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## Acknowledgements

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- Some of this material has been developed as part of a forthcoming paper by the Extreme Events working party
- Special credit due to Ralph Frankland (chair), Laura Hewitt, Parit Jakhria, Sandy Sharp and James Sharpe
- These slides are my (Andrew Smith's) views and not necessarily those of the working party

## What is Value-at-Risk?

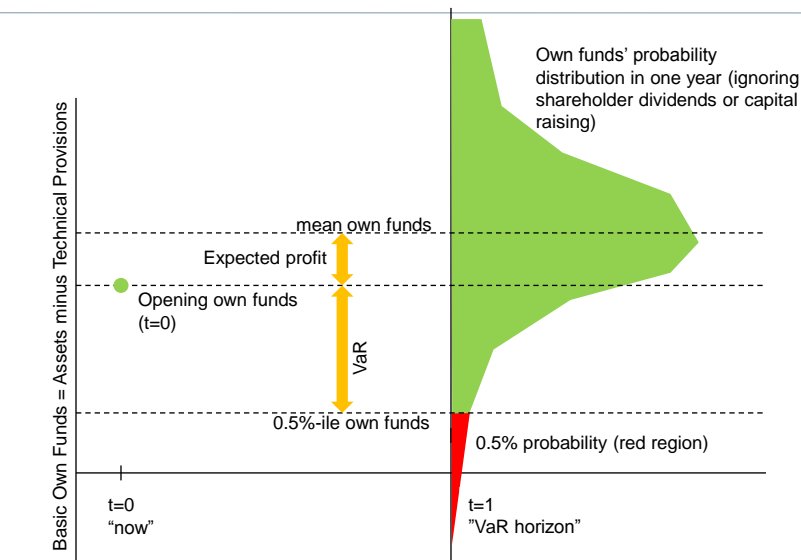
- Jorion (2007): “The worst loss over a target horizon such that there is a low, pre-specified probability that the actual loss will be larger”
- Legislative references: Insurance Solvency II Directive:

The Solvency Capital Requirement ... shall correspond to the Value-at-Risk of the basic own funds of an insurance or reinsurance undertaking subject to a confidence level of 99.5 % over a one-year period.

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## Calculating VaR Using Percentiles



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## Out-of-Model Experiences How to Respond?

We just had a \*very\*  
unlucky scenario

Improve model reliability  
by actively seeking out  
missing risks

Quantitative risk modelling  
is futile because the real  
risks are the ones we miss

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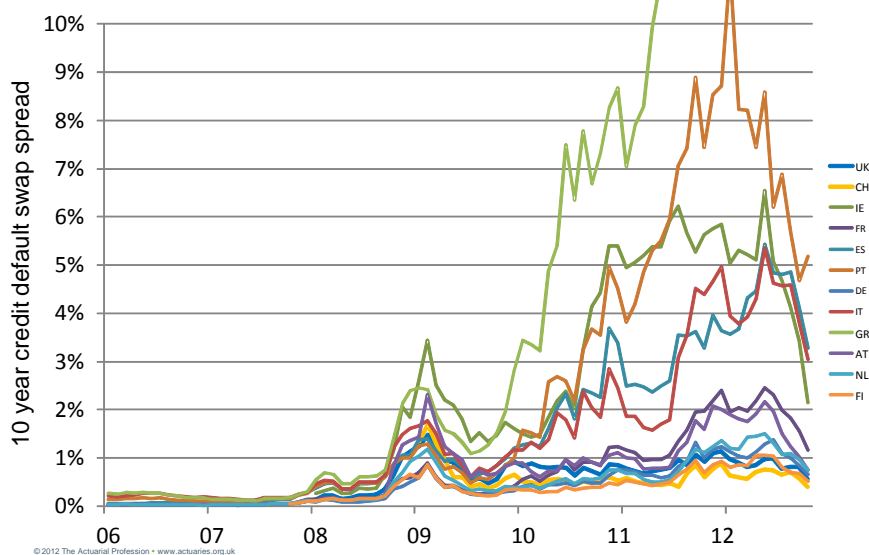
## Disaster Post Mortems and Communication of Model Scope

- Large market move disproves previously accepted model
- Unanticipated change in yield curve shape
- New basis risks, eg LIBOR vs OIS
- Switch of external model provider (ESG, Nat Cat)
- Unmodelled change in portfolio mix
- Loss of detail in model points
- Multiple causes (Cat in remote, politically unstable, region)
- Approximate modelling of reinsurance treaties
- Clarification of technical provision methodology

