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CONTINUOUS INVESTIGATION INTO THE MORTALITY OF ASSURED LIVES

Enquiry into the desirability of constructing a " Light " table of mortality

INTRODUCTORY NOTE

The following note describes an investigation into the possibility of constructing a " Light " table of mortality for assured lives, and of publishing monetary functions based upon it, for the use of life offices requiring a lighter mortality basis than the A1949-52 table. The conclusion was reached that the publication of such a table was unnecessary as a satisfactory reproduction of the experience of the lighter offices could be obtained from the A1949-52 table by making an appropriate rating-down in age.

It will be recalled that the publication of the A1924-29 table of assured lives' mortality was later followed by two supplementary tables known as the " A1924-29 Light " and the " A1924-29 Heavy " These were based on the 1924-29 data for groups of offices showing respectively lighter and heavier mortality than the all-offices average and the tables were intended to serve the needs of offices whose experience diverged substantially from the A1924-29 itself.

So far as is known, little use was made of the A1924-29 Heavy table. The A1924-29 Light table, however, was not only used for its intended purposes but also became extensively employed for premium calculations when declining mortality rendered the A1924-29 table increasingly out of date. Thus in the course of time, it came to serve a somewhat different purpose from that for which it was conceived.

When the A1949-52 table was in process of construction, it was again found that wide divergences existed among the mortality experiences of different offices and it was at first thought that corresponding Light and Heavy tables would prove helpful. On reflection, however, the need for a Heavy table appeared questionable. Rates of mortality are still falling and it is probably only a matter of time before the A1949-52 table will itself represent a heavy standard relative to the experience of the offices as a whole. It has to be remembered, moreover, that the period to which the basic data related included the exceptionally heavy year 1951. For these

reasons, therefore, it was considered unnecessary to pursue the question of constructing a " Heavy " table.

On the other hand, the desirability of publishing a " Light " table called for careful examination. Since the construction of the A1949-52 table, data for three more years had become available and it was thought that it would be more useful to examine the latest available information than to subdivide the original data for 1949-52. It was accordingly decided to make a preliminary investigation on data for the four years 1952-55, thus excluding the heavy year 1951. On examining the experience for individual offices, it did not prove feasible to segregate a small group of offices whose mortality experience was notably lighter than the remainder. It did, however, appear that the general distribution of offices according to level of mortality tended to fall naturally into a heavy section and a light section and that a separation of the data into these two sections would be practicable. The sections were approximately equal in the numbers exposed to risk, although twenty offices fell into the " heavy " category and forty-one into the " light " category.

The A1949-52 table was based upon the combined data of the medically examined and non-medical sections. Since, however, the medically examined section has for many years exhibited consistently lighter mortality than the non-medical section, it was decided that the data employed for exploring a basis for a " Light " table should be confined to the medically examined section.

Table 1 shows in quinary age groups, exposed-to-risk, actual deaths, expected deaths by the A1949-52 table and percentages of actual to expected deaths for medically examined lives, durations 2 and over, during the period 1952-55. These data are subdivided between forty-one light offices and twenty heavy offices, and are also presented for the combined experience of all sixty-one offices. For convenience, sub-totals are shown for three broad age-groups, i.e. under $45\frac{1}{2}$, $45\frac{1}{2}$-$74\frac{1}{2}$ and $75\frac{1}{2}$ and over. The percentages of actual to expected deaths in three broad groups are reproduced below :—

Age Group	41 light offices	20 heavy offices	All offices combined
Under $45\frac{1}{2}$	81·1	88·1	84·5
$45\frac{1}{2}$-$74\frac{1}{2}$	86·0	97·4	91·9
$75\frac{1}{2}$ and over	94·1	99·0	96·2
All ages	88·4	97·1	92·7

Two factors are reflected in the percentages in the final column viz :—

(i) secular changes in mortality between the period 1949-52 and the period 1952-55 (including the elimination of the heavy year 1951) and

(ii) restriction of the data in the present investigation to medically examined lives.

