among bachelors ov 45 to 54 shoud be les than among marryd men. The numbers ov bachelors at ages 45 and upwards being comparatively smal, I hav com to the conclusion that the abov divergencys must be treated as accidental irregularitys. I hav acordingly asumed that from the age ov 45 onwards the rate ov mortality among the bachelors is the same as among the males, and therefor the same as among the marryd.

The general conclusion to be drawn from our comparison is that, with the modifications pointed out, the two sets ov observations ar consistent with each other; and we may, with fair prospect ov succes, proceed to combine them so as to bring out the mortality among the marryd. For this purpos, we must first graduate the probabilitys ov deth, as found from Messrs. Bailey and Day's statistics. This has alreddy been done by Mr. Berridge in the 12th volume ov the *Jurnl*, and my first idea was to uze his results. Upon careful examination, however, I found his graduation not entirely satisfactory, and I was abl to adopt it only for the ages 34–49 inclusiv. At other ages up to 80, I hav substituted a graduation ov my oen; but from the age ov 81 onwards I hav taken the probabilitys of the H^{M} Tabl. At these advanst ages the statistics of Messrs. Bailey and Day seem to giv probabilitys ov deth which ar greatly too large. Possibly this may arise from their manipulation ov the original facts, hinted at in the folloing remark (vol. ix, p. 309): "As was unavoidabl, the numbers at the oldest ages wer somwhat arbitrarily delt with."

The figurs in colums 3 and 4 ov Table G sho how far Mr. Berridge's ajustment and mine respectivly agree with the original facts. It wil be notist that Mr. Berridge's graduation givs too hih a mortality at 15–19, and too lo a mortality at 20–24. In consequence ov this, the hiher mortality that prevails about the ages ov 23 and 24, as compared with those that precede and those that follo, is not so faithfully exhibited by Mr. Berridge's ajustment as by mine. He shos a maximum rate ov mortality, ·0108, at the age of 22, and a minimum of ·0085 at 34, whereas my table shos a considerably larger maximum, ·0116, at 23 and practically the same minimum, ·0084, at 33.

The folloing Tabl H (pp. 432-3) shos the unajusted probabilitys of deth among the Peerage males, and the ajustments by Mr. Berridge and myself; also my ajusted probabilitys for the Peerage Bachelors; and, lastly, the probabilitys of deth among the marryd, as actually employd in the calculations. At the ages 15 to 21 inclusiv, these ar the same as among the bachelors, also from age 45 to the end ov the tabl; at the remaining ages, 22 to 44 inclusiv, they wer obtaind by the proces now to be explaind.

Having got the ajusted rates ov mortality for the males and the bachelors, I supose, in Tabl I (p. 434), that 100,000 males ar under observation at the age ov 15, at which age they are al bachelors, and that they ar subject to the deth rate shoen in colum 4 of Tabl H; I thus get the number ov males remaining alive at each age, and the number dying in the folloing year, as givn in colums (3) and (4) ov Tabl I. Then suposing these bachelors to marry at the rate shoen in Tabl C, and those who remain unmarryd to dy at the rate shoen in colum 5 ov Tabl H, I get the efectiv numbers ov bachelors "at risk" and the number who dy unmarryd, as givn in colums (9) and (10) of Tabl I.

The probabilitys uzed in the calculations ar those denoted by q_1 and r_2 in the erlier part ov this paper, p. 415, and the number ov bachelors remaining after a year out ov l alive at any age can be found by the formula ${}^1l = l(1 - q_1 - r_2 + \frac{1}{2}q_1r_2)$.

Tabl H.—*Probability ov dying in a year, Peerage Males, Bachelors, and Marryd Men.*

Age.	Males. Unajusted.	Males. Ajusted by Mr. Berridge.	Males. Ajusted by Mr. Sprague.	Bachelors. Ajusted by Mr. Sprague.	Marryd.	Age.
(1)	(2)	(3)	(4)	(5)	(6)	(7)
15	·00408	·00473	·0050	Same as the males.	·0050	15
16	·00725	·00564	·0057		·0057	16
17	·00638	·00743	·0065		·0065	17
18	·00733	·00876	·0077		·0077	18
19	·00783	·00970	·0089		·0089	19
20	·00837	·01031	·0100		·0100	20
21	·01656	·01065	·0108		·0108	21
22	·00965	·01079	·0114	·0117	·0075	22
23	·01163	·01076	·0116	·0123	·0067	23
24	·00885	·01061	·0113	·0128	·0045	24
25	·01139	·01037	·0108	·0130	·0039	25
26	·01255	·01009	·0102	·0128	·0042	26
27	·00760	·00978	·0098	·0126	·0048	27
28	·00868	·00947	·0095	·0122	·0056	28
29	·00930	·00919	·0092	·0117	·0062	29
30	·01250	·00894	·0089	·0113	·0065	30
31	·00585	·00873	·0086	·0110	·0066	31
32	·01018	·00859	·0085	·0109	·0067	32
33	·00757	·00851	·0084	·0109	·0068	33
34	·00654	·00850	Mr. Berridge's ajustment adopted.	·0108	·0072	34
35	·00549	·00856		·0108	·0074	35
36	·01390	·00869		·0108	·0077	36
37	·00629	·00889		·0109	·0080	37
38	·00987	·00915		·0110	·0084	38
39	·00879	·00948		·0111	·0089	39
40	·01367	·00985		·0112	·0094	40
41	·00976	·01027		·0113	·0099	41
42	·01364	·01074		·0115	·0105	42
43	·01072	·01124		·0117	·0111	43
44	·00647	·01176		·0119	·0117	44
45	·01450	·01231		†	·01231	45
46	·01491	·01287		...	·01287	46
47	·01314	·01344		...	·01344	47
48	·01353	·01403		...	·01403	48
49	·01239	·01462		...	·01462	49
50	·01567	·01522	·0151	...	·0151	50
51	·01382	·01582	·0156	...	·0156	51
52	·01186	·01645	·0161	...	·0161	52
53	·01711	·01710	·0167	...	·0167	53
54	·02268	·01779	·0174	...	·0174	54
55	·01656	·01852	·0182	...	·0182	55
56	·02161	·01932	·0191	...	·0191	56
57	·01037	·02021	·0201	...	·0201	57
58	·02633	·02122	·0211	...	·0211	58
59	·02041	·02236	·0222	...	·0222	59
60	·02001	·02369	·0235	..	·0235	60
61	·02818	·02523	·0253	...	·0253	61
62	·02717	·02703	·0276	...	·0276	62

† From this point to the end ov the tabl, the probability ov deth for both bachelors and marryd is the same as for the males.

TABL H—(*continued*).

Age.	Males.			Bachelors.	Marryd.	Age.
	Unajusted.	Ajusted by Mr. Berridge.	Ajusted by Mr. Sprague.	Ajusted by Mr. Sprague.		
(1)	(2)	(3)	(4)	(5)	(6)	(7)
63	·03764	·02914	·0303	...	·0303	63
64	·02986	·03161	·0335	...	·0335	64
65	·04313	·03450	·0370	...	·0370	65
66	·03758	·03787	·0407	...	·0407	66
67	·03983	·04179	·0446	...	·0446	67
68	·05338	·04634	·0487	...	·0487	68
69	·05892	·05159	·0530	...	·0530	69
70	·06539	·05762	·0575	...	·0575	70
71	·06330	·06452	·0625	...	·0625	71
72	·05405	·07237	·0684	...	·0684	72
73	·07457	·08127	·0751	...	·0751	73
74	·08952	·09129	·0827	...	·0827	74
75	·11127	·09858	·0911	...	·0911	75
76	·09354	·10413	·1004	...	·1004	76
77	·09783	·10890	·1104	...	·1104	77
78	·11688	·11381	·1211	...	·1211	78
79	·10501	·11978	·1325	...	·1325	79
80	·10811	·12769	·1446	...	·1446	80
81	·17846	·13838	*	*	·1580*	81
82	·12500	·15261	...	...	·1714	82
83	·19535	·17108	...	...	·1859	83
84	·22353	·19432	...	...	·1989	84
85	·23077	·22271	...	...	·2099	85
86	·27369	·25644	...	...	·2197	86
87	·33846	·29544	...	...	·2312	87
88	·41026	·33940	...	...	·2393	88
89	·30000	·38775	...	...	·2532	89
90	·42857	·43967	...	...	·2795	90
91	·25000	·49415	...	...	·3127	91
92	·66666	·54999	...	...	·3513	92
93	...	...	...	...	·4158	93
94	...	...	...	...	·5073	94
95	...	...	...	...	·6370	95
96	...	...	...	...	·8163	96
97	...	...	...	...	1·0000	97

* From this point to the end ov the tabl, the probabilitys are those ov the H^M Tabl.

