The Actuarial Profession
making financial sense of the future

Pensions, benefits and social security colloquium 2011
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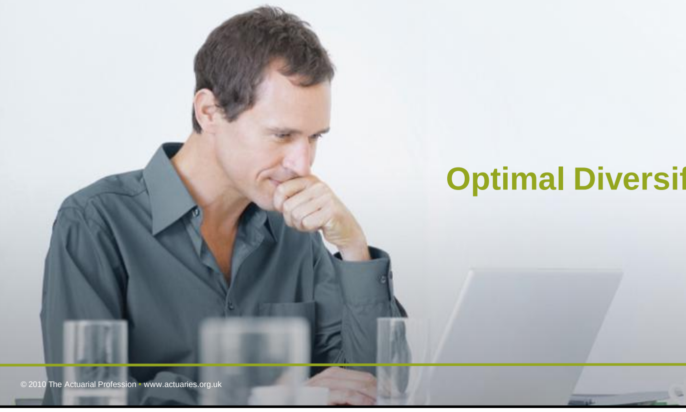

Optimal Diversification in Pension Funding

25-27 September 2011

Purpose of the research

- Increasing interest to mix pay as you go and funding techniques
- Balance of state and private pensions
- This mix can be done even inside the social security schemes (Sweden)
- Risk management approach in finance , in insurance … and … in pension : integration of risks in the decision process

- **<u>Purpose</u>** : theoretical justification of the diversification between PAYG and funding using portfolio theory arguments and choice of an optimal mix

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Outline

- 1. Introduction
- 2. Static Model
- 3. Pension as a Portfolio Problem
- 4. Binomial Model
- 5. Log normal Models

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1. Introduction

2 basic techniques in order to finance pension liabilities

PAY AS YOU GO	FUNDING
↓	↓
Pensions for retirees are paid by active people	Active people finance their own pension
Unfunded schemes	*Funded schemes*

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1. Introduction

	1°Pil. DB	1° Pil. DC	2°Pil. DB	2° Pil. DC
PAYG				
Funding				

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2. Static Model

Samuelson classical choice between pay as you go and funding :

Optimal macro economic choice between the 2 techniques

→ In a static environment , classical condition on the demographic and financial parameters

Illustration of this condition in a simple Overlapping Generation Model

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2. Static Model

The Overlapping Generation Model (OLG Model):

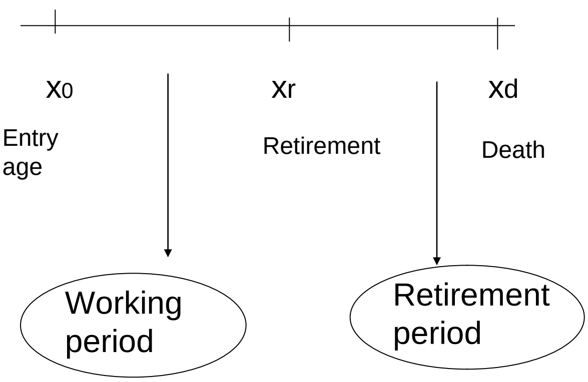
Stylization tool in order to capture the ***dynamic evolution*** of population in time with a focus on equilibrium between active people and retirees.

OLG Assumptions:

- Agents have finite lives
- They live in two periods :
  - they are "young" , then "old" , then dead
  - when one generation becomes old, another young generation is born .

2. Static Model

Model with 2 periods :



2. Static Model

Notations :

$$L(x,t) = \text{number of people aged } x \text{ at time } t$$
$$\pi = \text{contrib. rate on salary (DC plan)}$$
$$i = \text{financial rate of return}$$
$$s = \text{rate of increase of salary}$$
$$S(t) = \text{mean salary at time } t$$
$$P(t) = \text{mean pension at time } t$$
$$d = \text{demographic rate of increase}$$
$$p_{x_0} = \text{survival probability between } x_0 \text{ and } x_r$$

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2.Static Model

Demographic evolution :

Retired and active population at time t :

$$\boxed{L(x_r,t) = L(x_0,t-1)\,p_{x_0} = (L(x_0,t)/(1+d))\,p_{x_0}}$$

Longevity risk

Demographic effect

Retired population

Active population

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2. Static Model

Comparison of the replacement rate in pay as you go and in funding :

$$RR(t) = \text{replacement rate}$$
$$= \frac{\text{first pension}}{\text{last salary}}$$
$$= \boxed{\frac{P(t)}{S(t-1)}}$$