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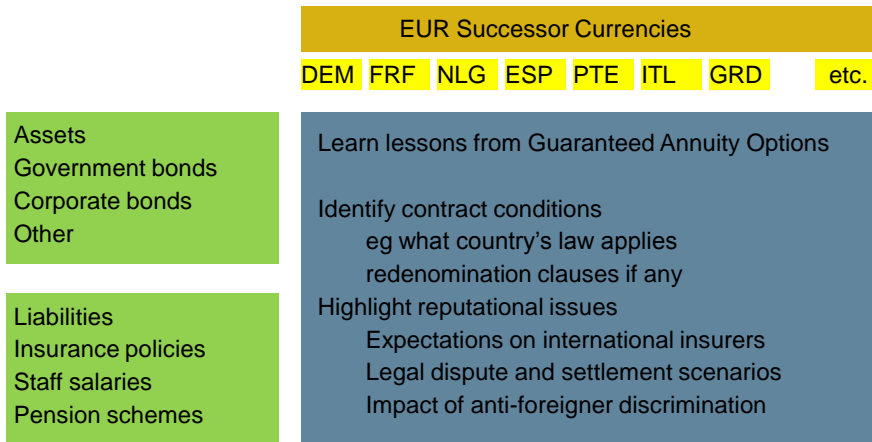
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## Lead Indicators: CDS Premiums on Govt Bonds



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## Euro breakup scenario – Redenomination Risk



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**Allowing for Known Unknowns:  
Model and Parameter Error**

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## Value-at-Risk is a hard problem because there are so many things you don't know

- There are 62 cities in China whose population exceeds one million people (source: Wikipedia)
- I picked 5 of these 62 cities at random and placed them in increasing size order. The populations were as follows:

City Name	Population
洛阳	1 265 000
大同	1 492 000
福州	1 860 000
郑州	2 280 000
商丘	7 362 472

- Problem: Estimate the population of the 6<sup>th</sup> largest Chinese city, 广州

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## Modelling

- Modelling is an inescapable part of Actuarial Life
- A model is necessarily a simplified representation of the real world!
- In this (Actuarial) context, we think of models as tools / processes that:
  - Use information from the past (history)
  - Together with knowledge about a particular problem (judgement)
  - To model future (uncertain) outcomes
  - (and hence help make decisions about the future)

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## Types of Uncertainties

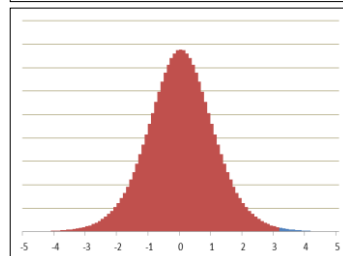
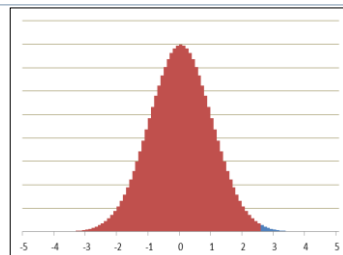
- A model is necessarily a simplified representation of the real world!
- The process of stripping down to the bare useful components and “calibrating” the resultant model has necessarily got a large amount of judgement / decisions associated with it
- These judgements / decisions manifest themselves in various different ways. Some of the ways we encounter decisions over the process of actuarial modelling are:
  - Choice of overall framework for the model
  - Choosing individual parts of the model (e.g. distribution)
  - Choice of calibration methodology
  - Choice of parameters, overriding certain parameters if necessary

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## Model certainty...

If we know true distribution, can just read off the relevant quantile e.g.

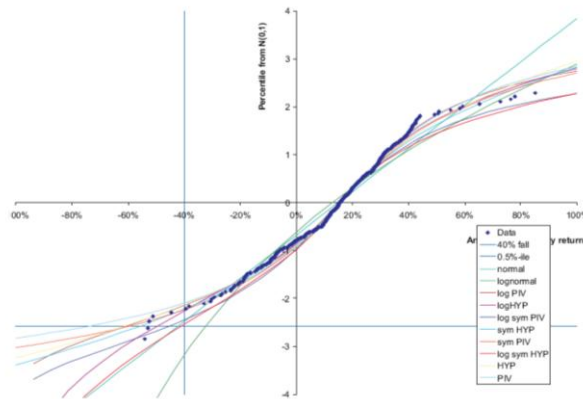
- Normal distribution with known  $\mu, \sigma$ 
  - 99.5<sup>th</sup> %tile is  $\mu + \Phi^{-1}(0.995)\sigma = \mu + 2.58\sigma$
- t distribution, 10 degrees of freedom
  - 99.5<sup>th</sup> percentile is 3.17
- etc...



