GIRO40

8 – 11 October, Edinburgh

Institute and Faculty of Actuaries

Long tail liabilities (LOBs)

- Correlations
- Accident year drivers
- Calendar year drivers
- Seemingly Unrelated Regressions(SUR)
- Single composite model for multiple LOBs
- Risk Capital Allocation
- One year ahead statistics(CDR)
- Economic Balance Sheet and risk diversification of SCR and Risk Margins

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Correlations between LOBs

- Three types of correlations
    - Process (volatility) correlation
    - Parameter (trend) correlation
    - Reserve distribution correlation

    Similar trend structure implying commonality in calendar year drivers or accident year drivers is a stronger relationships than correlations

- Cannot measure these correlations unless LOB trend structure and process variability (volatility) modeled accurately

- Most important direction is the calendar year

- Reserve distribution correlation << Process correlation

- Highest Process correlation we have seen is 0.6!

- Highest Reserve distribution correlation is 0.2!

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Correlations and other relationships between LOBs/Segments

Take-Away points:

- **Most long tail LOBs exhibit close to zero correlation**

- **Each company is different**

- **Each LOB/Segment is different**

- **Common accident year and calendar year drivers are stronger relationships than correlations**

- **Cannot assess the relationships between two loss development arrays unless the identified optimal model fits a distribution to each cell- the means on a log scale are related by the "trends" in the three directions**

- **A single composite model for multiple LOBs/segments involves Seemingly Unrelated Regressions (SUR) – Zellner 1962**

- **For 40 LOBs there are 780 pairwise correlations. Most are zero. We create clusters.**

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Correlation and Linearity

Correlation, linearity, normality, weighted least squares, and linear regression are closely related concepts.

The idea of correlation arises naturally for two random variables that have a joint distribution that is bivariate normal. For each individual variable, two parameters a mean and standard deviation are sufficient to fully describe its probability distribution. For the joint distribution, a single additional parameter is required the correlation.

If *X* and *Y* have a bivariate normal distribution, the relationship between them is linear: the mean of *Y*, given *X*, is a linear function of *X* ie:
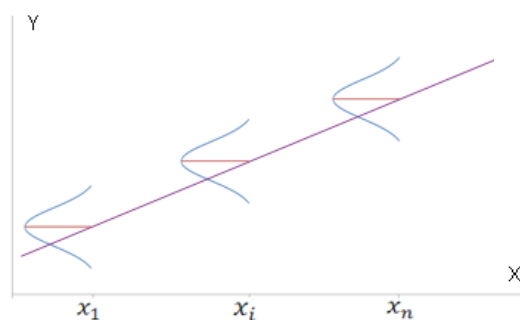
$$E(Y/X) = \alpha + \beta X$$

Correlation and Linearity

Weight = Y　　Height = X

For sub-populations of heights defined by $X = x_i$ the distribution of weights $Y|x_i$ is normal distribution with mean $\alpha + \beta x_i$ and variance σ^2.

Correlation and Linearity

The slope β is determined by the correlation ρ, and the standard deviations:

$$\beta = \rho\sigma_{\mathbf{Y}}/\sigma_{\mathbf{X}},$$

where $\quad \rho = Cov(X,Y)/\left(\sigma_{\mathbf{X}}\sigma_{\mathbf{Y}}\right).$

The correlation between Y and X is zero if and only if the slope β is zero.

Also note that, when Y and X have a bivariate normal distribution, the conditional variance of Y, given X, is constant ie not a function of X:

$$Var(Y/X) = \sigma^2_{Y|X}$$

Correlation and Linearity

If (Y, X) has a joint normal distribution then

$$Y|X = x \sim N(\alpha + \beta x, \sigma^2)$$

and

$$Var(Y) \geq Var(Y|X = x) = \sigma^2$$

Correlation and Linearity

This is why, in the usual linear regression model

$$Y = \alpha + \beta X + \varepsilon$$

the variance of the "error" term ε does not depend on X.

However, not all variables are linearly related. Suppose we have two random variables related by the equation

$$S = T^2$$

where T is normally distributed with mean zero and variance 1.

What is the correlation between S and T?

Correlation and Linearity

Linear correlation is a measure of how close two random variables are to being <u>linearly</u> related.

In fact, if we know that the linear correlation is +1 or -1, then there must be a deterministic linear relationship

$Y = \alpha + \beta X$ between Y and X (<u>and vice versa</u>).

If Y and X are linearly related, and f and g are functions, the relationship between $f(Y)$ and $g(X)$ is not necessarily linear, so we should not expect the linear correlation between $f(Y)$ and $g(X)$ to be the same as between Y and X.

(Answer to question on previous slide is zero)

Digression: A common misconception with correlated lognormals

Actuaries frequently need to find covariances or correlations between variables such as when finding the variance of a sum of forecasts (for example in P&C reserving, when combining territories or lines of business, or computing the benefit from diversification).

Correlated normal random variables are well understood. The usual multivariate distribution used for analysis of related normals is the multivariate normal, where correlated variables are linearly related. In this circumstance, the usual linear correlation (the *Pearson correlation*) makes sense.
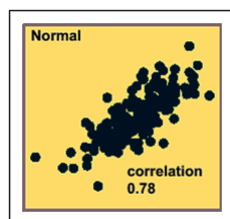
A common misconception with correlated lognormals

However, when dealing with lognormal random variables (whose logs are normally distributed), if the underlying normal variables are linearly correlated, then the correlation of lognormals changes as the variance parameters change, even though the correlation of the underlying normal does not.
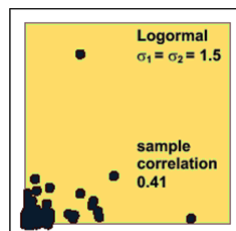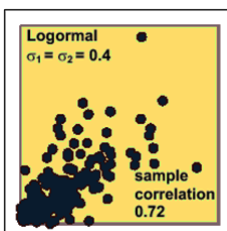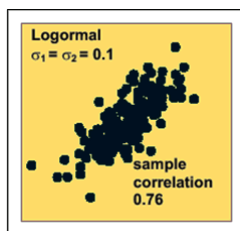
A common misconception with correlated lognormals

All three lognormals below are based on normal variables with correlation 0.78, as shown left, but with different standard deviations.

A common misconception with correlated lognormals

We cannot measure the correlation on the log-scale and apply that correlation directly to the dollar scale, because the correlation is not the same on that scale.

Additionally, if the relationship is linear on the log scale (the normal variables are multivariate normal) the relationship is no longer linear on the original scale, so the correlation is no longer linear correlation. The relationship between the variables in general becomes a curve:
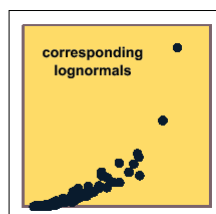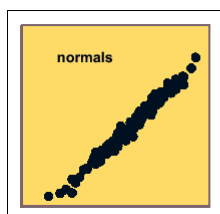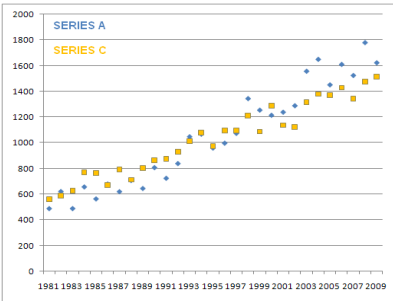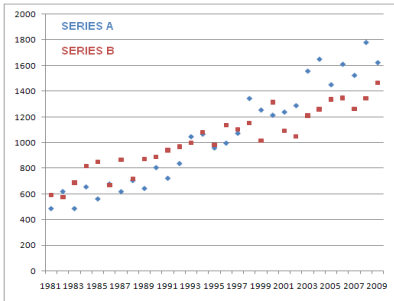
Correlation, Regression and Time Series

Correlations measured before and after regression can be very different. Hence if we want to assess the effective correlation between two series we must first remove trends (the predictable portion) and measure the correlation of the residuals (the random components.)


Consider the series A, B and C. Each has a linear trend, B and C appear quite similar. The correlation between A and B is 0.91 and between A and C is 0.97. Are A and B related? Are A and C related?

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De-trending the series

Removing trends from the series, in this case by linear least-squares regression separates the predicateble part from the random component.
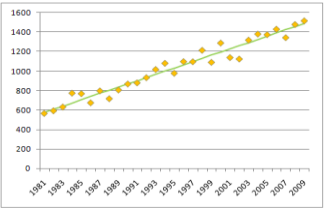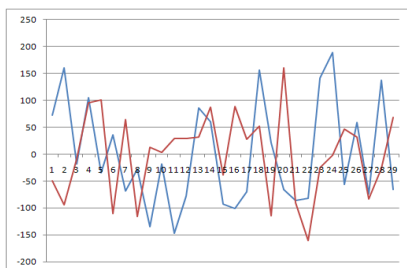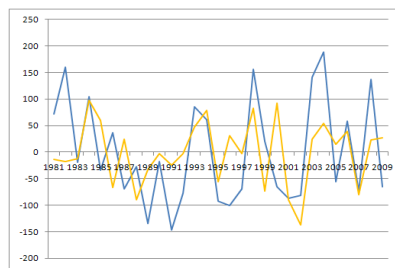

Series A

Series C

Series B

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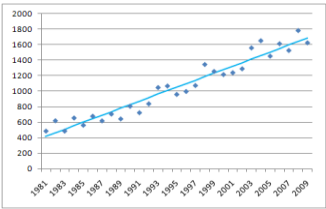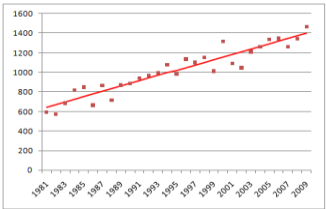
Compute the correlation of the residuals = the random component of each series

Residual or "Process" Correlation of A and B = -0.07

Residual or "Process" Correlation of A and C = 0.42

Conclusion: The series A and B merely share a common positive trend. There is no apparent causal or predictive relation between them. Series A and C exhibit a positive correlation. Information about the next value of C does have a significant bearing on prediction of the next value of A.

