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Contributions to the History of Insurance, and of the Theory of Life Contingencies. By FREDERICK HENDRIKS, ESQ., *Actuary of the Globe Insurance Company.*

(§ 24.) IN my previous sketches upon the subjects embraced in the above title, whilst, on the one hand, I was desirous to avoid burdening them with any apparently superfluous details, on the other hand, I clearly foresaw that the task would remain of following out, in at least another article, some needful explanations or justifications. To my former readers I therefore inscribe the following notes, in the anticipation of their being found not wholly unserviceable in the view just adverted to.

I may be permitted to mention that the publication of the treatise of De Wit on Life Annuities, has excited some attention in Holland; inasmuch as, whilst the love of scientific inquiry might alone have made it worth our while, as Englishmen, to forward a search for the hitherto missing document, this, in the case of our Dutch friends, acquires a stimulus in the degree of legitimate pride which they take in all that can illustrate the fame of their martyr-statesman, De Wit; and I make the assertion advisedly, that his Annuity Treatise, which the cursory reader might be disposed to look on as but a light matter in relation to that fame, is by no means so regarded by writers of eminence in Holland—men whose position and qualifications best fit them to express an opinion which is worthy of all respect on the subject.

The fresh details I am enabled to offer respecting this, in continuation of the evidence already adduced from other times and

countries, will be perceived to be usefully connected with the further facts which I shall have to present in the sequel.

The reader who recollects what is stated in the notes to sections 22 and 23 of my former paper, will understand that there was then fair presumptive evidence for assuming that the official intercourse between the Grand Pensionary De Wit and the Burgomaster Hudde led to a community of interest in establishing a properly devised national scheme of life annuities; and not only is this substantiated by subsequently acquired testimony; but the collaboration of these two remarkable men, in their studies as *financiers and actuaries*, is incontrovertibly proved by the discovery of the MS. drafts of several letters and memoirs on the subject, in the *autograph of De Wit*, preserved amongst the archives of the Netherlands, in the State collection at the Hague. For copies of these, (searched for since the publication of my paper on De Wit,) and certified by the Master of the Records, I am indebted to the polite attention of Mr. J. Heemskerk, member of the Second Chamber of the States General.

(§ 25.) To proceed with the evidence on the degree of importance attached in Holland to De Wit's treatise.—The Baron Collot d'Escury, in his work entitled *The Glory of Holland in the Arts and Sciences*,* writes in the following terms respecting De Wit (*vide vol. vi., p. 106*):—

“ He knew how to apply his scientific acquirements to the interests of the public, and even to reconcile his inclination for the sciences with the obligations of his official functions, and make it conduce to the utility of the country. Hence the so important Treatise upon the Value of Life Annuities in proportion to Redeemable Rents (*Over de Waerdye van Lyfrenten, naer proportie van Losrenten*), published in 1671, and the object of which was to demonstrate that it was more advantageous to the State to borrow by way of life annuities than by redeemable rents. It however reciprocally counselled the inhabitants to give the former the preference over the latter, in the supposition that they would know how to comprehend their domestic interests, *by employing industriously*, as he says, *the surplus or excess over the ordinary rents in the accretion of their capital*; the guarantee of such a course being secure in the then universally observed economy. From that moment people were led to scrutinize the rules of the calculus of probabilities; which were in existence, it is true, but the application of which could not be made so easily or with such certainty as at the present day, now that progress has been attained in the knowledge of the laws of mortality;† and this is why *De Wit acquired a great*

* H. Baron Collot d'Escury.—*Hollands roem in Kunsten en Wetenschappen*. 'S Hage en Amsterdam, gebr. v. Cleef. 1835.

† From the most remote times the Egyptians, Jews, Greeks, and Romans bestowed a scrupulous attention upon the births and deaths of mankind, although with the two

renown through this work, which demanded so much judgment and penetration. And it also to be regretted that the treatise of De Wit was impossible to be obtained anywhere, so that Leibnitz, being in this country, was unable to see it; which however appears to me incomprehensible, since in my judgment Hudde ought to have been able to give it to him. If this great man (De Wit) had devoted himself entirely to the cultivation of the sciences, without having to fill other duties which demanded all his care, he would have shone in them with the same brilliancy which history represents him to us as diffusing upon the theatre of politics—a lustre which has excited the respect and admiration of all Europe!”

(§ 26.) *Simons*, in his work on *John De Wit and his Age*,* has the following remarks (see pp. 42, *et seq.*):—

“Amongst the works which have been preserved to us through the care of Van Schooten, is the Treatise of De Wit, entitled by him *Elementa Curvarum Linearum*,—one of the most remarkable mathematical works of his time.

“Another work of De Wit, less known, but however more celebrated, is that on *The Value of Life Annuities in proportion to Redeemable Rents*, published in 1671 at the Hague, in folio, by Jacobus Scheltus, Printer in Ordinary to the States of Holland and West Friesland.

. . . . De Wit first gives the rules of the calculus of chances by simple examples, with their explanation, that is to say, to as great an extent as is necessary for his object. They are the rules which Huygens had already developed in his treatise, *De Ratiociniis in Ludo Aleæ*; but the application of these rules to the calculation of the value of life annuities is the work of De Wit, and he is, in my opinion, the first who made so useful an application of the doctrine of chances. The law of mortality was then almost entirely unknown; at that time one could only base such calculations upon more or less probable suppositions. If we

latter people the principal object was to learn the number of men fit for war; the early Christians doing the same, with the design of knowing the number of neophytes (*nieuw-bekeerden*), baptized persons, apostates, and ecclesiastical elders. It is true that the practice was continued subsequently on the spur of other circumstances; but the keeping of better established registers of births and deaths does not generally extend back beyond the commencement of the seventeenth century, as has been the case in this respect in our country. However, at the time of De Wit, a great enough advance had not yet been made in this subject to warrant congratulation on the possession of sufficient observations and calculations upon the proportion of deaths to births; and it was not until the last half of the eighteenth century that the laborious Süssmilch came forward in a voluminous work (which we are acquainted with in our country under the title of *De Goddelyke Orde, heerschende in de veranderingen van het menschelyke geslacht*), with the express view of providing against the want of such calculations; and hardly had this want been remarked, before the author had numerous imitators amongst his compatriots, which imitators have written in the like spirit. With us we had, in the same century, Kersseboom, Nicholas Struyck, and Houttuyn, who all applied themselves to this work. Struyck very modestly calls his treatise, which appeared in his *Introduction to Universal Geography*, ‘*Gissingen*’—i.e., *conjectures*—on account, says he, of the uncertainty of his researches; he constantly, however, expresses a hope that further astonishing things will be discovered therein. As regards the rest of this subject, read especially what relates to it in the *Historical Introduction to the Essay to establish Mortuary Registers*, &c. on the most appropriate footing (*Geschiedkundige inleiding van het Onderzoek naar de doelmatigste inrigting van Sterflijsten enz.*), for which Professor Suringar, of Amsterdam, obtained a prize of honour.—(C. d’E.)

* “Jan de Witt en zyn Tyd.”

consider the slender progress which the calculus of probability had then made, we must look upon the whole treatise as an eminent proof of the inventive genius of its celebrated author; and Hudde, who at the request of the Grand Pensionary had examined this work, and who certainly was a competent judge, declares that it is based upon solid mathematical principles.

“De Wit draws from his calculations a conclusion that each florin of annual life rent is worth, in proportion to redeemable rents, at four per cent. per annum, more than sixteen florins. In order to prove in a more precise manner the justice of his conclusions, De Wit had recourse to a method which is above all reflection, namely, the extraction from the *Registers* of the particulars of some thousands of lives upon which life annuities had been purchased, with notes upon the amount of life annuity paid to each of them, and in this manner he also demonstrates that each florin of annual life rent was worth at least sixteen florins.