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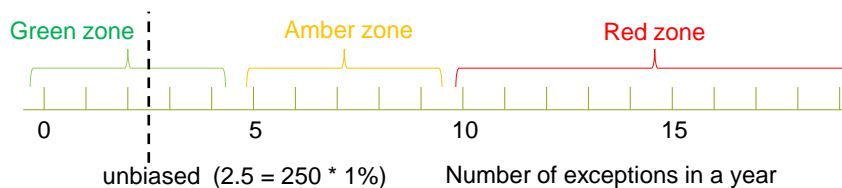
## vs. Model Uncertainty! (Frankland et al, 2008)



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## Longitudinal Validation under Basel How do you know your model is right?

- Bank regulation: 10 day VaR at 99% Confidence
  - Look back over last year (250 trading days, overlapping periods each looking 10 days back) in which both VaR and profit are updated



- What does this process test?
  - The “back test” includes implicit tests of model and parameter error as well as outcomes
  - Although it won't test risks that didn't materialise in the last year

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## Different Definitions of 1-in-200 event In the Presence of Parameter Error

- A1. We estimated the parameters using the [method of moments]. If these estimates are exact then €100m of capital is 99.5% certain to be sufficient. This calculation ignores the possibility of parameter estimation error.
- A2. We estimated the 99.5%-ile using the [method of moments]. If this method had been applied on many alternative historic scenarios, then on average the estimated 99.5%-ile is equal to the true 99.5%-ile (this is Fisher's concept of an unbiased estimate).
- A3. As A2, but in addition, our chosen method produces an estimate whose variance is lower than other methods (Fisher's concept of an efficient estimate)
- A4. We estimated the 99.5%-ile using the [method of moments]. We used the finite data available, but if we had unlimited data then our method would produce estimates that converge to the true 99.5%-ile (Fisher's concept of a consistent estimate)
- A5. We estimated upper and lower bounds for the 99.5%-ile using [chosen method]. In a large number of trials of alternative historic scenarios, this interval contains the true 99.5%-ile in 95% of the time, regardless of what the true parameters are. (Classical 95% confidence interval for the 99.5%-ile)

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## Yet More Definitions of the 1-in-200 event

- A6. We generated values for the parameters according to a prior distribution, and generated historic scenarios. We kept only those combined scenarios where the simulated data was close to our own history, which generated more scenarios for some parameters than others. Out of those scenarios, the average true 99.5%-ile was €100m and the most likely 99.5%-ile was €90m. (Bayesian mean or modal prediction)
- A7. We generated values for the parameters according to a prior distribution, and generated historic scenarios. We kept only those combined scenarios where the simulated data was close to our own history. Restricting attention to those close historic outcomes, the true 99.5%-ile was between €65m and €150m for 95% of cases. (Bayesian confidence interval)
- A8. We estimated the 99.5%-ile using [chosen method]. Using our method, and regenerating both past and future data, our estimated percentile exceeds the next observation 99.5% of the time, regardless of the true parameters. (Geissner's prediction interval)
- A9. As A8, but in addition our chosen method, on average, produces lower estimated percentiles than other methods. (Efficient prediction interval)
- A10. We generated values for the parameters according to a prior distribution, and generated linked historic and future scenarios. In each scenario we used [chosen method] to estimate the 99.5%-ile of the next observation. We kept only those combined scenarios where the simulated data was close to our own history, and out of those scenarios, our estimated percentile exceeds the next observation 99.5% of the time. [Bayesian prediction interval]

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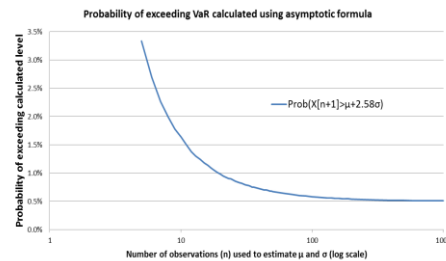
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## Example: Normal distribution Unknown parameters, n observations

- Standard unbiased estimates of mean and standard deviation
- i.e.  $\hat{\mu} = n^{-1} \sum_1^n X_i$ ,  $s^2 = (n-1)^{-1} \sum_1^n (X_i - \hat{\mu})^2$ ,  $\hat{\sigma} = s \cdot \sqrt{\frac{n-1}{2}} \cdot \Gamma(\frac{n-1}{2}) / \Gamma(\frac{n}{2})$
- Then  $\hat{\mu} + 2.58\hat{\sigma}$  is an unbiased estimate of  $\mu + 2.58\sigma$ , the quantile we are after

BUT

- It is not true that the next observation has a 0.5% chance of exceeding this estimate (or that 0.5% of the next  $m$  observations will exceed this level)
- $\mathbb{P}(X_{n+1} > \hat{\mu} + 2.58\hat{\sigma}) > 0.05\%$



