

Corporate Decisions: Beyond the Frontier

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Abstract

This paper shows how the powerful and flexible tool of stochastic modelling can be applied to a range of business decisions extending far beyond the asset allocation solutions that are common to many ALM studies. The example used to demonstrate these techniques is a non-life case study, but similar principles can be extended to many different business situations. At each stage of the analysis we consider the implications of modern financial theory on the management decision process together with a practical perspective on observed behaviour in the real world. Opportunities are taken to suggest directions into which further research may be of benefit to the actuarial profession.

1. Motivation

1.1 Recent years have seen significant developments in risk based capital (RBC) and asset-liability modelling (ALM) technology. This paper documents various ways in which the results of RBC and ALM can be used to assist corporate decision-making in Non-Life insurance. We examine how ALM fits in with more conventional decision tools for capital allocation, such as return on capital employed. We have used complex stochastic models in this evaluation based on the *GRAFT* equilibrium approach, developed out of the GISMO described in Christofides et al (1996). These models will be discussed in more detail in a workshop session specifically addressing the capital allocation issues.

1.2 Our conclusion is that, while RBC and ALM may give useful insights to management, the value that the conventional application of these techniques may generate for shareholders or policyholders can be at odds with the conclusions drawn by the managers. In particular, the managers can choose a strategy that improves the risk-reward profile of the company when viewed in isolation, but the value to the shareholder depends crucially on the other investments he holds in his portfolio. In this case, the benefits of the ALM exercise only arise if this conflict is understood and brought out into the open.

1.3 On a more positive note, we do have some concrete proposals as to the direction in which RBC and ALM could move to create genuine added value. However, this involves modelling factors such as management or underwriting skill, transaction costs and bankruptcy costs, which are outside the traditional armoury of actuarial techniques. We also need to understand tax better, and sharpen up our option pricing skills.

1.4 The overriding purpose of this paper is to stimulate discussion. Although we have tried to give a balanced view, at times we have been deliberately provocative in order to bring out commonly overlooked points. We look forward to a healthy debate on these important issues.

2. A Business Situation

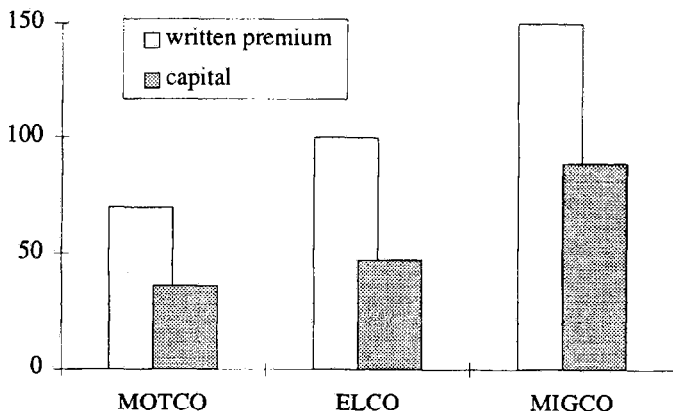
2.1 We consider 3 mono-line companies. One writes private motor, one writes employers liability, and the third writes pecuniary loss.

2.2 MOTCO is a private motor insurer. It chooses a high equity mix for its asset portfolio, which is used to finance aggressive premium scales - expecting a combined ratio of 105%.

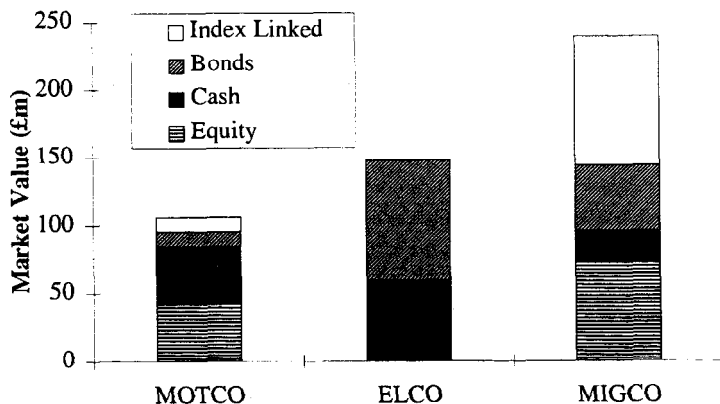
2.3 ELCO is an employers liability insurer. Its liabilities are more volatile than MOTCO, so it compensates with a more passive investment strategy, holding a mixture of cash and bonds.

2.4 MIGCO is a strongly capitalised pecuniary loss insurer. Like ELCO, its liabilities are volatile, but, unlike ELCO, the volatility is strongly correlated with the performance of the economy as a whole. Even so, MIGCO still invests heavily in equities, which it believes reflect the longer term real nature of the liabilities. A further substantial proportion is in index linked gilts, with the remainder in conventional gilts and a small amount in cash.

2.5 The chart below shows written premium and free reserves in £m.



2.6 The asset allocations are shown below, also in £m:



2.7 A merger has been proposed. We examine the rationale for the merger, in terms of capital required and the return which can be achieved on that capital. Having carried out the merger, we then examine various operational decisions, such as the split of different business lines, and the allocation of various classes of investment.

2.8 Our approach is to follow the conventional logic of RBC and ALM to guide corporate decisions. After each decision, we then examine in detail the position of shareholders and policyholders to see whether, in fact, either party is better off. We consider both the financial theory and empirical observations of real world behaviour.

3. Capital Allocation

3.1 The notion of capital enjoys a large number of mutually inconsistent definitions. For our purposes, we regard capital as comprising the excess of assets held over the liabilities of an insurer. This figure is commonly expressed as a fraction of the premium written during the preceding year.

3.2 As described in Hooker *et al* (1996), the legal definition of capital may differ from an economic definition. For example, in the calculation of solvency, not only is there an explicit requirement for capital (the EU minimum solvency margin) but also implicit requirements arising from margins in reserve calculation or, less commonly, in asset valuation.

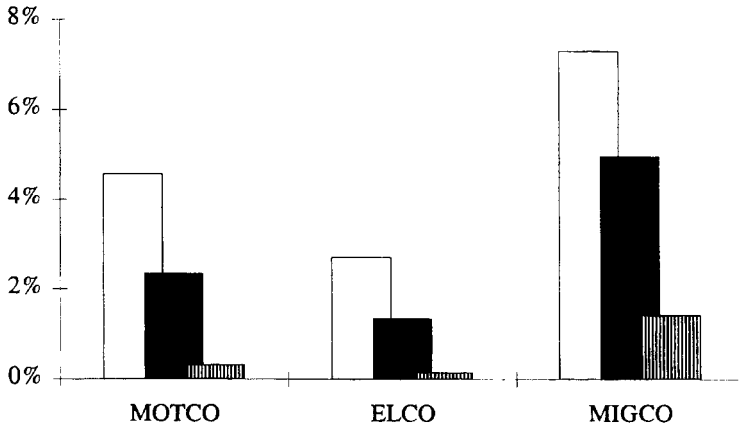
3.3 Hooker *et al* define capital to be the legal capital plus retained reserves plus or minus any margins in the valuation basis. In other words, capital is taken to be the market value of assets, less the present value of 'best estimate' projected liability cash flows. For any given insurance enterprise, we can then work out the actual amount of capital invested. There are still a number of loose ends here, in particular, defining what we mean by 'best estimate', and the appropriate rate of interest for discounting liabilities, but the idea is clear.

3.4 There is a danger here of circular arguments regarding return on capital. If economic capital is determined by reference to discounted cash flow calculations, it is a foregone conclusion that the ROCE (return on capital employed) will turn out to be equal to the assumed discount rate. Plainly, this is not then a helpful criterion for ranking projects. The resolution of this paradox is to count only the capital provided by shareholders (that is, excluding premium margins) when carrying out ROCE comparisons. Even so, we must take care that we are investigating genuine features of the business and not merely rediscovering our own assumptions.

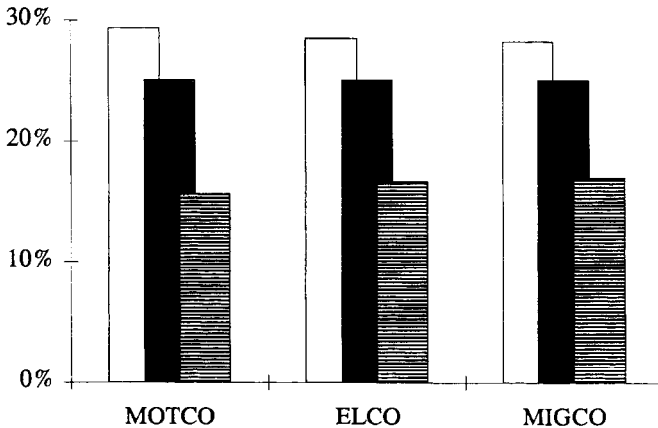
3.5 In practice, the capital definition employed in a particular investigation will reflect the constraints which are seen to be binding. For a weaker office, regulatory capital may be the most pressing constraint. Better capitalised insurers may prefer internal definitions which, they believe, reflect the business risk more accurately. Perhaps the commonest definitions employed are hybrid ones; capital may be allocated according to the probability of falling at some statutory hurdle on (or before) a specified future date. This requires the application of stochastic techniques, which we illustrate below.