“This Treatise upon the Value of Life Annuities is very rare, and seems to have been, even at its first appearance, difficult to be obtained; and although Leibnitz wished to possess it amongst his papers, it appears nevertheless that he could not obtain a sight of it. *Bernouilli* (at that time Professor at Groningen) wrote to Leibnitz—‘*Schediasma Pensionarii de Wit circa redditus ad vitam vix putem reperiri posse. Agnatus aliquis meus, qui nuper ex Batavis venit, se illud ibi reperire non potuisse mihi retulit.*’

“These two treatises are the only mathematical works which De Wit has left us; they suffice nevertheless to obtain for him a honourable rank in that science, and abundantly prove that, had it not been for pre-occupations of more paramount interest, he would have ranked amongst the first mathematicians of his day, and would perhaps have thus rivalled his countryman Huygens.’”

(§ 27.) The following extract, translated from the Baron *Collot d'Escury's* work* before cited, will be found most useful and important in arriving at a correct view respecting what will appear in the latter part of this paper as respects *Hudde's* share in the annuity calculations of De Wit:—

“Witsen has with justice called Hudde (whom Francius, in his discourses, once complimented as the Dutch Archimedes) ‘the incomparable mathematician.’ Was not this honourable title deserved by him on account of the calculations respecting the pressure of ships in the water, and to the making of which calculations gave rise the difference with Denmark, concerning the tonnage at which were to be estimated the vessels sailing to Norway to bring home timber? But there are other proofs, worthy of the praise which Hudde obtained in the midst of his fellow citizens, and especially among strangers.

His public functions, his activity in state and municipal affairs, may have hindered him, in the exact sense of the word, from publishing voluminous works. What we have learnt respecting him, what we know of him, suffices for our appreciating and ranking him amongst our most celebrated mathematicians. Those who know the works of Van Schooten,

* C. d'E. Hollands Roem, &c., 6th part, section 1, p. 100.

are acquainted with several productions of his youth, all bearing the evidences of a more than common penetration of mind. One of the subjects to which he most applied himself was 'Algebraic Equations,' a subject possessing an interest both great and general, and upon which one had, it is true, fixed rules at his time, but not sufficiently numerous, for which reason he essayed to increase them by new ones, and would perhaps have given them the needful extension, had he been able to bring out the great work which he had intended to publish in this regard. One may form some idea of it from the letters which Van Schooten communicated in 1659, and from what we may presume would have been contained in the Commentaries which he showed to Leibnitz, in the visit which the latter paid him at Amsterdam.* Has not a man who, like Hudde, from the year 1667 to his death in 1704, passed his life in the government of his country, applied his extraordinary knowledge with utility to his nation? This has been evidenced by *his calculations respecting life annuities* and loans of money which had to be raised, as well as by his labours for the good of the finances of the State. For when in 1678, by virtue of the proposition of the Grand Pensionary, a Government Commission was named (as has been done several times subsequently) to examine the condition of the pecuniary resources of the country, and to indicate the means of ameliorating them, he was chosen a member of this commission, whose report only requires to be gone through, for one to recognise in it his wise co-operation.† It is however to be regretted that Hudde, following the example of so large a number of our forefathers, seems not to have taken a sufficient care in the preservation of his manuscripts, otherwise we should certainly be acquainted with much more of his merit as a mathematician. It is said that the late celebrated Van Swinden entertained the project of making us more intimately acquainted with this illustrious countryman, but he was prevented by default of materials, probably lost, although it is presumable that they are still preserved somewhere or another, hidden or forgotten, until the time when a fortunate chance may one day lead to their discovery, which would be of the highest interest, although one might presume that the greater portion would consist of unfinished labours, in consequence of the active life of Hudde, who was twenty times burgomaster of Amsterdam."‡

* See the Commentaries of Van Schooten. The letters which the latter published under the title *Jo. Huddenii de Reductione Aequationum et de Maximis et Minimis*. Letter II. And Montucla, *Histoire des Mathématiques*, tom. ii., page 149, et seq.—(C.d'E.) The reader should also refer to Professor De Morgan's paper in the *Companion to the Almanac* for 1853, pages 12 and 13.—(F. H.)

† The resolution is of the 24th February, 1678, and the report was rendered to the Assembly of Holland of the twenty-fifth November following. The proposition, the resolution, and the report, are to be found in the Statements and Memoirs upon the Finances of Holland, published in 1797 by Messieurs van Herzele and Goldberg.—(C.d'E.)

‡ As remarks Professor van Lennep, in his Memoirs (*Memorabilia*) of the Athenæum of Amsterdam, page 106, where we read [*Huddeus*]=Joannes Hudde, anno 1667 Civitatis Amstelodamensis Scabinus, anno 1668 Senator, et circa idem tempus etiam Athenæi curator factus, mox effecit, ut Raulis Philosophiæ Professor Amstelodamum Leida vocaretur, qua de re d'Orvillius infra dicit, p. 41. Consul factus est demum Huddenus in illa rei publicæ conversione, anno 1672. Inde autem vicies fungens eo munere semper promovendis Athenæi commodis intentus fuit; et erat ipse vir, doctissimus, præsertim mathematicis, neque indignus, qui Archimedes nostras diceretur Francio in Oratione xxiii., p. 308, et hinc d'Orvillio, l. c. Juvenis scripta ediderat mathematica, de Reductione Aequationum et de Maximis et Minimis; atque eum multa in mathema-

(§ 28.) Having premised thus much respecting the historical and traditional views held until a recent period on the subject we have been examining, it now devolves on me to offer the following translations of the inedited letters to which I have before referred, as having been so obligingly transmitted for publication.

The letters tell their own tale so well, that there will be no reason for departing from the rule I have considered it right to adhere to in my former papers—namely, to put the reader in as *full possession as possible of facts, data, and collateral testimony*, and to leave his judgment unbiassed by extraneous arguments or dogmatic assertions.—

LETTER I.

“ The Hague, September 27, 1670.

“ SIR AND BROTHER,—Your missive of yesterday has reached me to-day, with the document belonging to M. Jean Heufft annexed thereto ; and I leave it to your judgment and discretion as regards the delay about the reimbursement of the portion recouped to Messrs. Wuytiers, on account of the great difference in bank money. But in case the probability of a fall in bank money be not considerably greater than that of a rise, it seems to me that the money of Messrs. Wuytiers should be paid them as early as possible ; at least, I am disposed to think they should already be advised of a redemption being near, which notice it seems to me that politeness and propriety demand. It will also be agreeable to me that M. Abraham Alewyn’s portion be redeemed in 6 months, and I shall then await fuller information that the aforesaid redemption has been effectually carried out.

“ I return you enclosed herewith your obligation of 3,200 florins, the interest upon which of 128 florins I have received, and credited you therewith. I have also caused the Receiver-General Pauw to be informed that the interest upon it will, in future, be received by me, so as better to prevent the redemption ; but I have not ventured to make the transfer of the obligation into my name, because I could not have affirmed with truth that it belonged to

ticus primum invenisse ipse testatus est Leibnitzius, qui anno 1674 Hollandiam invicens familiaritatem cum eo contraxerat. Illam autem scientiam suam Huddenus sæpe in utilitatem urbis patriæ adhibuit, ut in defæcendo per annum 1671, et seq. Amstela, quam operam ejus Francius celebravit dicta oratione et poematum, p. 243. Diem suum obiit 15th Aprilis, 1704, relictis plurimis scriptis mathematicis, quæ quominus absolveret ac vulgaret publica eum negotia prohibuerant. Scripta illa certum habeo adhuc alicubi condita seruari, unde optandum est, ut aliquando in lucem proferantur.”—(C. d’E.)

me as proprietor; but I hope that the aforesaid notification will have the same effect.

“ I could very usefully employ here the money which you have still in hand, after the said two redemptions; and if I persist in my resolution to redeem the portion of M. Sylvester Heereman of 25,000 florins, falling due the 1st November proximo, I should wish for a like capital to be contracted for several years, at much about the same period, at the rate and at the conditions which are perfectly expressed in the accompanying memoir, as well as regards the subscribers as the brokers; to which I have also added two formulæ of obligations, which could be prepared for this purpose, filled up in full, with the exception of the name and quality of the subscribers, so that after the conclusion of the transaction there may not remain any misunderstanding as to the extent of the obligations.