Subtracting now the number at any age in colum (9) from that in colum (3), we get the number ov marryd men "at risk", as givn in colum (12); and similarly the number ov deths of the marryd, as given in colum (13), is got by taking the difference ov the numbers in colums (4) and (10). Then dividing the numbers in (13) by those in (12), we get the annual deth rate to which the marryd ar liabl, as shoen in colum (6) ov Tabl H. From the way in which the figurs ar got it is clear that, when the deth

TABL I.

	MALES.			BACHELORS.							MARRYD.		
Age.	Chance ov Dying in a Year.	Remain.	Dy.	Chance ov Marry-ing in a Year.	Annual Rate ov Mor-tality.	Remain.	Marry.	At Risk. (7) − ½(8)	Dy Un-marryd.	Exit. (8)+(10)	At Risk. (3)−(9)	Dy. (4)−(10)	Age.
x	q_x	l_x	d_x	$(bmq)_x$	$(bdr)_x$	$(bl)_x$	$(bm)_x$	...	$(bd)_x$	...	...	$(md)_x$	x
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12	(13)	(14)
15	·0050	100,000	500	·0001	·0050	100,000	10	99995	500	510	5	0	15
16	·0057	99,500	567	·0004	·0057	99,490	40	99470	567	607	30	0	16
17	·0065	98,933	643	·0009	·0065	98,883	89	98838·5	642	731	94·5	1	17
18	·0077	98,290	757	·0020	·0077	98,152	196	98054	755	951	236	2	18
19	·0089	97,533	868	·0050	·0089	97,201	486	96958	863	1,349	575	5	19
20	·0100	96,665	967	·0100	·0100	95,852	959	95372·5	954	1,913	1292·5	13	20
21	·0108	95,698	1,034	·0300	·0108	93,939	2,818	92530	999	3,817	3168	35	21
22	·0114	94,664	1,079	·0500	·0117	90,122	4,506	87869	1,028	5,534	6795	51	22
23	·0116	93,585	1,086	·0600	·0123	84,588	5,075	82050·5	1,009	6,084	11534·5	77	23
24	·0113	92,499	1,045	·0700	·0128	78,504	5,495	75756·5	970	6,465	16742·5	75	24
25	·0108	91,454	988	·0760	·0130	72,039	5,475	69301·5	901	6,376	22152·5	87	25

rate among the bachelors is the same as among the males, that is to say, from age 15 to 21 inclusiv and from age 45 to the end ov the tabl, it must also be the same among the marryd. If, however, we divide the deths shoen at the erly ages in colum (13) by the numbers in colum (12), we shal not get the deth rate corectly; and in order to do so, we must calculate the numbers ov males and bachelors dying with greater accuracy, retaining several decimal places.

When we examin the deth rates ov the marryd which we hav thus got, we find that, at certain ages from 22 onwards, they differ in a very markt manner from the deth rates among the bachelors or the males generally. Whereas in the males ther is a maximum mortality at age 24, and in the bachelors at 25, it ocurs at 21 among the marryd. Whereas ther is a minimum mortality among the males at 34 and among the bachelors at 35, among the marryd it fals at the much erlier age ov 25. At this age the deth rate among the marryd is les than one third ov that among the bachelors: as the age increases up to 35, the deth rate among the bachelors gradually gros les, while that among the marryd increases; from 35 to 45, both deth rates increas, but that ov the marryd more rapidly; and from the age of 45 onwards, the rate is the same in the two classes.

Comparing the deth rate among the marryd with that among recently insured lives, as givn in my paper (p. 229), we see that at the ages ov 25 and 26 the two rates ar almost identical. It resulted from my investigations that the deth rate in the first insurance year was very nearly constant from the age of 20 to 40, being ·005 and ·0055 respectivly at those ages, and having its minimum (somthing les than ·0045) between the ages ov 25 and 30. It thus apears that marryge is as efficacios in reducing the deth rate as the medical examination made when a life is proposed for insurance. In both cases ther is a proces ov selection whereby the helthy ar separated from the unhelthy. If we consider the general body ov yung unmarryd men at, say 25, ther wil be among them a certain number in a bad state ov helth, in varios stages ov consumption, for instance. As a rule, these yung men wil have no desire to marry; and, if they wisht, they woud probably be considerd undesirabl husbands and be rejected. Those who marry being thus in good helth, and those in bad helth remaining bachelors, we hav a very lo rate ov mortality among newly marryd men, and a hih rate among those remaining unmarryd. The vitality ov the bachelors as a whole is reduced by the withdrawal from the body ov a large number ov helthy lives; and the deth rate among them is consequently increast.

The newly marryd men ar, as a whole, select lives, and the mortality among them is therefore liht. As in the case ov insured lives, the efect ov selection wil gradually wear off, and we hav seen that from the age of 45 the mortality among the marryd is no lihter than among the bachelors. It woud be interesting to extract from the records ov the peerage, statistics as to the mortality among newly marryd men. These woud sho how far the conclusions here drawn ar corect, and woud thro much liht on the question ov the wearing out ov the efect ov selection among insured lives. In these, we hav a disturbing element in the lapses and surrenders, which certainly caus the rate ov mortality to increas faster than it otherwise woud; but among the marryd ov the peerage familys we shoud hav no such disturbing caus. Most valuabl statistics bearing on the same question miht also be obtaind from the records ov the National Det offis; and it is to be hoped that, when next the Government publish their experience as to the grant ov life anuitys, they wil sho separatly the mortality among nominees ov each age at entry.

I obtaind then in the way abov described the figurs givn in colums (2), (4), (5), (6) of Tabl J, the Combined Marryge and Mortality Tabl; the radix, however, being increast to 1,000,000.

