



Institute
and Faculty
of Actuaries

Subject CS1 Actuarial Statistics Core Principles

Syllabus (*For exemptions via non-accredited courses only*)

for the 2023 exams

November 2022

Exemption application summary *(to be completed by the applicant)*

[illegible]

Please provide evidence of use of “R” or an equivalent statistical software in your university course see 1.1.6

CS1 – ACTUARIAL STATISTICS

CS1 – Actuarial Statistics	University Module Codes only	University Syllabus page Number
1. Random variables and distributions (20%)		
1.1 Define basic univariate distributions and use them to calculate probabilities, quantiles and moments		
1.1.1 Define and explain the key characteristics of the discrete distributions: geometric, binomial, negative binomial, hypergeometric, Poisson and uniform on a finite set.		
1.1.2 Define and explain the key characteristics of the continuous distributions: normal, lognormal, exponential, gamma, chi-square, t , F , beta and uniform on an interval.		
1.1.3 Evaluate probabilities and quantiles associated with distributions (by calculation or using statistical software as appropriate).		
1.1.4 Define and explain the key characteristics of the Poisson process and explain the connection between the Poisson process and the Poisson distribution.		
1.1.5 Generate basic discrete and continuous random variables using the inverse transform method.		
1.1.6 Generate discrete and continuous random variables using statistical software.* see above		
1.2 Independence, joint and conditional distributions, linear combinations of random variables.		
1.2.1 Explain what is meant by jointly distributed random variables, marginal distributions and conditional distributions.		
1.2.2 Define the probability function/density function of a marginal distribution and of a conditional distribution.		
1.2.3 Specify the conditions under which random variables are independent.		
1.2.4 Define the expected value of a function of two jointly distributed random variables, the covariance and correlation coefficient between two variables, and calculate such quantities.		

1.2.5 Define the probability function/density function of the sum of two independent random variables as the convolution of two functions.		
1.2.6 Derive the mean and variance of linear combinations of random variables.		
1.2.7 Use generating functions to establish the distribution of linear combinations of independent random variables.		
1.3 Expectations, conditional expectations.		
1.3.1 Define the conditional expectation of one random variable given the value of another random variable and calculate such a quantity.		
1.3.2 Show how the mean and variance of a random variable can be obtained from expected values of conditional expected values and apply this.		
1.4 Generating functions		
1.4.1 Define and determine the moment generating function of random variables.		
1.4.2 Define and determine the cumulant generating function of random variables.		
1.4.3 Use generating functions to determine the moments and cumulants of random variables, by expansion as a series or by differentiation, as appropriate.		
1.4.4 Identify the applications for which a moment generating function, a cumulant generating function and cumulants are used and the reasons why they are used.		
1.5 Central limit theorem – statement and application.		
1.5.1 State the central limit theorem for a sequence of independent, identically distributed random variables.		
1.5.2 Generate simulated samples from a given distribution and compare the sampling distribution with the Normal.		
2. Data analysis (10%)		
2.1 Data analysis.		

2.1.1 Describe the possible aims of a data analysis (e.g. descriptive, inferential and predictive).		
2.1.2 Describe the stages of conducting a data analysis to solve real-world problems in a scientific manner and describe tools suitable for each stage.		
2.1.3 Describe sources of data and explain the characteristics of different data sources, including extremely large data sets.		
2.1.4 Explain the meaning and value of reproducible research and describe the elements required to ensure a data analysis is reproducible.		
2.2 Exploratory data analysis.		
2.2.1 Describe the purpose of exploratory data analysis.		
2.2.2 Use appropriate tools to calculate suitable summary statistics and undertake exploratory data visualisations.		
2.2.3 Define and calculate Pearson's, Spearman's and Kendall's measures of correlation for bivariate data, explain their interpretation and perform statistical inference as appropriate.		
2.2.4 Use principal components analysis to reduce the dimensionality of a complex data set.		
2.3 Random sampling and sampling distributions.		
2.3.1 Explain what is meant by a sample, a population and statistical inference.		
2.3.2 Define a random sample from a distribution of a random variable.		
2.3.3 Explain what is meant by a statistic and its sampling distribution.		
2.3.4 Determine the mean and variance of a sample mean and the mean of a sample variance in terms of the population mean, variance and sample size.		
2.3.5 State and use the basic sampling distributions for the sample mean and the sample variance for random samples from a normal distribution.		
2.3.6 State and use the distribution of the t -statistic for random samples from a normal distribution.		