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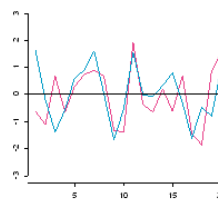
Correlation in time-series- not same as correlation between Y and X
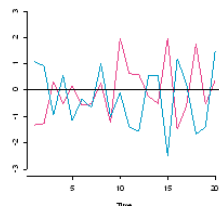# Loss Reserving is a study of time series by calendar year!
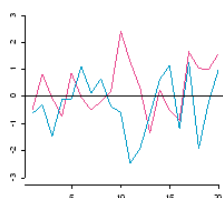
Series corr. = 0

Series corr. = 0.5

Series corr. = -0.5
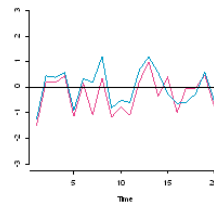
Series corr. = 0.8

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We call the correlation of the random component (after modeling the trend structure in the three directions) of two loss development arrays: process correlation
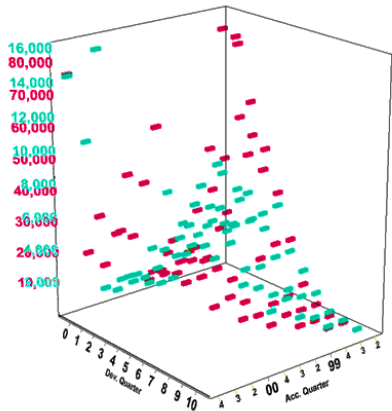
3D plot of data

These two triangular loss arrays have process corr. = 0.9 after modelling their respective trend structures.

Cannot detect from data plot.

Common calendar drivers: Gross vs Net

In Gross versus Net of Reinsurance data (E&O and D&O in example), common calendar year drivers are expected to be found since Net of Reinsurance is a subset of Gross. Trends, especially calendar and accident, are closely related. The comparable models are shown below:
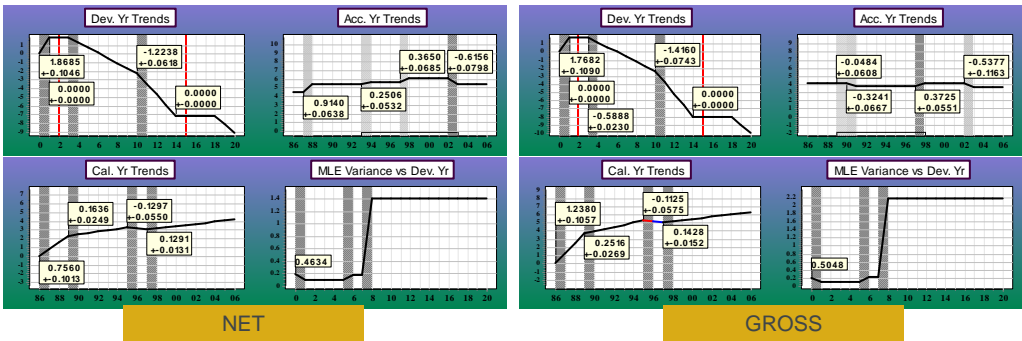
Common calendar drivers: Gross vs Net

The model trends are very similar; trend and volatility changes usually coincide. The critical trends in common are the calendar year trends (below) and accident year level changes. Common calendar year drivers are clearly visible as the trend changes occur at the same point.
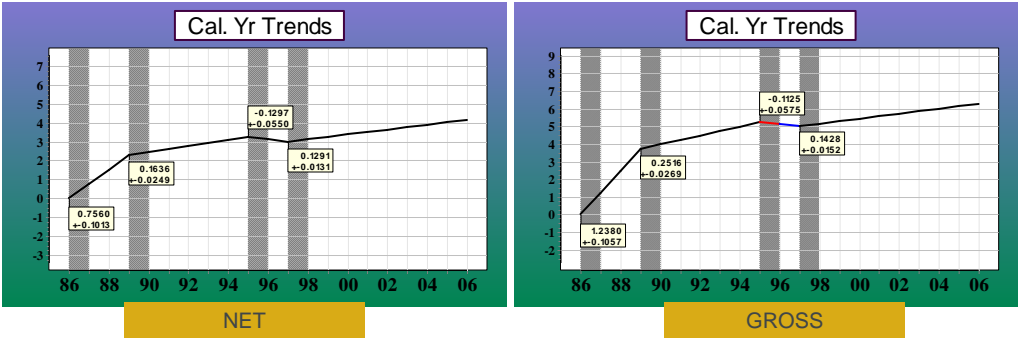
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Common calendar drivers: Gross vs Net

Blue line is trace of (single) calendar year (2006) along the accident years.

Process Correlation = 0.85

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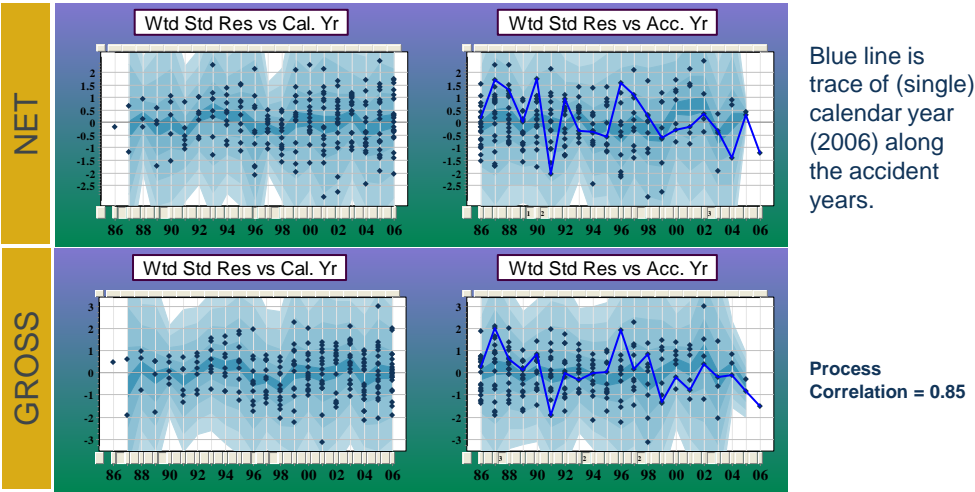
Common calendar drivers: Gross vs Net
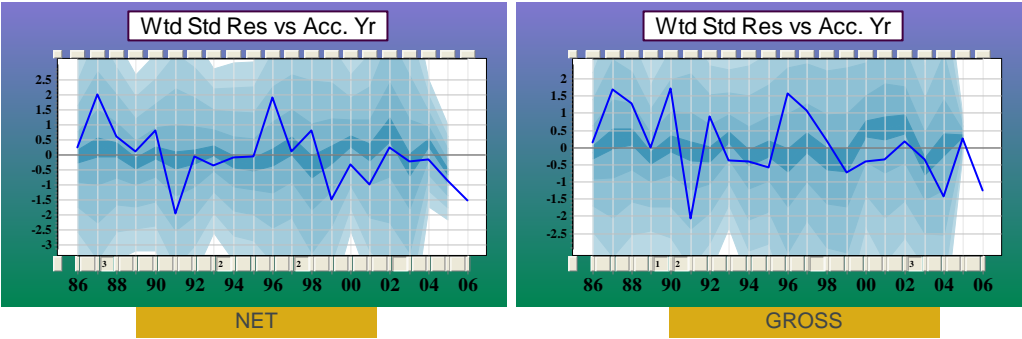
For the model described above, the residuals by accident year traced for the last calendar year are clearly correlated; when a value in a year is low/high in one segment it is usually low/high in the other segment also at the same time.

Common calendar drivers: Gross vs Net
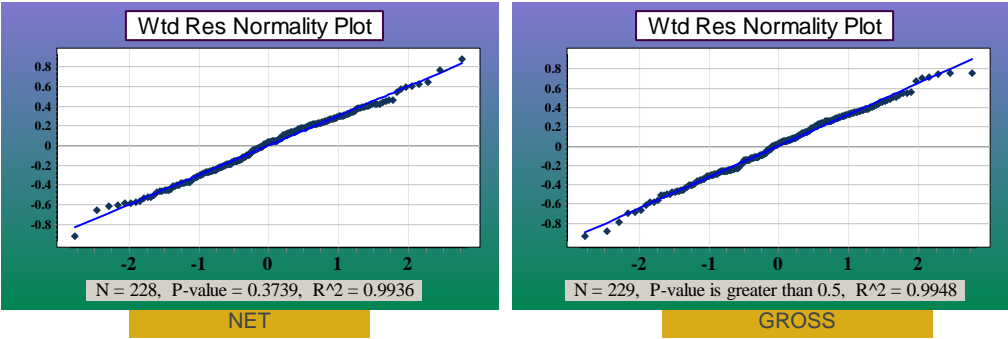
The residuals from both lines of business are statistically indistinguishable from two normal distributions.

Thus, the process correlation can be considered the volatility correlation between two normal distributions.

Common calendar drivers: Gross vs Net

Net vs Gross.
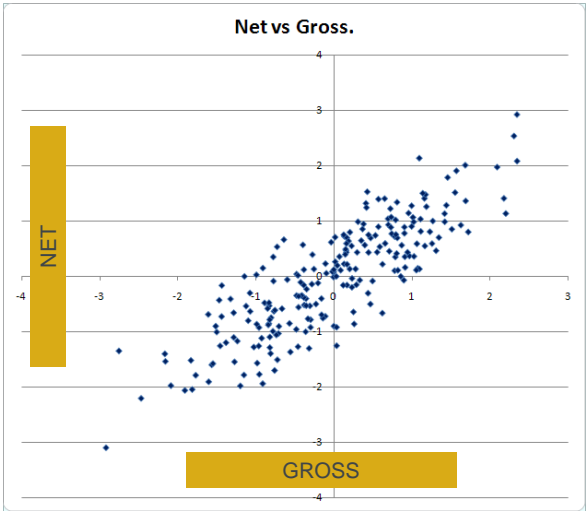
A scatter plot of the residuals, from the respective Gross and Net of Reinsurance models, exhibits a clear (linear) relationship; a correlation of 0.853.

Final Correlations

Final Weighted Residual Correlations Between Datasets

	Gross PL(I)	Net PL(I)
Gross PL(I)	1	0.853
Net PL(I)	0.853	1

3 iterations were executed
Residuals correlation difference
tolerance 0.010%

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A Tale of Two LOBs: LOB1 and LOB3

LOB 1

Cal. Yr Trends

0.0000
+-0.0000

0.1142
+-0.0330

LOB 3

Cal. Yr Trends

0.0000
+-0.0000

0.0790
+-0.0323
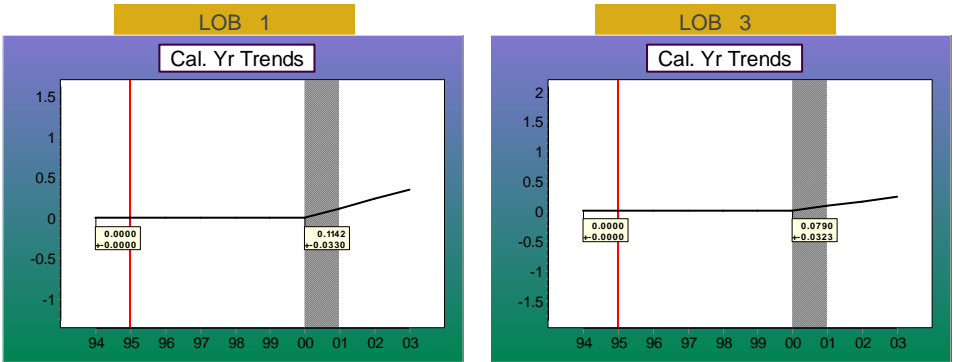
(Actually same line, different territories)

Both LOBs had a calendar year trend change in 2000

That should have been of concern!

