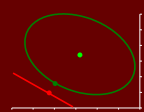**The Actuarial Profession**
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Risk Geographies
Understanding risk in n dimensions

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Agenda

- Introduction - background and context

- The Risk Geographies framework

- Worked example – illustration of the process

- Summary

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Introduction **–** background and context

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Aggregation within ICA – Current Best Practice

Correlation Matrix

The majority of firms are using a correlation matrix approach as the primary means to calculate diversified capital requirements

Advantages

- Easy to implement
- Simple to communicate to senior management and the board
- Enables the ICA to be built up from first principles
- Makes it easier to target the required confidence interval

Disadvantages

- A lack of data to set individual correlation assumptions
- Assumes risks are Normally distributed
- Will not allow for non-linearity between risks
- May double count the impact of management actions

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Aggregation within ICA – Current Best Practice

Scenario testing – FSA "Medium bang" approach

Scenarios are calculated assuming a lower confidence interval for each risk so that when run simultaneously it represents a 1-in-200-year level event

Advantages

- Should capture non-linearity impacts
- Will not double count the impact of management actions
- A subset of risks can be tested to target those areas where non-linearity is thought to exist

Disadvantages

- Difficult to assess the level of the reduced confidence interval
- Not clear that reducing the confidence of each individual stress by the same amount would generate the most onerous capital requirement at the required confidence interval

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Aggregation within ICA – Current Best Practice

Scenario testing – "Brainstorming" approach

Scenarios derived by considering possible adverse events and then brainstorming the knock-on effects within the business
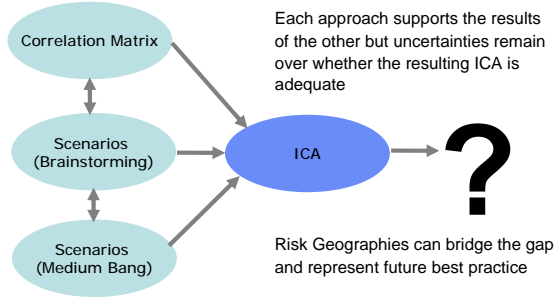
Advantages

- A good way to engage other areas of the business, senior management and the board
- Helps to demonstrate the "Use test"
- Can help firms to assess the particular risks and combinations of those risks to which the company is exposed

Disadvantages

- Extremely difficult to target these scenarios at the required confidence interval
- Resulting capital requirements are typically considerably lower than the correlation matrix approach
- Very little reliance can be placed on the result

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Aggregation with an ICA – Current Best Practice

Correlation Matrix

Scenarios (Brainstorming)

Scenarios (Medium Bang)

ICA

?

Each approach supports the results of the other but uncertainties remain over whether the resulting ICA is adequate

Risk Geographies can bridge the gap and represent future best practice

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The Risk Geographies framework

Risk Geographies – an overview

- A best practice framework for ICA
  - Can calculate the most onerous scenario at a given level of confidence

- The simplicity of the stress test and correlation approach

- The ease of communication of a scenario test

- Implicit allowance for non-linearity

- No new development of your models should be required for Risk Geographies
  - If you can calculate an existing ICA you are equipped to perform the necessary calculations

What is non-linearity?

- A very simple example
  - A with-profits contract
  - two **fully correlated** risks…

	Capital requirement
Fall in equity values	£100m
Persistency	£25m

What is the total capital requirement?
(the title of this slide gives a clue)

Response Function

Consider a model firm whose net assets are exposed to two risk drivers – for example interest rates and stocks. In this example, we have standardised the risk drivers to have mean zero and standard deviation 1.
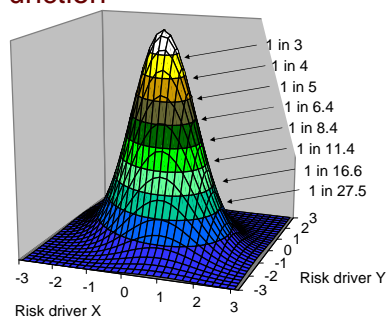
The response function expresses net assets as a function of risk drivers.
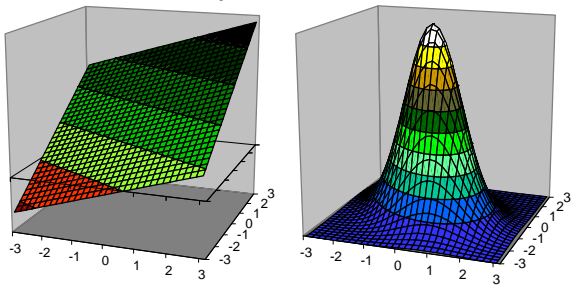

Likelihood Function

The likelihood function shows the probability density of various combinations of risk drivers. In this case, our likelihood function is a bivariate normal distribution with a characteristic bell-shape.

1 in 3
1 in 4
1 in 5
1 in 6.4
1 in 8.4
1 in 11.4
1 in 16.6
1 in 27.5

Where is Likely and Painful?

