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Constructing efficient stock market indices

- **Problems with Existing Equity Indices**
- Rehabilitating the Tangency Portfolio
- The Efficient Index: Implementation and Empirical Results

Problems with Existing Indices

Indices versus benchmarks

- There are two dimensions in allocation decisions for the construction of performance-seeking portfolios: broad asset allocation decisions versus benchmark portfolio construction decisions within a given asset class.
- In practice, the second step is almost always entirely trivialized through the use of market cap weighted indices as default choices for investment benchmarks.
- The words « index » and « benchmark » are often used interchangeably; yet they define *a priori* very different concepts:
  - An *index* is a portfolio that should represent the performance of a given segment of the market => focus on representativity.
  - A *benchmark* is a reference portfolio that is should represent the fair reward expected in exchange for risk exposures that an investor is willing to accept => focus on efficiency.

Problems with Existing Indices

Lack of Mean-Variance Efficiency

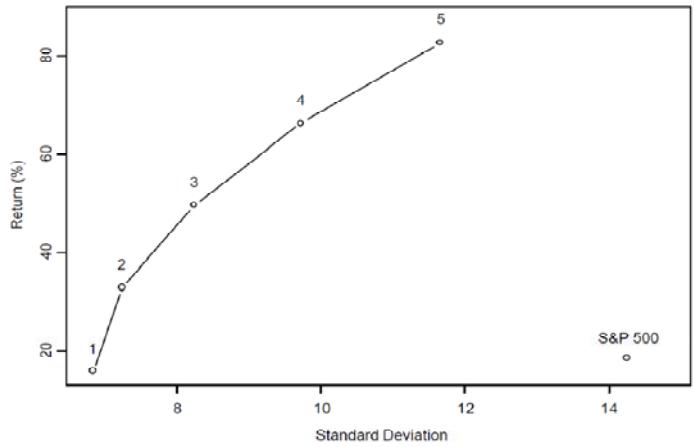
- The standard practice of using stock market indices based on market cap weighting schemes as investment benchmarks has recently faced renewed criticism.
- More than 15 years ago, a number of papers (e.g., Haugen and Baker (1991) and Grinold (1992)) have already offered empirical evidence that market-cap weighted indices provide an inefficient risk-return trade-off.

"Cap-weighted stock portfolios are inefficient investments. […] Even the most comprehensive cap-weighted portfolios occupy positions inside the efficient set."
(Haugen and Baker (1991))

"Market indices […] are if anything inside that [mean-variance] frontier"
(John Cochrane (2001))

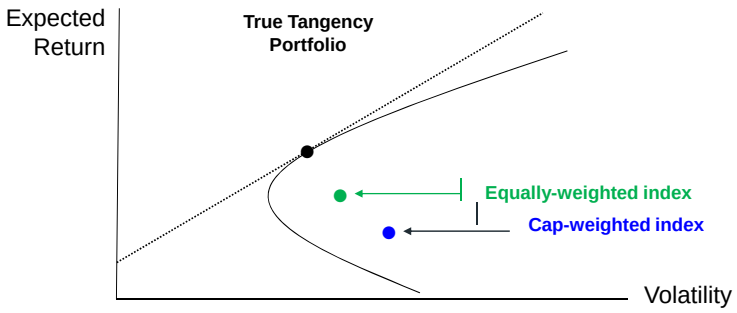
Problems with Existing Indices
Inefficiency - Empirical Arguments

- Cap-weighted index lies deep inside the ex-post efficient frontier.



Based on data for the period 1979-1998. The efficient frontier assumes a perfect forecast of the future covariance matrix and of the future mean return. Figure taken from Schwartz (2000), Figure 3, page 19.

Problems with Existing Indices
Cap Weighted versus Equally-Weighted Portfolios



Problems with Existing Indices
Inefficiency - Theoretical Arguments

- The poor risk-adjusted performance of cap-weighted indices should not come as a surprise given that the efficiency of the market portfolio is based on unrealistic assumptions:
  - Unlimited risk-free borrowing and short selling;
  - Homogenous preferences, expectations and horizons;
  - No frictions (taxes, transaction costs);
  - No non-tradable assets (social security claims, human capital, etc.).

Sharpe (1991) and Markowitz (2005) state that under real-world conditions the market portfolio may not be efficient

- Beside, even if the Capital Asset Pricing Model (CAPM) was the true asset pricing model, any given equity index is not a good proxy for the true market portfolio.

Problems with Existing Indices
Concentration - Effective Number of Stocks

- Cap-weighting leads to high concentration.