TABLE J.—*Combined Marryge and Mortality Tabl.*

Age.	Numbers Living.		Decrements caused by			Number ov Bachelors Marrying whos Marryges ar fruitful.
	Bachelors.	Marryd.	Bachelors Dying.	Bachelors Marrying.	Marryd Dying.	
x	$(bl)_x$	$(ml)_x$	$(bd)_x$	$(bm)_x$	$(md)_x$	$(bfm)_x$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
15	1,000,000	0	5,000	100	0	...
16	994,900	100	5,670	398	2	...
17	988,832	496	6,425	890	6	...
18	981,517	1,380	7,550	1,963	18	...
19	972,004	3,325	8,629	4,860	51	...
20	958,515	8,134	9,537	9,585	129	...
21	939,393	17,590	9,993	28,182	342	...
22	901,218	45,430	10,281	45,061	511	...
23	845,876	89,980	10,092	50,753	764	...
24	785,031	139,969	9,697	54,952	755	...
25	720,382	194,166	9,009	54,749	868	...
26	656,624	248,047	8,079	50,888	1,149	...
27	597,657	297,786	7,239	46,199	1,536	...
28	544,219	342,449	6,384	41,905	2,039	...
29	495,930	382,315	5,582	37,691	2,498	...
30	452,657	417,508	4,923	33,949	2,821	...
31	413,785	448,636	4,384	30,413	3,033	...
32	378,988	476,016	3,983	27,098	3,285	...
33	347,907	499,829	3,661	24,075	3,460	...
34	320,171	520,444	3,344	21,131	3,801	...
35	295,696	537,774	3,094	18,422	4,041	...
36	274,180	552,155	2,874	16,040	4,307	...
37	255,266	563,888	2,706	13,963	4,576	...
38	238,597	573,275	2,558	12,145	4,871	...
39	223,894	580,549	2,427	10,568	5,199	...
40	210,899	585,918	2,311	9,174	5,538	6,569
41	199,414	589,554	2,208	7,957	5,895	5,697
42	189,249	591,616	2,137	6,870	6,249	4,912
43	180,242	592,237	2,074	5,912	6,607	4,227
44	172,256	591,542	2,020	5,064	6,962	3,616
45	165,172	589,644	2,007	4,311	7,285	3,078
46	158,854	586,670	2,021	3,685	7,574	2,627
47	153,148	582,781	2,037	3,185	7,854	2,271
48	147,926	578,112	2,056	2,781	8,130	1,980
49	143,089	572,763	2,074	2,447	8,392	1,742
50	138,568	566,818	2,076	2,189	8,575	1,554
51	134,303	560,432	2,080	1,961	8,758	1,383
52	130,262	553,635	2,083	1,746	8,928	1,196
53	126,433	546,453	2,099	1,542	9,138	979
54	122,792	538,857	2,125	1,363	9,389	777
55	119,304	530,831	2,160	1,193	9,672	596
56	115,951	522,352	2,205	1,044	9,987	457
57	112,702	513,409	2,256	913	10,329	365
58	109,533	503,993	2,303	800	10,642	299
59	106,430	494,151	2,355	692	10,978	245
60	103,383	483,865	2,422	589	11,378	197
61	100,372	473,076	2,533	512	11,975	162
62	97,327	461,613	2,680	448	12,747	133

TABL J—(*continued*).

Age.	Numbers Living.		Decrements caused by			Number ov Bachelors Marrying whos Marryges ar fruitful.
	Bachelors.	Marryd.	Bachelors Dying.	Bachelors Marrying.	Marryd Dying.	
x	$(bl)_x$	$(ml)_x$	$(bd)_x$	$(bm)_x$	$(md)_x$	$(bfm)_x$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
63	94,199	449,314	2,848	377	13,620	106
64	90,974	436,071	3,043	318	14,614	83
65	87,613	421,775	3,236	263	15,611	64
66	84,114	406,427	3,419	219	16,546	50
67	80,476	390,100	3,585	185	17,402	39
68	76,706	372,883	3,732	161	18,163	31
69	72,813	354,881	3,856	138	18,813	25
70	68,819	336,206	3,953	124	19,335	20
71	64,742	316,995	4,043	104	19,816	15
72	60,595	297,283	4,141	91	20,338	12
73	56,363	277,036	4,230	73	20,808	9
74	52,060	256,301	4,303	63	21,199	6
75	47,694	235,165	4,342	48	21,426	4
76	43,304	213,787	4,346	39	21,466	3
77	38,919	192,360	4,295	27	21,238	2
78	34,597	171,149	4,188	21	20,727	1
79	30,388	150,443	4,026	12	19,935	...
80	26,350	130,520	3,810	8	18,874	...
81	22,532	111,654	3,561	2	17,645	...
82	18,969	94,011	3,250	...	16,109	...
83	15,719	77,902	2,921	...	14,478	...
84	12,797	63,424	2,545	...	12,614	...
85	10,252	50,810	2,152	...	10,664	...
86	8,100	40,146	1,779	...	8,818	...
87	6,321	31,328	1,462	...	7,244	...
88	4,859	24,084	1,163	...	5,763	...
89	3,697	18,321	936	...	4,639	...
90	2,761	13,682	771	...	3,824	...
91	1,989	9,858	622	...	3,083	...
92	1,367	6,775	480	...	2,380	...
93	887	4,395	369	...	1,827	...
94	518	2,568	263	...	1,303	...
95	255	1,265	162	...	806	...
96	93	459	76	...	375	...
97	17	84	17	...	84	...

The figurs in colum (3) wer obtaind by the formula $(ml)_{x+1} = (ml)_x + (bm)_x - (md)_x$. The number at any age in colum (7) was obtaind by multiplying the number in colum (5) by the corresponding probability in Tabl K (p. 438). Thus $(bfm)_x = (bm)_x (pfm)_x$. These probabilitys wer obtaind by me from an examination ov the statistics ov 339 marryges entered into, at or abov the age ov 40, by bachelors and widoers belonging to the Peerage familys, as explaind in a paper I lately redd before the Royal Society ov Edinburgh.

TABL K.—*Shoing the probability,* (pfm)$_x$, *that a Marryge enterd into by a Man ov the age ov* 40 *or upwards, wil be fruitful.*

x	$(pfm)_x$	x	$(pfm)_x$	x	$(pfm)_x$	x	$(pfm)_x$
40	·716	51	·705	62	·298	73	·118
41	·716	52	·685	63	·280	74	·104
42	·715	53	·635	64	·262	75	·090
43	·715	54	·570	65	·245	76	·076
44	·714	55	·500	66	·228	77	·063
45	·714	56	·438	67	·211	78	·050
46	·713	57	·400	68	·195	79	·037
47	·713	58	·374	69	·179	80	·024
48	·712	59	·354	70	·163	81	·012
49	·712	60	·335	71	·148	82	·000
50	·710	61	·316	72	·133		

I hav calculated by means ov this combined marryge and mortality tabl, the values, at 3 per-cent interest, ov varios benefits, as shoen in Tabl L (pp. 440-1), and I wil now explain the formulas I uzed for the purpos, and proov som interesting relations between the benefits. We naturally comence with $(ba)_x$, the value ov an anuity payabl so long as a bachelor now ov the age x shal continue alive and unmarryd. The probability ov his being alive and unmarryd at the end ov the nth year is clearly $(bl)_{x+n} \div (bl)_x$, and the value ov 1 to be payabl at the end ov the nth year subject to the givn condition, is $\dfrac{(bl)_{x+n}}{(bl)_x} v^n = \dfrac{(bl)_{x+n} v^{x+n}}{(bl)_x v^x}$.

Hence the value ov the required anuity is

$$\frac{(bl)_{x+1} v^{x+1} + (bl)_{x+2} v^{x+2} + \dots\dots\dots}{(bl)_x v^x}.$$

If for brevity we agree to uze Σu_x to denote $u_x + u_{x+1} + u_{x+2} + \dots\dots\dots$ to the end ov life, we hav $(ba)_x = \dfrac{\Sigma\{(bl)_{x+1} v^{x+1}\}}{(bl)_x v^x}$.

Next for an asurance payabl on the deth ov a bachelor unmarryd

$$(bA)_x = \frac{(bd)_x v + (bd)_{x+1} v^2 + \dots\dots\dots}{(bl)_x}$$

$$= \frac{(bd)_x v^{x+1} + (bd)_{x+1} v^{x+2} + \dots\dots\dots}{(bl)_x v^x} = \frac{\Sigma\{(bd)_x v^{x+1}\}}{(bl)_x v^x}.$$

So for an endowment payabl on marryge

$$(bm\mathrm{E})_x = \frac{(bm)_x v + (bm)_{x+1} v^2 + \dots\dots}{(bl)_x}$$

$$= \frac{(bm)_x v^{x+1} + (bm)_{x+1} v^{x+2} + \dots\dots}{(bl)_x v^x} = \frac{\Sigma\{(bm)_x v^{x+1}\}}{(bl)_x v^x}.$$

I calculated, then, the values ov

$$(bl)_x v^x,\ \Sigma\{(bl)_{x+1} v^{x+1}\},\ \Sigma\{(bd)_x v^{x+1}\},\ \Sigma\{(bm)_x v^{x+1}\},$$

which may be conveniently denoted by $(b\mathrm{D})_x$, $(b\mathrm{N})_x$, $(b\mathrm{M})_x$, $(bm\mathrm{EN})_x$; and aranged them in the usual columnar form, as shoen in Tabl M (pp. 442-3), and deduced from them the values ov the three benefits at each age, as givn in Tabl L. The anuity values wer chekt by the relation $(ba)_x = v(bp)_x\{1 + (ba)_{x+1}\}$.