An examination of the experience in the important middle age-group (i.e. $45\frac{1}{2}$-$74\frac{1}{2}$) has shown that the effect of these two factors is about equal so that of the 8% shortfall below the A1949-52 table 4% is due to secular changes and 4% to the restriction to medically examined lives.

As the offices have been deliberately subdivided into light and heavy groups, there is little that can be said about the significance of the differences between them. The experience of the light group is appreciably below the A1949-52 table and there would thus appear to be a *prima facie* case for preparing a Light table. It was accordingly decided to investigate the position further by devising a mortality curve which would broadly correspond to the experience of the light offices.

It seemed reasonable to expect that a satisfactory curve might be found by adopting the same formula as was employed in constructing the A1949-52 table with suitable variations in the values of some or all of the parameters. The A1949-52 formula was:

$$q_x = A + Bc^y/(Ec^{-2y} + 1 + Dc^y) \text{ where } y = x - 62{\cdot}5$$

It was found that a curve corresponding fairly closely with the light offices' experience could be developed by retaining the same value of c as in the A1949-52 table, keeping the origin at $x = 62{\cdot}5$, and deriving values of A, B, D and E in the following manner (the accented symbols refer to the A1949-52 values) :—

(i) $A = {\cdot}8A'$

(ii) $D + E = D' + E'$

(iii) $A + B/(D + 1 + E) = {\cdot}85(A' + B'/(D' + 1 + E'))$

(iv) $A + B/D = {\cdot}95(A' + B'/D')$

The parameter A represents the minimum value of q_x (which for practical purposes is the value at ages below 25); the function $A + B/(D + 1 + E)$ is the value of q_x at the origin (i.e. $x = 62{\cdot}5$); and

the function A+B/D is the maximum towards which q_x approaches asymptotically. It was felt that a series of simple equations between the parameter values of the two curves would give a clearer statement of the relation between the A1949-52 and the Light tables than would be obtained if the parameters of the new curve had been evaluated *ab initio*.

The values of the parameters which actually resulted are shown below, with the A1949-52 values set alongside for comparison.

	Light table	A1949-52 *table*
Origin	62·5	62·5
c	$(1{\cdot}0525)^2$	$(1{\cdot}0525)^2$
A	·00089	·00111
B	·01866	·02186
D	·02453	·02730
E	·02123	·01846

The resulting values of q_x for the Light table are given in Table 2 and a comparison of actual and expected deaths is given in Table 3. Although the curve is in no sense a graduation of the crude values of q_x there is close agreement between actual and expected deaths at ages below 70. At ages over 70 the expected deaths are rather less than actual deaths, indicating that the values of q_x are somewhat lighter than in the original experience ; but this is not a disadvantage in a table designed to exhibit light mortality and its effect on premium rates at the usual entry ages is slight.

Specimen values of q_x together with corresponding values from the A1949-52 table are shown below :

Age x	A1952-55 Light q_x	A1949-52 q_x	Age x	A1952-55 Light q_x	A1949-52 q_x	Age x	A1952-55 Light q_x	A1949-52 q_x
20	·00089	·00111	50	·00494	·00599	80	·09836	·11369
30	·00093	·00116	60	·01459	·01720	90	·22176	·25168
40	·00149	·00188	70	·03891	·04543	100	·40588	·44872

From age 40 upwards there is a fairly close resemblance between q_x on the Light table and q_{x-2} on the A1949-52 table. This is illustrated in the following table :

Age x	A1952-55 Light q_x	A1949-52 q_{x-2}	Age x	A1952-55 Light q_x	A1949-52 q_{x-2}	Age x	A1952-55 Light q_x	A1949-52 q_{x-2}
40	·00149	·00158	60	·01459	·01408	80	·09836	·09528
45	·00265	·00259	65	·02396	·02312	85	·15071	·14681
50	·00494	·00474	70	·03891	·03753	90	·22176	·21785
55	·00869	·00837	75	·06242	·06028	95	·30947	·30730