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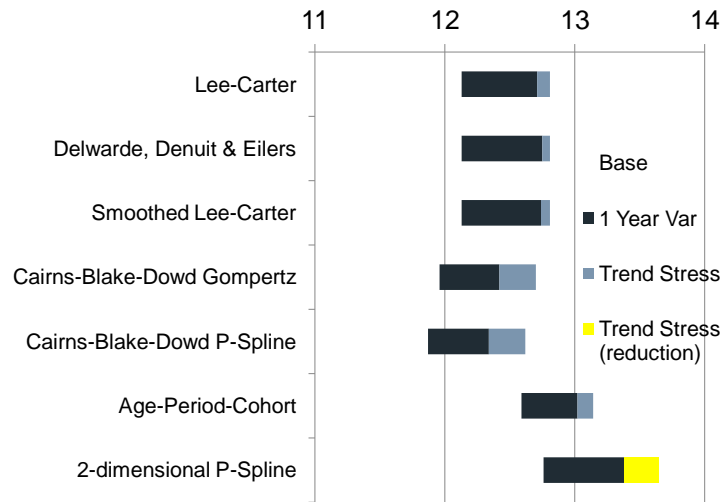
## Confidence intervals v prediction intervals

- |   |   |
|---|---|
| • Confidence interval                               | • Prediction interval   |
| • $\mathbb{P}(\theta \in [\hat{a}, \hat{b}]) = p_1$ | • $\mathbb{P}(X \in [\hat{a}, \hat{b}]) = p_1$                          |
| • $\mathbb{P}(\theta \leq \hat{b}) = p_2$           | • $\mathbb{P}(X \leq \hat{b}) = p_2$                                    |
| • Statement about parameters                        | • Statement about observations  |
| • Can be Monte Carlo tested                         | • Readily tested in practice  |
| • Parameter unobservable so untestable in practice  | • This is exactly the VaR backtest: count 'exceptions' in a time series |

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## Model Risk Example: Longevity 70 y/o Male Annuity (Richards et al, 2012)



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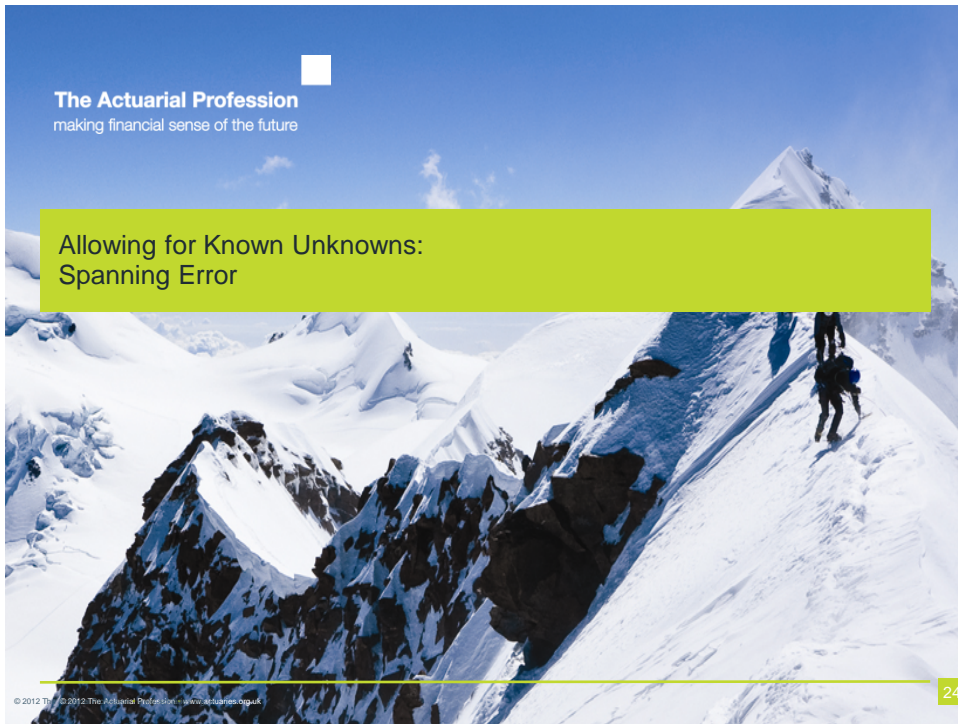
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## Practical issues

- Although the theory of parameter uncertainty appears to work well for certain univariate cases, it is difficult to scale - there are 100's to 1000's of choices being made in a typical life-office model
- There is still judgement required on the class of models
- We would need other methods for more generalised model choices e.g. time variation in returns, number of factors to model, etc...