The values ov the three benefits ar conected by a simpl relation. We hav

$$(bl)_x \times (ba)_x = (bl)_{x+1} v + (bl)_{x+2} v^2 + \dots\dots$$
$$(bl)_x \times (b\mathrm{A})_x = (bd)_x v + (bd)_{x+1} v^2 + \dots\dots$$
$$(bl)_x \times (bm\mathrm{E})_x = (bm)_x v + (bm)_{x+1} v^2 + \dots\dots$$

Now for any age z we hav

$$(bl)_z = (bl)_{z+1} + (bd)_z + (bm)_z.$$

Hence by adition we get

$$(bl)_x\{(ba)_x + (b\mathrm{A})_x + (bm\mathrm{E})_x\} = (bl)_x v + (bl)_{x+1} v^2 + \dots\dots$$
$$= (bl)_x v\{1 + (ba)_x\};$$

so that $\quad (ba)_x + (b\mathrm{A})_x + (bm\mathrm{E})_x = v + v(ba)_x$

and $\quad (b\mathrm{A})_x + (bm\mathrm{E})_x = v - (1-v)(ba)_x = \dfrac{1 - i(ba)_x}{1+i} \quad \dots\dots \quad (a).$

The analogy ov this relation to the familiar one between an ordinary asurance and an ordinary anuity, is obvios; and it becoms clear that the relation o't to exist, when we observ that $(ba)_x$ is an anuity to continue so long as a bachelor, x, remains alive and unmarryd, and $(b\mathrm{A})_x + (bm\mathrm{E})_x$ is the value ov an asurance to be paid at the end ov the year in which he shal cease to be a bachelor, either by deth or marryge. It is easy to see that the folloing relation also subsists:—

$$(1+i)\{(b\mathrm{M})_x + (bm\mathrm{EN})_x\} + i(b\mathrm{N})_x = (b\mathrm{D})_x.$$

In order to find the value ov the benefits $(bma)_x$, $(bm\mathrm{A})_x$, we must first find the values ov anuitys and asurances on the lives ov marryd men, which we wil denote by $(ma)_x$ and $(m\mathrm{A})_x$. The calculation ov these presents no peculiarity, but they ar got in the

TABL L.—*Values of certain Benefits depending upon the combined Contingencys ov Marryge and Mortality, calculated at Three per-cent Interest.*

x	Marryd.		Bachelors.				
	$(ma)_x$	$(mA)_x$	$(ba)_x$	$(bA)_x$	$(bmE)_x$	$(bmA)_x$	$(bfmA)_x$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
15	23·204	·29502	12·670	·14572	·45613	·17066	...
6	23·021	·30037	12·117	·14583	·47213	·17665	...
7	22·847	·30543	11·557	·14539	·48887	·18294	...
8	22·686	·31011	10·992	·14433	·50638	·18955	...
9	22·548	·31413	10·433	·14234	·52466	·19651	...
20	22·434	·31747	9·897	·13967	·54293	·20363	...
1	22·340	·32020	9·402	·13664	·56040	·21071	...
2	22·261	·32249	9·094	·13561	·57039	·21603	...
3	22·103	·32711	8·979	·13667	·57267	·21951	...
4	21·918	·33249	8·966	·13882	·57092	·22200	...
25	21·678	·33949	9·063	·14236	·56453	·22317	...
6	21·416	·34712	9·242	·14714	·55455	·22314	...
7	21·151	·35482	9·458	·15299	·54240	·22218	...
8	20·891	·36241	9·699	·15975	·52864	·22042	...
9	20·639	·36975	9·962	·16770	·51302	·21774	...
30	20·391	·37696	10·242	·17691	·49566	·21416	...
1	20·140	·38427	10·540	·18744	·47644	·20962	...
2	19·881	·39182	10·853	·19922	·45554	·20413	...
3	19·616	·39955	11·178	·21208	·43324	·19776	...
4	19·342	·40753	11·510	·22593	·40970	·19054	...
35	19·065	·41557	11·837	·24066	·38545	·18266	...
6	18·784	·42378	12·149	·25604	·36098	·17429	...
7	18·497	·43213	12·440	·27201	·33652	·16552	...
8	18·206	·44061	12·709	·28840	·31231	·15648	...
9	17·911	·44920	12·950	·30514	·28857	·14727	...
40	17·613	·45786	13·160	·32215	·26543	·13797	·09264
1	17·314	·46660	13·335	·33933	·24313	·12871	·08546
2	17·012	·47538	13·473	·35662	·22183	·11960	·07836
3	16·708	·48423	13·571	·37382	·20179	·11078	·07147
4	16·403	·49313	13·626	·39084	·18316	·10238	·06486
45	16·095	·50209	13·637	·40760	·16608	·09449	·05862
6	15·785	·51113	13·605	·42389	·15073	·08724	·05282
7	15·470	·52029	13·535	·43968	·13698	·08061	·04745
8	15·151	·52958	13·433	·45509	·12453	·07449	·04242
9	14·828	·53900	13·304	·47022	·11317	·06878	·03766
50	14·499	·54857	13·150	·48516	·10271	·06341	·03312
1	14·163	·55835	12·974	·50012	·09285	·05823	·02870
2	13·819	·56837	12·778	·51514	·08355	·05323	·02441
3	13·467	·57864	12·560	·53019	·07485	·04845	·02040
4	13·106	·58914	12·321	·54519	·06683	·04394	·01691
55	12·738	·59985	12·061	·56016	·05942	·03969	·01399
6	12·364	·61076	11·782	·57502	·05268	·03575	·01167
7	11·983	·62186	11·486	·58977	·04656	·03209	·00983
8	11·595	·63315	11·173	·60445	·04101	·02870	·00829
9	11·201	·64464	10·843	·61909	·03596	·02555	·00697
60	10·799	·65635	10·498	·63368	·03144	·02267	·00583
1	10·390	·66825	10·137	·64814	·02748	·02011	·00486
2	9·980	·68020	9·768	·66245	·02393	·01776	·00402
3	9·571	·69211	9·395	·67652	·02072	·01559	·00330
4	9·166	·70390	9·020	·69021	·01795	·01369	·00269

TABL L—(*continued*).

