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Controlling the growth of your hedge

Dynamic investment strategies working party

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June 2008, Manchester

Thanks to

- Matt Barnes
- Alvar Chambers
- Peter Ridges
- Richard McMahon
- Anthony Earnshaw

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Agenda

- The need for multi-period optimisation
- Utility formulation & solution
- Some examples from the finance literature
- Pension funding example & applications

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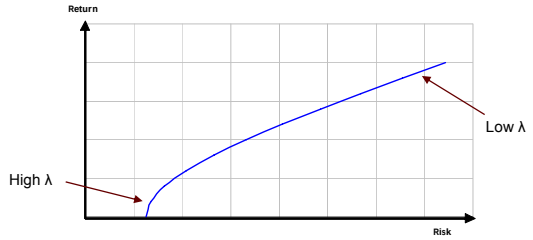
Multi-period optimisation basics

Single-period optimisation

- Maximise E(return)-λ.(Risk)



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Moving beyond mean-variance

- Risk budgets can obscure the investment goal
  - Eg target funding level or minimum guarantee
- Mean-variance can be unhelpful
  - Want to measure downside risk
  - Not a good framework for exploring strategies that change the distribution of outcomes (eg options)

These are related!

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Multi-period optimisation

- Maximise E(utility at horizon)
- Find investment allocation *process*
  - Now and at each future time
  - In each future state (dependent on history to date)
- Ability to rebalance makes problem massively more complex than just single allocation

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Recursive solution

- Solve Max E[U(w(T))] recursively:
  - Determine portfolio at T-1 that maximises E[U]
  - Calculate E[U] for this portfolio: call this V(w,T-1)
  - Determine portfolio at T-2 that maximises E[V]
  - Series of one-period problems
- In continuous-time setting can use martingale theory to separate problem in a different way

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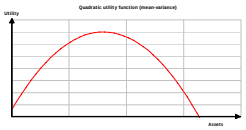
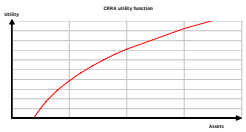
Examples from the finance literature

Which utility function?

- Quadratic function
  - As before
  - Has maximum-utility level
  - Results not intuitive
- Power law function
  - CRRA
  - Portfolios stable & intuitive
  - Common in literature





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Example portfolio problem

- Investor has finite lifetime and must choose
  - How to allocate his asset pool over time; and
  - How much to draw down from this pool
- Objective is to maximise expected lifetime utility
  - Discounted utility of cash spent + final pool
- Utility of the CRRA form ($w^{1-\gamma}$)

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Solution to Merton problem

- Asset allocation strategy is constant
  - Constant proportions in optimal growth portfolio and a bond expiring at death
- Formula for how much to consume over time

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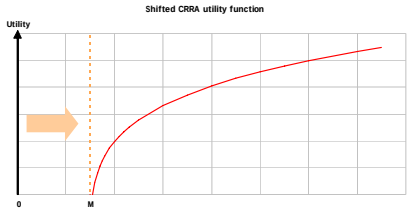
Minimum solvency constraint

- Shift the CRRA utility curve to the right:



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Solution

- Retain capital to meet minimum
- Invest remainder in the optimal (Merton) way:
  - Constant proportion in optimal mix of risky assets
- So risky allocation is proportional to the buffer above the minimum cushion
  - More commonly known as CPPI (constant proportion portfolio insurance)

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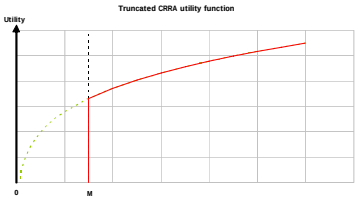
Alternative minimum constraint

- Minimum could be expressed differently
- Optimum strategy is then to add a put option to the Merton portfolio



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Summary so far

- CRRA utility function good base line
  - Risk & return preferences stable over time
  - Consistent with mean-variance intuition
- Simple program:
  - Express investor preferences in utility function
  - Determine optimal dynamic strategy
- Able to recover well-known strategies (CPPI, put options)