“ And if, contrary to my anticipation, such a contract should not suit, I beg you to have the same portion negotiated for twelve months at three per cent., upon the other formula not filled up, and added annexed hereto. Waiting for your answer in due time upon this subject, I conclude, and remain, &c. &c.

“ To M. Jean Deutz.”

“ MEMOIR ABOVE REFERRED TO.

“ To produce the accumulation of any given capital for minors or others, or to invest it for a certain number of years with a good and assured profit, a large sum of money is offered to be reinstated by a doubled capital at the end of 24 years; by the capital, and 76 per cent. gain in addition, at the end of 20 years; and, similarly, 62 per cent. profit at the end of 17 years; 53 per cent. at the end of 15 years; $40\frac{1}{2}$ per cent. at the end of 12 years; or 33 per cent. at the end of 10 years.

“ For perfect assurance of which, there shall be remitted as guarantee, and in pledge to the subscribers, a fifth more than the capital paid up, and this to be in settled obligations; and if it be for a somewhat long period, the said guarantee shall augment every 5 years by a same fifth of the original capital—that is to say, by exactly the fifth of the sum which shall have been remitted at first in obligations given in pledge; so that if the obligation be made for 24 years, one will have accumulated at the end of the 20th year a doubled capital in guaranteed obligations, and consequently the whole sum for reimbursement at the termination of 24 years.

“A similar proportion will be observed relatively to a smaller term of years, so that the subscribers of funds will always have in advance, in fixed and guaranteed obligations, a good portion more than they have furnished, or than they could lay claim to. And as to the broker’s allowance, it will be calculated at the rate previously in use, viz., one per mille per annum, and will be paid in advance on the spot, with discount in the same proportion as the rate for the capitals paid up; so that the broker will receive at once, and in advance, 1,200 florins in ready money, if the value of the investment be 100,000 florins, and contracted for 24 years; if it be for 20 years, he will receive 1,136 florins; for 17 years, 1,050 florins; for 15 years, 980 florins; for 12 years, 854 florins; and for 10 years, 758 florins: and for greater or lesser sums in the like proportion.

Moreover, a large sum in life annuities is offered under guarantee and pledge, as above, with the promise that if the person on whom the life annuity shall be constituted survive to the expiration of 12 years complete, the holder of the title is to be granted another full capital, and at the expiration of 24 years complete, 3 capitals, with this condition added, that should the subscribers of funds wish to be recouped their capital at the end of the first 12 years (the person on whom the life annuity is granted being still living), the annuitant, or he that has been receiving the aforesaid capital in life annuities, will be held bound to reimburse it him, less the rebate or deduction of a sixth of the capital, to provide for the diminution of the value of the invested rent caused by the increasing age of the person on whose life the annuity is constituted; at the expiration of 24 years with deduction of $\frac{1}{6}$ of the whole, and moreover the $\frac{1}{6}$ of the remaining $\frac{5}{6}$, that is to say, the $\frac{1\frac{1}{6}}{3}$ of the whole; at the expiration of 36 years or thrice 12 years, with deduction of $\frac{1\frac{1}{6}}{3}$ of the whole, and in addition also the $\frac{1}{6}$ of the remaining $\frac{2\frac{5}{6}}{3}$, that is to say, the $\frac{2\frac{1}{6}}{3}$ of the whole; at the expiration of four times 12 years, or 48 years, with deduction of $\frac{2\frac{1}{6}}{3}$ of the whole, and over above the same, the $\frac{1}{6}$ of the remaining $\frac{1\frac{2}{3}}{3}$, that is to say, $\frac{6\frac{6}{9}}{1\frac{2}{9}}$ of the whole; at the expiration of five times 12 years, or 60 years, the $\frac{1}{3}$ again above the aforesaid value; at the expiration of six times 12 years, or 72 years, similarly, and the $\frac{1}{4}$ of the remainder of the value in addition; at the expiration of 7 times 12 years, or 84 years, the same, and in addition $\frac{1}{5}$ of the remainder of the value; at the expiration of 8 times 12 years, or 96 years, in the same way, with $\frac{1}{2}$ of the

remaining value in addition ; at the expiration of 9 times 12 years, or 108 years (if the life be still existing), in the same way, and $\frac{2}{3}$ besides of the remaining value ; and in case of further life, $\frac{2}{3}$, $\frac{2}{3}$, &c.

"The power of redeeming the capital also remains constantly with the annuitant."*

LETTER II.

"The Hague, August 2, 1671.

"SIR,—Your esteemed missive of the last ult. has come to hand with the tables which accompanied it, and I feel myself bound to thank you heartily for the same, as also for the communication of the double question contained in your preceding letter.

"I have perfectly understood the estimation of the value of life annuities upon one life, computed from the life and death of 96 persons, included in the column for the age of 6 years, and I find it pertinent and conclusive ; but you will oblige me by informing me whether the computation of the value of 19 florins 16 stuyvers and 9 pennings of life annuities upon two lives, has also been made according to certain cases of several life annuities granted upon two lives ; or else whether it has been combined according to lot, or by some other method of drawing, or, in fine, whether you have computed the augmentation of 2 florins 12 stuyvers and 6 pennings by some certain calculation depending upon the primarily found value of the life annuity upon one life, and in what manner this calculation could be performed.

"But as there is a general persuasion that the life annuity upon two lives, at 17 years' purchase, is much more advantageous than that upon one life at 14 years' purchase ; and that it may even be, that the life annuity upon two lives, if sold at 18 years' purchase, would be even then preferred to that upon one life at 14 years' purchase ; as this might produce a notable advantage to the republic, it is, in my opinion, of the highest importance to leave people in this persuasion ; therefore I have not divulged it to anyone except yourself, that according to my calculation (since

* There is a marginal note in De Wit's handwriting against the first part of this Memoir, with the computations checked by rule of three in each case two or three times over, by different abbreviations. The substance of the note is to observe that "Upon 100,000 florins there being paid annually to the broker 1 per mille, or the sum of 100 florins, consequently 2,400 florins for 24 years, which being reduced to present value (in the proportion first referred to, i.e. as to doubling of capital) gives 1,200 florins ($200 : 100 :: 2400 : 1200$); for 20 years, $176 : 100 :: 2000 : 1136\frac{4}{11}$ florins; for 17 years, $162 : 100 :: 1700 : 1049\frac{3}{8}\frac{1}{1}$ florins; and so on, for each term of years mentioned in the body of the Memoir.

I remark that yours, upon two lives, is even lower still) the purchase upon one life at 14 years' purchase is more advantageous than that upon two lives at 17 years' purchase, and that is why I leave you to consider whether you do not judge it to be useful for the public good that this estimate should be absolutely hidden, and people left in their ancient persuasion, for the advantage of the State finances ; because I am convinced, on this subject, that they will not be put in the track by a calculation made in round numbers, and probably no one will make a precise calculation for them ; we may then act thus in every case and with a good conscience, so long a time as, when purchasing upon two lives, the country treats them still more advantageously than in the case of a revenue from an ordinary redeemable rent, at the usual rate.

"I submit myself besides in this respect to your better opinion and approval ; and I remain he who is truly yours, &c.

"P.S. I will have copied for you, at the first opportunity, the great table of discount and of reduction to present value.

"To M. Johan Hudde, Councillor and Ex-Sheriff*
of the City of Amsterdam."

LETTER III.

"The Hague, 7th August, 1671.

"SIR,—On the 2nd instant I acknowledged receipt of your missive of the last ult., as well as of the table and the estimate which accompanied it, adding thereto, that I should have been glad to have been informed of the method which you have followed in the calculation of the value of life annuities upon two lives ; and I shall be delighted to have your instructions on that head. In waiting for it, I hand you annexed a copy of the Table of Discount, which I promised you in my former letter, and with the help of which one may calculate, without much trouble, how much the life annuities in every class which you have extracted from the respective books have been worth in ready money to the annuitants.

"Since your departure I have thoroughly calculated ten different classes of lives, from the age of one year to ten years inclusive, and I find the same result as in the note enclosed.