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## Introduction: Spanning error in proxy models

- Insurers' assets and liabilities are complicated functions of millions of inputs whose future values are uncertain
- Insurers use "heavy models" to compute assets and liabilities as functions of the long list of inputs. In theory, we need a full stochastic projection to calculate the probability distribution of assets and liabilities
- Despite further (foreseeable) advances in computer calculation, a full stochastic projection of stochastic liabilities remains beyond the reach of most insurers. Instead, there is a widespread use of proxy models
- Proxy models have two main standard features:
  - A reduction in the number (or "dimension") of inputs, from millions to tens or hundreds
  - A set of selected "basis functions" from which a linear combination is selected to describe the assets or liabilities

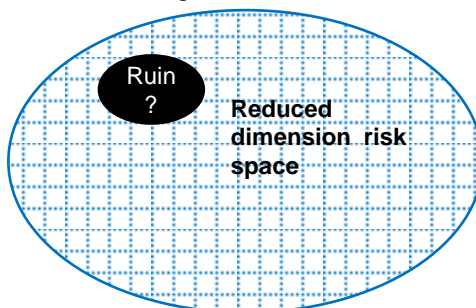
## Introduction: Spanning error in proxy models

- A reliance on proxy techniques assumes that accurate functional form approximations can be made to the heavy modelled “true” values. The following are typical examples where spanning failure can occur :
  - Missing material risks in the dimension reduction
  - The true function has discontinuities e.g. due to modelled stakeholder actions but all the basis functions are continuous
  - Regions of parameter values where assets collapse or liabilities explode, but none of the basis functions exhibit this behaviour
- These are all examples of spanning error, which is any mis-statement in the required capital that arises from spanning failure

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## Options for error analysis

- Out of sample checking runs are computationally intensive. To save computational time the runs may be:
  - focused on a particular region e.g. the suspected region of risk drivers having the largest impact on the SCR
  - a sparse covering of the entire risk space to more generally determine whether the reduced dimension model gives a correct “ruin region”



- It is possible to gain some insight into the task of fitting proxy models using models with closed form analytic solutions
- By flexing the reduced proxy model we can gain some insights into different product types

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## The toy model - Analytic formula models

The following assets and liabilities are modelled with a simplified set of term independent risk drivers. In addition we look at the error in the aggregate balance sheet

- **Liability models**
  - Term assurance
  - Annuity
  - Guaranteed equity option
- **Asset models**
  - Coupon paying Government and corporate bonds
  - Equity
  - Cash

We have given ourselves 3 liabilities, 3 risky assets and 9 risk drivers. All risk drivers have a shifted scaled log-normal distribution and the Toy Model technical provisions can be found analytically

## Toy model risk drivers

Risk driver	Term Assurance	Annuity	Guaranteed Equity	Government bond	Corp Bond	Equity
Risk free discount rate	X	X	X	X	X	
Equity price			X			X
Equity volatility			X			
Corp bond portfolio spread					X	
Liquidity premium *		X				
Mortality(Term)	X					
Mortality (annuity)		X				
Lapses (term)	X					
Lapses (Guaranteed equity)			X			

The most complex liability depends on 4 risk drivers and there is variation in the number of risk drivers

## Parameters and fitting

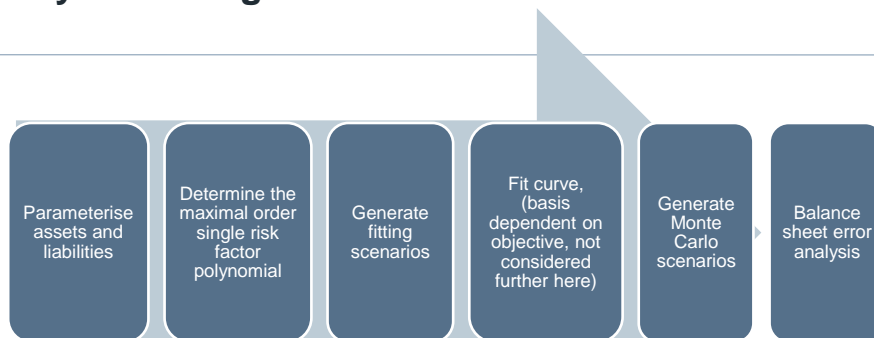
If we fit to the median and upper and lower quantile stressed values for each risk then we have three fitting points per risk. Assets and liabilities dependent on one risk driver are therefore limited to a quadratic curve fit.