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2. Static Model

Replacement rate in pay as you go :

Actuarial equivalence between contributions and benefits paid both at time t :

$$L(x_r, t)\, P(t) = L(x_0, t)\, \pi\, S(t)$$
$$\downarrow$$
$$\boxed{RR = \frac{\pi}{p_{x_0}}(1+d)(1+s)}$$

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2. Static Model

Replacement rate in funding :

Actuarial equivalence between present value of contributions and benefits for a fixed cohort:

$$L(x_r, t)P(t) = L(x_r - 1, t-1)\pi S(t-1)(1+i)$$

$$\boxed{RR = \frac{\pi}{p_{x_0}}(1+i)}$$

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2. Static Model

Replacement rate – diversification strategy:

a = proportion of the contribution invested in funding

1-a = proportion in payg
(with 0 < a < 1)

$$\boxed{RR(a) = \frac{\pi}{p_{x_0}}\{a(1+i) + (1-a)(1+s)(1+d)\}}$$

Same influence of longevity risk for payg and funding

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2. Static Model

Samuelson rule :

Pay as you go

$$RR = \frac{\pi}{p_{x_0}}(1+d)(1+s)$$

Funding

$$RR = \frac{\pi}{p_{x_0}}(1+i)$$

Conclusion :

if $(1+i) > (1+d)(1+s)$: 100% funding $(a=1)$

if $(1+i) < (1+d)(1+s)$: 100% pay as you go $(a=0)$

Diversification is never optimal… but no risks in this model !!!

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3. Portfolio

Classical Portfolio theory :

- Optimal choice between stocks and bonds depending on the risk aversion of the investor .
- Bonds and Stocks have different risk profiles

FINANCE	→	**PENSION**
Bonds		Pay as you go
Stocks		Funding

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3. Portfolio

Deterministic ⟶ **Stochastic**

a = proportion of the contribution invested in funding
(control variable)

$$RR(\omega) = \frac{\pi}{p_{x_0}}(a(1+i(\omega)) + (1-a)(1+s(\omega))(1+d(\omega)))$$

$$= \frac{\pi}{p_{x_0}} X(\omega)$$

Assumption :
p = deterministic
(no longevity risk)

General distribution
with dependency structure between:
- *financial risk (i)*
- *demographic risk (d)*
- *inflation risk (s)*

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3. Portfolio

Basic Random Variable :

$\boxed{X = (1-a)\, D.S + a I}$ =return of the mixed strategy

With : $D = 1 + d$; $S = 1 + s$; $I = 1 + i$

(3 positive random variables)

Dependency assumption:
- S and D independent (*salary and demography*)
- S and I dependent (*salary and returns*)

(correlation between I and D is an interesting question….).

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3. Portfolio

Risk Management - Mean variance analysis :

Optimization of the mean replacement rate but taking into account the risk through the variance.

The decision problem can be written as :

$$\begin{aligned} &\min_a \ \text{Var } X \\ &E(X) = X_0 \end{aligned}$$

Utility framework : for a fixed $\gamma > 0$ (risk aversion) :

$$\max_a \ U(X) = \max_a (EX - \frac{\gamma}{2}.\text{Var } X)$$

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3. Portfolio

Mean variance analysis :

Computation of E(X) and Var X

Mean :

$$\begin{aligned} EX &= (1-a)ED.ES + aEI \\ &= a(EI - ED.ES) + ED.ES \end{aligned}$$

$\nearrow$ with **a** if : $EI > ED.ES$

$\searrow$ with **a** if : $EI < ED.ES$

Samuelson rule !!
(γ =0)

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3. Portfolio

Variance :

The variance as a function of **a** is a quadratic form :

$$\boxed{\mathrm{Var}\ X = a^2(A + B - 2C) + 2a(C - A) + A}$$

With :

$A = \mathrm{Var}\ (D.S)$

$B = \mathrm{Var}\ (I)$

$C = \mathrm{cov}\ (D.S;I)$

$F = A + B - 2C = \mathrm{Var}\ (I - D.S)\ > 0$

Convex
with minimum

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3. Portfolio

Minimum variance :

$$\boxed{a_{min} = \frac{A - C}{A + B - 2C} = \frac{\mathrm{Var}\ (D.S) - \mathrm{cov}\ (D.S,I)}{\mathrm{Var}\ (I - D.S)}}$$

Short selling impossible in this problem.
Attainable minimum if :

$$\boxed{0 \leq a_{min} \leq 1}$$

… not so sure…..!