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A Tale of Two LOBs: LOB1 and LOB3
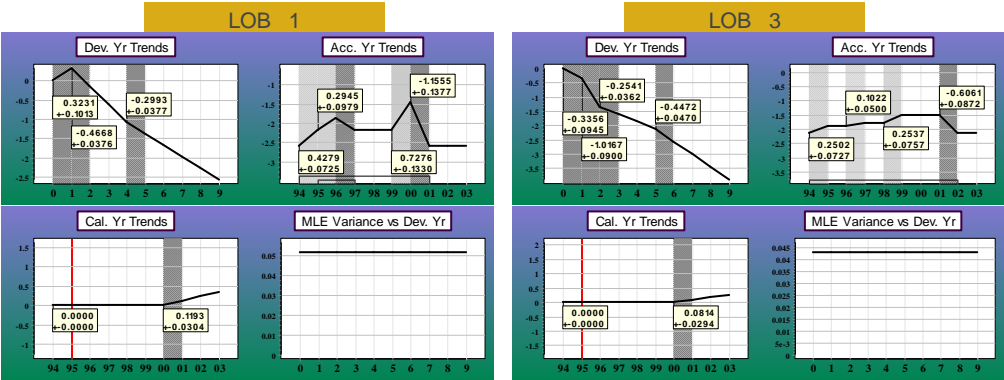
Full model display

Trends in each direction and variance of normal distributions

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A Tale of Two LOBs: LOB1 and LOB3

Volatility correlation = Process correlation = 0.35 = Correlation in normal distributed residuals
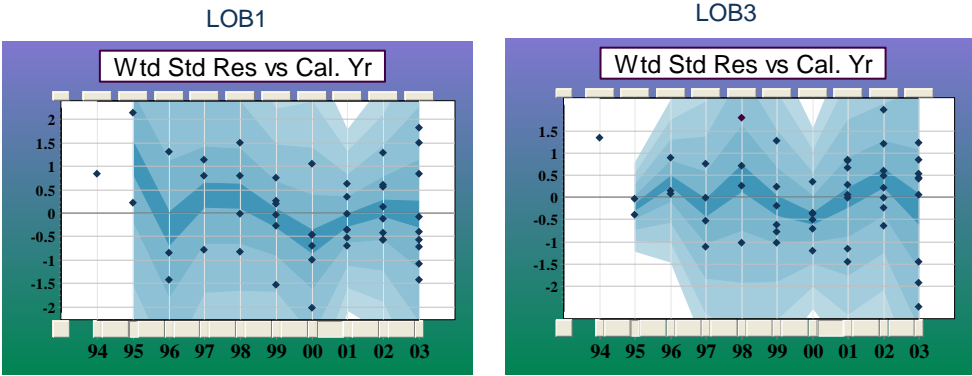
Note 98-00 common negative trend, 00-02 common positive trend
and 02-03 zero trend for LOB1 and negative trend LOB3.

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Regression in the presence of correlation Seemingly Unrelated Regressions (SUR) – Zellner (1962)

Model displays shown above correspond to two linear models, which are described by the following equations:

$$\begin{aligned} \mathbf{y_1} &= \mathbf{X_1\beta_1} + \mathbf{\varepsilon_1}, \\ \mathbf{y_2} &= \mathbf{X_2\beta_2} + \mathbf{\varepsilon_2}, \end{aligned} \qquad (1)$$

$$E\mathbf{\varepsilon_i} = \mathbf{0}, \ i = 1, 2; \ E(\mathbf{\varepsilon_1}, \mathbf{\varepsilon_2^T}) = \mathrm{cov}(\mathbf{\varepsilon_1}, \mathbf{\varepsilon_2}) = \mathbf{C}; \quad corr(\mathbf{\varepsilon_1}, \mathbf{\varepsilon_2}) = \mathbf{R}$$

Without loss of sense and generality two models in (1) could be considered as one linear model:

$$\begin{pmatrix} \mathbf{y_1} \\ \mathbf{y_2} \end{pmatrix} = \begin{pmatrix} \mathbf{X_1} & \mathbf{0} \\ \mathbf{0} & \mathbf{X_2} \end{pmatrix} \begin{pmatrix} \mathbf{\beta_1} \\ \mathbf{\beta_2} \end{pmatrix} + \begin{pmatrix} \mathbf{\varepsilon_1} \\ \mathbf{\varepsilon_2} \end{pmatrix} \qquad (2)$$

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Regression in the presence of correlation

Which could be rewritten as:

$$\mathbf{y} = \mathbf{X\beta} + \mathbf{\varepsilon}$$

For illustration of the most simple case we suppose that size of vectors y in models (1) are the same and equal to n, also we suppose that

$$E(\mathbf{\varepsilon_i}, \mathbf{\varepsilon_i^T}) = \mathrm{var}(\mathbf{\varepsilon_i}) = \mathbf{I_n}\sigma_i^2, \ i = 1, 2; \qquad \mathbf{C} = \mathbf{I_n}\sigma_{12}$$

In this case

$$\mathrm{var}(\mathbf{\varepsilon}) = \mathbf{\Sigma} = \begin{pmatrix} \mathbf{I_n}\sigma_1^2 & \mathbf{I_n}\sigma_{12} \\ \mathbf{I_n}\sigma_{12} & \mathbf{I_n}\sigma_2^2 \end{pmatrix}$$

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Regression in the presence of correlation

For example, when $n = 3$

$$\Sigma = \begin{pmatrix} \sigma_1^2 & 0 & 0 & \sigma_{12} & 0 & 0 \\ 0 & \sigma_1^2 & 0 & 0 & \sigma_{12} & 0 \\ 0 & 0 & \sigma_1^2 & 0 & 0 & \sigma_{12} \\ \sigma_{12} & 0 & 0 & \sigma_2^2 & 0 & 0 \\ 0 & \sigma_{12} & 0 & 0 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_{12} & 0 & 0 & \sigma_2^2 \end{pmatrix}$$

Regression in the presence of correlation

There is a big difference between linear models in (1) and linear model (2), as in (1) we consider models separately and could not use additional information, from dependency (process correlation) of these models, what we can do in model (2). To extract this additional information we need to use proper methods to estimate vector of parameters β. The estimation

$$\hat{\beta} = (X^T X)^{-1} X^T y$$

which derived by ordinary least square (OLS) method, does not provide any advantage, as the covariance matrix Σ does not participate in the estimations.

Only general least square (GLS) estimation

$$\widetilde{\beta} = (X^T \Sigma^{-1} X)^{-1} X^T \Sigma^{-1} y$$

could help to achieve better results.

Regression in the presence of correlation

However, it is necessary immediately to underline that we do not know elements of the matrix Σ and we have to estimate them as well. So, practically, we should build iterative process of estimations

$$\widetilde{\beta}^{\ (m)}, \ \widetilde{\Sigma}^{\ (m)}$$

and this process will stop, when we reach estimations with satisfactory statistical properties.

The SUR β is a (credibility) weighted average β_1 and β_2

Regression in the presence of correlation

There are some cases, when model (2) provides the same results as models in (1). They are:
1. Design matrices in (1) have the same structure (they are the same or proportional to each other.)
2. Models in (1) are non-correlated, in other words

$$\sigma_{12} = 0$$

However in situation when two models in (1) have common regressors model (2) again will have advantages in spite of the identical structure of the design matrices.

Model Displays for LOB1 and LOB3 for Calendar Years
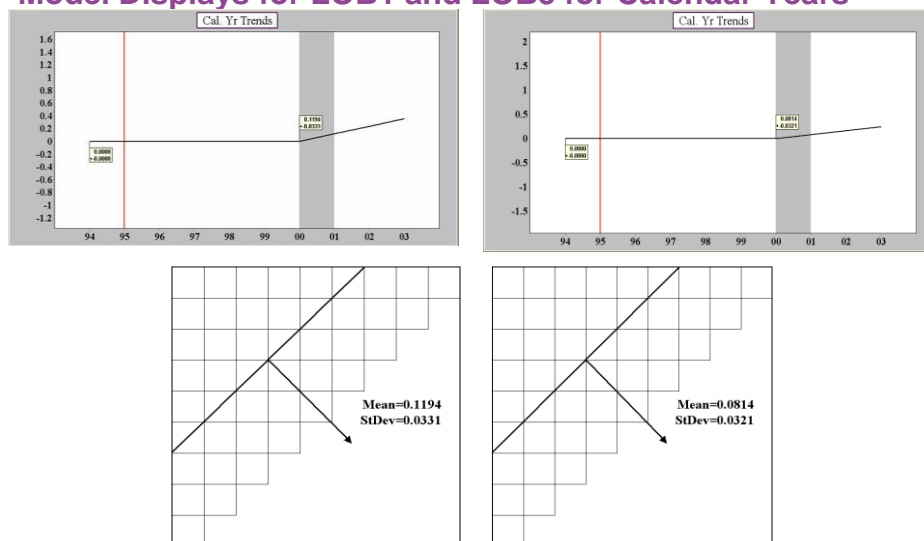
Model for individual iota parameters- they are correlated going forward

$$\hat{\iota}_1 \sim N(\mu_1, \ \sigma_1^2); \qquad \hat{\mu}_1 = 0.1194; \quad \hat{\sigma}_1 = 0.0331$$
$$\hat{\iota}_2 \sim N(\mu_2, \ \sigma_2^2); \qquad \hat{\mu}_2 = 0.0814; \quad \hat{\sigma}_2 = 0.0321$$

$$\begin{pmatrix} \iota_1 \\ \iota_2 \end{pmatrix} \sim N(\mathbf{\mu}, \mathbf{\Sigma}), \quad \hat{\mathbf{\mu}} = \begin{pmatrix} 0.1194 \\ 0.0814 \end{pmatrix}, \quad \hat{\mathbf{\Sigma}} = \begin{pmatrix} 0.001097 & 0.000344 \\ 0.000344 & 0.001027 \end{pmatrix}$$

$$\rho = corr(\iota_1, \iota_2), \qquad \hat{\rho} = 0.359013$$

Correlations are in the volatility component of a model

• Two lines are (positively) correlated when their results tend to miss their target values in the same way.

• This is what should concern business planners, because it affects the unpredictable component of the forecasts.

• What is predicable when it includes common trend patterns, as in the above example, does not count towards correlation, because its effects are already incorporated into the model and forecast.

• A forecast must include a volatility measure, ideally in the form of a loss distribution but at least in the form of a standard deviation.