x	MARRYD.		BACHELORS.				
	$(ma)_x$	$(mA)_x$	$(ba)_x$	$(bA)_x$	$(bmE)_x$	$(bmA)_x$	$(bfmA)_x$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
65	8·768	·71549	8·647	·70346	·01557	·01202	·00220
6	8·378	·72685	8·277	·71623	·01358	·01061	·00180
7	7·996	·73799	7·910	·72858	·01190	·00941	·00147
8	7·620	·74893	7·548	·74058	·01044	·00834	·00121
9	7·251	·75970	7·190	·75233	·00912	·00736	·00098
70	6·886	·77031	6·836	·76385	·00793	·00646	·00079
1	6·525	·78082	6·484	·77525	·00677	·00557	·00062
2	6·169	·79119	6·136	·78643	·00574	·00476	·00048
3	5·821	·80134	5·794	·79737	·00474	·00397	·00035
4	5·482	·81120	5·461	·80793	·00388	·00327	·00026
75	5·156	·82071	5·140	·81811	·00305	·00260	·00018
6	4·843	·82983	4·831	·82781	·00236	·00202	·00012
7	4·545	·83851	4·537	·83704	·00170	·00147	·00007
8	4·262	·84675	4·256	·84572	·00118	·00103	·00004
9	3·994	·85453	3·991	·85391	·00070	·00062	·00002
80	3·743	·86187	3·741	·86154	·00037	·00033	·00001
1	3·507	·86874	3·506	·86865	·00010	·00009	·00000
2	3·290	·87506	3·290	·87506	...	...	...
3	3·089	·88090	3·089	·88090	...	...	...
4	2·908	·88617	2·908	·88617	...	...	...
85	2·739	·89110	2·739	·89110	...	...	...
6	2·570	·89601	2·570	·89601	...	...	...
7	2·393	·90118	2·393	·90118	...	...	...
8	2·206	·90663	2·206	·90663	...	...	...
9	1·987	·91301	1·987	·91301	..	...	...
90	1·740	·92020	1·740	·92020	..	...	...
1	1·487	·92756	1·487	·92756	..	..	.
2	1·229	·93508	1·229	·93508	..	...	...
3	·951	·94317	·951	·94317	.	...	...
4	·677	·95116	·677	·95116	...	...	...
95	·415	·95878	·415	·95878	..	..	...
6	·178	·96568	·178	·96568	...	...	...
7	·000	·97087	·000	·97087	.	...	...

usual way from the probabilitys containd in colum (6) ov Tabl H, and their values ar givn in colums (2) and (3) ov Tabl L. Then the value ov $(bmA)_x$, an asurance to be payabl on the deth ov a person who is now a bachelor ov the age x, if he shal hav marryd, is got as follos. Out ov $(bl)_x$ bachelors now ov the age x, $(bm)_{x+n}$ marry in the $(n+1)$th year from the present time, or (on the average) at the age ov $x+n+\frac{1}{2}$; and asuming them al to marry in the midl ov the year, the sum to be then paid in order to provide an asurance ov 1 on the deth ov each, wil be $(bm)_{x+n}(mA)_{x+n+\frac{1}{2}}$, the present value ov which is found by multiplying by $v^{n+\frac{1}{2}}$. The total present sum therefore that is necesary to provide for insurances on the deth ov al the bachelors who marry out ov the $(bl)_x$, is

TABL M.—*Commutation Table, Three per-cent Interest.*

x	$(bD)_x$	$(bN)_x$	$(bM)_x$	$(bmEN)_x$	$(bmM)_x$	$(bfmM)_x$
	$(bl)_x v^x$	$\Sigma\{(bl)_{x+1} v^{x+1}\}$	$\Sigma\{(bd)_x v^{x+1}\}$	$\Sigma\{(bm)_x v^{x+1}\}$	$\Sigma\{(bm)_x v^{x+\frac{1}{2}} (mA)_{x+\frac{1}{2}}\}$	$\Sigma\{(bfm)_x v^{x+\frac{1}{2}} (mA)_{x+\frac{1}{2}}\}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
15	641,863	8,132,316	93529·44	292774·3	109541·0	...
6	619,989	7,512,327	90413·61	292712·0	109522·2	...
7	598,260	6,914,067	86983·17	292471·2	109448·1	...
8	576,539	6,337,528	83209·16	291948·4	109284·8	...
9	554,320	5,783,208	78903·50	290828·9	108930·1	...
20	530,708	5,252,500	74125·83	288138·0	108067·4	...
1	504,970	4,747,530	68999·22	282985·6	106399·9	...
2	470,339	4,277,191	63783·94	268277·6	101607·6	...
3	428,599	3,848,592	58574·64	245445·5	94083·6	...
4	386,184	3,462,408	53610·04	220478·3	85732·0	...
25	344,059	3,118,349	48978·71	194232·9	76784·5	...
6	304,473	2,813,876	44801·28	168846·1	67939·6	...
7	269,059	2,544,817	41164·20	145936·9	59779·2	...
8	237,865	2,306,952	38000·19	125744·4	52429·3	...
9	210,446	2,096,506	35291·16	107962·2	45822·4	...
30	186,488	1,910,018	32991·45	92434·0	39938·7	...
1	165,509	1,744,509	31022·31	78854·8	34693·7	...
2	147,175	1,597,334	29319·84	67044·3	30042·7	...
3	131,170	1,466,164	27818·14	56827·6	25940·2	...
4	117,197	1,348,967	26478·05	48015·1	22331·2	...
35	105,085	1,243,882	25289·65	40505·5	19194·7	...
6	94,601	1,149,281	24222·12	34149·3	16487·6	...
7	85,510	1,063,771	23259·38	28776·2	14153·9	...
8	77,598	986,173	22379·32	24235·1	12142·9	...
9	70,695	915,478	21571·62	20400·2	10411·4	...
40	64,653	850,825	20827·61	17160·5	8920·2	5989·185
1	59,351	791,474	20139·79	14430·1	7639·4	5072·091
2	54,685	736,789	19501·77	12130·9	6540·4	4285·191
3	50,566	686,223	18902·25	10203·5	5601·9	3614·165
4	46,918	639,305	18337·35	8593·3	4803·2	3043·162
45	43,678	595,627	17803·18	7254·1	4127·0	2560·310
6	40,783	554,844	17287·91	6147·3	3557·9	2154·014
7	38,174	516,670	16784·16	5228·8	3077·2	1811·256
8	35,798	480,872	16291·21	4458·1	2666·6	1518·489
9	33,619	447,253	15808·15	3804·7	2312·3	1266·237
50	31,608	415,645	15335·06	3246·5	2004·3	1046·919
1	29,743	385,902	14875·30	2761·7	1732·0	853·594
2	28,008	357,894	14428·08	2340·1	1490·9	683·646
3	26,393	331,501	13993·25	1975·6	1278·8	538·336
4	24,886	306,615	13567·84	1663·1	1093·6	420·744
55	23,475	283,140	13149·71	1394·9	931·8	328·514
6	22,151	260,989	12737·08	1167·0	791·8	258·513
7	20,903	240,086	12328·11	973·3	670·7	205·467
8	19,724	220,362	11921·87	808·9	566·0	163·588
9	18,607	201,755	11519·25	669·1	475·3	129·673
60	17,548	184,207	11119·53	551·6	397·8	102·224
1	16,540	167,667	10720·41	454·6	332·5	80·369
2	15,571	152,096	10315·17	372·7	276·5	62·660
3	14,632	137,464	9898·87	303·1	228·1	48·229
4	13,719	123,745	9469·30	246·3	187·8	36·957
65	12,828	110,917	9023·87	199·7	154·2	28·160
6	11,957	98,960	8563·76	162·3	126·9	21·461

TABL M—(*continued*).