Index	Nominal number	Effective number
S&P	500	94
NASDAQ	100	37
FTSE 100	100	28
FTSE AW Eurobloc	300	104
FTSE AW Japan	500	103

Average effective number based on quarterly assessment for the time period 01/1959 to 12/2008 for the S&P, 01/1975 to 12/2008 for the NASDAQ, and 12/2002 to 12/2008 for the other indices .

$$\tilde{n} = \frac{1}{\sum_{i=1}^{n} w_i^2}.$$

$\tilde{n}$ is the reciprocal of the Herfindhal index, a commonly used measure of portfolio concentration

$\tilde{n} = 1$ if $\exists\ i$ such that $w_i = 100\%$

$\tilde{n} = n$ if $w_i = 1/n\ \ \forall\ i = 1,\ldots, n$

$\Rightarrow$ hence the interpretation as effective number of stocks

Problems with Existing Indices

Efficiency versus Representativity

- Overall, commercial indices are not **efficient** or **well-diversified** portfolios because they have never meant to be efficient or well-diversified.
- The main objective of these indices is to **represent** the stock market, thus neglecting investors' need for the most efficient risk-return trade-off.
- Alternative weighting schemes have been suggested to generate indices that better represent the economy: characteristics-based indices that weight stocks according to their economic footprint, as opposed to their market cap.
- These indices focus on representativity, and do not explicitly aim at improving the risk-reward ratio (efficiency).

Rehabilitating the Tangency Portfolio

Back to the Basics of Portfolio Theory

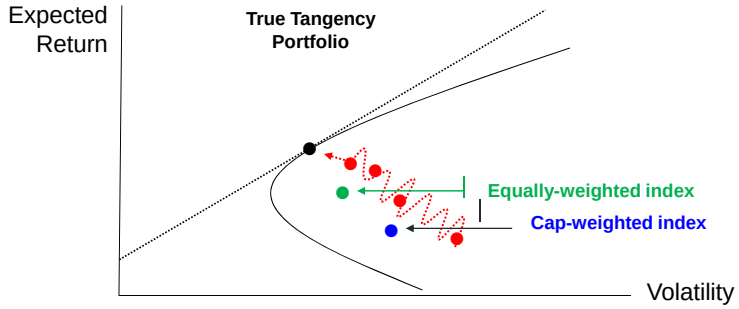
- For a rational investor, the goal is not to have the portfolio with the highest representativity; the goal is instead to obtain the best risk-adjusted performance.
- In the end, if one cares for a high reward-to-risk ratio, one should aim at maximizing the reward-to-risk ratio, which requires:
  - estimate of risk parameters
  - estimate of expected return parameters
- Are we really ready to believe that absolutely nothing meaningful can be said about covariance and expected return parameters?

Rehabilitating the Tangency Portfolio

Designing Investable Proxies for MSR Portfolios

*The true tangency portfolio is a function of the (unknown) **true** parameter values*

- $w_{MSR} = f(\mu_i, \sigma_i, \rho_{ij})$



*Implementable proxies depend on **estimated** parameter values*

- $\hat{w}_{MSR} = f(\hat{\mu}_i, \hat{\sigma}_i, \hat{\rho}_{ij})$

Rehabilitating the Tangency Portfolio

Estimating Covariance Parameters

- Statistical techniques can be used to generate decent risk estimates.
- The key challenge is curse of dimensionality
- Factor models help to reduce dimensionality. Correlations across stocks are represented by their exposure to common risk factors and the correlation across these common risk factors.
- Based on an equity factor model, we can estimate the entire covariance matrix for a universe of constituent stocks.

Rehabilitating the Tangency Portfolio

Estimating Expected Return Parameters

- Statistics is close to useless in terms of expected return estimation (Merton (1980)), but economic analysis & (perhaps better?) common sense can help.
- Common sense: *risk-return tradeoff* implies that expected return parameters should be positively related to risk parameters.
- Economic analysis can help identify the relevant risk indicator the functional form of the risk-return relationship:
  - Systematic risk (CAPM) buta lso specific risk should be rewarded (Merton (1987)) (*);
  - Higher moment risk is also rewarded (many references).

(*) See also Barberis and Huang (2001) Malkiel and Yu (2002), Boyle, Garlappi, Uppal and Wang (2009) .

