

CONTINUOUS INVESTIGATION INTO THE MORTALITY OF ASSURED LIVES

MEMORANDUM ON A SPECIAL INQUIRY INTO THE DISTRIBUTION OF DUPLICATE POLICIES

THE Continuous Investigation into the Mortality of Assured Lives is based on *policies* and not on *lives*. Consequently the death of a policyholder carrying n policies appears as n deaths in the data. In the absence of information on the distribution of policies per life assured, it has been difficult to make satisfactory estimates of the standard deviations needed to test differences between actual and expected deaths. For previous estimates and techniques, see Redington and Michaelson (*Transactions 12th International Congress*), Daw (*J.I.A.* 72, 174), Solomon (*J.I.A.* 74, 94), Beard and Perks (*J.I.A.* 75, 75) and Beard (*J.I.A.* 77, 382).

2. Accordingly, acting on a suggestion originally put forward by Perks (*J.I.A.* 77, 427) the Joint Mortality Investigation Committee decided to carry out a special investigation into the distribution of these *duplicate* policies. Offices which contribute data to the investigation were invited to complete a return in respect of policies becoming claims by death during 1954, indicating at each age the numbers of lives having 1, 2, 3, ... policies. The return related to policies at durations 3 and over which qualified for inclusion in the Continuous Investigation. There was no subdivision according to class of business. In the event, returns were received from 62 out of 63 contributing offices, the exception being an office which represents a little over 1 per cent of the data.

3. The special investigation was limited to the study of duplicate policies *within* offices. It would have been difficult to trace duplicates occurring in different offices, *i.e.* as the result of an individual life being assured with more than one office.

4. The numbers of lives with n policies in the same office, for the combined data, are shown in quinary age groups in Table 1. Table 2 shows the same particulars in a different way; in each age group, and for each value of n , the number of lives with n policies is shown as a percentage of all the lives in that age group. It will be seen that the percentage of lives with only one policy starts in the nineties at the youngest age group, falls steadily to a minimum of a little over 80% in the group 45-49, and then rises gradually until it again exceeds 90% from age 85 onwards. Conversely, the percentages with two and three policies have their maximum in the age group 45-49. For the larger numbers of policies, the data are too scanty for any discernible trend to be noted, apart from the fact that the weight of the data (such as it is) is toward the middle of the table.

5. It was to be expected that the number of lives with only one policy would be relatively high at the youngest ages, and would tend to drop as the lives became older; also perhaps that this trend would cease after the ages when endowment assurances started to mature. But the precise level of the percentages, and the point at which the minimum occurred, could scarcely be guessed (let alone estimated), and for this reason it is thought that the table

might prove useful if published. It is also thought that any attempt to make allowance for duplicates would have to be on an age basis; the table shows quite clearly that the same allowance could not accurately be made at every age.

6. If this investigation is to be put to practical use, it will be necessary to translate the above into a suitable measure to indicate how the variance of a deviation is affected by the presence of duplicates. Daw (*J.I.A.* 77, 261) and Beard and Perks (*J.I.A.* 75, 75) have shown that the ratio of the variance where there are duplicates to a straightforward binomial variance is

$$\frac{Nqm_2 - Nq^2m_1^2}{Npqm_1}$$

where m_1 and m_2 are the first and second moments about zero of the duplicates distribution,

which reduces to $m_2/pm_1 - qm_1/p$. The effect of taking p as unity and q as zero is to understate both the positive and the negative terms, and the error tends to cancel out. The 'variance ratio' has been calculated for part of the data, first using the above formula accurately (employing p 's and q 's from the new 1949-52 Table) and secondly assuming that the ratio may be taken as m_2/m_1 ; the error in the latter approximation is practically negligible and it has therefore been considered sufficient, since graduated p 's and q 's are not available for the 1954 experience, to take the variance ratio as m_2/m_1 . On this basis, the variance ratios have been calculated in age-groups from the total data and are shown below. A further series of ratios is also shown, derived from the data for non-Industrial offices reporting not less than 300 policy claims. This further series is intended as a maximum estimate of the variance ratio, it having been ascertained that non-Industrial offices show a higher incidence of duplicates within the experience than Industrial offices and that, within the non-Industrial offices, the incidence of duplicates increases with size of office. The values of m_1 and m_2 , and the variance ratios are as follows:

All offices				'300 and over' non-Industrial offices			
Age group	m_1	m_2	Variance ratio	Age group	m_1	m_2	Variance ratio
Under 25	1.051	1.154	1.10	Under 35	1.103	1.462	1.33
25-29	1.093	1.280	1.17				
30-34	1.155	1.534	1.33				
35-39	1.182	1.612	1.36				
40-44	1.217	1.799	1.48				
45-49	1.258	2.097	1.67				
50-54	1.222	1.887	1.54				
55-59	1.197	1.730	1.45				
60-64	1.172	1.656	1.41				
65-69	1.176	1.676	1.43				
70-74	1.198	1.835	1.53				
75-79	1.178	1.727	1.47				
80-84	1.147	1.533	1.34				
85-89	1.101	1.342	1.22				
90 and over	1.106	1.380	1.25				
All ages	1.186	1.725	1.45	All ages	1.223	1.937	1.58

Table 1. Numbers of Lives, in quinary age groups, with *n* Policies in the same office included in the 1954 Death Claims (Assured Lives, All Sections, Durations 3 and over, All offices)

Nearest age	<i>n</i>								Total number of lives
	1	2	3	4	5	6	7	Over 7	
Under 25	37	2	—	—	—	—	—	—	39
25-29	97	10	—	—	—	—	—	—	107
30-34	202	26	2	2	—	—	—	—	232
35-39	276	40	8	1	—	—	—	—	325
40-44	606	92	20	4	2	1	—	—	725
45-49	1,113	191	45	5	5	3	—	2	1,364
50-54	1,942	307	55	15	8	—	—	2	2,329
55-59	2,480	371	69	12	4	1	—	1	2,938
60-64	2,122	261	45	8	4	2	2	1	2,445
65-69	1,383	171	26	8	3	2	2	—	1,595
70-74	1,431	151	36	12	7	4	3	—	1,644
75-79	1,460	154	26	11	11	—	1	1	1,664
80-84	1,316	135	23	8	2	1	—	—	1,485
85-89	781	61	8	3	—	—	—	—	853
90-94	289	20	4	—	—	—	—	—	313
95 and over	66	5	2	—	1	—	—	—	74
All ages	15,601	1,997	369	89	47	14	8	7	18,132

Table 2. Percentage of Lives in each age group with *n* Policies in the same office included in the 1954 Death Claims (Assured Lives, All Sections, Durations 3 and over, All offices)

Nearest age	<i>n</i>							
	1	2	3	4	5	6	7	Over 7
Under 25	94.87	5.13	—	—	—	—	—	—
25-29	90.65	9.35	—	—	—	—	—	—
30-34	87.07	11.21	.86	.86	—	—	—	—
35-39	84.92	12.31	2.46	.31	—	—	—	—
40-44	83.59	12.69	2.76	.55	.28	.14	—	—
45-49	81.60	14.00	3.30	.37	.37	.22	—	.15
50-54	83.38	13.18	2.36	.64	.34	—	—	.09
55-59	84.41	12.63	2.35	.41	.14	.03	—	.03
60-64	86.79	10.67	1.84	.33	.16	.08	.08	.04
65-69	86.71	10.72	1.63	.50	.19	.13	.13	—
70-74	87.04	9.18	2.19	.73	.43	.24	.18	—
75-79	87.74	9.25	1.56	.66	.66	—	.06	.06
80-84	88.62	9.09	1.55	.54	.13	.07	—	—
85-89	91.56	7.15	.94	.35	—	—	—	—
90-94	92.33	6.39	1.28	—	—	—	—	—
95 and over	89.19	6.76	2.70	—	1.35	—	—	—
All ages	86.04	11.01	2.04	.49	.26	.08	.04	.04