x	$(bD)_x$	$(bN)_x$	$(bM)_x$	$(bmEN)_x$	$(bmM)_x$	$(bfmM)_x$
	$(bl)_x v^x$	$\Sigma\{(bl)_{x+1} v^{x+1}\}$	$\Sigma\{(bd)_x v^{x+1}\}$	$\Sigma\{(bm)_x v^{x+1}\}$	$\Sigma\{(bm)_x v^{x+\frac{1}{2}} (mA)_{x+\frac{1}{2}}\}$	$\Sigma\{(bfm)_x v^{x+\frac{1}{2}} (mA)_{x+\frac{1}{2}}\}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
67	11,106	87,854	8091·91	132·1	104·5	16·345
8	10,278	77,576	7611·55	107·3	85·7	12·397
9	9,472	68,104	7126·10	86·4	69·7	9·268
70	8,692	59,412	6639·18	68·9	56·1	6·841
1	7,939	51,473	6154·40	53·7	44·2	4·891
2	7,214	44,259	5673·08	41·4	34·3	3·435
3	6,514	37,745	5194·41	30·9	25·9	2·306
4	5,842	31,903	4719·73	22·7	19·1	1·512
75	5,196	26,707	4250·98	15·8	13·5	·925
6	4,580	22,127	3791·63	10·8	9·3	·545
7	3,997	18,130	3345·36	6·8	5·9	·287
8	3,449	14,681	2917·13	4·1	3·6	·141
9	2,941	11,740	2511·71	2·1	1·8	·054
80	2,476	9,264	2133·40	·9	·8	·017
1	2,056	7,208	1785·82	·2	·2	·002

$$(bm)_x v^{\frac{1}{2}}(mA)_{x+\frac{1}{2}} + (bm)_{x+1} v^{\frac{3}{2}}(mA)_{x+\frac{3}{2}} + \ldots\ldots$$

and the value ov the asurance required is therefore

$$\frac{(bm)_x v^{\frac{1}{2}}(mA)_{x+\frac{1}{2}} + (bm)_{x+1} v^{\frac{3}{2}}(mA)_{x+\frac{3}{2}} + \ldots\ldots}{(bl)_x},$$

$$\text{or } (bmA)_x = \frac{(bm)_x v^{x+\frac{1}{2}}(mA)_{x+\frac{1}{2}} + (bm)_{x+1} v^{x+\frac{3}{2}}(mA)_{x+\frac{3}{2}} + \ldots\ldots}{(bl)_x v^x}$$

$$= \frac{\Sigma\{(bm)_x v^{x+\frac{1}{2}}(mA)_{x+\frac{1}{2}}\}}{(bl)_x v^x}.$$

In precisely the same way we may proov that

$$(bma)_x = \frac{\Sigma\{(bm)_x v^{x+\frac{1}{2}}(ma)_{x+\frac{1}{2}}\}}{(bl)_x v^x}.$$

In order to find the value ov an asurance on the deth ov a person who is now a bachelor, if he shal hav contracted a fruitful marryge, we hav only, in the formula for $(bmA)_x$, to substitute $(bfm)_x$ insted ov $(bm)_x$. Thus,

$$(bfmA)_x = \frac{\Sigma\{(bfm)_x v^{x+\frac{1}{2}}(mA)_{x+\frac{1}{2}}\}}{(bl)_x v^x}.$$

We can establish a simpl relation between $(bma)_x$, $(bmA)_x$, and $(bmE)_x$; for we hav

$$(bl)_x(bmA)_x = (bm)_x v^{\frac{1}{2}}(mA)_{x+\frac{1}{2}} + (bm)_{x+1} v^{\frac{3}{2}}(mA)_{x+\frac{3}{2}} + \ldots\ldots$$

also $(bl)_x(bma)_x = (bm)_x v^{\frac{1}{2}}(ma)_{x+\frac{1}{2}} + (bm)_{x+1}v^{\frac{3}{2}}(ma)_{x+\frac{3}{2}} + \ldots\ldots$

Now, observing that, whatever the age, $(1+i)(mA)+i(ma)=1$,

we get $(bl)_x\{(1+i)(bmA)_x+i(bma)_x\} = (bm)_x v^{\frac{1}{2}} + (bm)_{x+1}v^{\frac{3}{2}} + \ldots\ldots$

$$= (bl)_x\sqrt{1+i}(bmE)_x,$$

whence $$(1+i)(bmA)_x+i(bma)_x = \sqrt{1+i}(bmE)_x \quad . \quad . \quad . \quad (\beta)$$

This equation expresses that an endowment ov $\sqrt{1+i}$ payabl at the end ov the year in which a bachelor shal marry, or ov 1 at the instant ov marryge, wil be suficient to provide an anuity ov i for his life after marryge and an asurance ov $1+i$ on his deth after marryge.

If we now eliminate $(bmE)_x$ from the two equations we hav obtaind, we shal get a third which is worthy ov notis. Multiplying (a) by $\sqrt{(1+i)}$ and adding it to (β), we get

$$\sqrt{(1+i)}(bA)_x + (1+i)(bmA)_x + i(bma)_x = \frac{1-i(ba)_x}{\sqrt{(1+i)}}\ ;$$

whence $$(bA)_x + \sqrt{(1+i)}(bmA)_x = \frac{1-i\{(ba)_x+\sqrt{(1+i)}(bma)_x\}}{1+i}\,.$$

This equation shos us that the asurance $(bA)_x+\sqrt{(1+i)}(bmA)_x$ corresponds to the anuity $(ba)_x+\sqrt{(1+i)}(bma)_x$. This sugests that these quantitys may be the values ov the asurance and the anuity on the life of x independent ov marryge; but I hav not yet been abl to satisfy myself whether this is the case or not.

I calculated then the values of

$$\Sigma\{(bm)_x v^{x+\frac{1}{2}}(mA)_{x+\frac{1}{2}}\},\quad \Sigma\{(bfm)_x v^{x+\frac{1}{2}}(mA)_{x+\frac{1}{2}}\},$$

(which I denote by $(bmM)_x$ and $(bfmM)_x$; *see* colums (6) (7) ov Tabl M) and thence found $(bmA)_x$ and $(bfmA)_x$. I had som douts as to whether it was worth while to print the values in Tabl M, but I decided to do so becaus they wil be useful to any person who may wish to calculate the values ov temporary and deferd benefits depending on marryge as wel as mortality.

One purpos for which the results obtaind in this paper wil be useful is the calculation ov the singl premium for insurance agenst issue to a bachelor ov any age. Insurances ov this kind hav for many years past been granted by a few life offices at what they considerd safe rates of premium, but I am not aware that any aproximatly accurat calculation ov the risk ataching to them has ever before been made. The premiums resulting from my investigations, even after they hav been heavily loaded, wil, I believ, be much loer than those hitherto charged; and whether my results ar acted upon or not, it wil be satisfactory to actuarys to be abl to estimate more accuratly than hitherto the risk of these exceptional transactions.

NOTE ON THE AJUSTMENT OV THE PROBABILITYS OV MARRYGE OR DETH.

In ajusting any series ov numbers, two conditions hav to be observd, (1) that the ajusted series shal proceed regularly without any breaks or jumps, and (2) that it shal on the whole agree with the original series. We can determin whether the ajusted series has suficient regularity, by differencing the numbers and observing whether ther ar any obvios irregularitys in the first and second differences; but a stil better plan is to exhibit those irregularitys to the eye by plotting down the ajusted numbers as the ordinats ov a curv. As regards our second condition, the test we aply must depend on the nature ov the numbers we ar dealing with. When these ar the probabilitys ov dying (or marrying) in a year, the most satisfactory test is to calculate the expected deths (or marryges) acording to the ajusted probabilitys, and compare these with the actual deths (or marryges) for quinquennial or other intervals.

The application ov these principls leads at once to the method ov graduation which I hav adopted in the foregoing paper. I first ov al plot down on cros-ruled paper the ungraduated probabilitys, either for each year or for quinquennial averages, as may be found more convenient, and then draw by hand a curv which, while proceeding with regularity, follos the general progression ov the points. Having got this curv, it is easy to estimate its ordinats for each age, or the ajusted probabilitys. I then calculate by means ov these the expected deths (or marryges) and see how they compare with the actual. If during any quinquennium, or other interval for which the figurs ar compared, the deths (or marryges) acording to the graduated probabilitys are more numeros than the actual, the ordinat ov the curv requires to be reduced, and to be increast if the expected deths (or marryges) are too few. These corections then ar to be aplyd to the curv by hand, taking care stil to preserv due regularity in its figur; and from the corected curv thus obtaind the probabilitys can agen be estimated, thus getting a second aproximation; and the process can be repeated as often as may be tho't desirabl. Comparing the ajusted with the unajusted probabilitys year by year, we shal find that the latter generally proceed very irregularly, and the expected deths (or marryges) at particular ages ar somtimes greater and somtimes les than the actual. When we take quinquennial groops ov ages, we find the irregularitys ov the original facts ar greatly diminisht, especially when the numbers observd ar large; and we shoud aim at obtaining an exact agreement in each quinquennium between the expected and actual deths (or marryges). This, however, especially when the numbers observd ar smal, wil somtimes be found incompatibl with preserving the proper degree ov regularity in the curv; and we must then be satisfyd with obtaining an agreement between the total expected and actual deths (or marryges) when two or three quinquenniums ar combined. The trifling irregularities in the progression from age to age which may stil remain, ar best remoovd by an aplication ov the method ov differences.