Rehabilitating the Tangency Portfolio

On the Relationship between Downside Risk & Expected Returns

Evidence that stock downside risk is related to expected returns:

Authors	Risk Measure	Relation
Zhang (2005)	Skewness	+
Zhang (2005)	Skewness	+
Boyer, Mitton and Vorkink (2009)	Skewness	+
Tang and Shum (2003)	Skewness (but not kurtosis)	+
Connrad, Dittmar, Ghysels (2009)	Skewness (but not kurtosis)	+
Ang et al. (2006)	Downside correlation	+
Huang et al (2009)	Value-at-Risk (EVT)	+
Bali and Cakici (2004)	Value-at-Risk (Historical)	+
Chen et al. (2009)	Semi-deviation	+
Estrada (2000)	Semi-deviation	+

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- **The Efficient Index: Implementation and Empirical Results**

Empirical Tests

Methodology

- Our objective is to go back to the basics of Modern Portfolio Theory to generate a proxy for the tangency portfolio.
- Such a portfolio may provide investors with a more efficient way of extracting the equity risk premium from the stock market.
- We use the link between expected stock returns and downside risk to estimate expected returns.
- For practical reasons, we also wish to control portfolio turnover and only update portfolio weights if the weight changes are substantial.(*)

(*) See Leland (1999), or Martellini and Priaulet (2002).

Empirical Tests

Long-Term US Results

Index	Ann. average return	Ann. std. Deviation	Sharpe Ratio	Information Ratio	Tracking Error
Efficient Index	11.63%	14.65%	0.41	0.52	4.65%
Cap-weighted	9.23%	15.20%	0.24	0.00	0.00%
Difference (Efficient minus Cap-weighted)	2.40%	-0.55%	0.17	-	-
p-value for difference	0.14%	6.04%	0.04%	-	-

The table shows risk and return statistics portfolios constructed with using the same set of constituents as the cap-weighted S&P 500 index. Rebalancing is quarterly subject to an optimal control of portfolio turnover (by setting the reoptimisation threshold to 50%). Portfolios are constructed by maximising the Sharpe ratio given an expected return estimate and a covariance estimate. The expected return estimate is set to the median total risk of stocks in the same decile when sorting on total risk. The covariance matrix is estimated using an implicit factor model for stock returns. Weight constraints are set so that each stock's weight is between 1/2N and 2/N, where N is the number of index constituents. P-values for differences are computed using the paired t-test for the average, the F-test for volatility, and a Jobson-Korkie test for the Sharpe ratio. The results are based on weekly return data from 01/1959. We use a calibration period of 2 years and rebalance the portfolio every three months (at the beginning of January, April, July and October).

Empirical Tests

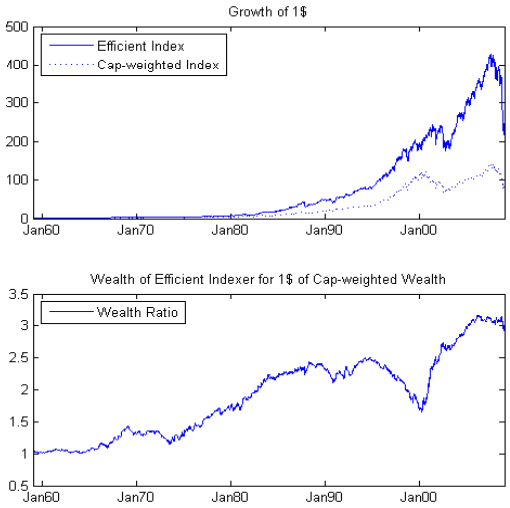
Results – Turnover and Concentration

Index	Annual one-way turnover	Excess turnover vs. Cap-weighted	Average Effective constituents	Effective constituents to nominal constituents
Efficient Index	23.10%	18.41%	382	76%
Cap-weighted	4.69%	0.00%	94	19%

The table shows the resulting turnover measures for Efficient Indexation portfolios that have been implemented using the controlled reoptimisation with a threshold value of 50%. The table indicates the effective number of constituents in the efficient index and in the cap-weighted index, computed as the inverse of the sum of squared constituent weights. This measure is computed at the start of each quarter and averaged over the entire period. The results are based on weekly return data from 01/1959 to 12/2008.

Empirical Tests

Results – Evolution of Wealth



- Prolonged lower returns occurred in the bull market of the late 1990s.
- This underperformance happened as the cap-weighted index returned in excess of 20% annual.
- Even in this period, efficient indexation had lower volatility than cap-weighting.

Empirical Tests

Results – Robustness Checks

Risk and return in different decades

	Ann. average return		Ann. Volatility		Sharpe ratio	
"Decade"	Cap-weighting	Efficient Indexation	Cap-weighting	Efficient Indexation	Cap-weighting	Efficient Indexation
1999-2008	-1.22%	3.47%	18.98%	18.04%	-0.23	0.01
1989-1998	19.16%	16.43%	12.84%	12.45%	1.07	0.89
1979-1988	16.32%	20.82%	16.02%	15.82%	0.42	0.71
1959-1978	2.96%	4.24%	16.02%	15.47%	-0.20	-0.13
1959-1968	10.33%	14.29%	10.65%	10.05%	0.62	1.05

The table shows risk and return statistics when dividing the sample in periods of ten years. The results are based on weekly return data from 01/1959 to 12/2008.