"The number which you have recently proposed respecting the ratio between the chances of two players at (the game of) *quinque*

* Oudt Schepen—Gallicè, Ancien Echevin. Query—Ex-Treasurer?

novem agrees literally with the one which I had already calculated in that regard, and included in the note which was recently shown to you here ; so that we may be mutually confident that there is no error in the computation. In conclusion, I remain, &c., &c.*

“ To M. Johan Hudde, Councillor and Ex-Sheriff
of the City of Amsterdam.”

LETTER IV.

“ The Hague, 24th August, 1671.

“ SIR,—Having, contrary to my anticipation, remarked in your notice (which you had conveyed to me on the 15th instant) that your computation upon the class of lives of the age of 7 years only brings out 15 florins 2 stuyvers 13 pennings as the value of one florin of annual life rent, upon the 77 lives constituting the aforesaid class and taken one with the other, I have had a large number of classes calculated, and a greater number of lives, and never in any class has the value been under 16 florins.

“ After the closing of the Assembly of their Noble and Mighty Lordships, I calculated the aforesaid class with the extremest care, at first for terms of complete years and with the discount at interest of 4 per cent. proportioned thereto (and this in the same manner as all your calculations are performed), and I have obtained as the result, 15 florins $15\frac{1}{2}$ stuyvers, as I beg you to remark in the calculation annexed under No. 1 ; and having subsequently computed it in half-yearly terms, I have obtained 15 florins $18\frac{11}{100}$ stuyvers as the result, as you will remark in No. 2, enclosed herewith ; and as it may as easily have happened that the annuitants, when they have reached the termination of their lives, and been registered for complete years, shall have died in the last six months of the year following as in the first six months of the same year, we ought, in order to estimate the matter with the utmost precision, to compute how much the said further value

* The note referred to is not copied in the minute.

The above short letter affords the most conclusive of proofs that De Wit's and Hudde's studies respecting annuities were entered upon consentaneously with or rather immediately consequent upon their attention being directed to the discoveries of the laws of chance or probability, exemplified in the then unpublished but already celebrated labours of Pascal and Fermat, on those laws applied to games of hazard ; and which were the more familiar to *De Wit* and *Hudde* through the treatise of their countryman *Huyghens* (*De Ratiociniis in Ludo Aleæ*).

We may be sure it would have rejoiced the great Hanoverian Philosopher (whose words I quoted in my former paper) if he could have seen this direct illustration by De Wit of the truth of that favorite theory of his to which those words advert.

An explanation of the problem of the game of *Quinque Novem* will be found in the work of *Remond de Montmort*, p. 173.

amounts to, by presupposing that out of each even number, 2, 4, or 6, there die respectively 1, 2, or 3 lives in the last six months of their year of decease, according as the division may be conveniently made free from fractions; which having also done, I have then found that one florin of annual rent would have produced for each of the aforesaid 77 lives, taken one with the other, a value of a little more than 16 florins, as you will observe more in detail in the calculation herewith annexed and numbered No. 3.

This is what has induced me to have the 9 other classes also calculated, each with discount and in half-yearly terms; and the result has been found such as you will be so good as to remark in the note herewith enclosed, No. 4, and to observe that on one side or the other a slight mistake has been made in the classes of lives aged 2, 3, 8, and 9 years; but as I have not personally computed the latter classes, I should not venture to speak of them so positively as of the preceding class of lives of the age of 7 years, which I have myself calculated by all the methods above indicated; however, in case no error has been committed in the other aforesaid classes, all those classes, the one taken with the other, have each the value of 17 florins 9 stuyvers; to which must further be added 2 stuyvers for the augmentation arising from what, as above said, one may justly presuppose from there dying in the last six months of the year of decease as many lives as in the first six months. All these annuitants may then be presupposed to have enjoyed a half-year's more of annuity; so that they will, in effect, each have drawn 17 florins 11 stuyvers for the value of 1 florin of annual rent (in the case when they have been paid in half-yearly instalments).

"Your missive of the 9th instant reached me in due course. I thank you for it very sincerely, and remain, &c.

"To Johan Hudde, Councillor, &c."

N. B.—The notes (above referred to) numbered 1, 2, 3, and 4, are not annexed to the draft of this letter. The following memorandum forms a postscript:—

"I also remark, that in your table ascending to the age of 80 years, the last number is always taken a little too small, which progressively amounts to this much—that my table for complete years produces, for the 80th year, 2,304,980, whilst the same item in the computation of yours is only 2,304,848 or (*minus*) 123,* which cannot, however, cause but a very little difference in the life annuities."

* *Sic.*

LETTER V.

"The Hague, October 20, 1671.

"SIR,—Since your departure from here I have made some more mature reflections, both upon what you have communicated to me respecting the proportion found from experience in the mortality* of lives from year to year, and upon the method of calculating a life annuity upon *two lives*, which I have briefly explained to you; and, as regards the first subject, I remark (contrary to my expectation) that the equation of mortality from year to year may with confidence be defined as ascending from the age of 50 to that of 75 years; and it is found that, if one chooses several lives, aged 50 years, without inquiring whether they be then in a good or bad state of health—that is, taken promiscuously as their lives are then found to be—they die almost exactly (at least without any sensible difference) as follows:—

From 50 to 55 inclusive,	$\frac{1}{6}$;
„ 55 „ 60 „	$\frac{1}{5}$;
„ 60 „ 65 „	$\frac{1}{4}$;
„ 65 „ 70 „	$\frac{1}{3}$;
„ 70 „ 75 „	$\frac{1}{2}$.

(And thus up to such time during each year, upon an equal number of persons; so that my conclusion, deduced from very natural and known principles, and which I have extended to 50 years, is not only neither applied and taken for too great a number of years, nor for too long a space of time; but it may be confidently applied, according to experience, for 70 years, and some years beyond, as from the age of 2 or 3 years up to that of 75 years.)

From 75 to 80 inclusive,	$\frac{2}{3}$;
„ 80 „ 85 „	$\frac{2}{3}$;
„ 85 „ 90 „	$\frac{7}{8}$;
„ 90 „ 100 „	$\frac{1}{1}$.

"I have not yet made any research relatively to the preceding years in respect to application of the principles; but I will with pleasure communicate to you, in due course, the result of my investigations on that subject.

"In the meanwhile, I observe that you also compute the 50th year, reckoning from the age of 6 years—that is to say, to 56 years; and that, of 30 persons still surviving out of the 80, 5 die in the 5 following years, which only makes $\frac{1}{6}$ for that period;

* *Affsterven*—deaths, decrements of life.

whilst my experience, carefully made upon several lives, gives $\frac{1}{3}$ as the deaths for the same period, even in beginning one year sooner; similarly, also (according to your foregoing fixing of time), a life annuity invested upon one life of 50 years of age, without distinction of constitution or corporal complexion, would then have a value of 11 florins 7 stuyvers, being computed upon the table of complete years; whilst, after having had calculated several subjects of the same category, according to the actual deaths experienced, amounting in the aggregate to nearly 750 in number, I perceive that the most advantageous class does not exceed 11 florins 0 $\frac{2}{3}$ stuyvers; that the lowest, although it comprises 271 subjects, does not amount to more than 9 florins 11 $\frac{2}{3}$ stuyvers; and that, taken one with the other, the annuity is not above 10 florins 10 $\frac{1}{2}$ stuyvers; notwithstanding that being further calculated upon the table of half-years, as you have based it on lives of 50 years, the value exceeds by a whole florin that which experience has proved.

“As respects the method of calculating a life annuity upon two lives, such as I have forwarded to you, it is erroneous, because it depends, *inter alia*, upon the foundation that the probability of decease of one life, upon which the annuity is supposed to have been purchased, would be equally great in each of the 80 years; which is absolutely true upon the presupposition of your aforesaid principle, if the said life be considered by itself, and having no relation with another; but if this same life be placed in such case that it has to be compared with another, and which survives it, the same presupposition then becomes entirely false; because it is almost certain that in such case the aforesaid life will die in the first years, and almost impossible that he should chance to die in one of the last years, because, as respects dying in the first years, he would almost certainly be the survivor, since he is already placed in the supposition as being likely to die the first, to such a degree that in case the said life chance to die at the end of the first year, it is 79 to 1 against his being the first who dies; and in case of his happening to demise at the commencement of the last year, it is again 79 to 1 that he is the survivor; so that the case being already assumed that the said life be the survivor, the first and the last years are no longer in the least degree equal relatively to the day of his death; and consequently, also, we cannot base thereon any equal chance, as has, however, been established in the aforesaid estimate.