It follows that a similar relationship may be expected in the monetary values derived from the two tables. That this is in fact so may be seen from the following table of premiums based upon interest at 3% :

Age x	A1952-55 Light P_x	A1949-52 P_{x-2}	A1952-55 Light $P_{x:\overline{15}|}$	A1949-52 $P_{x-2:\overline{15}|}$	A1952-55 Light $P_{x:\overline{30}|}$	A1949-52 $P_{x-2:\overline{30}|}$
20	·00798	·00799	·05267	·05278	·02104	·02116
30	·01139	·01132	·05277	·05286	·02160	·02165
40	·01708	·01684	·05359	·05357	·02379	·02369
50	·02689	·02643	·05667	·05649	·02999	·02965
60	·04414	·04332	·06484	·06438		
70	·07638	·07483	·08656	·08531		

In view of the close resemblance, revealed in the foregoing table, between premiums based on the Light table and on the A1949-52 table rated down two years it seems clear that the preparation and publication of monetary functions based on the A1952-55 Light table would not be justifiable. Offices which feel that they need a lighter standard than the A1949-52 should obtain it by a uniform rating down in age, the precise extent of which will vary with the individual experiences of the offices concerned.

Durations 0 *and* 1

The experiments so far described relate to the data at durations 2 and over. Variations in the relationships between the rates of mortality at durations 0 and 1 and the ultimate (or " 2 and over ") rates cannot affect the conclusions to any appreciable extent. Nevertheless, for the sake of completeness, the data for durations 0 and 1

have been examined. Owing to the small number of deaths at these early durations, it has been necessary to combine the data in three broad age-groups. The results are given in Table 4.

As the data for the " Light " experience were restricted to medically examined lives, it was to be expected that a greater degree of selection would be exhibited than in the A1949-52 table, which was based on medical and non-medical data combined. That this is in fact the case can be seen from Table 4. In the oldest of the three age-groups, however, non-medical data at durations 0 and 1 are relatively sparse and it might be expected that the two experiences would resemble each other more closely in the degree of selection revealed. As the reverse appears to be true it would seem possible that the offices included in the " Light " experience may have a more stringent standard of selection than the offices as a whole.

One result of this more pronounced selection is that the actual deaths are less than the deaths expected by the A1949-52 table with a two-years' rating down in age. But the financial effect of this difference is small and not sufficient to alter the main conclusion of this enquiry.

Conclusion

A Light table of mortality has been constructed and it is found that monetary functions based upon it correspond fairly closely to similar functions derived from the A1949-52 table with a rating down in age of two years.

The constructed " Light " table shows lower mortality than the actual experience at ages over 70 and the approximation of rating down two years in age in using the A1949-52 table itself gives values somewhat less than by the constructed table. Thus the rating down in age in using the A1949-52 table to represent the " light " experience should be a little less than two years. However, offices must, in any event, have regard to their own experience in deciding upon a choice of table and it has been decided not to proceed with the preparation of monetary functions based upon the "Light" table but to recommend that offices which may require a lighter standard than the A1949-52 table should use that table with an age adjustment appropriate to their particular circumstances.

TABLE 1

Medically examined assured lives 1952-55 : *exposed-to-risk and deaths at durations* 2 *and over with expected deaths according to the A*1949-52 *table, subdivided between* 41 *light offices and* 20 *heavy offices.*