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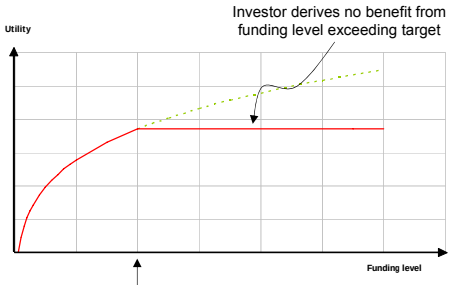
Liability targets

Liability target



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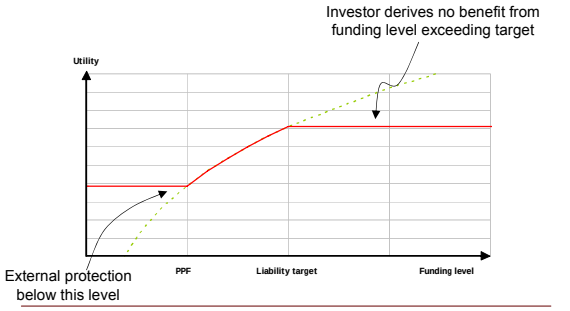
More realistic utility curves (?)



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Optimal strategy is option-like

- Maximise E[U(w(T)]
- Optimal strategy is
  - Merton-style portfolio (with liability hedge rather than bond expiring at time T)
  - Plus options on this portfolio to sell upside / take more downside risk
- Or a delta hedge of the payoff from this strategy
  - Strike prices of options don't evolve through time

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Applications

Sample applications

- We have explored two main applications:
  - How should a funding strategy evolve through time?
    - Evolution of the liability hedge
    - Spread of the risk budget through time
  - How do options help?
    - Look at commonly-available options rather than the theoretical options that fall out of the modelling

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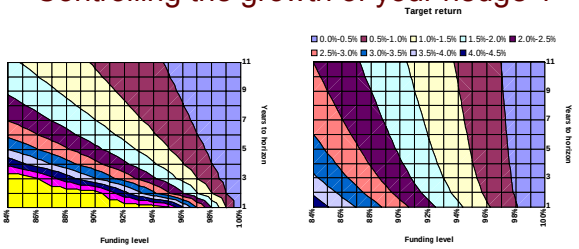
Controlling the growth of your hedge 1



- Common approach (left) is to target sufficient return to exactly hit 100% funding at horizon
- Optimal-utility strategy (right) involves taking more risk early on and less risk as the target horizon approaches

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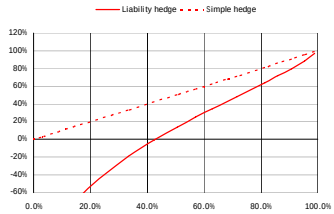
Controlling the growth of your hedge 2



- Optimal strategy also tells us how much liability to hedge
- Graph shows situation with 5 years to go – hedge less than funded liability as significant risk that funding level will fall

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Controlling the growth of your hedge 3

- 90% funded, 5 year planning horizon
- Compare 2 rules: (1) target 100% (2) optimise utility
- Useful to look at different statistical metrics

	Simple target strategy	Optimal utility strategy
Wealth-equivalent utility	94.8%	95.3%
Spread of funding level	13%	9%
Expected worst 5% deficits	40%	33%

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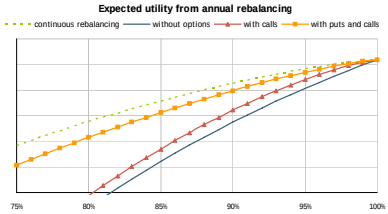
The benefit of options



- Adding one-year options to the mix of available assets can result in significant utility increase (equivalent to funding level increase)
- Shape of utility function (upside / downside) shows which options are going to be useful

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Conclusions

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- Dynamic strategies can be analysed fairly easily
  - Lots of flexibility to determine the utility function – broad features probably fairly easy to determine
  - Focus on what the client objectives really are rather than building complicated models
- Provides a coherent framework in which to:
  - Determine rebalancing strategies (trigger points etc.)
  - Consider the benefits (or otherwise) of option strategies

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