3.6 Working out the actual capital held raises the theoretical question of how much capital there ought to be in the business. This question is typically approached from the policyholder's perspective, for example with a focus on the 'ruin event' in which the company's solvency margin is completely exhausted. From the perspective of a policyholder who has already paid his premium, it is desirable to keep this probability as low as possible. In practice, it is not usual to measure credit risk at an individual policy level; instead a probability of ruin constraint (say a maximum of 5%, over a 5 year period) is placed on the office as a whole. This probability reflects not just the outcome at some fixed horizon, but also counts insolvencies at any point before the horizon.

3.7 We have calculated the ruin probabilities for these three companies over time horizons of 10, 5 and 2 years, as shown below:

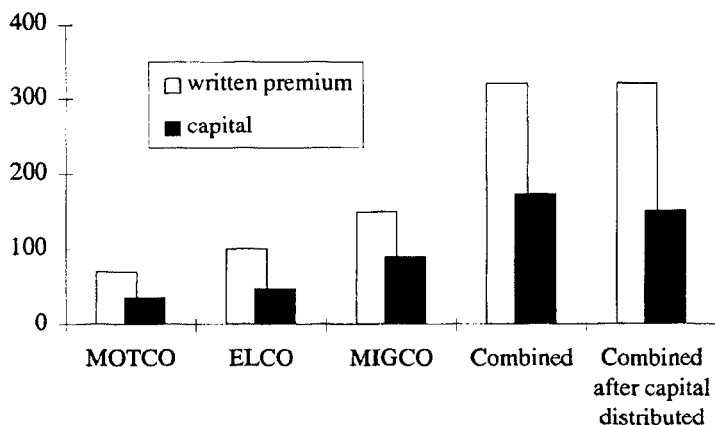


3.8 Managers will often concentrate on less extreme definitions of adverse events for internal purposes; for example, they may consider an adverse event to be one where regulatory capital falls below 30% of written premium at some point during the projection, for example. A higher probability constraint, say 25%, may be acceptable for such events, depending on the availability of additional capital should it be required. For our three companies, the probability of impairment, over 10, 5 and 2 years, is as shown:

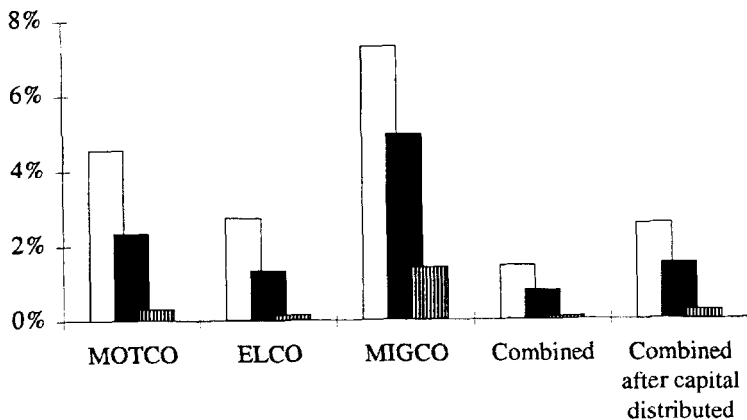


3.9 These examples have deliberately been chosen in such a way that the impairment probability within 5 years is exactly 25%. The management objectives for these companies have been assumed to be identical, and to this standard the companies are equally solvent. However, as noted from the previous chart, the ruin probabilities are different. A rating agency may provide a further perspective. An interesting question is how the cost of capital might differ between these monoline companies. We now look at what would happen if these companies were combined - plainly we expect the combined company to be more resilient because of the capital pooling, but by how much?

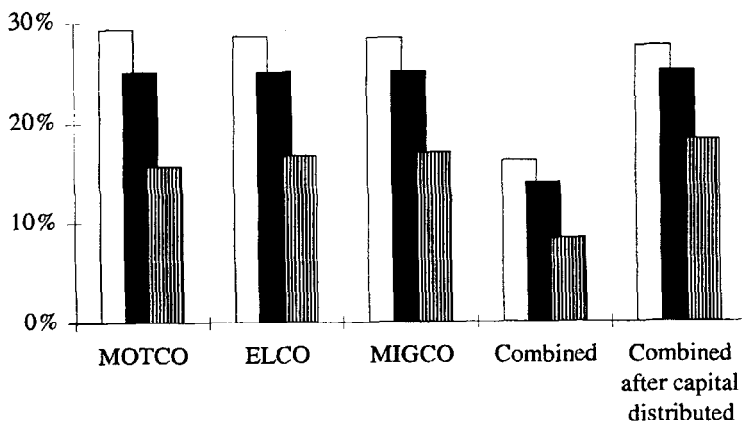
3.10 An alternative way of looking at this is to say that if we regard a 25% probability of impairment to be acceptable, then the combined company is overcapitalised. Not all the capital of the combined company is required to keep the impairment probability down to 25%. The remainder could, for example, be distributed to shareholders, or put to other use. This leads us to consider five cases, as shown below:



3.11 Allowing for this distribution of capital, the relevant ruin probabilities then appear as follows (again, over 10, 5 and 2 years):



3.12 The corresponding impairment probabilities are as follows:



3.13 For many purposes it is helpful to know not only the aggregate capital required but also to allocate this capital down to each line of business. Increasingly, managers are looking to set profit targets for individual business areas that relate to the underlying cost of capital. But how do we allocate capital? In this example we have a problem because although we set up the example so that each

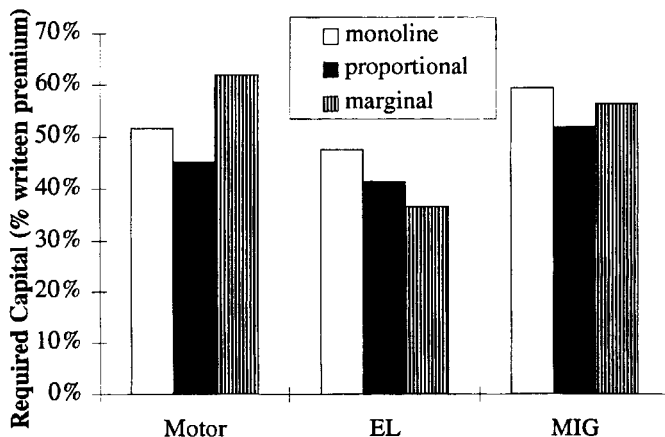
business had exactly the 'right' amount of capital prior to the merger, there are some diversification benefits from the merger which also need to be allocated down. Who gets credit for the diversification benefit?

3.14 An easy option would be to continue to regard the businesses as stand alone entities. The diversification benefit has arisen from management decision making and they might argue that by keeping the surplus pool to re-allocate according to the strategic direction they wish the company to take makes perfect sense. This implies that components of the business have to be viable on a stand-alone basis, but does not preclude possible subsidy from the management capital pool if they regard this as an appropriate use of capital.

3.15 Another way of approaching this problem would be to allocate the diversification savings back down in proportion to the original capital. However, this may well produce distortions that lead to inappropriate business decisions. As these capital savings are due to diversification, we might hope to tie the savings to the diversification achieved by each class. For example, we might argue that most of the risk in the EL book is liability risk, which is likely to be diversified within the merged company; on the other hand, both MIGCO and MOTCO are taking substantial exposure to the equity market; no diversification benefit can be expected here from the merger.

3.16 A more sophisticated method of capital allocation examines the *marginal* increase in capital required for the merged business as a result of an extra £1 of written premium in each line. This marginal calculation does reflect the diversification credit more logically; as we suspected, the proportional method undercharged MIG and overcharged EL. We have assumed in this calculation that the asset mix used is proportional to that inherited from the original 3 companies.

3.17 The above three methods of calculating capital are illustrated in the chart below:



3.18 An interesting feature of the latter method is that if we calculate total (ground up) capital requirements using the marginal capital rates, we get back to the overall requirement for the new office as a whole. This is a useful feature of the approach because it gives us an additive model of capital within the business. Management are often seduced by such simplicity. Note that this feature occurred despite our rather awkward definition of capital (which was a mixture of regulatory and economic capital, calculated by simulation with a 'ruin at any time' definition). The industrious reader will demonstrate mathematically that this reconciliation always works, provided that the overall capital requirement is first degree homogeneous in the vector of written volumes.

3.19 The practical difficulty with this marginal approach is that future management decisions may significantly affect the marginal capital rate with consequent implications for the underlying business areas. In our example, consider what would happen if another monoline company was taken over - would it be "fair" to penalise an existing business line if this resulted in an increase in marginal capital required?

3.20 An implicit assumption when using these numbers is that the probability of the adverse event is a fair measure of risk - in particular that the relationship of that event to other external events is immaterial. For example, MOTCO and ELCO had the same probabilities of impairment, so we might think that a prospective investor would regard both as equally risky. However, if they have read the finance text books, diversified investors should regard ELCO as less risky in a portfolio context. This is because ELCO's risk, which arises mainly from underwriting, is arguably well diversified within the shareholders' other (non-insurance) investments. On the other hand, most of the MOTCO's risk arises from the stock market itself, so may well be less easily diversified. Whether shareholders do, in practice, take such considerations into account when constructing their portfolios is a hotly contested point. The outcome of this debate has significant implications for company management and the interpretation of asset-liability studies. We will return to this subject later.