Risk Drivers	Fitting Points	Parameters in proxy function		
		Linear	Quad	Cross
1	3	2	3	-
2	9	3	5	6
3	27	4	7	10
4	81	5	9	15
5	243	6	11	21

### Features of the model:

- Annuity with a discontinuous first derivative
- Guaranteed equity bond has optionality.
- We can investigate dimensionality (to a certain extent)

## Toy modelling

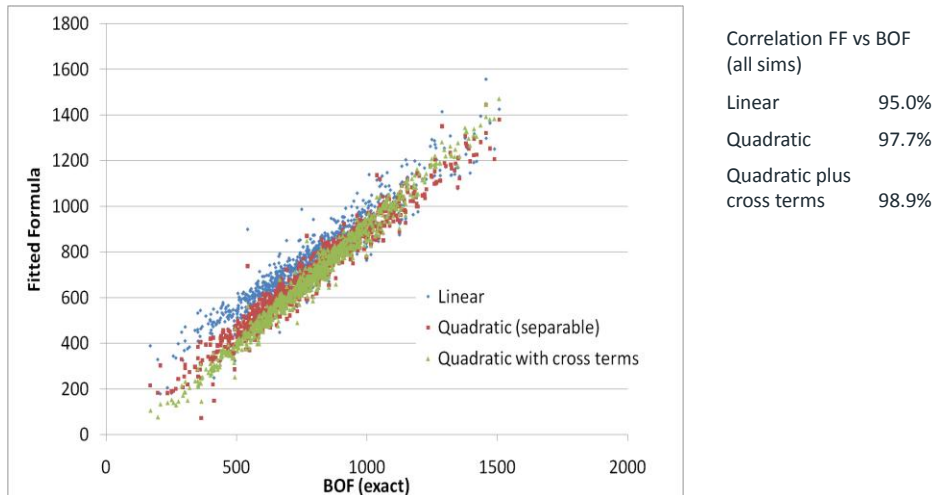


### Simulation methodology

- Specify the risk driver ( shifted exponential distribution used)
- Simulate outer scenarios
- Value portfolio



## Experimental results



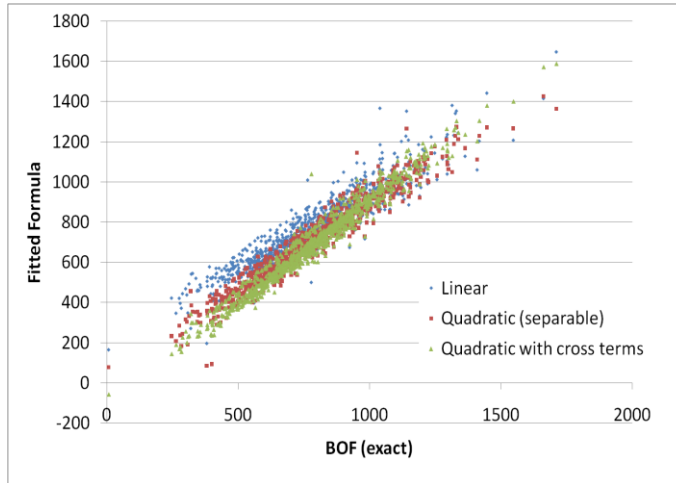
## Discontinuities and management actions

- Fitting without management actions

### Equity example:

- If the purchase of an equity put option is triggered to prevent a fall below 25% but this isn't included in the modelling then the quality of the fit is reduced. (Particularly in the tail)
- The same could be true in real life balance sheets for any un-modelled management actions e.g. executive bonuses
- This highlights the importance of a rule which is modelled in the stress tests but not in the original tech provision calculation
  - it might be argued that the discontinuity is an unintended consequence of inconsistencies in different parts of the model
  - if you capture the action in the TP calculation then the liability is continuous and the fit is better

## Experimental results including modelled management action



Correlation FF vs BOF  
(all sims)

Linear	93.9%
Quadratic	96.9%
Quadratic plus cross terms	98.4%

## Some Lessons From the Proxy Model Example

- Importance of cross terms
  - Without the flexibility to use cross terms the accuracy of fits is severely reduced
- In our example bringing the fitting points in from the tails increases the R squared but actually reduces the accuracy of the SCR (c.f. the “true” – no proxy- value)”
- Can we seek a point where the accuracy is best? corresponding in some sense to a 99.5% confidence
- There is a trade-off between the measures we can easily use to fit (R-squared, equivalent to OLS) and the measures we would theoretically like to use (minimax)
- Discontinuities and areas of blow up will need to be modelled as an additional layer and cannot be well modelled using polynomial functions