- Prolonged lower returns occurred in the bull market of late 1990s.
- This happened as the cap-weighted index returned 20% annual.
- In this period, efficient indexation still had lower volatility.
- Similar results obtained for Eurobloc, UK, Asia ex-Japan and Japan.

Conclusion

- Cap-weighted indices are not **efficient** or **well-diversified** portfolios because they were never meant to be; the main objective of these indices is to **represent** the stock market, thus neglecting the need for the most efficient risk-return trade-off.
- Alternative weighting schemes do not explicitly aim at improving the risk-reward ratio either.
- The efficient index uses robust estimates of expected returns and covariance as inputs in a maximisation of the reward-to-risk ratio.
- Out-of-sample reward-to-risk ratios are higher than for the value-weighted index.
- Performance is consistent across different time periods and geographical zones.

References

- Bali, Turan G., and Nusret Cakici, 2004, Value at Risk and Expected Stock Returns. Financial Analysts Journal, 60(2), 57-73.
- Barberis, N., and M. Huang, 2001, Mental Accounting, Loss Aversion and Individual Stock Returns, Journal of Finance, 56, 1247-1292.
- Barberis, N. and M. Huang, Stocks as lotteries: The implications of probability weighting for security prices, 2007, working paper.
- Boyer, B., and K. Vorkink, 2007, Equilibrium Underdiversification and the Preference for Skewness, Review of Financial Studies, 20(4), 1255-1288.
- Boyer, B., T. Mitton and K. Vorkink, 2009, Expected Idiosyncratic Skewness, Review of Financial Studies, forthcoming.
- Chen, D.H., C.D. Chen, and J. Chen, 2009, Downside risk measures and equity returns in the NYSE, Applied Economics, 41, 1055-1070.
- Connrad, J., R.F. Dittmar and E. Ghysels, Ex Ante Skewness and Expected Stock Returns, 2008, working paper.
- Cochrane, John H., 2005, Asset Pricing (Revised), Princeton University Press
- El Bied, S., L. Martellini, and P. Priaulet, 2002, Competing investment strategies in the presence of market frictions, USC working paper.
- Estrada, J, 2000, The Cost of Equity in Emerging Markets: A Downside Risk Approach, Emerging Markets Quarterly, 19-30.
- Grinold, Richard C. “Are Benchmark Portfolios Efficient?”, Journal of Portfolio Management, Fall 1992.
- Haugen, R. A., and Baker N. L., “The Efficient Market Inefficiency of Capitalization-weighted Stock Portfolios”, Journal of Portfolio Management, Spring 1991.

References

- Leland, H., 1999, Optimal asset rebalancing in the presence of transaction costs, U.C. Berkeley University, working Paper.
- Malkiel, B., and Y. Xu, 2002, Idiosyncratic Risk and Security Returns, working Paper, University of Texas at Dallas.
- Markowitz, H. M., “Market efficiency: A Theoretical Distinction and So What?”, Financial Analysts Journal, September/October 2005.
- Martellini, L., and P. Priaulet, 2002, Competing methods for option hedging in the presence of transaction costs, with P. Priaulet, Journal of Derivatives, 9, 3, 26-38.
- Martellini, L., and V. Ziemann, Improved estimates of higher-order comoments and implications for portfolio selection, Review of Financial Studies, forthcoming.
- Merton, Robert, 1987, A Simple Model of Capital Market Equilibrium with Incomplete Information, Journal of Finance, 42(3).
- Schwartz, T., 2000, How to Beat the S&P500 with Portfolio Optimization, DePaul University, working paper.
- Sharpe, W.F., 1991 , “Capital Asset Prices with and without Negative Holdings”, Journal of Finance, 46.
- Tang, Y., and Shum, 2003, The relationships between unsystematic risk, skewness and stock returns during up and down markets, International Business Review.
- Tinic, S., and R. West, 1986, Risk, Return and Equilibrium: A revisit, Journal of Political Economy, 94, 1, 126-147.
- Tobin, J., 1958, Liquidity Preference as Behavior Towards Risk, Review of Economic Studies, 67, 65-86.
- Zhang, Y., 2005, Individual Skewness and the Cross-Section of Average Stock Returns, Yale University, working paper.