4. Return on Capital

4.1 Most insurers believe capital is a scarce resource. It could be allocated to a number of competing areas. Different business areas may offer different *returns on capital employed*, though with different degrees of variability. The techniques of ALM can be applied to this area to help understand the key issues. Management are keen to allocate surplus capital to those areas offering the highest marginal return. Longer term, this information can help them shape the business towards those lines with the highest ROCE. Of relevance here is the issue of business scalability - inevitably at some point there will be diminishing marginal returns.

4.2 Different methods exist for calculating the return on capital in a given simulation. Each have their own merits and potential difficulties. However, if we are using the results to compare different businesses then it is the differences between them that are more important than the absolute values.

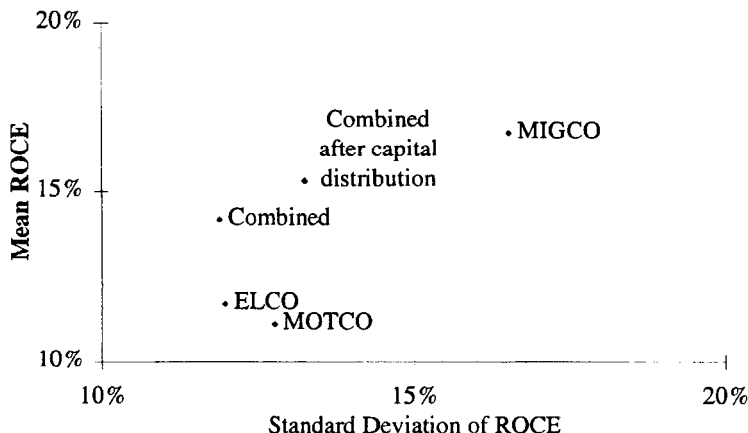
4.3 Perhaps the simplest method is to measure the amount of capital invested at the start and end of a project, calculate intermediate cash flows, and determine the internal rate of return. This is akin to the concept of a money-weighted return. The attraction of this method is that it avoids having to set a discount rate to apply to future cashflows. The downside is that, for complicated cashflows, there may be no solution to the IRR problem.

4.4 An alternative approach is to measure the amount of capital invested every time a cash flow occurs, and calculate the cumulative return over each sub period. This approach equates more closely to a time-weighted rate of return. It will always provide a solution, but is computationally more difficult. There are also conceptual issues to contend with - the method implicitly assumes that capital is freely available when required and can be handed back when not required. In practice this is unlikely to be the case.

4.5 An approach favoured by some finance theorists is to discount future cashflows at an appropriate cost of capital. This is determined by reference to the volatility of expected cashflows and the capital market line. Its chief merit is that it distils the ROCE into a single number, whereas the methods in 4.3 and 4.4 result in a distribution of outcomes from the simulations. Management often find it easier to compare two numbers rather than two distributions (though it might be thought that it is the task of senior management of an insurance company to be happy to do so). However, they also tend to be suspicious of CAPM and finance theory in general, so this method may not always be easy to sell.

4.6 Other variations on ROCE calculations are possible. The most obvious distortions in the above calculations are caused by known biases in the statutory reserving bases for the liabilities. One would perhaps wish to strip out any such margins before calculating returns. It is also possible to calculate returns based on distributable profit, or on dividends actually paid, rather than emerging cash flows within the insurer. Inevitably, when working in a large organisation, various (possibly conflicting) definitions will arise within the group.

4.7 Using the IRR method, based on actual cash flows, we can plot the mean ROCE against the standard deviation of ROCE as shown below: The implication of the chart is that ELCO dominates MOTCO on this measure. MIGCO offers higher returns but with more risk. The merged business looks impressive on this ROCE measure, with a higher mean than ELCO and MOTCO but lower risk than either.



4.8 This is an example of a 'risk-return' plot. Generally, we would like to combine low risk with high return, which means that we are pushing up and to the left. The *efficient frontier* is the boundary which determines the highest possible return for a given level of risk (or conversely, the lowest possible risk for a given level of return). An efficient portfolio choice will hold a combination of the businesses that lies on the frontier.

4.9 There are a number of limitations to the ROCE approach, which we now consider. These largely focus around the problem that, in practice, capital cannot instantaneously be re-allocated from one line of business to another. Instead, capital is tied up for a period after the business is written - generating so-called *sequential dependence*. There is an element of sunk cost which is essentially irreversible. This means that allocation of capital today may preclude a much more profitable investment tomorrow. It may in fact be better to retain excess capital based, for example, on forecasts of the premium cycle. Such considerations also have significant implications for dividend policy.

4.10 One approach for getting around the problem of sequential dependence of different lines of business has been outlined by Bride and Lomax. Their idea (based on Myers, 1977) is to redefine the notion of a *project* for which one calculates the return. The Bride and Lomax project concept includes future growth opportunities arising from a given decision, so that all consequences of a given change in business (for example, impact on renewals) are taken into account. This contrasts with the traditional product-centred approach where one year's new business is essentially treated as a product in its own right.

4.11 In addition, moving capital around is not necessarily linear. For example, if an economic definition of capital is employed, then this would need to include any profit margins in the premiums written. This means that £1 of capital allocated to motor may suddenly become £1.10 when reallocated to MIG.

4.12 A further complication arises because of the effect of decreasing marginal returns. In order to expand a book of business, rates may need to be cut. It is unreasonable to suppose that the same loss ratio distribution would apply if the business volume were to double. Lines of business which apparently achieve a high ROCE are effectively exploiting past investment in a warm customer base. Allocating more capital to these lines may not achieve proportional profits on a marginal basis.

4.13 It can be argued that the variability of ROCE is also a poor measure of true risk because it is overly influenced by essentially arbitrary reporting rules. For example, let us suppose we are examining a suitable investment portfolio for reserves corresponding to some long tail line of business. Let us suppose further that the actual cash amounts are reasonably predictable, so that the main risk relates to the investment side. It may seem self-evident that the lowest risk investment in this context is a matching portfolio of bonds; however, if the reserves are not discounted then a gilt-based strategy may show a volatile profit stream. Investment in 100% cash would seem less volatile, as the imposition of an arbitrary time horizon naturally fails to take into account reinvestment risk after that time horizon. However, unless the office is particularly stretched from a solvency perspective, it seems odd to argue that the economic minimum risk portfolio should depend on whether or not the reserves are discounted for accounting purposes. This whole topic, of seeking the "best" risk-reward plot on an economic basis, subject to the constraint of meeting acceptable criteria on an accounting basis, forms a whole subject in its own right. We have not expanded on it here, but this is an area that a real-life user of ALM needs to have a proper understanding of.

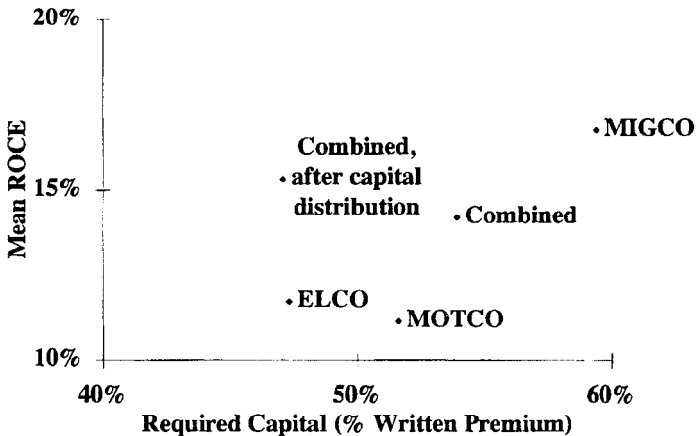
4.14 An alternative way of making decisions, rather than maximising the return on capital, is to set a required hurdle cost of capital - which is often in the range 13-15% depending on the perceived riskiness of the line of business concerned. This method is equivalent to that described in 4.5, though presented slightly differently. The hurdle rates are often translated into underwriting targets, but some care in interpretation is required here - setting a target loss ratio of 70% does not guarantee that the statistical distribution of ultimate claims has a mean equal to 70% of premium.

4.15 More formally, this cost of capital could be taken into account when calculating premium rates using a profit test construction. In theory, there are significant complexities to this approach which should be taken into account - one should allocate capital for the entire period of loss development. Both the amount allocated and the cost could vary over this period. In practice, it seems common to allocate capital simply over the term of the contract; this assumes that ultimate losses are known for certainty as soon as all the premium is earned.

4.16 One potential problem with this approach is the double-counting of risk adjustments. It appears self evident that a shareholder will require higher returns to justify commitment to riskier lines of business (although we will later qualify this statement). Perhaps this needs to be reflected in a higher required ROCE. The trouble is that this risk has already been counted once in allocating more capital in the first place. There may also be implicit margins loaded into the various actuarial assumptions. In practice there is a tendency for 'margins for risk' to pop up all over the place, with rather little economic clarity as to why they are there. This highlights the importance of having a clearly articulated methodology that is internally consistent.