“I have therefore followed another method, which is annexed hereto, and which I submit to your examination, albeit, however,

that such method is founded upon your aforesaid presupposition ; and as I am persuaded that the said presupposition attributes to the lives, towards the last years, more vitality than nature grants them, that is why also the life annuity on two lives calculated thereupon must be augmented a little too much in value. In finishing my letter I remain, yours, &c., &c.*

“ To M. Hudde.”

LETTER VI.

“ 22nd October, 1671.

“ SIR,—I forwarded you yesterday a more detailed calculation respecting the value of the life annuity upon two lives ; but although I look upon that method as correct, a mistake has nevertheless been made in the order of the items, each multiplied item having to be increased by the item in the table, which would augment the value discovered by about one stuyver ; on which account I will shortly send you a revised Memoir, and I will also have recalculated the aforesaid value in another manner, subjected to rather larger multiplications, to see if all be in good agreement.

“ To M. J. Hudde, &c.”

LETTER VII.

“ 27th October, 1671.

“ SIR,—Since the departure of my last letter of the 22nd, your two letters of the 21st and 22nd instant have respectively come to hand, with the table of life annuities calculated upon two lives, in the selected class of 96 lives all aged 6 years, and also the fuller demonstration of the provisional hypothesis ; namely, that out of 80 young lives, about 1 dies annually. I feel myself bound to thank you most particularly for it, and likewise for the information respecting the error found in your calculation of the value of an annuity upon two lives, as estimated upon the aforesaid provisional hypothesis.

“ I informed you in my last letter of the 22nd instant that I had found the value of an annuity upon two lives by another method, embracing rather longer multiplications ; since then I have more maturely examined the same subject, and have besides so perfected the said method by the employment of several abbreviations, that I think it is at present one of the most general and easy rules which I imagine can be found in the nature of the thing, and

* N.B.—The calculations referred to are wanting.

applicable not only to the exact calculation (without loss of the least fraction of a penny) of the value of a life annuity upon two lives, but also upon 3, 4, 5, 6, or as many more lives as one may wish, which rule is briefly included in the annexed explanation, with the urgent request that you will try its application, and take an opportunity of communicating to me your sentiments thereon. Both the reason and the demonstration of the said method will assuredly present themselves without fail to your mind—to you who have already reflected upon the subject, and have so perfectly comprehended it—and which I would willingly enlarge upon, if one could put it as briefly into writing as one can concisely demonstrate it verbally and by ocular reference to the tables. However, if you think it worth the trouble, I will with pleasure respond to your wish upon the subject, and in that case would establish thereon an argument *à priori*, although I have found it *à posteriori*, like in almost all inventions.

“Following the aforesaid rule, the value of the life annuity upon 2 lives, computed according to your said provisional hypothesis of 80 lives, and taking the first to die during the year, has been found to be 20 florins 15 stuyvers $15\frac{8}{3}\frac{6}{1}\frac{4}{2}\frac{4}{5}$ pennings, which is a little more than $1\frac{1}{4}$ penning above the value communicated in your last letter, which approximates very closely, but ought not to differ at all if we had both operated with precision.

“I shall receive with pleasure the method you have promised me, and upon which the calculation is made according to the case of the aforesaid provisional hypothesis, whereby, and also by the further communication of your more exact estimates upon the said subject, you will singularly oblige him who is, and at all times will remain, yours, &c., &c.

“To the Sheriff Hudde.”

“The Hague, 27th October, 1671.

The Memoir annexed to the above, and therein referred to, is as follows:—

“General method for accurately calculating the value of a life annuity, constituted upon 2, 3, 4, or upon 5 or upon as many more lives as may be desired, in the given or selected case of the decease of several lives, according as they have been successively found to have survived or died; or, according to a general and well-proportioned hypothesis, based upon the experience of ordinary mortality, so that if a life annuity constituted upon 2, 3, 4, 5 or more of the aforesaid deceased persons had been bought and sold at

the price of the aforesaid value which is sought, the buyer and the seller would have speculated with precisely the same advantage as though they had respectively bought and sold the exact value upon one life of the chosen or established worth of life annuities constituted upon one only of the aforesaid lives.

“Let there be given an average and well-proportioned case of the decease of several lives, as, for example, of 8 young lives (that number being given in order to avoid here too great a complication), and who are found to have lived as follows—the first to have become defunct 7 full years from the well-established date at which I suppose has been bought or invested upon each of the aforesaid lives a life annuity of 10,000,000 little pennings (*kleyne penninxhens*), making together one florin per annum; the second life 15 years; the third 24 years; the fourth 33 years; the fifth 41 years; the sixth 50 years; the seventh 59 years; the eighth 68 years; all complete years, as in the case of the first life; then this life annuity, calculated entirely for full years and for one of the aforesaid lives, taken one with the other, is found to have had a value of rather more than 17 florins $4\frac{2}{3}$ stuyvers, as is demonstrated in the subjoined calculation.

“It is now required to be known how much ought to have been paid, in the proper proportion, for a life annuity constituted upon 2, 3, 4, or 5 or more lives.

“Place the numbers, expressing the number of years* which each life respectively has lived, the one following the other in natural order, as hereunder; namely, the smallest number (which may also be 0) the first; the nearest to it, second; and so on to the end, as has been done hereunder in the before described case for the first line.

7	.	15	.	24	.	33	.	41	.	50	.	59	.	68
		1	.	2	.	3	.	4	.	5	.	6	.	7
				1	.	3	.	6	.	10	.	15	.	21
						1	.	4	.	10	.	20	.	35
								1	.	5	.	15	.	35
										1	.	6	.	21
												1	.	7
														1

If you now wish to ascertain the value of the life annuity upon 2 of the said lives, place unity, or 1, under the second of the numbers proposed, 2 under the third, 3 under the fourth, and so on to

* That is to say, for each life separately; therefore, if 2, 3, or even several lives have survived the same number of years, one must also arrange the number of their years twice, thrice, or more times, one beneath the other.—(De W.)

the end. If you wish to ascertain the value of the annuity upon 3 of the aforesaid lives, place unity, or 1, beneath the third of the numbers proposed, 3 beneath the fourth, 6 beneath the fifth, 10 beneath the sixth, and so on; namely, always the number formed by the cumulation (one upon the other) of 2, 3, 4, &c. of the numbers above employed in the estimate of the annuity upon two lives.

“And if you wish to know the value of the life annuity upon 4 of the aforesaid lives, place unity, or 1, under the fourth of the numbers proposed, 4 under the fifth, 10 under the sixth, 20 under the seventh, and so on; namely, in each case as great a number as is formed by the cumulation (one upon the other) of 2, 3, 4, &c. of the numbers above made use of in the calculation of the life annuity upon 3 lives; and so on, and in the same order, relatively to the life annuity upon 5, 6, 7, or more lives, as many as may be required, comprised in the case proposed or adopted.

“This preparation being then made, I say, as a general conclusion, that the required value of a life annuity upon 2, 3, 4, 5, or a larger number of lives, is rightly and exactly equal to the value of a life annuity upon one life of a class of as many lives as form, in the aggregate, the aforesaid numbers placed beneath or alongside, which lives have also respectively survived as many years since the purchase of the annuity as are indicated above or over against such numbers following that which has been established; namely, that the value of the life annuity upon 2 lives, in the aforesaid selected or proposed case, is rightly and precisely equal to the value of a life annuity upon one life of a class of 28 lives, of which one life has lived 15 complete years; 2, each 24 years; 3, each 33 years; 4, each 41 years; 5, each 50 years; 6, each 59 years; and 7, each 68 years; so that the value of the life annuity upon 3 lives, in the aforesaid case, is exactly equivalent to that of a life annuity upon one life of a class of 56 lives, of which 1 has lived 24 full years; 3, each 33 years; 6, each 41 years; 10, each 50 years; 15, each 59 years; and 21, each 68 years.