DISCUSSION.

The PRESIDENT (Mr. A. H. Bailey) said—I am sure we shall all concur with Mr. Sprague in the regret which he expresses, that Mr. Day should have left off after getting into the middle of a subject which he had made peculiarly his own. The combined probabilities of mortality and marriage enter much into the calculations affecting widows' funds and other similar funds which are becoming increasingly popular. Whether wisely or not wisely, is not now the question. Governments, boards of directors, and large employers of labour, seem to be very much enamoured with some of the schemes which are to put the world in general and the employees of those bodies in particular to rights. And consequently actuaries are now very frequently applied to to form such schemes, for which purpose such materials as Mr. Sprague has given in his paper are essentially necessary. Some of the conclusions at which he arrives are, I think, based upon very insufficient data. I think we cannot reason at all upon numbers of marriages and deaths such as he gives about the ages 45 and 50. When he speaks about the mortality of married men and bachelors, he says that you will find from the ages of 45 to 49 there are 9 deaths; and 12 in the next quinquennium, and also 12 in the next. No conclusions can be drawn from such numbers as these. When Mr. Sprague gets to the question of selection, I doubt whether the unhealthy young men of whom he speaks, have no desire to marry; and I think that if their pecuniary circumstances are entirely satisfactory, they are not considered such undesirable husbands; and I am sure, as regards the other sex, it has been a subject of observation that heiresses are generally only children, and most frequently their fathers and mothers are dead, and died young; and certainly therefore they are not very eligible subjects of life assurance. But in the matrimonial market the demand for heiresses very much exceeds the supply. Mr. Sprague says, "Most valuable statistics bearing on the same question might also be obtained from the records of the National Debt Office." I do not think the National Debt Office has given very much information about the conjugal condition of annuitants. With regard to graduation, I am a disciple of the late Professor De Morgan, never having found any process of adjustment satisfactory to me. I have always objected to this process when it is assumed that it is a correction based on the principle that the laws of mortality or marriage can be represented by a regular curve. The probabilities of dying from the ages of 20 to 24 are greater than from 25 to 29; and in Anglo-Indian mortality there is evidence to show that, up to a certain period, as the age increases the probability of dying diminishes. If this peculiarity is to be graduated away, you are altogether destroying what I believe to be an essential element in Anglo-Indian mortality. And so in many other cases, and in all scientific procedures, we must be guided by observation and experiment; and observation and experiment in this subject do not show that the law of mortality follows any regular curve.

Mr. A. DAY—I wish publicly to say how extremely glad I am that Mr. Sprague has taken up what I had left undone. I owe an apology to the Institute for having left so much unaccomplished when I had become morally responsible for its completion, and when I had fully intended to redeem my engagement. But the circumstances have very much changed since I prepared my papers which appeared in the 10th and 12th volumes of the *Journal*. At that time we were almost without information as to the probability of marriage and at the same time we were getting a great number of risks contingent upon failure of issue before us. I was then living in a sort of atmosphere of contingent reversions, but now I see nothing of the same class of cases. That is my principal excuse for not having gone on with the investigation. It was of value to me in those days, and is not of practical value to me now. Mr. Sprague has been kind enough to speak of the plainness with which the facts have been put before the Institute in my previous papers; and I do not know that I have any fault to find with his criticisms, except that I do not agree with him in saying that it is "beyond all question" that there should be no deduction in respect of the deaths in forming the tables of the probability of bachelors marrying.

Mr. MACFADYEN—I wish to refer to what our President has said about Mr. Sprague's statement concerning the National Debt Office. I do not think that Mr. Sprague intended it to be understood that the "most valuable statistics bearing on the same question that might be obtained from the records of the National Debt Office", were statistics referring to the immediate subject of his paper, but rather to the effect on selection of the lapses and surrenders mentioned in the previous sentence. [The PRESIDENT—There is nothing about lapses and surrenders in the records of the National Debt Office.] Quite so; but it is their absence there, and their presence in ordinary assurance offices, that would make the statistics of the National Debt Office so valuable. When preparing my paper on surrender values five years ago, I used the Government annuities to measure the force of exit selection, and even now I do not know where else to find material equally good for this purpose.

Mr. C. J. BUNYON—Our President rather controverted Mr. Sprague's remark as to the expected mortality which might occur among married men as compared with those who were not married.

I should have thought it was almost an obvious fact that the marriages operate as a selection almost in the same way as a medical examination itself. There are and have been societies that have carried on a successful business without medical examination; and the very fact that a man has placed himself before a dozen or twenty directors, and shown a blooming countenance, and said that he was about to marry, was sufficient to pass him. And it has been found that the mortality experience amongst such persons has been more favourable than among those passed by all the doctors. Therefore, I should have said that the result which Mr. Sprague shadows out, is only what we might have expected from past experience and natural probability. Some fifteen or sixteen years ago I formed a table precisely similar to Mr. Sprague's, showing the probability of death and marriage amongst 10,000 unmarried people, up to the age of 60, only my table was not a table of bachelors but of spinsters. The marriage rate was very low at the ages of 15 or 16, culminated at 22, and then rapidly reduced.

Mr. WALFORD said, that while appreciating the ingenuity of the method proposed in the paper, and the labour involved in its preparation, he believed that no single table would be applicable generally. He thought that each widows' fund has not only its own specific death rate but also its own marriage rate, the latter depending very often upon the pensions and other inducements held out to the members. It was, however, an advantage, to have the mind familiarized with such joint problems as are treated in the paper. He was under the impression that Mr. Ansell's *Statistics of Middle Class Families* might have furnished some data in connection with the present subject.

Mr. G. W. BERRIDGE—Both in my own graduation and that of Mr. Sprague there is no attempt in the adjusted curve to do away with the peculiarities of the original facts. Those facts show a retrogression of the mortality for about 10 years—that is to say, the mortality at the age of 22 is greater than the mortality at 32, and therefore the figures were difficult to graduate by any of the usual methods. It is rather curious, that having criticized my graduation so closely, Mr. Sprague has failed to see that I have used his graphic method between the ages of 3 and 15. I did not attempt any elaborate explanation of it, but I mentioned it in the paper.