5. Analysis of the Merger

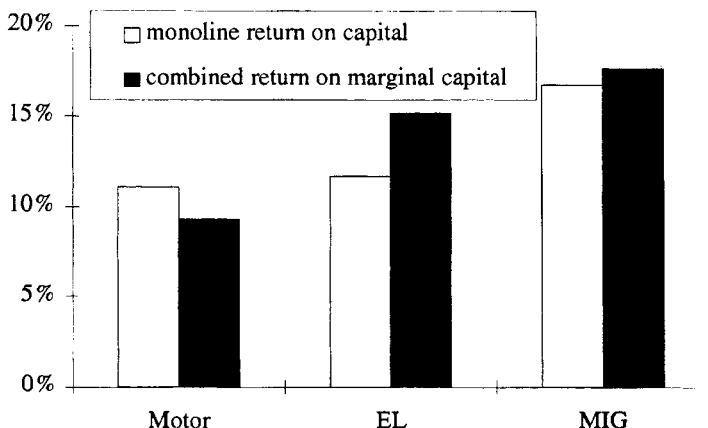
5.1 We can use our stochastic model to analyse the effects of the merger. The starting point for our analysis is to look at the risk (in terms of capital requirements) and also the expected ROCE over a 5 year period. These are shown below:



5.2 This clearly shows how the merger both reduces risk (in terms of required capital) and also improves the average return. It would seem clear that shareholders have gained significantly from the merger.

5.3 If not all of the excess capital is distributed, but some is retained on the balance sheet, then the policyholders are also better protected as the ruin probabilities have also fallen. So it would seem that everyone has gained from the merger - there are no losers at all. These results are not peculiar to the insurance industry - taking this argument to its logical conclusion, there would seem to be strong social benefits from all companies merging into a combined *World Industries PLC*. Of course this is an oversimplification, and other externalities, such as the consumer benefits of competition, need to be taken into account. However, if we fail to take such issues into account in ALM, as is commonly done, the likely conclusion is that mergers will generally look attractive.

5.4 We can carry this reasoning further by examining how the profitability of each line of business contributes to the profitability of the combined operation of the business as a whole. To do this, we allocate profits by line, and also capital by line according to marginal usage. The ROCE is then calculated by each line. The results are shown below:



5.5 These figures suggest that while the profitability of EL and MIG have both benefited from the merger, motor has actually got worse. The reason for this is that we have assumed the same asset allocations are still hypothecated to each liability line; the heavy equity backing of the motor portfolio concentrates risk compared to the other lines, attracting a heavy marginal charge. On this basis, it would seem better to exclude MOTCO from the merger.

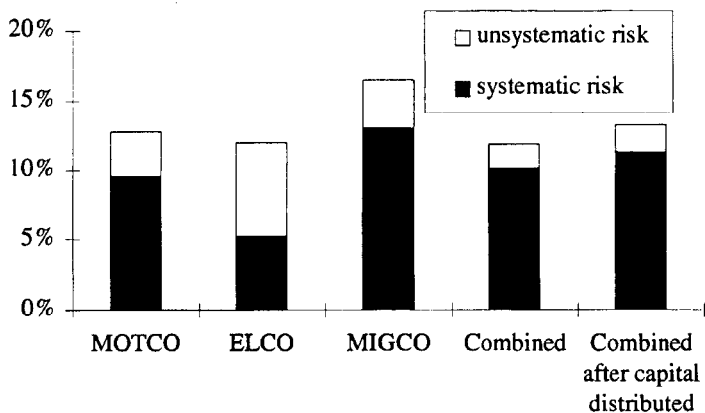
5.6 We should now inject a note of caution. A traditionally minded finance director might point out that shareholders and policyholders enjoy competing claims on a common pool of assets. The merger has rearranged those claims, but has not created any more assets. It then seems odd to argue that both shareholders and policyholders can simultaneously have gained from the transaction.

5.7 In particular, the supposed diversification benefits we are crediting to shareholders may have been open to them anyway simply by holding shares in all three companies. It is not clear why the shareholder would gain from combining these three companies, since in the event of a large claim, all three could go down together, whereas under the old regime, limited liability applied to the three companies individually.

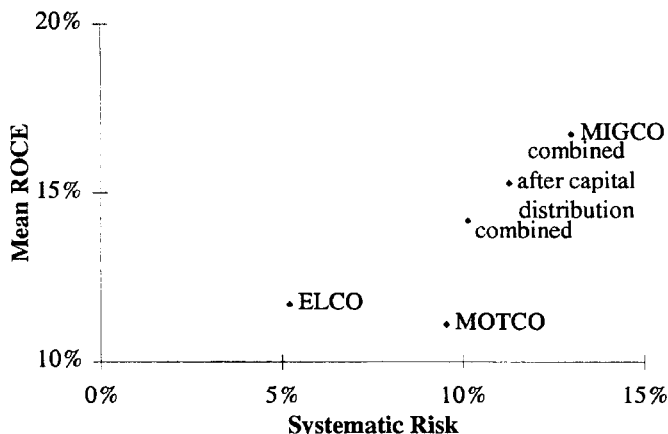
5.8 Perhaps the shareholder who did not hold all three companies has gained from the diversification? Again, this is unconvincing; if the shareholder had a good reason to prefer one of the original companies to the other two, he will not thank managers for frustrating his portfolio selection and forcing him to hold all three.

5.9 The complex stochastic models involve a lot of probability and statistics, and we are not questioning the statistical result that the ROCE could both rise in mean and reduce in variance as a consequence of the merger. The economic step which is being glossed over is the assumption that shareholders feel better off as a result. In other words, the implicit assumption is that the distribution of ROCE is the right thing to be looking at from the shareholders' perspective. Modern financial theory would suggest that, in a stochastic context, we can find measures which better represent the shareholders' interests; sadly, these measures tend to give support to the traditionally minded finance director, contradicting the supposed rocket science of ALM.

5.10 So the shareholder who has read the textbooks will take a different approach. He will make a distinction between different types of risk - systematic and unsystematic risk (also known as non-systematic, specific or diversifiable risk). The former represent risks that cannot be diversified away within a portfolio eg. equity market risk. The latter can be diversified away, examples including stock specific risk and company specific underwriting risk. The standard deviation of return on capital can be split into two portions, a systematic part correlated with other financial markets, and unsystematic risk specific to the company under consideration and which can be diversified simply by investing in other companies. For the companies under consideration, the split between systematic and unsystematic risk is shown below:



5.11 We can see that in this context, while much of the risk of MOTCO and MIGCO is systematic, ELCO contains a large dose of unsystematic risk. While the merger results in diversification of unsystematic risk, the systematic risk is conserved. A more meaningful risk-return plot would show return against systematic risk:



5.12 Thus, according to the text book, shareholders will see little gain from the merger. The systematic risk of the merged company is just an average of the constituents, with no gain for diversification. All the apparent risk reduction is a reduction in unsystematic risk, which the shareholder would have diversified anyway. The systematic risk is not eliminated by the merger - in fact, it increases if capital is distributed because the profits are more highly geared. The improvement in mean ROCE achieved by the merger is merely a fair compensation for the fact that the earnings have poorer quality. It is the same compensation as the shareholder would have got from gearing up his own portfolio. There is then, in theory, no overall gain to shareholders from the merger.

5.13 Financial theory suggests that the merger does not create value of itself while the same assets are still being held to meet the same liabilities. The merger can only create value if something economic changes as a result. For example, if the new company is better positioned to take advantage of profitable business opportunities than the individual entities, then value could be created. Perhaps the new entity has the resources to eliminate competitors. There may also be expense savings. Possibly management resources can be better employed.

5.14 In order for us to judge whether, in fact, value has been created, we need to model the expense savings, oligopoly profits, business opportunities and effectiveness of management resources. To date few asset liability studies have managed to address these issues.

5.15 If we are to accept the above economic views, then there are profound implications for ALM studies. The key question here is how the theory is borne out in practice. It would seem unlikely that the shareholders have adequate information about a company upon which to make their portfolio choices. In practice this is precisely the information that the managers of the business are searching for! It therefore cannot be well disseminated in the market. The separation of risk into systematic and unsystematic components relies on estimated correlations between assets and liabilities; such correlations are notoriously difficult to estimate with any confidence. The extent to which individuals make rational portfolio choices is also open to debate. Further, the text book theory itself starts to break down if we take account of market frictions such as taxes and costs of bankruptcy. Maybe as quality of information improves through time the theory will be borne out in practice - meanwhile there is a pressing need for management to understand the dynamics of the businesses they control. Whether this knowledge can be translated into shareholder value will continue to be debated.

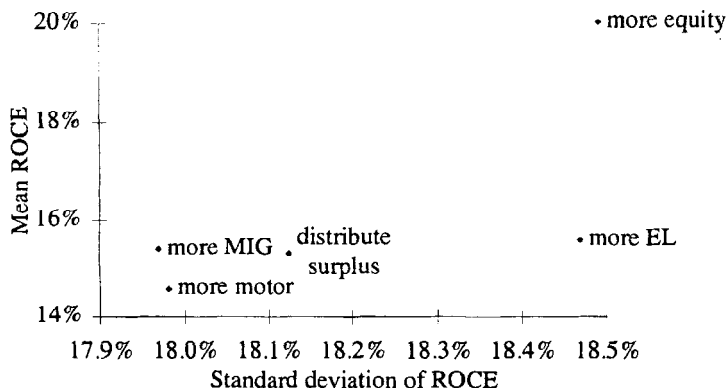
6. Uses for the Surplus

6.1 We have identified the fact that, following the merger, the capital held by the company may be more than is needed to support the current business. We have investigated the effect of distributing this capital to shareholders. However, that is not the only use for capital - it could perhaps be retained and put to better use within the company.