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Common accident year drivers: SAD and SAM
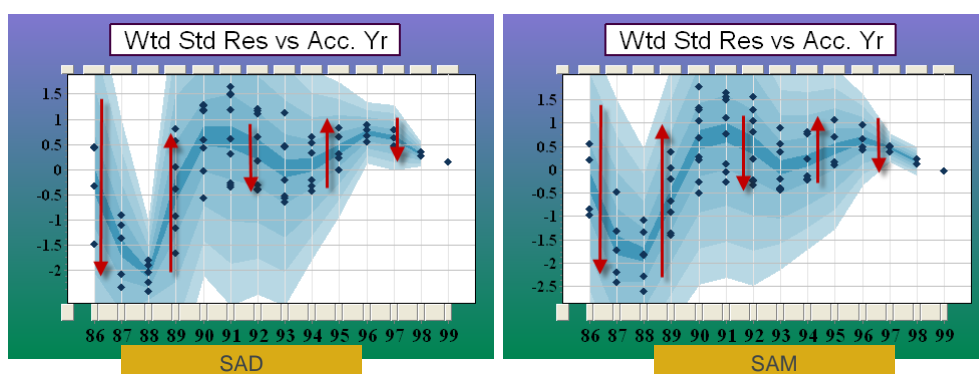
A model which does not take into account the changes in accident year levels shows a marked similarity in the fluctuations of residuals in the accident direction.

This is **not** correlation!

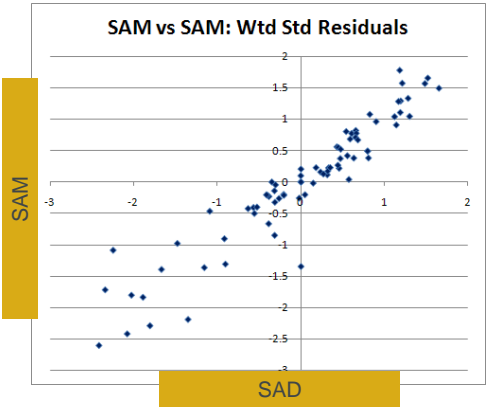15 October 2013   38

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Common accident year drivers: SAD and SAM

Final Weighted Residual Correlations Between Datasets		
	SAD:PL(I)	SAM:PL(I)
SAD:PL(I)	1	0.956
SAM:PL(I)	0.956	1
3 iterations were executed Residuals correlation difference tolerance 0.010%		

SAM vs SAM: Wtd Std Residuals

If the common accident year movements are ignored and the average accident year level fitted to both segments, then a very high spurious correlation measure of 0.96 is obtained.

The residual displays with scatterplot for SAD and SAM are shown for this model. The correlation is very high, but it is largely spurious - there are distinct changes in level across the accident years which were ignored in this model.

Common accident year drivers: SAD and SAM

Accounting for accident year level (trend) changes

Acc. Yr Trends — SAD

Acc. Yr Trends — SAM
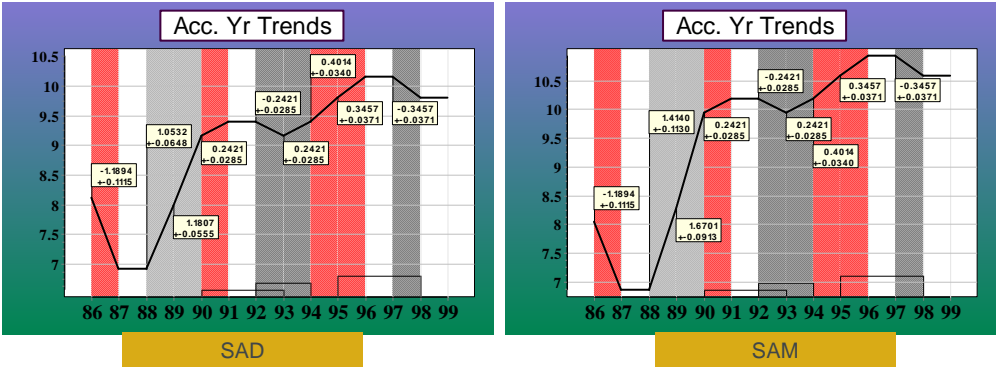
The red bars indicate common parameters between the segments. Although the calendar and development year parameters vary slightly, the accident year parameters move synchronously thus making the mean ultimates vary synchronously (but this is not correlation).

Common accident year drivers: SAD and SAM

Final Weighted Residual Correlations Between Datasets

	SAD:PL(I)	SAM:PL(I)
SAD:PL(I)	1	0.249
SAM:PL(I)	0.249	1

4 iterations were executed
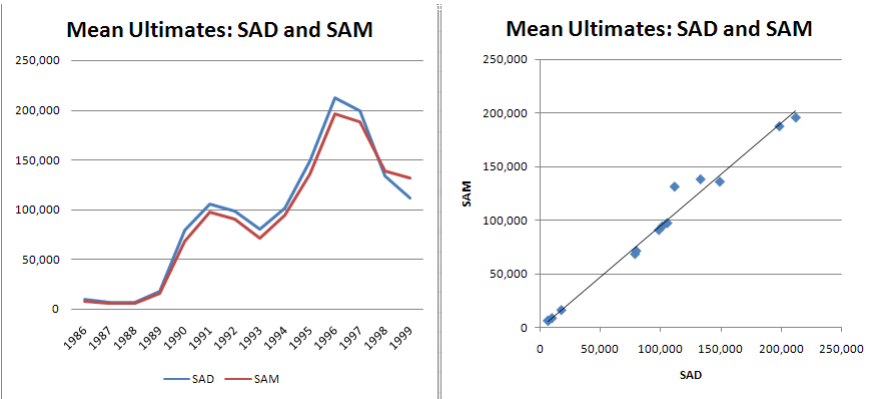Residuals correlation difference
tolerance 0.010%

SAM vs SAM: Wtd Std Residuals
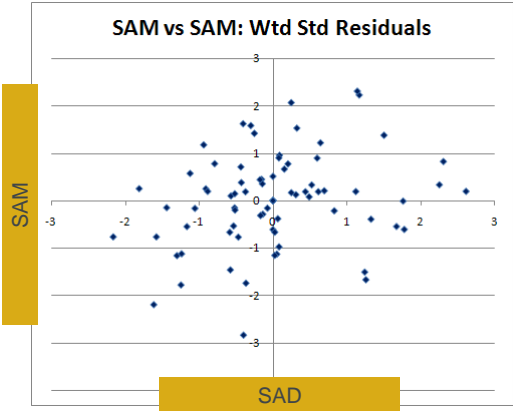
Both sets of residuals test well for normality and have no indications of non-randomness so the process correlation (0.249) is the volatility correlation between two normal distributions.

Common accident year drivers: SAD and SAM

Mean Ultimates: SAD and SAM

Mean Ultimates: SAD and SAM

• The accident year levels moving together is a much stronger relationship than volatility correlation.
• The mean ultimates move synchronously (left) and a graph of the mean ultimates of SAM versus the mean ultimates of SAD (right) shows an almost perfect linear relationship.
• The reserve distribution correlation is only 0.086! The reserve correlation is the correlation in the losses *not* explained by the means – and therefore is the critical measure when evaluating risk diversification.
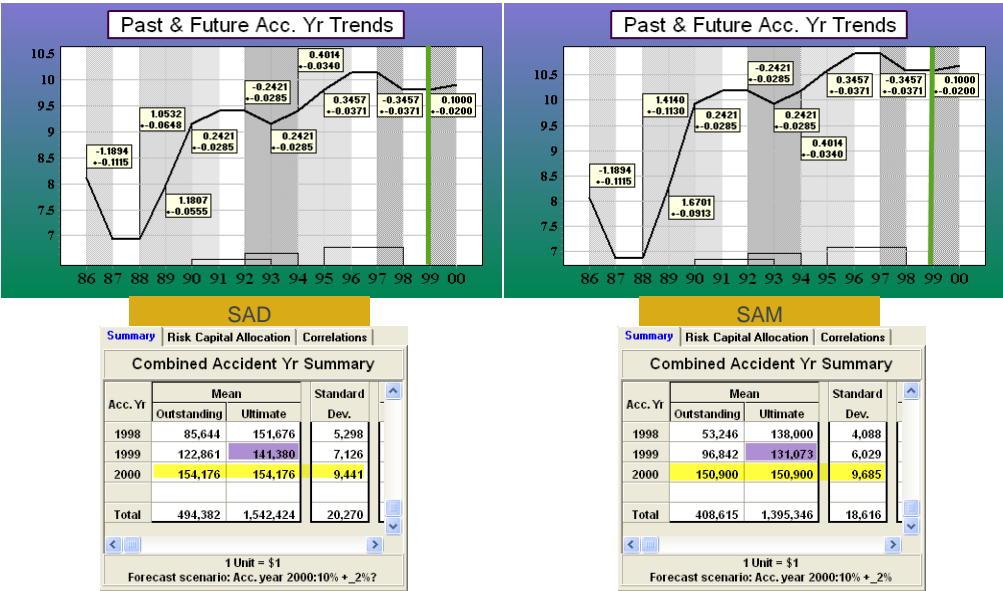
Common accident year drivers and pricing future accident years

• The linear relationship in mean ultimates is important when forecasting future underwriting (accident) years.

• If the accident year level for one segment is expected to increase by 10%+_2%, then the other segment is also likely to increase by 10%+_2% in the same accident year.

• The relationship in the mean parameter estimates is not volatility (risk) correlation and does not indicate lack of diversification.

• The movement in means may be able to be related to internal or external drivers - and risk exposure can be managed.

• Correlation in risk is significantly harder to manage as it invokes correlation in the random component - variation which is not readily able to be connected to any internal or external drivers.

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Common accident year drivers and pricing future accident years
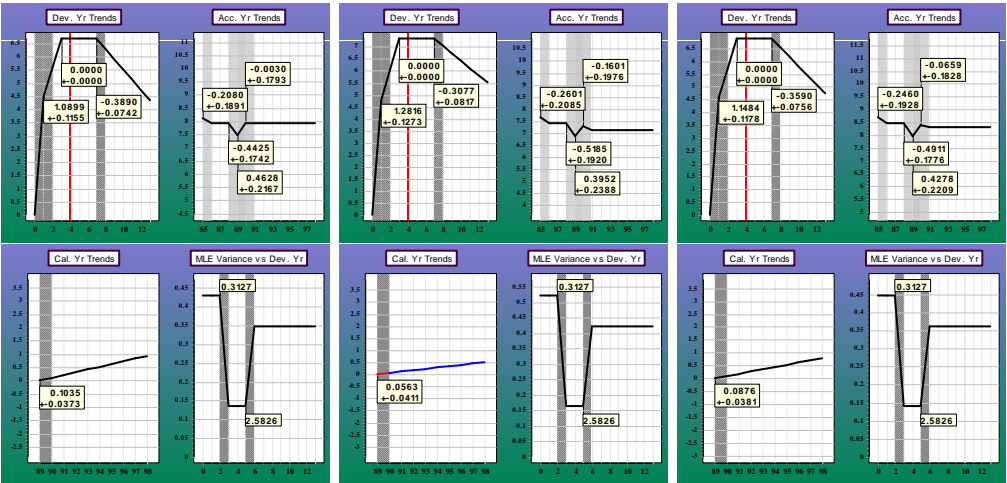

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Common accident year and common calendar year drivers

- Common drivers are a stronger influence than correlation.