“That the life annuity upon 4 lives, in the case proposed, is exactly equal to the value of a life annuity upon one life of a class of 70 lives, of which 1 has lived 33 full years; 4, each 41 years; 10, each 50 years; 20, each 59 years; and 35, each 68 years. “And so on, consecutively, with reference to 5, 6, 7, &c. lives, as is constantly indicated by the subjoined numbers and the superposed years.

“The calculation of all this may likewise be effected by means of a simple addition, without multiplication, by an easy method, as

may be more precisely judged of in the annexed calculation ; and when the numbers bear a continuous proportion to each other, or when they include the sums of numbers which are in a constant ratio to each other (as is found when we apply the above general rule to the hypothesis that in 80 years more or less, according to the constitution of the annuity, there dies in each year one person out of as many persons as there are years given), one may further find or make use of other abbreviations for the facilitation of the work.

Lives. —		Lived, Years.		Value reduced to present worth.		Or upon 2 lives.
1		68		232,635,074		232,635,074 *
1		59		225,284,296		457,919,370
1		50		214,821,846		672,741,216
1		41		199,930,518		872,671,734
1		33		181,476,457		1,054,148,191
1		24		152,469,631		1,206,617,822
1		15		111,183,874		1,317,801,696
1		7		60,020,547	7)	5,814,535,103 †
				8) 1,377,822,243	4)	830,647,872
				172,227,780		207,661,968 ‡

Gives, upon one life, 17 florins $4\frac{2}{5}$ stuyvers.

And in the same way that by means of the 8 items of the annuity upon 1 life, the value has been found of the life annuity upon 2 lives, by adding the first 7 results, in the aforesaid manner, and dividing their sum by 28; so, by the 7 items of the estimate of the same life annuity upon 2 lives, is found the proportionate value of the life annuity upon 3 lives, by adding the first 6 results and dividing their sum by 56; and so on consecutively for the value of the life annuity upon 4 lives, by adding the first 5 of the 6 items resulting from the latter operation, and dividing their sum by 70; and so on consecutively to the end, as it has here been calculated."

(§ 29.) Before again adverting to the progress virtually accomplished in the labours of De Wit, we must briefly review the state

* Add to the first item of the first column, the second item; to the second item, the third item; and so on, to the seventh.

† This sum includes 7 times the first item, 6 times the second, 5 times the third, 4 times the fourth, 3 times the fifth, twice the sixth, and once the seventh; *ergo*, according to the rule, let it be divided by 28, and it gives the value of the annuity upon two lives; viz., 20 florins $15\frac{3}{10}$ stuyvers.‡

The above two notes are from De Wit.

of popular information on the subject of life annuities, as it existed at the period we are considering; and we must moreover bear in mind, that in endeavouring to ascertain the views popularly entertained upon it in his country in the seventeenth century, we are searching for the then highest general standard of intelligence upon the matter; and we need no better proof of this than the fact that the example of Holland in *all* financial, commercial, and economical operations, was most studiously enjoined for the imitation of this country by the first authorities of that time—by such men, for instance, as Child, Culpeper, Petty, and others of their stamp.

For our present purpose a glance at certain pamphlets which appeared in the years 1670 and 1671 (and of which I annex the titles*) will perhaps be found sufficient. These pamphlets—7 in number, and which refer to the towns of Kampen, Middelburg, and Groningen—are fair specimens of the schemes which the Dutch municipalities employed for raising capitals by the grant of life annuities.

One of the points of information they convey is, that it had long been the practice in Holland for life annuities to be granted, to nominees *of any age*, in the constant proportion of double the rate of interest allowed on stock; that is to say, if the towns were borrowing money at 6 per cent., they were willing to allow 12 per cent. by way of life annuity; if at 5 per cent., the life annuity granted was 10 per cent.; and so on. The States of Holland and West Friesland seem not to have held out quite such favourable terms to life annuitants. De Wit, in the commencement of his Treatise, refers to their then rate of charge being 14 years' purchase, whilst redeem-

* I. Vertoogh, en Calculatie, waer mede, klaer en blijckelijck wort vertoont, en aangewesen, de Beneficien en voordeelen, die nyt dese Lyf-Renten by Overlevingh, in diervoegen, als by de Stadt Kampen werden opgenomen, soo wel voor de Opnemers, als voor de Gevers, staen getrocken en genooten te werden. Gestelt in vergelijckinge met Los-Renten tegens 4 ten hondert, en ordinaris Lijf-renten tegens 8 ten hondert. *Small 4to. Amsterdam, 1670. 8 pp.*

II. Naerder Conditionen en Voorwaerden waer naer de Heeren Burgermeesteren, Schepenen en Raedt der Stadt Kampen hebben geresolveert op te nemen een somme van hondert duysent guldens op Lijfrenten by Overlevingh. Te weten; op een Collegie of Compagnie van 400 Persoonen, tegens 4 per cento's Jaers: Oock de versekeringh die voor de prompte betalinge sal werden gegeven; als mede de ordre die in't administreren, en betalen van de Lijf-renten sal werden geobserveert. *Small 4to. Amsterdam, 1670. 8 pp.*

III. Lyste van het Collegie ofte de namen der Persoonen, op wiens leven de Heeren Regeerders der Stadt Campen, hebben opgenomen ten behoeve ende tot laste van haer gemelte Stadt, door t' Administratie van Jacob van Dael, gesworen Makelaer der Stadt Amsterdam, hondert duysent Caroli guldens, ofte veertigh duysent Rijcxdaelders, op lijfrenten by Overlevingh, Gesloten den primo November Ouden, ofte den elfden dito Nieuwen—stijl, Anno 1670. De Gecommitteerde van't Collegie, Adriaen van Loon, Isaac Jan Nys, Joost de Smith, Johan van Beuningen. *Small 4to. Amsterdam, 1671. pp. 16.*

IV. Conditionen en Voorwaerden waer naer de Heeren Burgermeesteren ende Raedt tot Middelburg in Zeelandt, hebben geresolveert, te doen verhandelen, &c. Een Capitael

able annuities were at 25 years' purchase. The state life annuity was consequently $1\frac{1}{2}$ year's purchase dearer than the ordinary rule would have given. This, and other remarks in De Wit, to some extent warrant the inference that experimental investigation or observation had previously been made of the financial or mortality results of such transactions.

A favourite financial plan of that time was the raising of funds by the grant of survivorship annuities on the Tontine plan. An association, consisting of a given number of lives or nominees, sometimes with, and at other times without, stipulation as to age, was formed; each member subscribing an equal sum, to be paid into the coffers of the contractors, who guaranteed the payment of interest on the whole amount until the extinction, by death, of the entire association,—the survivors from year to year, and eventually the last survivor, enjoying the annual proceeds, but without final restitution of capital.

The advantages to those who raised funds by such means are too manifest to require referring to, but it will not be uninteresting to give, from the pamphlets numbered I. to III. in the preceding note—(and which, as well as the others, may be unique copies),—an example of the attractive way in which these transactions were brought under the notice of the public, and of the ingenious method whereby comparative and assumed possible results were tabulated for the benefit of discerning subscribers.