6.2 One alternative use of capital would be to write more business, in any one of the three lines. This will create value if the business is profitable.

6.3 In practice, it may not be desirable, or even possible, instantaneously to grow any one line of business. However, a similar effect can sometimes be achieved by cutting back on the reinsurance programme, thus ensuring that the existing capital base works harder. The reinsurance program itself is something that can be explored using a (potentially very complex) stochastic model, and this is currently an area of expanding interest and activity. We have not explicitly considered reinsurance levels in this example.

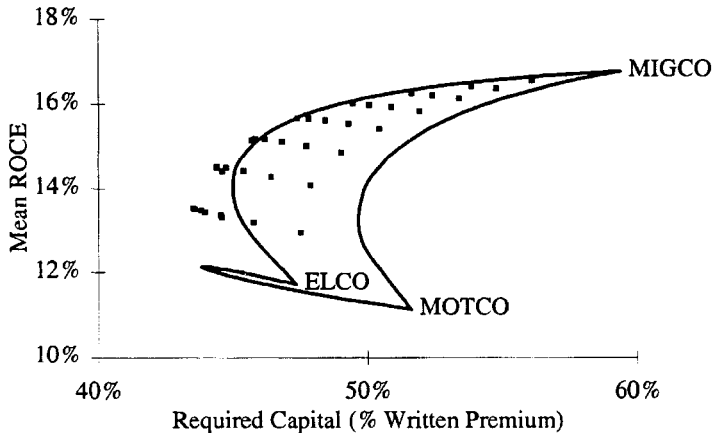
6.4 The improved capital position could, alternatively, be used to gear up the investment strategy of the company as a whole, by investing more in equities for example. Liquidity constraints may now be less pressing, allowing use of higher yielding assets. The effect of these strategies on the distribution of ROCE is shown below. The "best" option would appear to be to expand the MIG business. Writing more EL looks to be the least rewarding strategy (a contrast with earlier observations in 4.7). The gearing of assets into equities offers higher returns but also increased risk. Interestingly, increasing motor business looks to be relatively rewarding. Earlier in the paper we questioned whether MOTCO had a place in the merger - this highlights the need to separate the effects of the underlying business from the asset strategy.



7. Choosing a Liability Mix

7.1 We have considered the possibility of a merger, as an alternative to running companies separately. This is a special case of considering a composite insurer which writes several lines of business in various proportions.

7.2 We can investigate such issues by considering the risk (measured by required capital) and mean ROCE of various business mixes. The result of such an investigation is shown below:



7.3 In this plot, each dot corresponds to a different mix. The curves represent mixes of two liability classes. In each case, the original asset mixes are hypothecated to each business class.

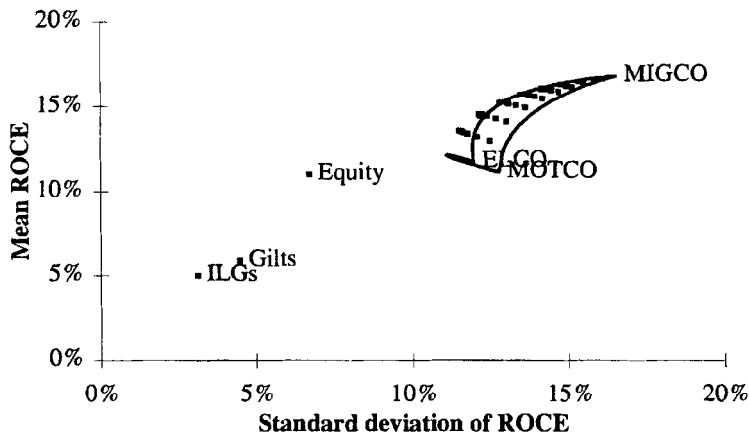
7.4 The curves joining different lines of business show the effect of combining just two lines, omitting the third. The curvature of these lines to the left indicate the benefits of diversification - illustrating the fact that, for example, a mixture of EL and motor may require less capital than either considered alone.

7.5 Examining the constituents of the efficient frontier helps us understand some key features of the businesses. We had previously questioned whether MOTCO really had a place in the merger, on account of its high capital requirements. At the higher risk end, it would indeed seem that there is little role for motor business, but lower risk companies might still optimally write some motor business

7.6 Many commentators believe in market cycles, where prospective loss ratios change in a more or less sinusoidal pattern over time. It is true, then we would expect an optimal liability strategy to be a dynamic one, where volumes rebalance towards the most attractive rates at any point in time. Such optimisation involves considerable additional complexities which we have not considered in our simple example.

7.7 Similar pictures can be used to evaluate reinsurance options, which, in this context, behave similarly to negative inward business. The attractiveness of reinsuring high severity low frequency events depends heavily on the objective functions applied. For the case in point, where capital is measured with regard to a 25% probability of impairment, rare events are likely to have relatively little effect and so the cost of such reinsurance is likely to outweigh the benefits. If, however, we were monitoring ruin probabilities, reinsurance may appear to have a larger role. One of the perpetual frustrations of ALM is that the risk parameters set are often quite arbitrary (as we have seen in our own example). By changing assumptions and objectives it is possible to justify a wide variety of courses of action. One of the key values of the modelling process is helping to quantify how different the assumptions need to be to negate the value of a particular strategy.

7.8 From the shareholders perspective, the variability of ROCE may be a better measure of risk than capital required. In this case, we can again plot the set of outcomes from a range of strategies:



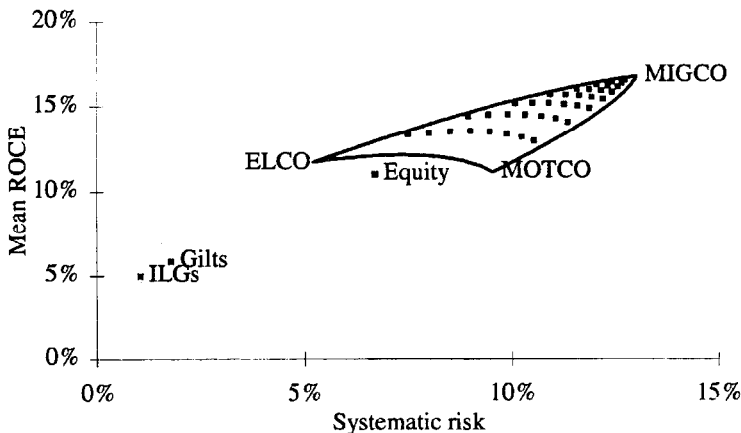
7.9 One benefit of considering the distribution of ROCE is that it enables insurance business to be compared with alternative uses of capital, such as direct investment in securities. Insurance business often appears high risk and high return relative to other investments, because the liabilities can be viewed as a form of debt which is being used to gear the asset portfolio.

7.10 This chart is also a bit misleading, because equity investment is not an exclusive alternative to investing in insurance. We can do both, by writing insurance business and investing the premiums in equities. The equivalence of investing inside or outside a company is a dominant theme in modern corporate finance. While the expected returns of an insurer can undoubtedly be improved by an equity strategy, it is also open to the shareholder to implement an equity strategy himself. He gets the same equity return either way; the choice of which route is better may depend on second order issues such as tax or transaction costs. The choice does **not** depend on the equity risk premium.

7.11 For poorly capitalised insurers, we discover a new motivation for equity investment. In effect, the insurance business can be considered as a form of debt, so the insurance company itself can behave like a geared equity investment. But while a personal debt is still enforceable in the event of poor equity performance, a limited liability company may simply go into liquidation. Insurance may then provide a favourable means of geared equity investment which comes with a free put option on the downside. However, if policyholders recognise the implicit credit risk and demand lower premiums, then the put option is not free and there is no overall gain to shareholders.

7.12 Of course, individuals making investment decisions will want to take proper account of risks and returns. The notion that the equity risk premium should not affect *corporate* investment policy may seem iconoclastic, but is in fact simply a reflection of the fact that £100 of equity is worth the same as £100 of gilts (that is, £100) whether held by a company or by private individuals. This is one of the major areas where established ALM comes into conflict with modern financial economics.