- However, they are not typically found outside closely related losses.

- For example, Gross versus Net of Reinsurance (Net of Reinsurance is a subset of Gross so common drivers are expected), layers (layers are subsets of ground up losses), and segments of the same line. In this respect, detection of common drivers is as important as understanding correlations.

- The two effects must be correctly distinguished and adjusted for as management strategies of these risk components differ.

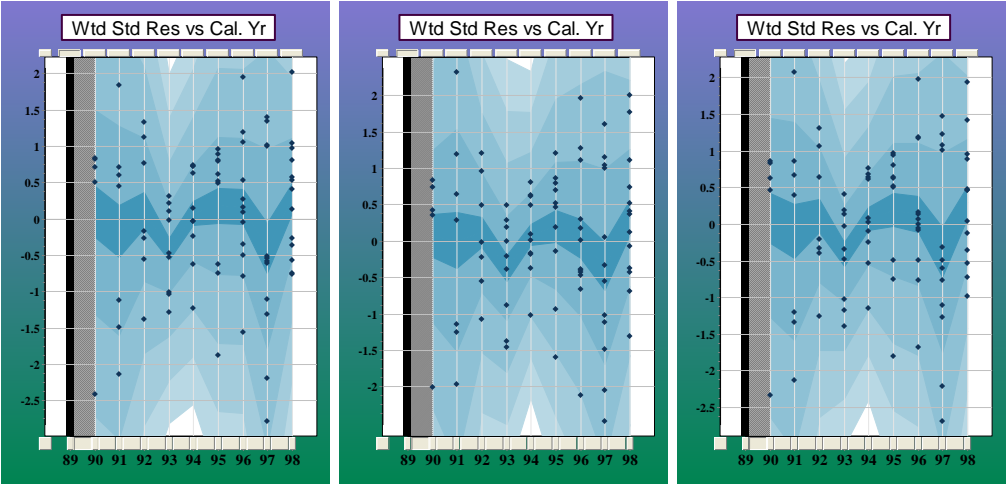15 October 2013   45

Layers Lim1M, Lim2M and 1Mxs1M; Lim2M=Lim1M+1Mxs1M

The trend structure is the same for each layer (Left to right 1M, 1Mxs1M, 2M)

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Layers Lim1M, Lim2M and 1Mxs1M; Lim2M=Lim1M+1Mxs1M

Very high process correlations (Left to right 1M, 1Mxs1M, 2M)

Layers Lim1M, Lim2M and 1Mxs1M; Lim2M=Lim1M+1Mxs1M

Tables of process correlations (linear) and calendar year parameter correlations (linear)

This type of equivalent trend structure and high parameter and process correlations has not been observed for two LOBs

Iota Correlations				
		1M:PL(I)	1Mxs1M:PL(I)	2M:PL(I)
Dataset	Period	1989~1998	1989~1998	1989~1998
1M:PL(I)	1989~1998	1	0.945646	0.992496
1Mxs1M:PL(I)	1989~1998	0.945646	1	0.977333
2M:PL(I)	1989~1998	0.992496	0.977333	1

| Reserve Forecast Distributions Correlations Between Datasets (Totals)			
	1M:PL(I)	1Mxs1M:PL(I)	2M:PL(I)
1M:PL(I)	1	0.939207	0.991686
1Mxs1M:PL(I)	0.939207	1	0.974411
2M:PL(I)	0.991686	0.974411	1

Spurious correlation

Two LOBs are simulated independently each with its own unique trend structure. The only material difference in the LOBs is that one LOB has a calendar year trend of 10%, the other of 20%. Each has a -30% development year trend.

A correct model of the underlying data process, would recognise that each LOB has a separate trend for each direction and a process correlation of zero - since this is how the data were generated.
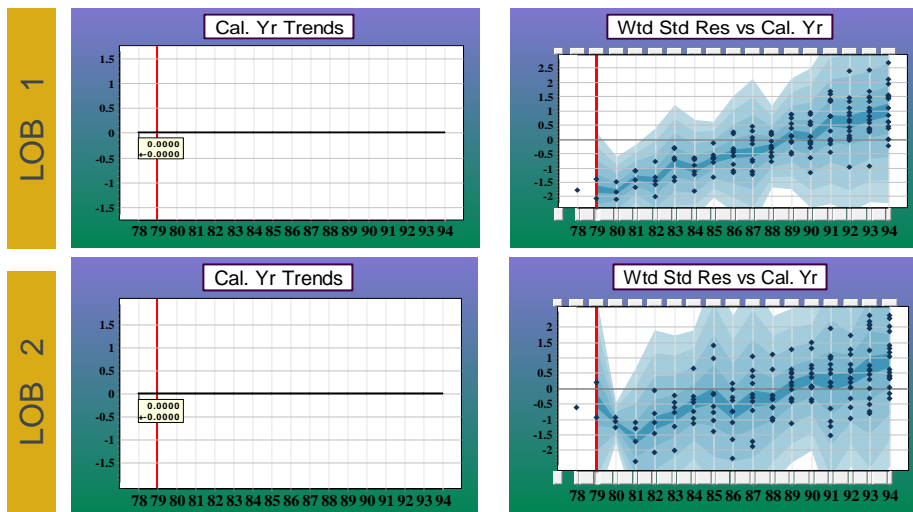
If an incorrect model is used, one that does not describe the calendar year trends, then a spurious correlation would be detected, as an artefact of unaccounted-for structure in the data.

Spurious correlation
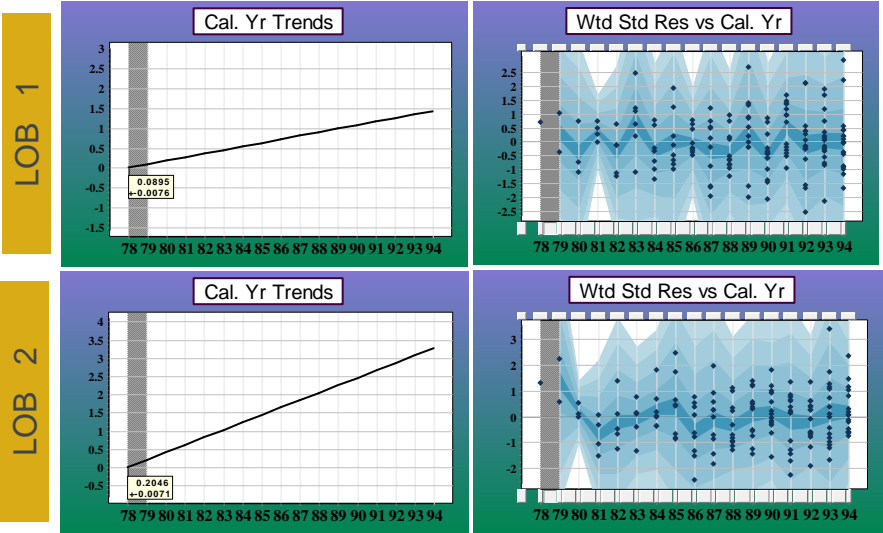
Correct model picks up true calendar year trend; process correlation is zero!

Spurious correlation

Incorrect model fails to pick up calendar trend; measures 98% correlation!
But this is not correlation since each sample is not random. They have structure.

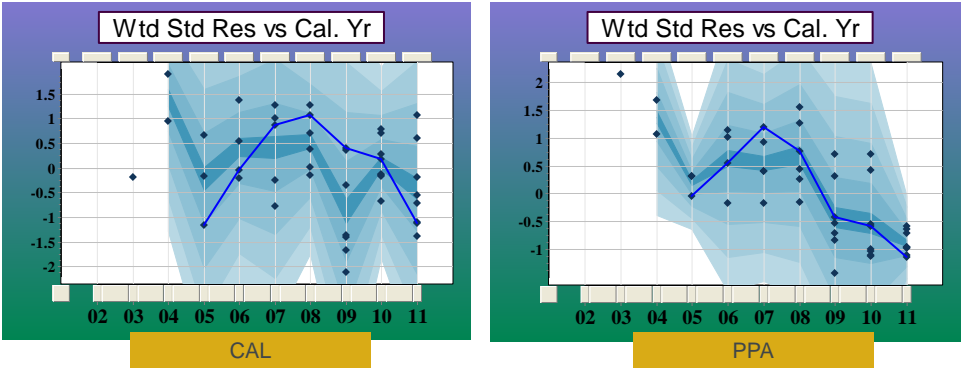# Spurious correlation between Industry PPA and CAL data

As was shown in the previous case study, spurious correlation is introduced by failing to detrend the data in the three directions. The correlation measured was spurious as there were trends in the data not described in the models. Once these trends were accounted for, the process correlation was statistically insignificant.

Spurious correlation between Industry PPA and CAL data due to wrong model

Paid Losses for the Industry PPA and CAL data from AM Best (2011) are modelled using the Mack method. The residuals are shown by Calendar year for CAL and PPA with the trace line for accident year 2004 highlighted.
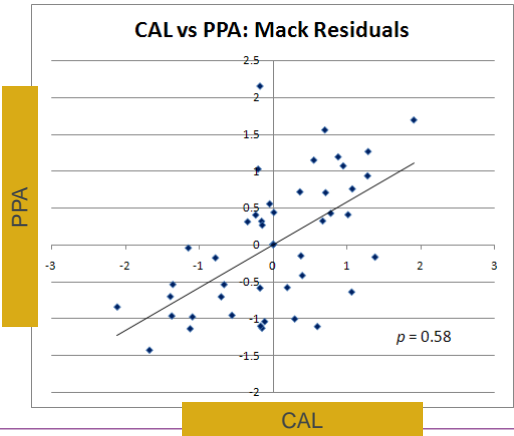
Spurious correlation between Industry PPA and CAL data

Although the residual correlation is strong the indication is misleading. The observed correlation is due entirely to limitations of the model.