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3. Portfolio

Minimum variance :

Particular cases :

CASE 1 : *no correlation between D.S and I* :

GDP *Return on asset*

$$a_{min} = \frac{A}{A+B} = \frac{Var(D.S)}{Var(D.S) + Var(I)} < 1$$

→ *Attainable minimum*

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3. Portfolio

Minimum variance :

Particular cases :

CASE 2 : negative correlation between D.S and I :

$$a_{min} = \frac{A-C}{A+B-2C} = \frac{Var(D.S) + |\ cov(D.S,I)\ |}{Var(D.S) + Var(I) + 2|\ cov(D.S,I)\ |}$$

→ *Also attainable*

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3. Portfolio

Minimum variance :

Particular cases :

CASE 3 : positive correlation between D.S and I
(?? Normal economical situation ?)

$$a_{min} = \frac{A - C}{A + B - 2C} = \frac{Var(D.S) - |\ cov(D.S,I)\ |}{Var(D.S) + Var(I) - 2|\ cov(D.S,I)\ |}$$

→ *Could be negative !!!!*

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3. Portfolio

Optimal choice based on utility function :

$$U(X) = EX - \frac{\gamma}{2}.Var\,X$$

$$= -\frac{\gamma}{2}F.\,a^2 + a((EI) - (ED)(ES) - \gamma.(C - A)) + (ED).(ES) - \frac{\gamma}{2}.A$$

$$= \boxed{-\alpha.a^2 + \beta.a + \delta}$$

< 0 → Concave with a unique max !!! → Theoretical Solution : OK

… but … 0 < a < 1 ??? → Practical Solution: ???

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3. Portfolio

Theoretical optimal diversification level :

$$a_{OPT} = \frac{Var(D.S) - cov(D.S,I)}{Var(I - D.S)} + \frac{1}{\gamma}.\frac{E(I - D.S)}{Var(I - D.S)}$$

$$\boxed{= a_{min} + \frac{1}{\gamma}.\Delta} \qquad (\gamma > 0)$$

First particular case :

if E I= ED.ES (same mean return for funding and payg) :

$$a_{OPT} = a_{min}$$

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3.Portfolio

Practical optimal diversification level :

Additional natural constraint : $\boxed{0 \leq a_{OPT} \leq 1}$

Different situations depending on a min :

Case 1 : $\boxed{0 < a_{min} < 1}$

$EI > E(D.S)$ → $a_{opt} \in [a_{min}, 1]$

Funding optimal ..but very risky
Other possible mixed strategies

$EI < E(D.S)$ → $a_{opt} \in [0, a_{min}]$

Pay as you go optimal ..but very risky
Other possible mixed strategies

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3. Portfolio

Case 2 : $a_{min} \leq 0$

$EI > E(D.S)$ → $a_{opt} \in]0,1[$

Every strategy can be chosen !!

$EI < E(D.S)$ → $a_{opt} = 0$

No diversification
Only pay as you go

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3.Portfolio

Case 3 : $a_{min} \geq 1$

$EI > E(D.S)$ → $a_{opt} = 1$

No diversification
Only funding

$EI < E(D.S)$ → $a_{opt} \in]0,1[$

Every strategy can be chosen!!

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4. Binomial Model

Numerical illustration :

Binomial model with complete independence ; 8 scenarios

	scenario 1	scenario 2	prob sc 1	prob sc2	Mean
d	0%	2%	0,5	0,5	0,010
s	2%	3%	0,5	0,5	0,025
i	4%	6%	0,5	0,5	0,050

Samuelson rule on mean values :

$$(1.01).(1.025) < 1.05$$

? Funding at 100% optimal ?? …..and the risk ???