## Progress Continues on Hard Problems

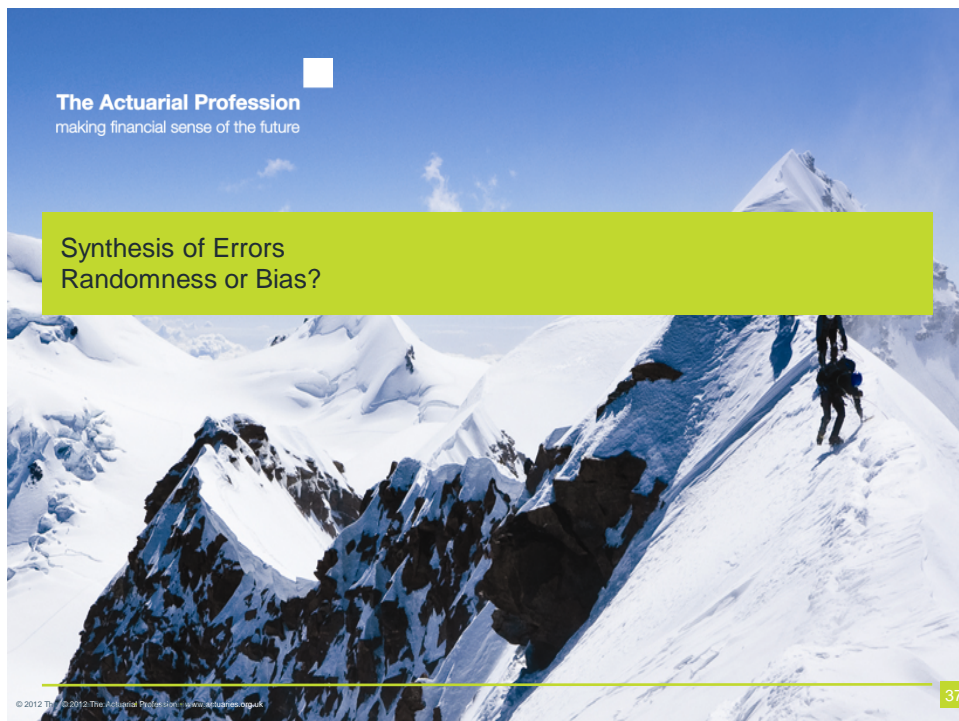
- The relationship between R squared and the maximum error
  - Location of fitting points and the error for the actual distribution
  - Compare the quality of proxy fits based on R-squared versus minimax
  - We would like to derive minimax statements for the toy example such as :
    - A: “The true function and the proxy function differ by at most  $\epsilon x$  over a given joint risk driver range. This range has a (real world probability) of at least 99.9%”

therefore

  - B:” The 0.5%-ile of the true function lies between 0.4%-ile of the proxy function-  $\epsilon x$  and 0.6%-ile of the proxy function +  $\epsilon x$
- which leads to statements such as “the use of proxy models introduces an error of no more than 20% in the SCR”

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## Potential sources of error in VaR Calculations (the well-known examples)

Category	Example	Bias
Random	Draw from an experiment whose distribution is not in dispute. Textbook examples: coin toss, drawing coloured balls from an urn.	
Parameter error	Estimation of parameters from finite samples	Portfolio optimisation finds strategies where returns are over-stated or risks under-stated
Model error	Chosen mathematical model family does not contain the process that generated the data	Complexity bias (eg use normal distribution instead of fat tails, linear AR1 instead of non-linear heterosecastic, dimension reduction, commercial pressure)

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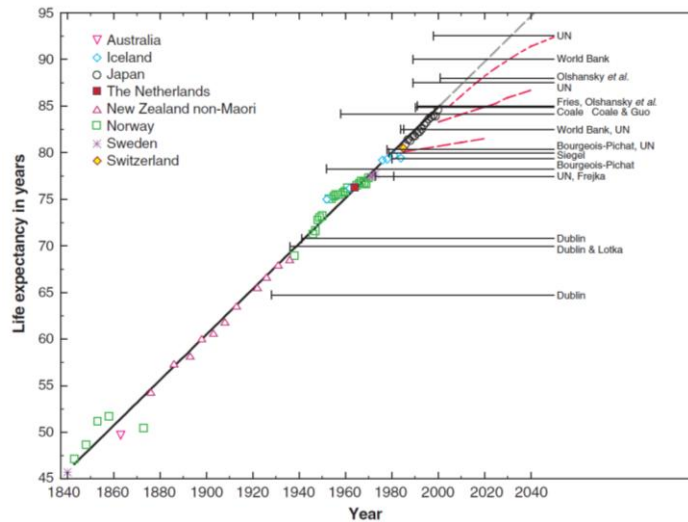
## Less-discussed sources of error Did these contribute to AIG/Fortis Exceptions?