CAL vs PPA: Mack Residuals

$p = 0.58$

Spurious correlation between Industry PPA and CAL data

The observed correlation is due entirely to limitations of the model.
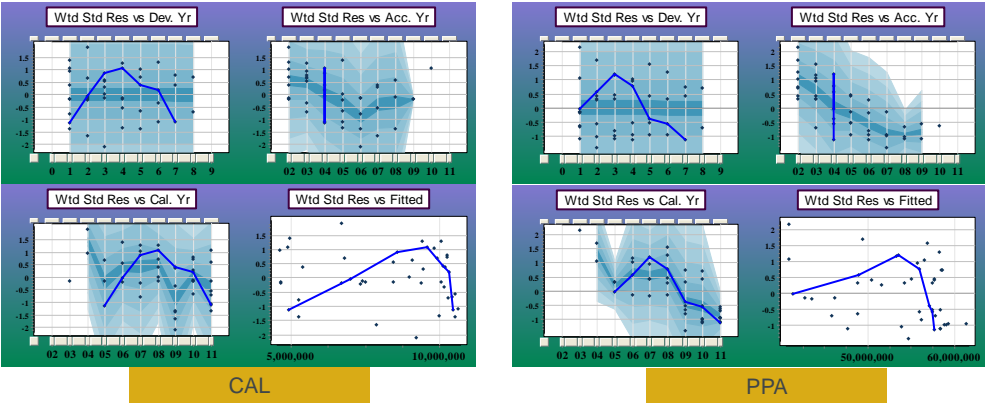
The calendar year residuals show the Mack method over-fits the recent data - producing a common negative trend in both residual displays.

Models for PPA and CAL

No LOBs have the "same" tend structure and most LOBs have zero process correlation. Consider Private Passenger Automobile and Commercial Auto Liability
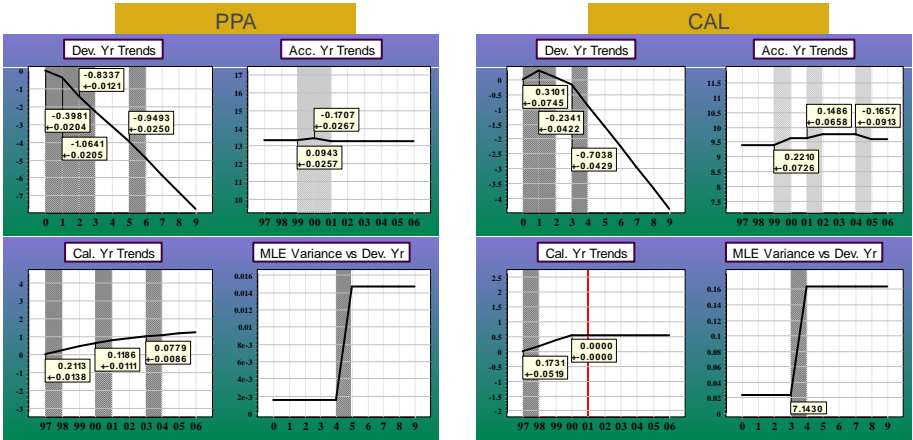
The two lines have very different trend structure and process variance!

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Process correlation is zero

PPA and CAL have different trend structure and zero process (validation) correlation

Blue lines represent trace of calendar year 2006
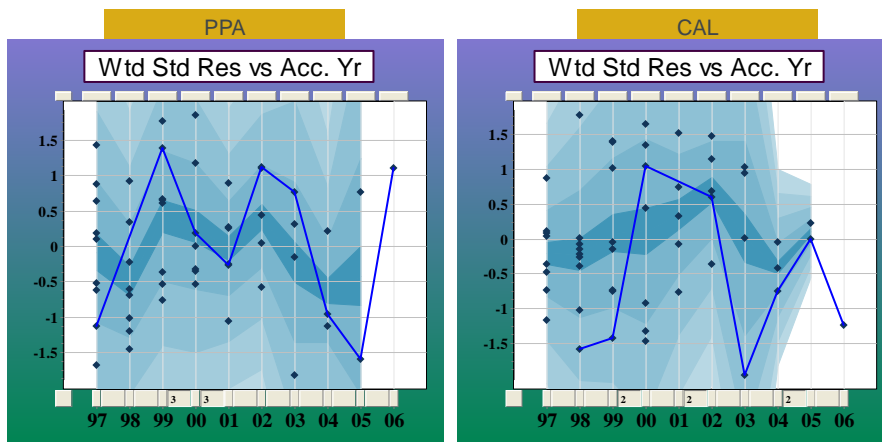
Note zero process correlation.

Reserve distribution correlations between two distinct LOBs - a very different story

• Highest process correlation observed between two different LOBs is about 0.6 (in our experience)

• But Reserve distribution correlation is typically lower.

• Trend structures for two LOBs typically different

• Parameter correlations low or zero

• See Private Passenger Automobile (PPA) versus Commercial Auto Liability (CAL)

Correlations- and other relationships

There are five types of relationships.

1. Process Correlation (correlation between two sets of (random) residuals)
2. Parameter Correlations
3. Same trend structure (especially along the calendar years)- common calendar year drivers. This is stronger than correlations.
4. Common accident year drivers- major implications for pricing future accident years. This relationship is also stronger than correlations.
5. Reserve distribution correlations by total, accident years and calendar years
The optimal single composite model may also involve cross dataset parameter constraints.

#1 induces #2. However, #3 is the 'worst' kind of relationship you can have between two LOBs as it results in very little, if any, risk diversification. It means that in terms of future calendar year trends the two LOBs move together, that is, a trend change in one LOB means a trend change in the other LOB. If two LOBs satisfy #3, then #1 and #2 are typically not far from 1.

Fortunately, #3 we have only observed between layers of the same LOB, between segments of the same LOB, and between net of reinsurance and gross data (of the same LOB). #1, #2, #3 induce #5. #5 is typically much less than #1 in the absence of #3.

#4 results in mean ultimates by accident year moving synchronously. The relationship in mean ultimates may be close to linear- this is stronger than correlations and has implications for pricing. Synchronous mean ultimates are already incorporated in the reserving model. Sometimes only one or two accident years move synchronously due to a major event like Katrina. The process correlation about the new levels (trends) is usually low.

It is important to recognize that you cannot measure the relationship between two LOBs unless you first identify the trend structure and process variability in each LOB. It is only in the Probabilistic Trend Family (PTF) modelling framework that you can identify a parsimonious model that separates the trend structure in the three directions from the process variability. The data triangle (real data) is regarded as a sample path from the identified model that fits (different) normal distributions to each cell. When you simulated triangles from the identified model, they are indistinguishable in respect of statistical features from the real data.

Updating, monitoring, variation in mean ultimates one year hence (CDR) and consistent estimates of prior year ultimates
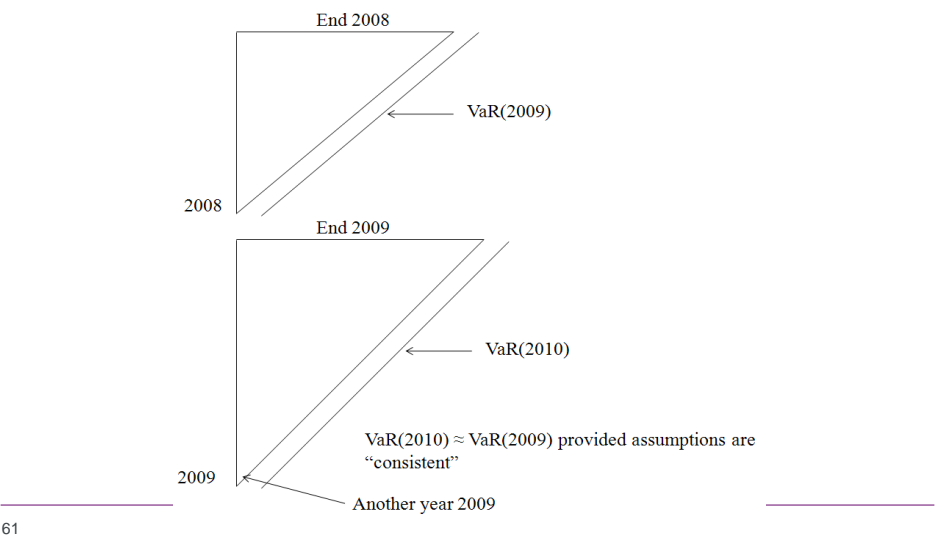
61

Consistent estimates of prior year ultimates and SII metrics on updating
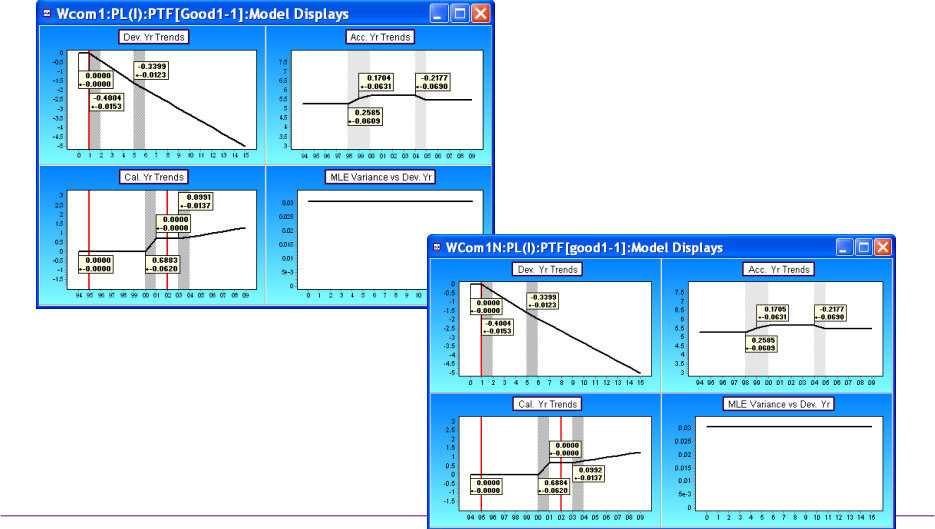
62

Consistent estimates of prior year ultimates on and SII metrics updating

Calendar year trend has not changed statistically on updating

63

Consistent Estimates of prior year mean ultimates on updating only under certain conditions

64   At end 2008, ultimate 2008=64.9+_5.8, at end 2009 66.2+_4.22

With identified optimal parametric distribution models that are tested from the data, it is relatively straightforward to compute the CDR. Note Pythagoras's theorem, viz., Var[Ult.]=E[Var[Ult.|CY1]]+Var[E[Ult.|CY1]]
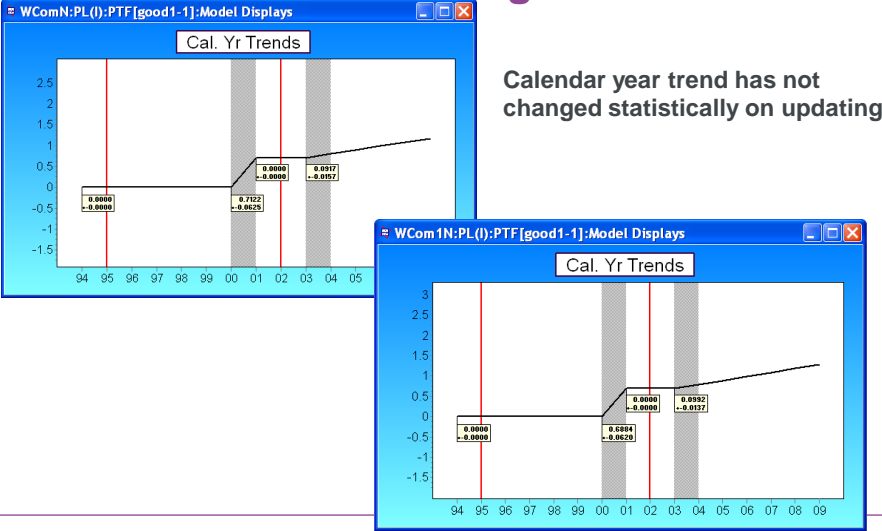**Variation in mean ultimate one year hence is represented by Var[E[Ult.|CY1]]. Variance of (distribution of) Ultimate = Mean Conditional Variance + Variance of Conditional Mean**