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4. Binomial Model

Numerical illustration :

Mean variance analysis :

a	E[X]	Var [X]	
0	1,035	0,000131	→ Payg
0,1	1,037	0,000107	
0,2	1,038	0,000088	
0,3	1,040	0,000073	
0,4	1,041	0,000063	
0,5	1,043	0,000058	
0,6	1,044	0,000057	→ Min var
0,7	1,046	0,000061	
0,8	1,047	0,000069	
0,9	1,049	0,000082	
1	1,050	0,000100	→ Funding

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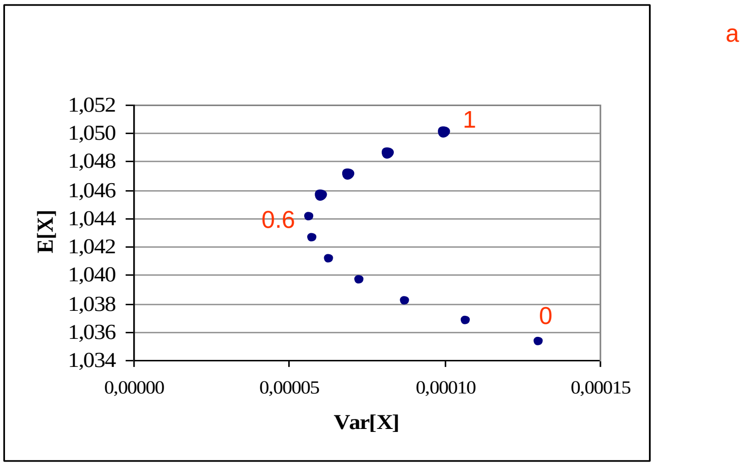
4. Binomial Model

Numerical illustration :



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5. Log normal Models

EXAMPLE : correlated log normal model :

$$D = e^{X} = e^{N(\rho,\sigma_d^2)}$$

$$S = e^{Y} = e^{N(\mu,\sigma_s^2)}$$

$$I = e^{Z} = e^{N(\delta,\sigma_I^2)}$$

With : - X independent of Y and Z
- Y and Z correlated :

$$\mathrm{corr}(Y,Z) = \eta$$

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5. Log normal Models

- **Optimal mix between funding and PAYG :**

$$a_{OPT} = \frac{Var\,(D.S) - cov(D.S,I)}{Var\,(I - D.S)} + \frac{1}{\gamma} . \frac{E(I - D.S)}{Var\,(I - D.S)}$$

$$= \frac{Var(D.S) - cov(D.S,I) + (EI - ED.ES)/\gamma}{Var\ I + var(D.S) - 2\,cov(D.S,I)}$$

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5. Log normal Models

- Moments of multivariate lognormal distributions :

$$EI = e^{\delta+\sigma_I^2/2}$$

$$var\,I = e^{2\delta+\sigma_I^2}(e^{\sigma_I^2} - 1)$$

$$var\ (D.S) = e^{2(\mu+\rho)+\sigma_d^2+\sigma_s^2}(e^{\sigma_d^2+\sigma_s^2} - 1)$$

$$cov(D.S,I) = e^{(\mu+\rho+\delta+(\sigma_d^2+\sigma_s^2+\sigma_I^2)/2)}(e^{\eta\sigma_I\sigma_s} - 1)$$

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5. Log normal Models

- **Minimum and optimal mix :**

$$a_{min} = \frac{e^{2(\mu+\rho)+\sigma_d^2+\sigma_s^2}(e^{\sigma_d^2+\sigma_s^2}-1) - e^{\mu+\rho+\delta+(\sigma_d^2+\sigma_l^2+\sigma_s^2)/2}(e^{\eta\sigma_l\sigma_s}-1)}{b}$$

$$\text{with } \; b = e^{2\delta+\sigma_l^2}(e^{\sigma_l^2}-1) + e^{2(\mu+\rho)+\sigma_d^2+\sigma_s^2}(e^{\sigma_d^2+\sigma_l^2}-1) - 2e^{\mu+\rho+\delta+(\sigma_d^2+\sigma_l^2+\sigma_s^2)/2}(e^{\eta\sigma_l\sigma_s}-1)$$

$$a_{opt} = a_{min} + \frac{1}{\gamma b}(e^{\delta+\sigma_l^2/2} - e^{\rho+\sigma_d^2/2}.e^{\mu+\sigma_s^2/2})$$

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Future research

1. Multi period model

2. Realistic distributions for the various risks and calibration ; problem of correlation

3. Funding with several assets

4. Value at risk approach

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THANK YOU

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