The following is a translation, with an abridgment of the figures, of the scheme for the town of Kampen, dated 1670. It will be sufficiently intelligible to the readers of this paper, so that explanations can be conveniently spared.—

van hondert twintigh duysent guldens by Overlevinge tegens $4\frac{1}{2}$ per cento in't Jaer. Ende dat op het Leven van een Collegie of Compagnie van 240 Persoonen. *Small 4to. Amsterdam, 1671. pp. 8.*

V. Conditien en Voorwaerden waer naer de Heeren Burgermeesteren en Raedt in Groningen hebben geresolveert, te doen negotieren, &c. Een Capitael tegens 4 per cento van vijf duysent ses hondert en vijf-en-twintigh guldens jaerlijcsche Lijfrenten by Overlevingh. En dat op het Leven van een Collegie ofte Compagnie van 500 Persoonen, alle van ongelimiteerden Ouderdom. *Small 4to. Amsterdam, 1670. 8 pp.*

VI. Vertoogh en Uytrekeningh, waer mede Klaer en duydelijk wort betoont en aengewesen, dat de Lijfrenten by Overlevingh, invoegen als onlangs voor de Stadt Campen, ende nu noch tegenwoordigh voor de Stadt Groningen, en andere Steden zijn ende werden genegotieert; voor de Contribuanten ofte Inleggers, better en voordeeliger zijn, als ordinaris Lijfrenten tegen 8, ja sels tegen 10 ten hondert. Als mede, Dat die te prefereren zijn voor de Collegien, die hare Capitalen aenleggen in Actie op de Oost-Indische Compagnie. *Small 4to. Amsterdam, 1671. pp. 20.*

VII. Conditien en Voorwaerden Waer naer de Heeren Burgermeesteren en Raedt in Groningen noch hebben geresolveert, te doen negotieren en opnemen, &c. Een Capitael van hondert en tien duysent guldens, op Lijfrenten by Overlevingh, tegens 5 per cento's Jaers En dat op het Leven van een Collegie ofte Compagnie van 200 Persoonen, alle van den Ouderdom van 25 Jaren en daer boven. *Small 4to. Amsterdam 1671. pp. 10.*

“CALCULATION OR DURATION-OF-LIFE-RECKONING,

“From which to find, and have clearly before one’s eyes, the remarkable profits which may be drawn and enjoyed, by the purchasers or investors, from life annuities by way of survivorship, in a college or company consisting of a number of 400 persons, aged one year and upwards, at 4 per cent., calculated on the basis that from 1 to 60 years, in each 12 years, the half of the number survive, and thenceforward, from 60 to 66 years, in each year one, and from 66 years to 76 years one every 2 years, and the last to 80 years; and placed in comparison with redeemable rents at 4 per cent., and ordinary life annuities at 8 per cent., as is more fully set forth in the annexed demonstration respecting the undertaking by the city of Kampen.

“Column No. 1 indicates the years.

“No. 2, the redeemable rents at 4 per cent. upon 250 florins, with the capital added up for each year.

“No. 3, the ordinary life rent at 8 per cent. upon 250 florins, all years added together.

“No. 4, the augmentation, or how much the life annuity is elevated by means of survivorship in each year.

“No. 5, how much per cent. the life annuities produce each year by means of survivorship.

“No. 6, the number of persons who, from the age of 1 to 60 years, in each 12 years, and thenceforward for each year, up to the age of 80 years, are assumed to remain still in existence.

“No. 7, the yearly life rent by survivorship upon 250 florins of capital added together for every year.”

N. B.—The Table above referred to is printed in the original, *in full* for each of the 80 years; but the Abstract annexed in the opposite page will answer every purpose of information upon the tenor of the scheme.

Amongst the pamphlets before quoted is one containing a complete list of the names of the 400 nominees of the Kampen Tontine. They seem to have been all, or nearly all, under age, and many were probably in the first years of infancy. The names of the parents are given; and, with the single exception of “Charles II., King of Spain and the Indies,” the lives are those of residents in Holland,—(in frequent cases 5, 6, or 7 of a family)—and include the children of many of the municipal authorities, clergymen, and officials, and some domestic servants.

The

1	2	3	4	5	6	7
Year.	Redeemable Rent at 4 per Cent. upon 250 Florins.	Ordinary Life Annuity at 8 per Cent. upon 250 Florins.	Augmen- tation per Cent.	Rent per Cent.	Persons.	Life Annuity by Survivorship at 4 per Cent. upon 250 Florins.
	Florins. 250 10	Florins.			400	Flor. Stuv. Pfen.
1	260 10	20 20	$\frac{1}{6}$	$4\frac{1}{6}$	.	10 8 9 10 16 9
2	270 10	40 20		$4\frac{1}{3}$	..	21 5 2 11 5 0
3	280 10	60 20		$4\frac{1}{2}$	..	32 10 2 11 13 3
4	290	80		$4\frac{2}{3}$	..	44 3 5
12	370 10	240 20	$\frac{1}{2}$	$8\frac{1}{2}$	200	172 3 1 21 5 0
13	380	260		..	..	193 8 1
24	490 10	480 20	1	17	100	428 15 12 42 10 0
25	500	500		..	..	471 5 12
36	610 10	720 20	2	34	50	1,142 1 13 85 0 0
37	620	740		.	..	1,227 1 13
48	730 10	960 20	$4\frac{1}{3}$	$68\frac{1}{3}$	25	2,568 14 8 170 16 9
49	740	980		.	..	2,739 11 1
60	850 10	1,200 20	$12\frac{1}{12}$	$145\frac{1}{11}$	12 11	5,462 0 2 363 12 12
61	860	1,220		.	..	5,825 12 14
65	900 10	1,300 20	$38\frac{2}{21}$	$266\frac{2}{3}$	6	7,741 10 5 666 13 5
66	910 10	1,320 20		$266\frac{2}{3}$	6	8,408 3 10 666 13 5
	920	1,340	.	.	..	9,074 16 15
74	990 10	1,480 20		800	2	17,341 4 1 2,000 0 0
75	1,000 10	1,500 20	800	1,600	1	19,341 4 1 4,000 0 0
76	1,010 10	1,520 20		1,600	1	23,341 4 1 4,000 0 0
77	1,020 10	1,540 20		1,600	1	27,341 4 1 4,000 0 0
78	1,030 10	1,560 20		1,600	1	31,341 4 1 4,000 0 0
79	1,040 10	1,580 20		1,600	1	35,341 4 1 4,000 0 0
80	1,050	1,600				39,341 4 1

The pamphlets adduce several very apposite reasons for recommending such methods of investment as worthy of the consideration and patronage of the prudently economical, and as admirably suited to the wants of those who had to provide for the after-lifetime of young families. We may readily understand that such schemes presented but an imperfect substitute for the important social benefits arising from Life Insurance Companies; but at the same time, and viewing the matter as compared with the practice of the present times, it may well occur to us to inquire whence it has arisen that in England we have such strong prejudices against *Tontine Associations*,—based, perhaps, on the assumed *rationale* that they are too selfish and speculative to be encouraged,—and this, too, when such associations as Burial Clubs are tacitly countenanced,—to the detriment of the morality of our labouring population, and sometimes to the promotion of the worst phases of human nature.

To return, however, from this digression;—I may mention that my desire in quoting the foregoing Table of Computations was to show that the idea of a life table, arranged in the form of the number surviving to a given age out of a specified number living at any age, originated, roughly though it may have been, in these Tontine annuity schemes. The projectors in their announcement state—(with reference to the column No. 6 of the Table)—that they are aware, that anything like a precisely accurate calculation of the proportion dying out of a given number is not attainable, but “the assumed numbers had been framed upon a new elaborate scrutiny and observation, and there was no ground for assuming that the results to be experienced would range either below or above the anticipated number of survivors.”*

(§ 30.) The state of information on the subject of life annuities was thus of a very imperfect and neglected kind at the period when it was taken up and made a theme of correspondence and study by De Wit and Hudde. They had to ascertain facts, deduce results, and create a method of computation; and after the oppor-

* We need not, of course, take this statement of the Tontine projectors as to their *calculations*, except with the “grain of salt,” with which experience tells us we must season our belief in worldly matters; but nevertheless the fact of the remark existing in a paper getting on for two centuries old is worth some notice.

I should also mention, in passing, that in the pamphlet respecting the Groningen plan (see No. VI. in last note) the credit of the invention is honestly and properly given to the “sagacious,” or more literally “sharp-witted” Italians, and the statement is further made of its having been but recently introduced into Holland (“eerst by de Scherp-sinnige Italianen uytgevonden, ende nu onlanghs door ons op de Hollandtsche wijze geschildt en in't werck gestelt”).

N.B. It would be desirable to ascertain whether previous Tontine annuity schemes, as, for instance, those instituted by *Tonti*, in France or elsewhere, and prior in date to 1670, include any tables assimilating to that quoted in this paper.

tunities now offered of verification of this statement; and which are so amply presented in the letters from De Wit to Hudde, edited for the first time in this paper, it would be a work of supererogation to introduce much further detail.