WC1 New:Composite DS:MPTF[Good1-1]:Reserve Forecast Summaries

PL(I)

Acc. Yrs | Cal. Yrs | Observed vs Mean Estimate | (%) Differences | Summary Graphs | Forecast Settings |
Summary | Risk Capital Allocation | Correlations |

Accident Yr Summary

Acc. Yr	Mean		Standard	CV		Cond. on Next Cal. Per.	
	Outstanding	Ultimate	Dev.	Outstanding	Ultimate	E[SD[Ult|Data]]	SD[E[Ult|Data]]
1994	64,094	4,367,010	11,945	0.19	0.00	0	11,945
1995	183,446	5,904,943	25,739	0.14	0.00	14,625	21,181
1996	348,907	6,811,128	42,393	0.12	0.01	27,437	32,317
1997	610,713	9,346,243	68,453	0.11	0.01	46,357	50,367
1998	941,013	10,645,174	101,389	0.11	0.01	70,024	73,324
1999	1,722,604	16,661,780	192,216	0.11	0.01	134,377	137,441
2000	3,224,708	24,724,755	323,235	0.10	0.01	229,618	227,501
2001	4,977,958	35,702,799	487,335	0.10	0.01	346,575	342,609
2002	7,756,800	43,404,360	756,603	0.10	0.02	536,924	533,067
2003	12,254,513	49,679,159	1,213,615	0.10	0.02	858,082	858,229
2004	17,786,533	58,024,535	1,818,109	0.10	0.03	1,280,371	1,290,803
2005	19,286,752	54,283,669	1,952,756	0.10	0.04	1,350,644	1,410,325
2006	29,230,395	62,411,590	2,980,888	0.10	0.05	2,043,089	2,170,595
2007	43,451,297	69,742,015	4,481,674	0.10	0.06	3,052,683	3,281,238
2008	54,355,368	64,970,760	5,709,779	0.11	0.09	3,866,919	4,201,013
Total	196,195,101	516,679,920	15,211,543	0.08	0.03	9,847,780	11,593,631

1 Unit = £1
Forecast scenario: Si

SD   Mean Conditional SD   CDR

The CDR is Var[E[Ult.|CY1]]

With identified optimal parametric distribution models that are tested from the data, it is relatively straightforward to compute the CDR. Note Pythogras's theorem, viz., Var[Ult.]=E[Var[Ult.|CY1]]+Var[E[Ult.|CY1]]

Anoglous to One Way ANOVA
Total SS= Within Group SS+ Between Group SS

All:CDS:MPTF[Good-1]:Reserve Forecast Summaries

Agg MBI+PL | Aggregate | Cars | PL | T&O | MBI | Cars_H:PL(I) | Cars_H:Lost PL(I) | Cars_H:NII PL(I) | Cars_H:ME PL(I) | Cars_nH:PL(I) | Cars_nH
(%) Differences | Summary Graphs | Clusters
Summary by Datasets | Acc. Yrs | Cal. Yrs | Observed vs Mean Estimate | Incurred Losses
Summary | Risk Capital Allocation | Correlations |

Accident Yr Summary

Acc. Yr	Mean		Standard	CV		Cond. on Next Cal. Per.	
	Outstanding	Ultimate	Dev.	Outstanding	Ultimate	E[SD[Ult|Data]]	SD[E[Ult|Data]]
1999	7,757	237,366	1,267	0.16	0.01	872	920
2000	11,612	256,535	1,750	0.15	0.01	1,226	1,248
2001	17,374	265,118	3,229	0.19	0.01	2,322	2,244
2002	21,915	252,631	3,129	0.14	0.01	2,230	2,195
2003	31,166	272,209	4,725	0.15	0.02	3,242	3,437
2004	55,956	349,458	7,265	0.13	0.02	5,197	5,077
2005	92,608	487,089	11,259	0.12	0.02	8,006	7,917
2006	132,188	477,713	17,977	0.14	0.04	13,599	11,758
2007	177,426	503,984	21,409	0.12	0.04	18,013	11,571
2008	202,713	507,123	22,130	0.11	0.04	19,721	10,041
2009	255,236	419,641	26,382	0.10	0.06	23,045	12,843
2010	244,140	321,048	30,701	0.13	0.10	26,278	15,875
2011	332,395	356,913	47,337	0.14	0.13	38,978	26,860
Total	1,582,487	4,706,829	113,328	0.07	0.02	85,799	74,040

1 Unit = $1,000
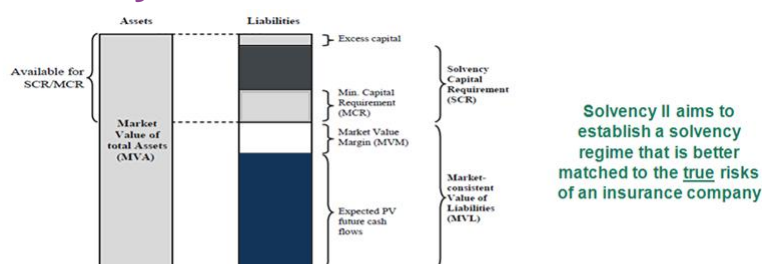Forecast scenario: Reasonable Combination   Sqrt(CDR)

Example of risk diversification of SCR and Risk Margins

• SII metrics for the aggregate of real life six LOBs compared with SII metrics for the most volatile LOB to illustrate amongst other things risk diversification of SCR and (MVM (Risk Margin) component) of TP (Fair Value of Liabilities).

• Undiscounted reserves for the aggregate of six LOBs

   = (approx) Technical Provisions +Solvency Capital Requirement (SCR)

   = total in Economic Balance Sheet,

   using a risk free rate of 4% and a spread of 6%.

• No need for additional capital in this example due to risk diversification SCR and MVM.

• Conditions for consistent estimates of prior accident year ultimates and SII risk measures on updating?

• We will explain how to avoid model error "distress".

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Solvency II – Economic Balance Sheet

Solvency II aims to establish a solvency regime that is better matched to the true risks of an insurance company

Ann Hagen in "Solvency II : Brave new world:

"Doing the job

Under Solvency II, the way that work is carried out will change. For example, Solvency II is likely to require different actuarial techniques from the ones currently used. Technical provisions will be estimated as a probability-weighted average of expected future cash flows, taking into account the time-value of money and including a risk margin. Many of us are estimating claims reserves using traditional deterministic actuarial techniques, primarily relying on incurred claims data. Under Solvency II, not only will we need to discount these reserve estimates, requiring projected payment patterns, we will also need to demonstrate a deep understanding of the uncertainty of those reserves. We will additionally be required to apply the same approach to evaluating unexpired risk liabilities currently allowed for in the unearned premium reserves."

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Solvency II one-year risk horizon:
* satisfies 3 conditions
* decomposing the directives
* What are the basic elements?

• Risk Capital is raised at the beginning of each year and any unused capital is released at the end of the year;

• The analyses are conditional on the first (next) calendar year being in distress (99.5%);

• At the end of the first year in distress, the balance sheet can be "restored" in such away that the company has sufficient technical provisions (fair value of liabilities) to continue business or to transfer the liabilities to another risk bearing entity.
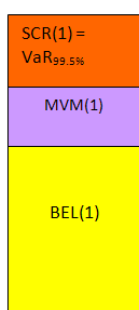
An important consideration is that **fungibility** by calendar year is only in the forward direction

Risk Capital – One Year risk Horizon

Simplest Case: Only One Year Runoff

$L_1 =$ projected losses for the year. This is a random variable.

$$BEL(1) = \frac{E(L_1)}{(1+d)^{0.5}} \quad \text{Where } d = \text{interest rate. Losses are paid}$$

uniformly through year, so we discount for half a year.

$$SCR(1) = VaR_{99.5\%}(L_1), \text{ i.e. } Pr\big(L_1 \le E(L_1) + SCR(1)\big) = 0.995$$

MVM(1) is the cost incurred in having risk fund of *SCR(1)* available for the year. It is paid to capital provider at end of year and so is discounted by a full year.

$$MVM(1) = \frac{SCR(1)*s}{(1+d)}, \text{ if the interest on the risk fund is paid directly to capital}$$

provider, or $MVM(1) = \frac{SCR(1)*(s+d)}{(1+d)}$, otherwise.

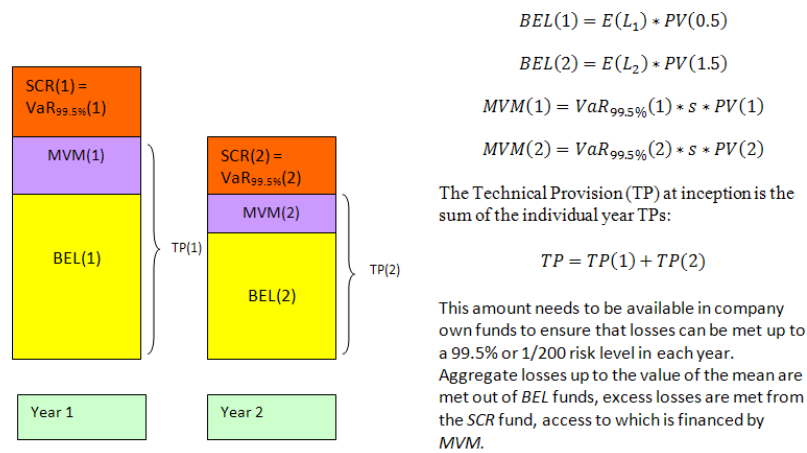$TP(1) = BEL(1) + MVM(1)$. This is the Technical Provision and must be held in company own funds.