7.13 The enlightened shareholder who has read the textbooks will want to compare the various liability mixes according to the systematic risk, as shown below:

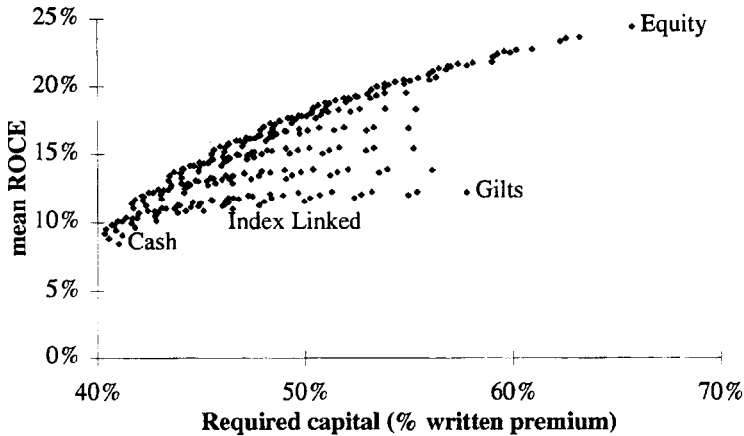


7.14 This chart shows a rather different picture - after taking out the diversifiable risk, ELCO is now looking attractive relative to UK equities. Of course, it is important not to take this too far - you need to hold some UK equities in order to be able to ignore the unsystematic risk in ELCO. By contrast, MOTCO now looks very unattractive.

7.15 Of course, all the curvature we expect to see in efficient frontier plots has now virtually disappeared. The efficient frontier contains only combinations of two investments at a time. This is what we would expect - by considering only systematic risk we strip out all benefits of diversification. The slight non-linearity which remains is a consequence of the non-linear definition of required capital and compounding effects within the ROCE calculation.

8. Asset Allocation

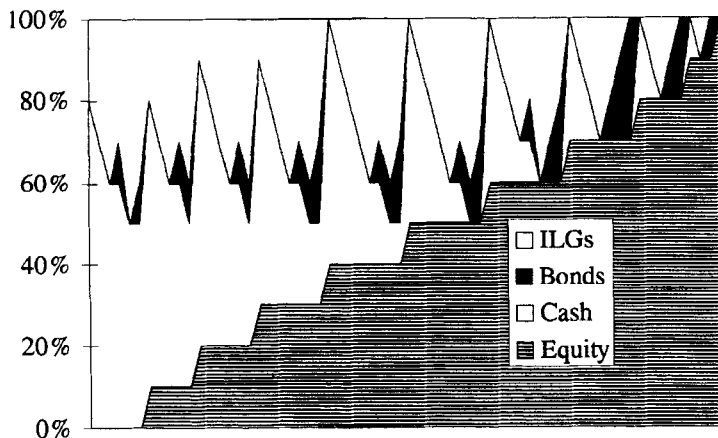
8.1 Many ALM studies begin and end with an assessment of the appropriate asset mix to hold that corresponds to the liabilities. We have demonstrated that the powerful and flexible tool of stochastic modelling can be used to gain insight into many areas of business decision making, of which asset allocation is but one. The standard tool for asset allocation has become known as the *asset liability efficient frontier*, or ALEF for short. This technique holds liabilities constant, and quantifies the consequences of different asset mixes on capital requirements and expected returns. An ALEF for our combined company is shown below:



8.2 Alternative presentations are also possible, for example, by measuring risk in terms of variability of ROCE, or concentrating on systematic risk.

8.3 Strictly speaking, the ALEF is the top surface of this scatter plot. This plot has been calculated using various prior specified asset mixes; it is also possible to apply optimisation routines to produce a smoother picture.

8.4 The scatter plot shows what risks and returns are possible; in order to implement this, we need to know what portfolios are represented on the efficient frontier. This can be shown in an area chart, which displays the asset mixes corresponding of each point on the ALEF (moving from low risk at the left hand side, to higher risk on the right):



8.5 Many of the conclusions from this chart are pretty intuitive - as we move to higher risk portfolios, we hold less cash and more equity. Perhaps the relatively limited scope for conventional bonds is the most surprising result - since such investments form the bulk of most non-life asset portfolios in practice.

8.6 The irregularity of this chart is a reflection of the fact that markedly different portfolios may lie close together on the efficient frontier. This means that the results of an ALEF may be highly sensitive to the assumptions adopted. The efficient frontier itself is nothing but a replay back to us of our underlying assumptions. It follows that if we change our assumptions, many of which have a range of possible values, then the composition of assets on the frontier will also change.

8.7 The most practical way to resolve this dilemma is to regard the efficient frontier with some suspicion. What we are really seeking is a portfolio of assets that is reasonably efficient (ie. close to the frontier) but one that continues to be reasonably efficient as we change the underlying assumptions. In this way we resolve some of the issues surrounding parameter uncertainty.

8.8 Once again, we return to the issue of whether an ALEF genuinely reflects the interests of shareholders. Conventional economics would suggest that shareholders should, themselves, be calculating ALEFs for their personal financial planning. In this exercise, we would not expect each share to lie on the ALEF, but the combined portfolio should.

8.9 We might then question why insurers should try to put themselves on an ALEF without reference to other assets their shareholders might own. One possible answer to this is that, unlike shareholders, managers may find it hard to diversify unsystematic risk. For example, if a company fails and the DTI forbids its directors from acting as directors again, any other directorships held will not soften

the blow. The cynic might argue that there is an incentive for managers to invest shareholders' money in ALM to cover their own interests, even though the outcome may be of little or no benefit to shareholders (or indeed, to policyholders).

8.10 A more mainstream perspective would be to argue that it would be unreasonable and almost certainly infeasible for a company to take account of the risk preferences of their entire book of shareholders. The company acts towards its own view of the appropriate assets and liabilities to hold and the presence of an existing set of shareholders suggests that this view is supported. Where things become more difficult is if the company changes its strategic direction - in reality we observe that gains or falls in share price reflect the degree to which the change of plans are supported by the shareholders.

8.11 In practice, we observe that very few firms choose to hold the minimum risk asset position, but instead choose to take additional risk on the asset side of the balance sheet by holding equities. There may be a number of rational explanations for this. It may be the case that the regulatory capital they are required to hold exceeds the underlying economic capital they regard as appropriate. If the firm is prevented from writing more business by the regulators, the natural conclusion is to seek to maximise asset returns subject to the economic capital constraint. Another possibility would be if the firm does hold *genuine* surplus capital then it may wish to retain this for future usage rather than pass capital back to the shareholders. Such an approach recognises the opportunity cost of capital. In such circumstances it may be appropriate to gear the assets until the capital can be utilised, especially if policyholders or brokers overlook the increased *credit risk implied by such a strategy*.

8.12 The combined effect of many companies holding equities on the balance sheet is rather curious. Each company will have its own core business, be it widgets or insurance. Each company also has the opportunity to diversify its core business by investing in the stock market. When this added up in the shareholders portfolio, of, say 100 stocks, the net effect is that the core businesses are represented once each, but the stock market is sampled 100 times over.

8.13 Of course, the stock market only consists of those same core businesses again. So the cross-investment arguably has little effect on the diversified shareholder, except to complicate his arithmetic when examining economic exposures. However it is frustrating for the speculator who is bullish about insurers and bearish about widgets if he cannot get exposure to insurers without obtaining an indirect exposure to widgets.

8.14 Insurers are not the only example of cross investment - the same feature occurs to a much greater degree in the UK because of defined benefit pension schemes. Such money circulation primarily benefits the Inland Revenue, who can erect tax gates at various stages of the cycle.

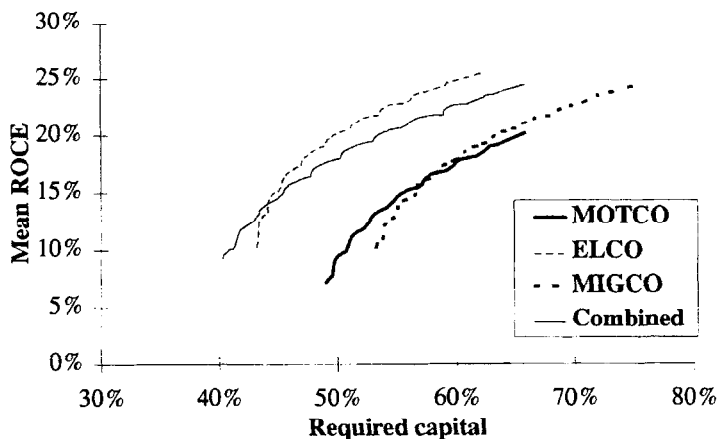
9. Choosing between Asset and Liability Risk

9.1 It is now time to put the pieces together to find out whether shareholders money is better spent on writing more business or gearing up investment policy.

9.2 There is an important distinction between asset and liability allocations which needs to be pointed out here. Asset allocations can, by and large, be carried out quickly and at relatively low cost. By contrast, changes in business mix or corporate structure tend to be expensive to execute, and may take considerable time.

9.3 It is therefore natural to identify a particular corporate strategy not with a single point on the risk-return plane, but with a whole ALEF, corresponding to different asset mixes. One then compares alternative structures by examining how the ALEF changes. This takes into account the fact that any change in corporate structure may be accompanied by an asset reallocation.

9.4 In our example, the ALEFs before and after the merger are as follows:



9.5 This shows a different picture from when assets and liabilities were examined separately. On the basis of ROCE, MIGCO appeared by far the most valuable. But here, ELCO looks the most attractive for most of the risk-return plane, with the combined company featuring at the low risk end. In other words, for ELCO's shareholders, most of the gains from the merger could have been obtained, presumably at lower cost, simply by rearranging the asset portfolio.