2.3.7 State and use the F distribution for the ratio of two sample variances from independent samples taken from normal distributions.		
3 Statistical inference (25%)		
3.1 Estimation and estimators.		
3.1.1 Describe and apply the method of moments for constructing estimators of population parameters.		
3.1.2 Describe and apply the method of maximum likelihood for constructing estimators of population parameters.		
3.1.3 Define the following terms: efficiency, bias, consistency and mean square error.		
3.1.4 Define and apply the property of unbiasedness of an estimator.		
3.1.5 Define the mean square error of an estimator and use it to compare estimators.		
3.1.6 Describe and apply the asymptotic distribution of maximum likelihood estimators.		
3.1.7 Use the bootstrap method to estimate properties of an estimator.		
3.2 Confidence intervals and prediction intervals.		
3.2.1 Define in general terms a confidence interval for an unknown parameter of a distribution based on a random sample.		
3.2.2 Define in general terms a prediction interval for a future observation based on a model fitted to a random sample.		
3.2.3 Derive a confidence interval for an unknown parameter using a given sampling distribution.		
3.2.4 Calculate confidence intervals for the mean and the variance of a normal distribution.		
3.2.5 Calculate confidence intervals for a binomial probability and a Poisson mean, including the use of the normal approximation in both cases.		

3.2.6 Calculate confidence intervals for two-sample situations involving the normal distribution and the binomial and Poisson distributions using the normal approximation.		
3.2.7 Calculate confidence intervals for a difference between two means from paired data.		
3.2.8 Use the bootstrap method to obtain confidence intervals.		
3.3 Hypothesis testing and goodness of fit.		
3.3.1 Explain what is meant by the following terms: null and alternative hypotheses, simple and composite hypotheses, type I and type II errors, sensitivity, specificity, test statistic, likelihood ratio, critical region, level of significance, probability value and power of a test.		
3.3.2 Apply basic tests for the one-sample and two-sample situations involving the normal, binomial and Poisson distributions, and apply basic tests for paired data.		
3.3.3 Apply the permutation approach to non-parametric hypothesis tests.		
3.3.4 Use a chi-square test to test the hypothesis that a random sample is from a particular distribution, including cases where parameters are unknown.		
3.3.5 Explain what is meant by a contingency (or two-way) table, and use a chi-square test to test the independence of two classification criteria.		
4 Regression theory and applications (30%)		
4.1 Linear regression		
4.1.1 Explain what is meant by response and explanatory variables.		
4.1.2 State the simple regression model (with a single explanatory variable).		
4.1.3 Derive the least squares estimates of the slope and intercept parameters in a simple linear regression model		
4.1.4 Use appropriate software to fit a simple linear regression model to a data set and interpret the output:		
Perform statistical inference on the slope parameter.		

○ Describe the use of measures of goodness of fit of a linear regression model. ○ Use a fitted linear relationship to predict a mean response or an individual response with confidence limits. ○ Use residuals to check the suitability and validity of a linear regression model.		
4.1.5 State the multiple linear regression model (with several explanatory variables).		
4.1.6 Use appropriate software to fit a multiple linear regression model to a data set and interpret the output.		
4.1.7 Use measures of model fit to select an appropriate set of explanatory variables.		
4.2 Generalised linear models		
4.2.1 Define an exponential family of distributions. Show that the following distributions may be written in this form: binomial, Poisson, exponential, gamma, normal.		
4.2.2 State the mean and variance for an exponential family, and define the variance function and the scale parameter. Derive these quantities for the distributions above.		
4.2.3 Explain what is meant by the link function and the canonical link function, referring to the distributions above.		
4.2.4 Explain what is meant by a variable, a factor taking categorical values and an interaction term. Define the linear predictor, illustrating its form for simple models, including polynomial models and models involving factors.		
4.2.5 Define the deviance and scaled deviance and state how the parameters of a generalised linear model may be estimated. Describe how a suitable model may be chosen by using an analysis of deviance and by examining the significance of the parameters.		
4.2.6 Define the Pearson and deviance residuals and describe how they may be used.		
4.2.7 Apply statistical tests to determine the acceptability of a fitted model: Pearson's chi-square test and the likelihood-ratio test.		
4.2.8 Fit a generalised linear model to a data set and interpret the output.		

5 Bayesian statistics (15%)		
5.1 Explain the fundamental concepts of Bayesian statistics and use these concepts to calculate Bayesian estimators.		
5.1.1 Use Bayes' theorem to calculate simple conditional probabilities.		
5.1.2 Explain what is meant by a prior distribution, a posterior distribution and a conjugate prior distribution.		
5.1.3 Derive the posterior distribution for a parameter in simple cases.		
5.1.4 Explain what is meant by a loss function.		
5.1.5 Use simple loss functions to derive Bayesian estimates of parameters.		
5.1.6 Derive credible intervals in simple cases.		
5.1.7 Explain what is meant by the credibility premium formula and describe the role played by the credibility factor.		
5.1.8 Explain the Bayesian approach to credibility theory and use it to derive credibility premiums in simple cases.		
5.1.9 Explain the empirical Bayes approach to credibility theory and use it to derive credibility premiums in simple cases.		
5.1.10 Explain the differences between the two approaches and state the assumptions underlying each of them.		
Assessment Assessment consists of a combination of a one-hour and forty-five-minute computer-based data analysis and statistical modelling assignment and a three-hour and fifteen-minute written examination. <i>2021 If you undertook a University assessment which is a requirement of your program during the COVID-19 pandemic, please note the IFoA will not consider modules or results where the assessment period of that module/exam was over an extended time. For example, if your assessment period for a module or exam was 12-24 hours</i>		

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