Mr. R. P. HARDY—The subject which Mr. Sprague proposes to investigate has been on two occasions under my consideration, and is now again for a time, and I was in hopes that we should have some further light thrown upon this very difficult question, but was disappointed to find that Mr. Sprague has no further materials beyond Mr. Day's of some years' standing. As to the particular set of problems which Mr. Sprague seems to have chiefly in view, I think there are very few existing assurances against issue—probably not a thousand altogether. The treatment of widows' funds is, I think, one of the most difficult questions that can come before an actuary. Not only do there seem to be individual problems, but the particular experience of a fund, when looked at from year to year, seems to be constantly varying; and not only would it not be safe to draw serious deductions from so limited a body of facts as those given in the paper, but one should be very careful in applying even the data deduced from the experience of a fund under examination. On the question of the ages at which males marry, there is no doubt that in one large dependency of the Crown the rate of marriage in the last fifteen years has been rapidly rising; and it practically amounts to this now—that all marry. I am aware that this is not the general experience of mankind; but when we consider the powerful circumstances influencing the will, we shall not be altogether surprised. If a man is relieved from the necessity of having to make a provision for his family, I think it only reasonable to suppose that he will marry as often as the opportunity is afforded to him. For every male dying there is very nearly always a widow left to be a burden and encumbrance upon the fund. Mr. Sprague does not show us the marriage rate of spinsters. That is influenced by some special circumstances. The culminating point I have found a little higher than Mr. Bunyon takes—there being a tendency in society in these days to defer marriage. If you examine the statistics carefully one by one, you will find the rate of marriage very much depends upon whether the young lady has a father living or not. Girls in a family with the father in a good position, marry much more quickly than the orphan daughters living with their pensioned mothers. Then with regard to the re-marriage of widows. Mr. Brown investigated the question at some length, and it will be found that there are strong influences, I will not say actually preventing re-marriage, but leading a woman to think twice over it. The forfeiture or suspension of the pension, and the breaking up of the home, no doubt, operate as deterring influences. When we remember the very small number of cases to which these calculations have been applied, it is not desirable to attempt the affectation of precision. The method which I am forced to adopt is the method of limits—that is, allowing every male a female nominee at the time of his death, and to take the difference between that and the ordinary contingent annuity, and to adjust between the two according to the circumstances. Although I admit that this is a very rough and ready plan, I do not suppose it is likely to be very much more out than results carefully brought out from such a small body of facts as those of the paper. I think the peculiarity shown in the mortality of bachelors has entirely disappeared at that age when issue assurances are practically effected.

Mr. F. G. P. Neison said that in the old Indian funds it made a very material difference whether a pension was lost or not by re-marriage. If we compare the rate of marriage of widows in the Bombay fund, in which one-half of the pension is retained, as against the percentage of re-marriage under the other procedure, we find a very material difference. He had recently had before him one of the largest widows' and orphans' funds in connection with some mining operations in the North of England, embracing 80,000 members altogether, and it was necessary to determine the rate of re-marriage among the widows. Having been supplied with the statistics of 16 or 17 years of the society, he found that in the time of the high prices of coal in 1873 the proportion of re-marriages among the widows was extraordinarily high, but in the last few years, the re-marriage rate has gone down immensely. Out of the large number of widows in the fund, hardly one re-married, the reason being that the women preferred, in the depressed state of the coal trade, to keep the £5 pension rather than run the risk of entering the bond of marriage with an impoverished miner.

Mr. SMITH—There is much good sense in Mr. Sprague's suggestion that Mr. Huie's table may possibly be erroneous in regard to the age of marriage. The latter says, "that the probability of marriage of clergymen is greatest at about the age of 38", but the fact is that the clergy find it necessary to marry sooner, as is the case with medical men, for various reasons which must be obvious. I know something of the marriage statistics of two Scotch ecclesiastical bodies. In the Scotch Episcopal body the number of unmarried men is very small. Mr. Trollope's character of Mr. Quiverfull seems to hold good there. I had also to do something, a good many years ago, with the statistics of one of the smaller Dissenting bodies. The majority were bachelors, and they died bachelors, under the operation of an exceptionally heavy death rate. I was informed that they were a very poor body, made up of tradesmen's sons and similar classes, who, when they had a delicate member of the family, said, "Oh, we must put him into the ministry." These did not get married, and the mortality was alarmingly high.

Mr. M. N. ADLER—The investigation of widows' funds has not only recently engaged the attention of actuaries, but 50 and even 100 years ago these questions came before men of science, and especially so in Germany, where tables of some interest have been published. Brune's well-known tables are based upon the experience, commencing more than a century ago, of the Berlin Widows' Fund, and show the mortality of both male and female life. These records would, I believe, supply also some interesting statistics as to marriages of men and women. Mr. Sprague states that the results obtained from the annual reports of the Registrar-General and the census reports cannot be safely adopted in estimating the financial position of a widows' fund, and he thinks that Mr. Day's peerage statistics should be the basis on which such calculations are to be made. But, however valuable these statistics are for the other purposes he mentions, as the members of a widows' fund live in a very different position to those of the peerage, their experience, in the absence of special features, would best agree with the facts brought out from the census reports as to the general population. As to Mr. Sprague's graphic method of graduation, he thought it should not be generally applied, as we should not satisfy ourselves with merely the measurement of our eyes in framing and graduating tables of mortality and observations generally.

MR. SPRAGUE'S REPLY.

Mr. Bailey expresses the opinion that some of the conclusions I have arrived at are based upon very insufficient data. It may possibly be true that the data I have dealt with are not sufficiently numerous to warrant our feeling any great confidence in the conclusions I have drawn from them. My object, however, has been to show how the best conclusions can be drawn from the existing data, such as they are; and the conclusions I have drawn, seem to me so reasonable in themselves, and so consistent with each other, that I confidently anticipate that the results of further inquiries, made with more extensive data, will be to confirm rather than discredit my conclusions. The instance Mr. Bailey takes is not a happy one. He points out that in the statistics as to bachelors there are only 9 deaths at the ages of 45 to 49; 12 in the next quinquennium, and 16 in the next; and adds that no conclusions can be drawn from such numbers as these. But he has failed to observe that this is the exact conclusion at which I arrived myself in the paper, and that accordingly, from the age of 45 onwards, I discard the probabilitys of death derived from the statistics as to bachelors, and assume that the bachelors will be subject to the same rate of mortality as the married men. It is perfectly true that the number of observed facts is smaller than could be desired, but I hold the opinion strongly that a small number of carefully observed facts, when properly treated, will at all events furnish a better basis of calculation than mere conjecture. Mr. Bailey's remarks upon the marriage probability of heiresses have, of course, no bearing whatever upon anything contained in my paper. The statistics that are open to the public do not inform us as to the fortunes of the young people of both sexes; but from the nature of the case the number of heiresses will always bear a very small ratio to the total number of marriageable women. Mr. Macfadyen has given the correct answer to Mr. Bailey as to his remarks upon the statistics to be obtained from the records of the National Debt Office. The remarks of Mr. Bunyon render it unnecessary for me to say anything further as to the nature of the selection that takes place in marriage and its effect upon the rate of mortality; and Mr. Berridge has very clearly explained that the methods of adjustment adopted by him and by myself, have no tendency to remove out of sight any peculiarities in the progression of the original facts. Mr. Bailey in his remarks appears to have lost sight of the circumstance that the rate of mortality, as deduced from unadjusted observations, very frequently proceeds most irregularly as we pass from one age to the next. It is the object of a graduation to get rid of these irregularities, which we believe to be simply accidental and a consequence of the smallness of the numbers observed. My method of graduation does this, but does not remove peculiarities in the progression of the rate of mortality such as those he mentions. On the contrary, it faithfully preserves every feature that has the least claim to be considered an essential element in the law of mortality.

Mr. Hardy remarks that there are probably not a thousand insurances against issue in existence altogether. These, however, are generally for large amounts, and in the aggregate they form a somewhat important branch of the business of the British life insurance companies. I am furthermore of opinion that this branch of business has not been studied so completely as its importance deserves. The insurances that have been granted hitherto have mostly been to cover the risk of a man at present married to a wife in good health and past child-bearing age leaving issue by a future wife; and the business, so limited, has (I believe) been remarkably profitable to the companies. Very little comparatively has been done in the way of insuring against the risk of issue to a man who is now either a bachelor or a widower, or a married man with a wife still of child-bearing age. I have satisfied myself that the risk of a bachelor leaving issue is very much less than has hitherto been conjectured, and the question now arises whether it would not be good policy for the companies to accept such risks at considerably lower premiums than has hitherto been their practice, in the hope that an increase of the business may more than recompense them for their immediate loss of profit.

It did not fall within the scope of my paper to enlarge upon the subject of the graduation of mortality tables, but I thought it would be desirable to describe briefly the method which I have adopted with, as it seems to me, considerable success; and I invite my brother actuaries to try the graphic method for themselves before they condemn it as a rough unscientific method of procedure.