We seek to determine combinations of factor values which are simultaneously likely and painful.

Birds-Eye View: Response

Net assets > 0

Net assets < 0

Bird's Eye View: Likelihood

Risk Geographies framework

Likelihood locus
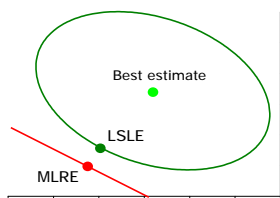
Best estimate assumptions

Ruin locus

Can you identify

Most Likely Ruin Event (Malorie)?
Least Solvent Likely Event (Leslie)?

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Risk Geographies framework

Best estimate

LSLE

MLRE

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Worked example – illustration of the process

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Simple example : with-profits fund

Realistic balance sheet	£m
Assets	**1,200**
Asset shares	800
Cost of guarantees	63
Liabilities	**863**
Surplus	**337**

- Non-linearity from guarantee costs
- 3 risk factors - (joint) normally distributed
    - Equity values
    - Fixed interest yields
    - Persistency
- Can have as many factors as you like / need

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Correlation matrix approach

- Usual approach aggregates capital requirements
    - Doesn't automatically produce scenarios
    - Assumes linearity
- Board are presented with the following assumptions to approve

Risk	Stress test	Correlations		
		Equity values	Fixed interest yields	Persistency
Equity values	40% fall	100%	-30%	50%
Fixed interest yields	1.9% fall	-30%	100%	0%
Persistency	8% fall	50%	0%	100%

- Results:

	£m	
ICA assuming linearity	100	Using medium bang at 94th percentile approach
Non-linearity adjustment	26	
Total ICA	**126**	

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Applying Risk Geographies to ICA

- How do you identify an event at a given percentile for a multi-dimensional distribution?

Joint normal distribution is itself a normal distribution

From standard deviations and correlations we can derive overall standard deviation

Likelihood locus – points of equal probability density that are required number of standard deviations from best estimate

(i.e. for 99.5%ile, 2.58 sd's)

- Calculating ICA involves identifying Least Solvent Likely Event (LSLE)
- Also have the Most Likely Ruin Event (MLRE) but this example focuses on LSLE

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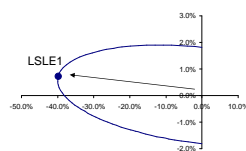making financial sense of the future

Applying Risk Geographies to ICA – how does it work?

- Risk geographies identifies the most onerous scenario at the required confidence level – the LSLE – the scenario that minimises net assets or solvency

- Iterative search process

- Should not require significant model changes

- Convergence generally takes place in 2-5 steps, depending on the number of factors involved and their non-linearity

Process – the first estimate of the LSLE

- **Calculate LSLE** - identify the point consistent with your stress test & correlation approach

- **Calculate solvency at LSLE 1**
  1. Can assume "linearity" exactly as correlation matrix approach does
  2. Can also feed scenario into ICA models

What does the LSLE 1 scenario look like?

	LSLE 1 scenario	%ile level
Equity values	37% fall	99.2%
Fixed interest yields	0.1% rise	45%
Persistency	6% reduction	97.2%

- Can demonstrate that this scenario is at the 99.5% level overall (because it lies on the likely locus)
- And then use it to calculate the surplus in the LSLE 1 scenario…
- Note % level not the same for each risk

Surplus in the LSLE 1 scenario

	Base	LSLE 1	
		Assume linearity	Scenario
Assets	863	701	701
Liabilities	863	801	822
Surplus	**0**	**-100**	**-121**

- Demonstrable link between Risk Geographies and correlation matrix approach
  - Assuming linearity, get same result
  - Identifies scenario that underpins correlation matrix approach

- Impact of "non-linearity" from feeding the scenario into the ICA model

- Advantages of a scenario
  - Easier to visualise compared to matrix multiplication of capital requirements?
  - Can think through what management would do in a given scenario?

Then repeat…

…until results converge

Comparing the results

	Correlation matrix approach		Risk Geographies	
Calculation	Matrix multiplication of capital requirements	£100m	Derive scenario and recalculate balance sheet	£121m
Allowance for non-linearity	Approximation using 94%ile medium bang	£26m	Accurate allowance	
Total ICA		**£126m**		**£121m**

Summary

Understanding and communicating risk

- Risk Geographies highlights scenarios that drive capital requirements

- Can think through management response
  - Hedging
  - Reinsurance
  - Investment strategy
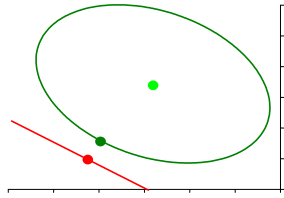 - Contingent capital

- …then calculate a new LSLE and start again!

Risk Geographies – in summary

- A best practice framework for ICA
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- The simplicity of the stress test and correlation approach

- The ease of communication of a scenario test

- Implicit allowance for non-linearity

- No new development of models should be required for Risk Geographies
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Risk Geographies