We will also let, *PV(k;d)*, or *PV(k)* be used to abbreviate the Present Value factor $\frac{1}{(1+d)^k}$

SCR(1) = VaR$_{99.5\%}$

MVM(1)

BEL(1)

TP(1)

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Risk Capital – One Year risk Horizon

Next Simplest Case: Two Year runoff, No correlation

$$BEL(1) = E(L_1) * PV(0.5)$$

$$BEL(2) = E(L_2) * PV(1.5)$$

$$MVM(1) = VaR_{99.5\%}(1) * s * PV(1)$$

$$MVM(2) = VaR_{99.5\%}(2) * s * PV(2)$$

The Technical Provision (TP) at inception is the sum of the individual year TPs:

$$TP = TP(1) + TP(2)$$

This amount needs to be available in company own funds to ensure that losses can be met up to a 99.5% or 1/200 risk level in each year. Aggregate losses up to the value of the mean are met out of *BEL* funds, excess losses are met from the *SCR* fund, access to which is financed by *MVM*.

Capital flow: Uncorrelated future calendar years
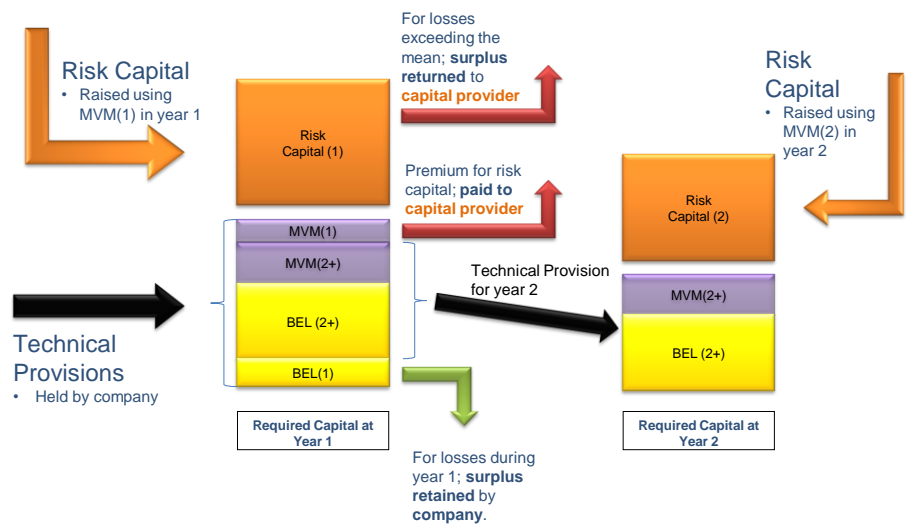
Risk Capital – One Year risk Horizon
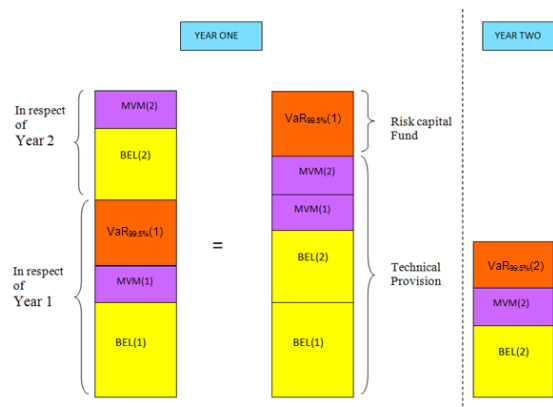
Two-year picture of accounts: In year 1 we require reserves to meet paid loss liabilities for years 1 and 2 and we also need to able to fund the cost of access to the risk capital funds for years 1 and 2, however we only need access to the year 1 risk fund. When year 2 begins our accounts reset, since any cost over-runs from year 1 were paid out of the risk fund and do not degrade our prepared reserves for year 2. Provided the loss over-run is below RC(1) = $VaR_{99.5}$(L1).

Risk Capital – One Year risk Horizon

•This is fine, except for one thing:

　What if the distribution for the losses in year 2 has changed conditional on the losses in year one?

•Simply put, the previous picture assumes there is no correlation between the distributions for years 1 and 2. In other words, whatever the outcome observed after year 1 we are going to remain fixed on our previous course, full steam ahead
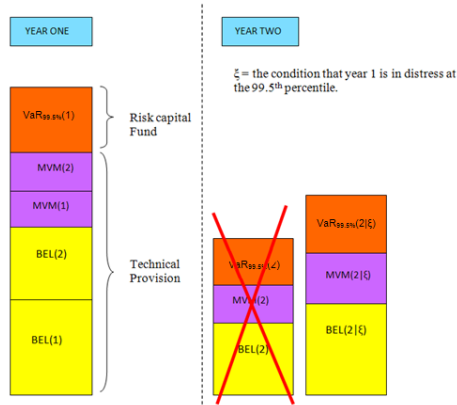
Typically calendar year distributions are positively correlated.

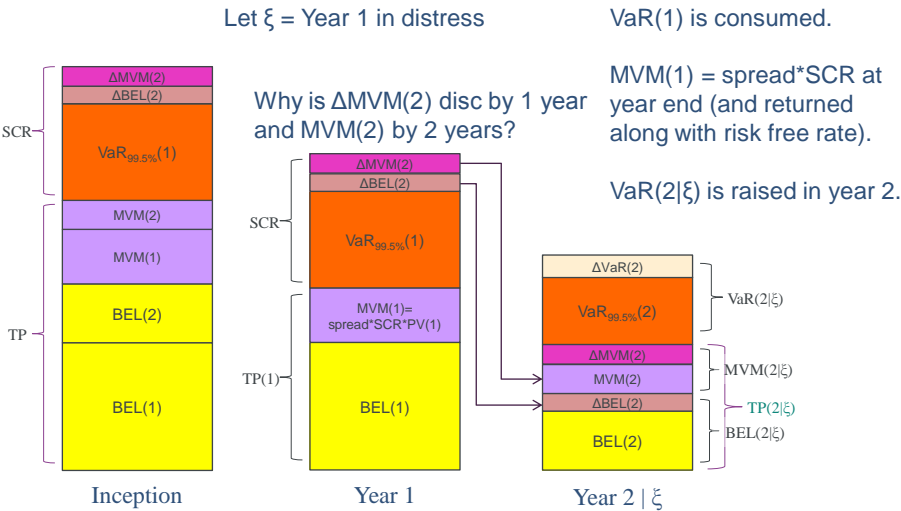**The correlations are driven by parameter uncertainty**.
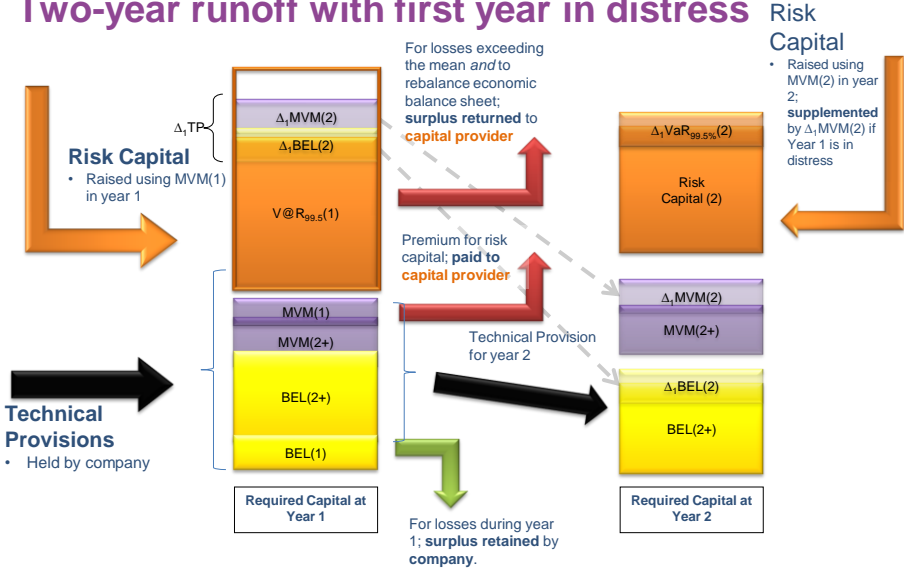
Risk Capital – One Year risk Horizon

If year 1 is in distress at the 99.5th percentile, then our risk fund carries us over into year 2, but the conditional distributions are now different. Year 2 now must be re-evaluated in the light of conditional distributions and these increase the size of the BEL and the MVM, the cost of holding the risk fund. We need to include these adjustments in the year 1 risk fund

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Two-year runoff with first year in distress.

Let ξ = Year 1 in distress

Why is $\Delta MVM(2)$ disc by 1 year and MVM(2) by 2 years?

VaR(1) is consumed.

MVM(1) = spread*SCR at year end (and returned along with risk free rate).

VaR(2|ξ) is raised in year 2.

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Capital flow:
Two-year runoff with first year in distress

N-year run-off (Correlated)

$$TP = \Sigma BEL(k) + \Sigma MVM(k); \; k = 1 \text{ to } n$$
$$SCR = VaR_{99.5}(1) + \Sigma[\Delta BEL(k) + \Delta MVM(k)]; \; k = 2 \text{ to } n$$
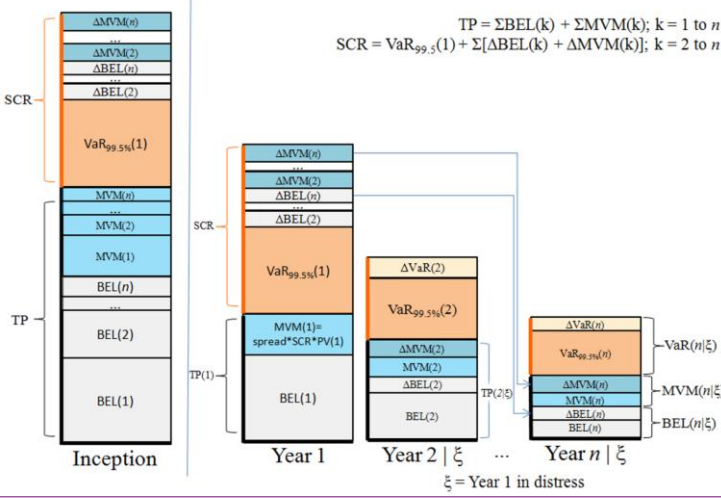
Two-year runoff with first year in distress

• There is sufficient risk capital SCR and Fair Value to withstand a distressed first year at 99.5% confidence and restore Fair Value at beginning of the second year.

• An important consideration is that fungibility by calendar year is only in the forward direction.

   Consistent metrics on updating from year to year- under what conditions?

   See also E&Y GNAIE paper (2007)

"Market Value Margins for Insurance Liabilities in Financial Reporting and Solvency Applications , October 1, 2007"

What Causes Distress in the first year?

1. "Inflation parameters" going forward. For example under the assumption 10%+_3%, a 60% trend is distress.

2. Process volatility- large values from the tail of lognormal distributions.

3. Combinations of 1. and 2.

4. Which LOBs contribute more to distress than others?

• Process volatility

• Parameter uncertainty

• "Size" of LOB