Whether the facts were correctly scheduled, or rightly extracted, arranged, and certified, is a question which it would be impossible to solve; but we are not on that account justified in any doubt that experimental observation and collection of the indications of mortality at various ages, and in different classes of lives, were the principles in which the acuteness of De Wit and Hudde recognized the only true foundations for their labours.

The Treatise (or rather, Report) presented to the States was avowedly on the basis of approximative presuppositions; it would not, therefore, be fair to suppose that even De Wit himself intended it *practically* for more than an exemplification of a correct mode of applying the doctrine of chances to the calculation of life contingencies, and as a means of enforcing upon the States that he had ground for thinking it unwise on their part to grant the rate of annuity which they proposed, and quoted in their resolution of the 25th April, 1671, upon a negotiation of one million of florins by life annuities at 14 years' purchase upon one life, and 17 years' purchase upon two lives.

In following out some concluding researches on this point, I find that there was a Supplement appended to the Treatise of De Wit, and of which it will be necessary to give a translation in full, affording, as it does, the most important testimony as to his examination of the actual results of mortality experience amongst annuitants. It is as follows, and is well worthy of perusal:—

“SUPPLEMENT.

“As daily experience clearly proves that a great many persons are neither disposed nor competent to apply their judgment in such a manner to a complicated although unfailing course of reasoning, as fully to understand its force and the conviction it carries, and as with such persons examples have greater influence than reasonings of the kind, I find it incumbent on me here to add, for fuller satisfaction, that since the proof of the foregoing demonstration I have had very carefully extracted from the registers of your Lordships *some thousands of cases* of persons upon whose lives annuities have been purchased, with the memoranda up to the last due-dates to which the life annuities have been paid to each; and after a careful estimate that the respective annuitants have drawn upon each life,

in ready money, at 4 per cent. per annum discount, the same value in half-yearly instalments as in rents, from the time of purchase to the last day of the aforesaid due-dates inclusive, I have found that the said rents, invested on a fair number of young lives (as, for example, one hundred or thereabout), have, taken one with the other, always produced to the annuitants in ready money, from the time of purchase, a value of more than sixteen florins of capital arising from one florin of annual rent; that is to say, each florin of rent which they have successively drawn upon the aforesaid lives, all reduced to ready money, at the discount of 4 per cent. per annum, and for as long a time as each half-year of rent has had to be paid since the purchase, has invariably produced upwards of sixteen florins; so that, this being calculated upon considerably more than a hundred different classes, each class consisting of about one hundred persons, it has not been found in any one instance that the aforesaid rents which had been drawn at the above discount had attained less than sixteen years' purchase, but they were frequently much higher, even up to eighteen years' purchase. Thus one finds with wonderment, that in practice, when the purchaser of several life annuities comes to divide his capital which he intends to invest upon several young lives—upon ten, twenty, or more—this annuitant may be assured, without hazard or risk of the enjoyment of an equivalent, in more than sixteen times the rent which he purchases. All these particular calculations can, if needed, be demonstrated to your Noble and Mighty Lordships by him who is, and all times will be,

“Your Noble and Mighty Lordships’

“Very dutiful Servant,

(Signed)

“JOHAN DE WIT.

“The Printer to your Noble and Mighty Lordships will print the aforesaid calculation and demonstration, some copies of which will be remitted in course to the members.”*

(§ 31.) De Wit's system of computing an annuity value required only a consideration of the probability of a person aged x years dying at the age of $x+1$, $x+2$. . . to $x+z$ years. The expression,

* This sufficiently accounts for the rareness of the work in its separate form as a Report or Treatise. The members of the Council of State were few in number, and we are not aware that they had any incentives for preserving a paper which to them possessed only a passing interest. Should a copy ever be met with, it will be well to look to the paging, as it may either be complete in itself or run on consecutively with the paging of the other Resolutions. The Printer was Jacobus Scheltus, typographer in ordinary to the States.

$$\frac{l_{x+n+1}-l_{x+n+2}}{l_x},$$

or the probability of a life aged x surviving n years or half-years, and then dying in the $(n+1)$ th year or half-year, can be correctly traced in his calculation; the course of which may be symbolized in our most elementary notation as:—

$$\left(\frac{l_{x+1}-l_{x+2} \cdot \frac{1-(1+i)^{-1}}{i}}{l_x} + \frac{l_{x+2}-l_{x+3} \cdot \frac{1-(1+i)^{-2}}{i}}{l_x} \right. \\ \left. + + \dots \frac{l_{x-1}-l_x \cdot \frac{1-(1+i)^{-x-1}}{i}}{l_x} \right) = a.$$

This formula is identical with the fourth method of *Tetens* (described in Volume I., p. 9, of the *Assurance Magazine*): viz., in the Schleswig Professor's Notation,—

$$\Delta^1 A \int \frac{1}{r} + \Delta^2 A \int \frac{1}{r^2} + \Delta^3 A \int \frac{1}{r^3} + + \Delta^{x-1} A \int \frac{1}{r^{x-1}} = A \lambda \bar{a}.$$

When De Wit observed that it was as likely that a man should die in any one half-year as in any other half-year of the first 100 half-years of the first period, it is not at all in evidence that he either implied or was considering the probability of a person who *had attained an age of* $x+n$, $x+n+1$, to $a+z-1$ years, dying in the $x+n+1$, $x+n+2$, to $x+z$ th year. On the contrary, this was not required by his method, and the hypotheses, presuppositions, and comparisons of mortality proceeded on what we may call a formula of antecedent probability, that a life, now aged x , should *survive to the age of* $x+n$, $x+n+1$ to $x+z-1$ years, and *then die* in the $x+n+1$, $x+n+2$ to $x+z$ th year.

This will afford a full justification of many passages in De Wit's Treatise, and particularly as to his numerical comparisons of decrements at the various ages; and although he certainly has several ambiguities of expression (which, however, may be greater in the translation than in the original), we may nevertheless maintain the opinion, that the valuable and distinguishing marks of progress through the whole of his instructive and important Treatise more than counterbalance its apparent deficiencies in occasionally not distinguishing between the shades of difference implied in the terms "living to," "surviving," or "dying in" specific years of life—terms which are obvious enough to a modern reader, and would, in all likelihood, have been so to De Wit, had it fallen to his province to pursue his researches further into the theory of

life contingencies. The letters here published will prove that at least he had both the industry and the will to do so. The last Memoir, in which he attempts to ascertain a method for the computation of the value of an annuity on the life of the longest survivor of two, three, or a greater number of lives, falls however far short of the object it was erroneously supposed to have attained, for such a calculation involves more complex probabilities than could be imagined or solved in the course of his reasonings, such as we see them in the essay to apply the combinatory formula, or arithmetical triangle of Pascal, to the problem.

We must concur in regretting that some of the computations referred to in the letters to Hudde are wanting, and that Hudde's correspondence and papers are to all appearance lost. This much, however, is fully proved,—that De Wit's chief and earnest endeavour, was to indoctrinate, for the advantage of his fellow citizens and the State, some *correct method* for deducing estimates of value from the ever-fluctuating indications presented in the data of the mortality experienced in various classes of lives. In pointing out such a method he was by no means unsuccessful; and, in reviewing the words of his Treatise, we must not lose sight of its being a State paper,—intended to be made plain to a legislative body,—and not to be elaborated in the style of a dissertation written for purely abstract scientific purposes.

It is no discredit to De Wit and Hudde that, in the infancy of the science of probabilities, and at a time when its language was but just forming, they should have fallen into some contradictions when they came to apply presuppositions or postulates as to the relative mortality at various ages. We are not called upon to criticise these minutely, because, under the peculiar circumstances adverted to, such a standard of measurement would hardly be fair. Rather let us concede that the then novel and ingenious views, and the many germs of truth contained in De Wit's Treatise and other labours on the subject of life annuities, did honour to his discrimination as a mathematician, and to his judgment as a statesman; and that he may be considered as the first who perceived that a new-found science, which was but beginning to attract the attention of philosophers of his day, could be applied not solely to the investigation of the hazards of players at ignoble games of cards and dice, but also to the business of life and to the good of the commonwealth.

F. H.

London, 1st December, 1852.