9.6 The same techniques can be employed to compare other changes in corporate structure; for example, in the reinsurance programme. It is self evident that in most cases purchasing reinsurance will reduce risk and return; the question is whether the effect has been obtained more cheaply than simply adopting a more defensive asset orientation. Again, such charts are useful in considering alternative risk transfer vehicles, such as securitisation, insurance derivatives or financial reinsurance, where consistency is required both with the standard processes for evaluating reinsurance and also with the investment process.

9.7 It is worth pondering for a minute the amount of calculation involved in a full ALEF investigation covering both assets and liabilities. Suppose we consider 100 mixes and 200 asset mixes. This already gives 20,000 scenarios to consider, each of which needs to be put through, say 5,000 simulations. But the work is not over, because the capital allocation is an iterative procedure, solving for the specified impairment probability. If this requires 20 iterations for each scenario, then we have a total of 2,000,000,000 ruin probability evaluations to contemplate. In practice, we can use some clever tricks to avoid this computational burden. To start with, we need to do as much as possible analytically, and prune bits of unused code (for example, accounting outputs). We can use variance reduction techniques to choose our simulations cleverly (instead of independently) so reducing the number required. We can also eliminate some scenarios as obviously not on the efficient frontier after a few simulations, or by domination arguments in comparison with other scenarios. We have also employed various approximations and interpolations. This brain intervention means we can start to ask (and answer) questions which are completely beyond the reach of sledgehammer Monte Carlo investigations.

10. Dialogue with Shareholders

10.1 ALM has often been seen as a tool in company management, to improve understanding of the business and hence improve decision making. We think there is another benefit where work to date has only scratched the surface.

10.2 While the information from an ALM exercise may help managers to make decisions, the same information is potentially valuable to shareholders wanting to construct their portfolios. For example, it is currently unclear whether shareholders have enough information to look through companies they own to ascertain the underlying economic exposures. However, if shareholders did have access to such information, it would be much easier for them to avoid unwanted risk concentrations. They would prefer to invest in companies that provide such information.

10.3 There is some evidence to suggest that fund managers add value to the economy as a whole by forming a superstructure above individual companies, and allocating capital efficiently to profitable projects. We are starting to see stochastic investigations in prospectuses for company launches; it is natural to suppose that this will increase, as it maximises the value which can be derived from the superstructure.

10.4 In this context, it might not be sensible to incorporate optimisation within ALM, given that our model inevitably misses out some features of the real world. Instead, we should restrict attention to a small number of strategies which make business sense, disclose these more fully, and expose projects to the rigours of the market.

10.5 If we are helping shareholders to construct optimal portfolios, we need to take account of the choices open to the shareholder. For example, there are many ways in which a shareholder can achieve equity exposure. It would seem less intrusive to leave this decision to shareholders themselves, rather than force their hand, for example by holding equities on company balance sheets. On the other hand, a shareholder can only get insurance exposure by buying insurance companies. This is a motivation for concentrating on core skills, and letting the shareholder diversify on his own account.

10.6 This argument is strengthened when we realise that the insurance markets contain significant barriers to entry. It is reasonable to suppose, therefore, that at some times of the cycle there may be excess oligopoly profits to be had, which the shareholder cannot exploit directly. By contrast, the barriers to investing in capital markets are very low. So, economic theory suggests that oligopoly profits are not likely to exist. In particular, there is no reason to suppose that an insurer might actively manage an equity portfolio better than the shareholder can do himself.

10.7 This fundamental distinction between assets and liabilities is not, we believe, widely appreciated. Indeed, one of the supposed benefits of ALM is to enable managers to integrate asset and liability risk and return within a common framework. Using such tools, managers have figured out, correctly, that it is easier to create high returns by investing aggressively than by careful underwriting. The dawning of this realisation has heralded a new era of equity investment by insurers. It is, however, debatable whether insurance shareholders are better off as a result.

10.8 In the context of ALM, it is instructive to consider the possible consequences of shareholder irrationality on optimal management behaviour. A number of fund managers do use risk control procedures which attempt to allow for correlations between stocks in determining a portfolio risk. Individual stocks might then be viewed in terms of their influence on the portfolio as a whole, that is, their systematic risk. However, anecdotal evidence suggests that simpler investors may use stock variability, for example, as a measure of risk. Such a shareholder would presumably require the same reward for both systematic and unsystematic risk. This view would seem to lend support to traditional ALM which does not separate the two kinds of risk.

10.9 However, there are further implications. As already noted, financial markets tend to reward systematic risk; extracting rewards from the unsystematic risk in insurance business is much harder - some would say impossible. Traditional ALM, often used as the basis for increasing equity exposure, naturally ends up taking the easy route of passing on systematic risk because the rewards follow without hard work.

10.10 The shareholder who cannot tell the difference between systematic and unsystematic risk then ends up with more of the strain of risk which he cannot diversify. In this situation, it is questionable if management are really creating shareholder value. The more astute shareholders that recognise what is being passed through can compensate by adopting a short equity position.

10.11 Whether shareholders are in a position to understand these issues from the market flow of information is questionable. It may also be the case that the regulatory capital framework naturally encourages insurers to gear investment risk. Nevertheless, caution would be advised when changing investments on balance sheet. Capital markets tend to be unkind to companies whose policies fly in the face of investors preferences as revealed by market investment choices. Such behaviour on the part of companies may lead to adversarial relationships between capital providers and managers from which nobody gains.

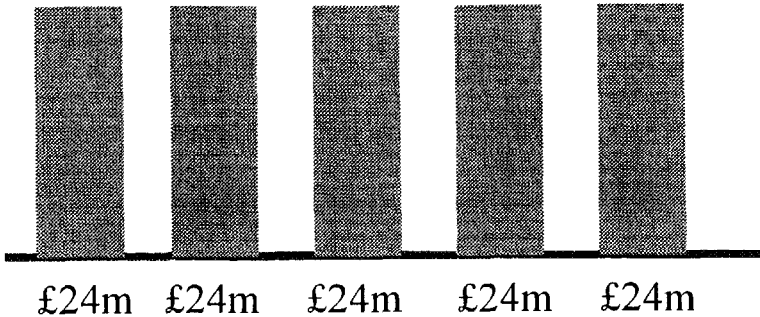
10.12 If shareholders do fail to separate systematic and unsystematic risk, this will have easily testable consequences. For example, if shareholders discount future receivables according to their total variability, then investment trusts should trade at a substantial premium to net assets. Historically, most of them have traded at a discount.

11. Capital Structure

11.1 One further issue we could ask our stochastic model to address is the optimal capital structure for the balance sheet. However, firstly there are some theoretical issues to consider.

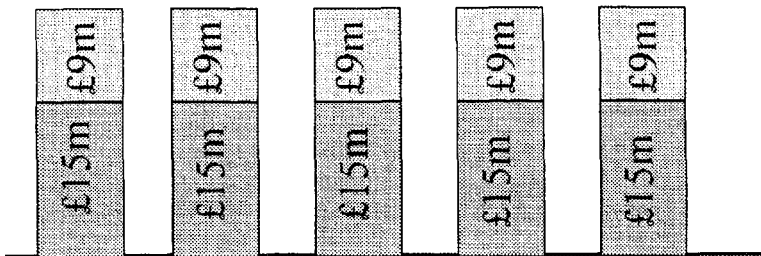
11.2 The Modigliani-Miller (MM) theorem is perhaps one of the most discouraging results of financial economics. It states that the value of a firm is not affected by its capital structure; for example, by whether it is debt or equity funded, or how it decides to invest its assets.

11.3 This flies in the face of much conventional wisdom of corporate finance. For example, consider a company which generates constant expected cash flows in perpetuity:



11.4 If the cost of equity capital is 15% pa then this company is worth $\text{£}24\text{m}/15\% = \text{£}160\text{m}$.

11.5 Now suppose we issue £100m of debt with a 9% coupon, and use the money raised to buy back shares. Operating profits remain as before, except that bond holders now take their 9% cut. The new situation can be seen as follows:



11.6 We can now value the firm again. The value of equity now seems to be $\pounds 15\text{m}/15\% = \pounds 100\text{m}$. So the total value of the firm (equity plus debt) seems to have gone up from $\pounds 160\text{m}$ to $\pounds 200\text{m}$.

11.7 Of course, the catch with this wealth creation wheeze, which Modigliani and Miller were the first to spot, is that the quality of earnings has been spoiled by the increased gearing. Shareholders may require a higher return now, say 25% which conveniently brings the firm value back down to the $\pounds 160\text{m}$ we started with.