Category	Example	Bias
Cyclical (point in time estimates)	Mis-identification of hidden state variables, excluding "irrelevant" historic periods	Symmetric dampeners, judgements about underlying investment value and correction of distorted or illiquid markets
Data	Incomplete or inaccurate	Falsification or selective submission of data. Underwriting bias such as winners curse. Exaggerate benefit of lessons learned or effectiveness of recently imposed controls.
Exposure (proxy model)	Mis-statement of asset and liability sensitivity to combined moves in risk drivers	Constructing hedges to minimise stated VaR; devising "easy" stress test that are known to pass. Lack of preparation for out-of-test stresses.
Computation	Roundoff in floating point calculations; differential equation discretisation, simulation sampling error	Debug effort focuses on commercially unacceptable output.

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## The Repeated Failure of Expert Judgement (Oeppen & Vaupel, 2002)



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## Bias and Risk Culture

- Organisations differ in their approach to risk
- Lead predictors of bias may include:
  - Organisation and governance structure
  - Safety in speaking up during the risk discovery process
  - Confidence in management approach
  - Response to increase in stated risk
  - Prestige of independent challenger
- Understanding risk culture can give insight into likely biases and how they may be remediated

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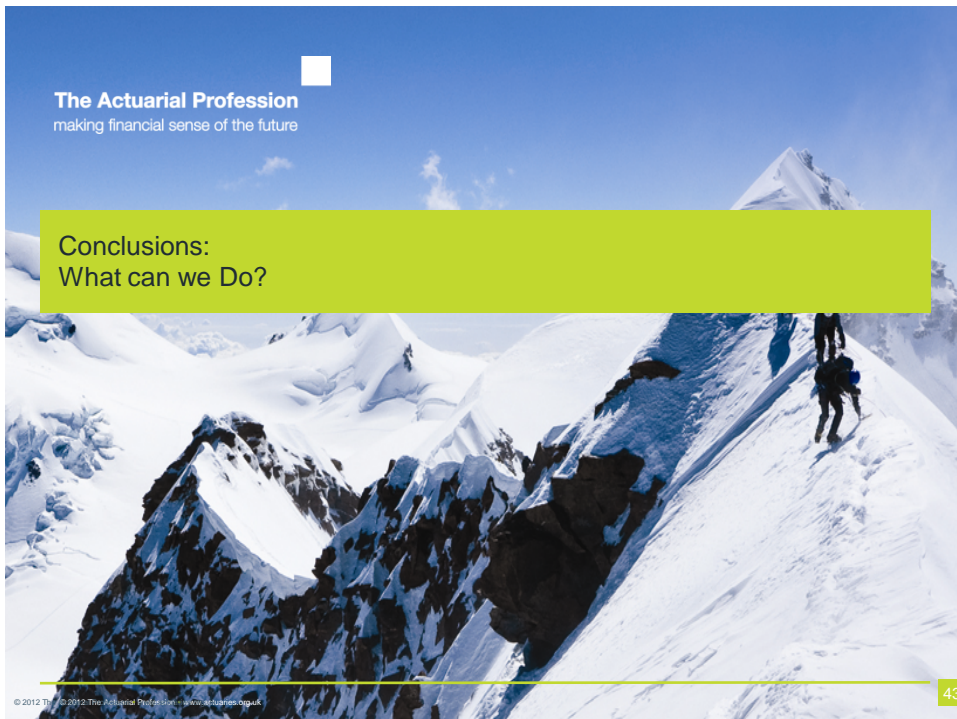
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## Choosing a “Best Methodology” involves balancing stakeholder concerns

Stakeholder	Example of possible concerns
Policyholder	Benefit security Cost of insurance cover
Corporate manager	Reported return on capital Management flexibility
Regulator	Market confidence Financial stability
Shareholder	Share price growth Dividends
General public	Amplitude of economic cycle Bail-outs
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**Conclusions:  
What can we Do?**

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## Conclusions: What Can We Do? Three Things to take Away

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- Foster an open risk culture that encourages discovery and discussion of unmodelled or emerging risks.
- Take advantage of existing statistical techniques for parameter and model uncertainty.
- Be aware of biases and seek to address them in corporate culture as well as statistics.

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## Questions or comments?

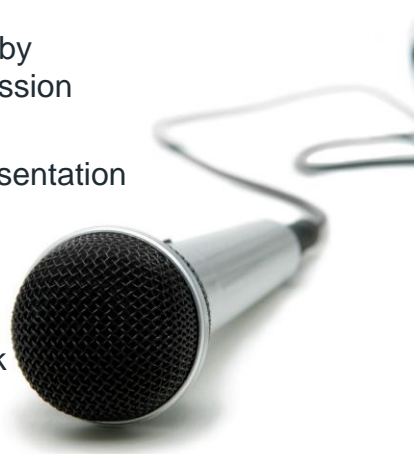
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The views expressed in this presentation are those of the presenter.

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