Age Group	41 Light Offices				20 Heavy Offices				All Offices Combined			
	Exposed to risk	Actual deaths	Expected deaths by the A1949–52 table	100 A/E	Exposed to risk	Actual deaths	Expected deaths by the A1949–52 table	100 A/E	Exposed to risk	Actual deaths	Expected deaths by the A1949–52 table	100 A/E
Under $20\frac{1}{2}$	4,861	2	5	40·0	5,602	4	6	66·7	10,463	6	11	54·5
$20\frac{1}{2}$–$24\frac{1}{2}$	38,231	42	43	97·7	37,796	46	42	109·5	76,027	88	85	103·5
$25\frac{1}{2}$–$29\frac{1}{2}$	128,458	98	146	67·1	140,002	136	159	85·5	268,460	234	305	76·7
$30\frac{1}{2}$–$34\frac{1}{2}$	245,095	244	301	81·1	248,255	259	304	85·2	493,350	503	605	83·1
$35\frac{1}{2}$–$39\frac{1}{2}$	354,568	463	555	83·4	346,681	440	543	81·0	701,249	903	1,098	82·2
$40\frac{1}{2}$–$44\frac{1}{2}$	485,400	992	1,220	81·3	461,309	1,063	1,158	91·8	946,709	2,055	2,378	86·4
Under $45\frac{1}{2}$	1,256,613	1,841	2,270	81·1	1,239,645	1,948	2,212	88·1	2,496,258	3,789	4,482	84·5
$45\frac{1}{2}$–$49\frac{1}{2}$	502,473	1,830	2,260	81·0	470,703	1,971	2,115	93·2	973,176	3,801	4,375	86·9
$50\frac{1}{2}$–$54\frac{1}{2}$	405,023	2,790	3,198	87·2	387,309	3,085	3,064	100·7	792,332	5,875	6,262	93·8
$55\frac{1}{2}$–$59\frac{1}{2}$	270,445	3,046	3,589	84·9	301,767	3,992	4,043	98·7	572,212	7,038	7,632	92·2
$60\frac{1}{2}$–$64\frac{1}{2}$	153,555	2,832	3,351	84·5	212,216	4,358	4,622	94·3	365,771	7,190	7,973	90·2
$65\frac{1}{2}$–$69\frac{1}{2}$	86,791	2,701	3,099	87·2	100,643	3,444	3,549	97·0	187,434	6,145	6,648	92·4
$70\frac{1}{2}$–$74\frac{1}{2}$	68,482	3,523	3,943	89·3	61,407	3,502	3,504	99·9	129,889	7,025	7,447	94·3
$45\frac{1}{2}$–$74\frac{1}{2}$	1,486,769	16,722	19,440	86·0	1,534,045	20,352	20,897	97·4	3,020,814	37,074	40,337	91·9
$75\frac{1}{2}$–$79\frac{1}{2}$	49,692	4,200	4,479	93·8	38,696	3,394	3,490	97·2	88,388	7,594	7,969	95·3
$80\frac{1}{2}$–$84\frac{1}{2}$	25,697	3,379	3,539	95·5	19,312	2,651	2,651	100·0	45,009	6,030	6,190	97·4
$85\frac{1}{2}$–$89\frac{1}{2}$	9,517	1,812	1,934	93·7	6,764	1,406	1,371	102·6	16,281	3,218	3,305	97·4
$90\frac{1}{2}$–$94\frac{1}{2}$	2,430	636	692	91·9	1,652	467	472	98·9	4,082	1,103	1,164	94·8
$95\frac{1}{2}$–$99\frac{1}{2}$	293	98	111	88·3	217	69	82	84·1	510	167	193	86·5
Over $99\frac{1}{2}$	29	13	14	92·9	13	6	7	85·7	42	19	21	90·5
Over $74\frac{1}{2}$	87,658	10,138	10,769	94·1	66,654	7,993	8,073	99·0	154,312	18,131	18,842	96·2
All ages	2,831,040	28,701	32,479	88·4	2,840,344	30,293	31,182	97·1	5,671,384	58,994	63,661	92·7