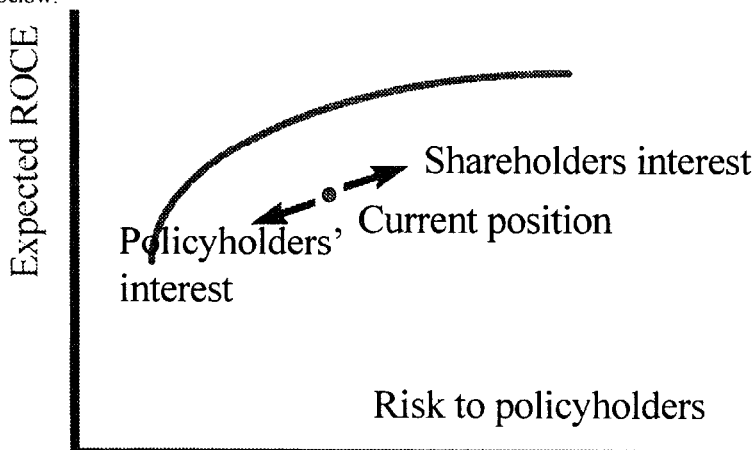
11.8 This always has to happen, argued Modigliani and Miller, because the quality of the cake is determined by the ingredients, not how you carve it up. Similar considerations rule out added value from changing investment policy, because holding equities on the balance sheet, for example, dilutes earnings quality which in turn increases the cost of capital, thus negating the effect of the higher expected profits. In simple terms, this happens because $\pounds 100$ of equities is worth $\pounds 100$ whether held privately or on a company balance sheet.

11.9 Of course the problem for ALM is that the MM theorem rules out the possibility of added value overall precisely where we are trying to create it - in rearranging the capital structure or investment policy of insurers. So, who believes this theory works in practice? Naturally, one would expect ALM experts to be more than sceptical. In fact we also find little support for the theory from the academic community. The presence of market frictions, particularly taxes and costs of bankruptcy tend to invalidate the MM results. In the real world, we observe many companies changing their capital structures - changing debt into equity, equity into debt, fixed debt into floating debt etc. Do shareholders really adjust their personal portfolios after each of these changes? In addition, there may be some bodies of shareholders who would feel temperamentally inclined to accept the higher risk associated with more volatile earnings in return for the chance of higher returns, and they would be prepared to pay a higher price for the company in paragraph 11.7.

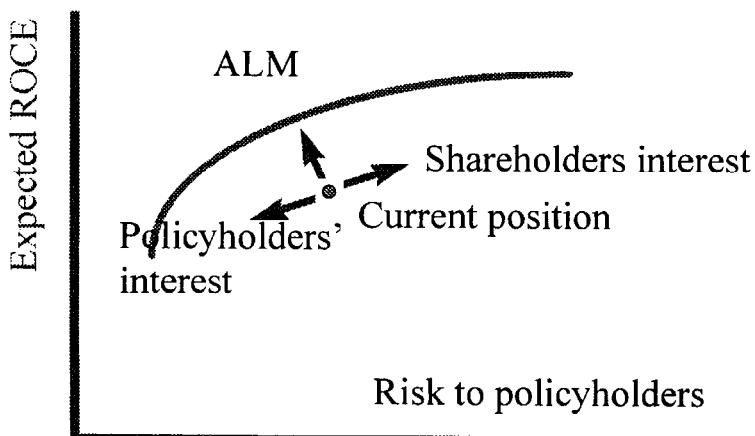
11.10 There clearly are some ways in which investment policy, for example, can affect the quality of the cake. For example, a strategy which produces highly volatile performance relative to the liabilities may destroy value by diverting management attention from creating value in the insurance business. As Jagger & Mehta (1997) point out "The excess returns available from riskier investments compensate for risk taken in a portfolio context (systematic risk) but not for avoidable losses arising from imprudent management". Shareholders and policyholders clearly have a common interest in avoiding the direct and indirect costs associated with bankruptcy.

11.11 In addition, while there may be only limited scope for overall gains in value when investments are rearranged, we can reallocate claims on those assets by asset liability management. For example, shareholders would like to extract as much as possible from policyholders, while passing on as much risk as possible. Policyholders, of course, want the opposite.

11.12 These tensions are illustrated by the efficient frontier diagram below:



11.13 Established ALM methodology tries to determine an efficient frontier which trades off risk (to policyholders) against return (to shareholders). If the current strategy lies off the efficient frontier, it may be possible to increase return without increasing risk, benefiting shareholders. Similarly, it may be possible to decrease policyholder risk whilst maintaining expected returns. More likely, however, the ALM result will change the balance of risks. Management will judge the appropriateness of this behaviour.



12. Applications of Real Option Theory

12.1 Some of the most exciting recent work in finance has concerned so-called 'real options': that is, *un-exploited future opportunities for future growth*. The excellent text book by Dixit & Pindyck (1994) covers this ground well. In the non-life context, Bride & Lomax (1994) touch on many of these issues.

12.2 The central idea behind the real options approach is that many projects have an element of sunk cost, but also flexibility of timing. At any point, the manager can decide either to launch the project or wait for new information.

12.3 Suppose I have a pet project which I want to pursue. Each day, I calculate the NPV of the project, but unfortunately it comes out negative. Then one day, I wake up and calculate my daily NPV, and get a positive number. I sprint to the finance director to get support for my project.

12.4 If the finance director is rational, he will probably tell me to wait. If he goes for each project the first day it has a positive NPV, he will end up initiating a large number of projects with only marginally positive values. If, on the other hand, he forces each applicant to wait another month, more information has emerged, and our finance director can choose between projects that are very positive (which he accepts) and those which are very negative (which he rejects). Thus, the average profitability per project, and hence shareholder value, is dramatically enhanced.

12.5 This goes some way to explaining why hurdle rates of return employed in industry tend to be much higher than naive economic theory suggests. A project needs to be very profitable before the time value of starting now outweighs the value of waiting for more information. Discounted cash flow analysis effectively ignores the option value of waiting, and so is only applicable to 'now or never' decisions. As the new methods take root, we expect to see a decline in the use of measures such as ROCE to evaluate corporate strategies. It will be important for the actuarial profession to fully understand these issues if it wishes to maintain its claims to expertise in capital projects.

12.6 Dixit & Pindyck use their real options theory to explain a wide range of social and other phenomena - for example why government cutting interest rates does *not* stimulate investment. In effect, the value of waiting for new information increases with the volatility of interest rates, so if the government makes interest rates volatile, it causes projects to be deferred. They also consider more far-fetched applications to marriage/divorce and suicide!

12.7 The only way to value these options of waiting is to use a form of stochastic model. This seems an ideal value-added application of ALM, for example in determining the timing of entry into a new market according to the market cycle.

12.8 However, the kind of stochastic model we need for pricing real options may be rather different from what we currently have. In particular, we need to allow for dynamic feedback triggering option exercise at a point in time. For this, we need to be able to calculate contemporary conditional distributions at a point in time - which either requires an exponentially branching tree of simulations or tractable analytical properties, such as those provided by GRAFT or the random walk model. In addition, in option pricing, the fit to current market prices transcends more traditional notions of statistical adequacy based, for example, on data-mined time series.

12.9 Real option techniques also provide insight into capital and dividend policy. Some insurers retain a substantial proportion of earnings; others pay large dividends in good times and then make rights issues when capital is short. In a frictionless world, there would be no point in staying well capitalised; capital can be brought on board when needed, at a market cost, and distributed when excess to requirements. In reality, moving capital around incurs taxation and other transaction costs, and contains inherent in-built delays. This provides a motivation for keeping capital on the balance sheet to ensure optimal exercise of timing options. However, capital on the balance sheet also attracts tax on investment returns and gratuitously enhances the credit risk provided to policyholders. ALM provides a way of balancing option values and transaction costs to find an optimal approach to capital management.

12.10 Another application of the real options theory is to ruin theory. Shareholders and policyholders have competing claims on the assets of an insurer. The position of shareholders is protected to some degree by statute, but shareholders also have some valuable options. For example, within limits, shareholders may demand higher or lower dividends, and so can control the capitalisation of the company. Equally well, if the company is short of capital, shareholders have an option to inject more in the hope of continuing profits from future business.

12.11 This means that financial ruin is not so much a random event arising from lack of cash, but rather the exercise of an option by shareholders. The willingness of shareholders to support a company will depend on its prospective future profitability, not only on its past record. This suggests that many definitions of ruin used in ALM need updating. From the shareholders' perspective, ALM could be a valuable tool in deciding when to pull the plug.

13. Conclusions

13.1 ALM might now be considered a mature science, with a fair weight of complex mathematics and impenetrable literature behind it. The ability of ALM to cram a hard disk full with statistical output is now beyond dispute. The biggest challenge now is interpreting this avalanche of figures intelligently to lead to better decisions than we made before.

13.2 ALM is still not a unified or self-contained body of knowledge and no one method of interpretation seems uniquely preferred in all circumstances. Different techniques promoted under the ALM banner may produce conflicting answers. These answers may also conflict with financial theory and with current or established business practice.

13.3 Financial theory itself is going through a period of radical development. This has important implications for the way we implement ALM and use it to assist business decisions. A more thorough understanding of financial economics may help us remove some of the arbitrariness and inherent contradictions in current ALM practice.

13.4 Stochastic modelling is a powerful and flexible tool that can help us understand the dynamics of a business. There remains vast untapped potential for adding value using such techniques. The benefits include improving management information, reducing transaction costs, saving tax, reducing bankruptcy risk, providing useful information to shareholders and evaluation of real options embedded in projects.

Disclaimer

Any opinions expressed in this paper are those of the authors alone, and are not necessarily those of our employers, the Institute and Faculty of Actuaries, or any working parties of which we are members.

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