TABLE 2

Light Offices 1952-55 : *Values of* q_x *at durations* 2 *and over*

Age	q_x	Age	q_x	Age	q_x
20	·00089	50	·00494	80	·09836
1	·00089	1	·00556	1	·10740
2	·00089	2	·00624	2	·11714
3	·00089	3	·00699	3	·12758
4	·00090	4	·00780	4	·13877
25	·00090	55	·00869	85	·15071
6	·00090	6	·00967	6	·16341
7	·00091	7	·01074	7	·17687
8	·00091	8	·01191	8	·19110
9	·00092	9	·01319	9	·20607
30	·00093	60	·01459	90	·22176
1	·00094	1	·01613	1	·23814
2	·00096	2	·01783	2	·25514
3	·00098	3	·01968	3	·27277
4	·00101	4	·02172	4	·29090
35	·00105	65	·02396	95	·30947
6	·00110	6	·02643	6	·32841
7	·00117	7	·02913	7	·34762
8	·00125	8	·03209	8	·36700
9	·00136	9	·03534	9	·38645
40	·00149	70	·03891	100	·40588
1	·00165	1	·04281	1	·42518
2	·00184	2	·04709	2	·44425
3	·00207	3	·05176	3	·46301
4	·00234	4	·05686	4	·48135
45	·00265	75	·06242	105	·49921
6	·00301	6	·06848		
7	·00342	7	·07507		
8	·00387	8	·08223		
9	·00438	9	·08998		

TABLE 3

Light Offices 1952-55 (*durations* 2 *and over*): *Comparison of actual deaths with deaths expected by graduated values of* q_x

Age-group	Actual deaths	Expected deaths	100 A/E
$20\frac{1}{2}$–$24\frac{1}{2}$	42	34	123·5
$25\frac{1}{2}$–$29\frac{1}{2}$	98	117	83·8
$30\frac{1}{2}$–$34\frac{1}{2}$	244	240	101·7
$35\frac{1}{2}$–$39\frac{1}{2}$	463	442	104·8
$40\frac{1}{2}$–$44\frac{1}{2}$	992	976	101·6
$45\frac{1}{2}$–$49\frac{1}{2}$	1,830	1,847	99·1
$50\frac{1}{2}$–$54\frac{1}{2}$	2,790	2,668	104·6
$55\frac{1}{2}$–$59\frac{1}{2}$	3,046	3,036	100·3
$60\frac{1}{2}$–$64\frac{1}{2}$	2,832	2,854	99·2
$65\frac{1}{2}$–$69\frac{1}{2}$	2,701	2,652	101·8
$70\frac{1}{2}$–$74\frac{1}{2}$	3,523	3,388	104·0
$75\frac{1}{2}$–$79\frac{1}{2}$	4,200	3,867	108·6
$80\frac{1}{2}$–$84\frac{1}{2}$	3,379	3,074	109·9
$85\frac{1}{2}$–$89\frac{1}{2}$	1,812	1,695	106·9
$90\frac{1}{2}$–$94\frac{1}{2}$	636	612	103·9
$95\frac{1}{2}$–$99\frac{1}{2}$	98	100	98·0
	28,686	27,602	103·9

TABLE 4

Light Offices 1952-55 : *Analysis of the experiences at durations* 0 *and* 1

Duration t	Age group	Actual deaths in the 1952-55 " Light " experience	Comparison with age $(x-2)$ at the corresponding duration t in the A1949-52 table		Comparison with the " Light " table at durations 2 and over		Weighted mean percentage of $q_{[x-t]+t}$ to q_x from the A1949-52 table
			Expected deaths	100 A/E	Expected deaths	100 A/E	
0	Under $45\frac{1}{2}$	101	130	78	182	55	57
	$45\frac{1}{2}$–54	76	90	84	181	42	51
	$55\frac{1}{2}$–$69\frac{1}{2}$	36	56	64	121	30	47
	Total	213	276	77	484	44	52
1	Under $45\frac{1}{2}$	134	143	94	177	76	71
	$45\frac{1}{2}$–$54\frac{1}{2}$	103	129	80	191	54	69
	$55\frac{1}{2}$–$59\frac{1}{2}$	84	94	89	149	56	66
	Total	321	366	88	517	62	69