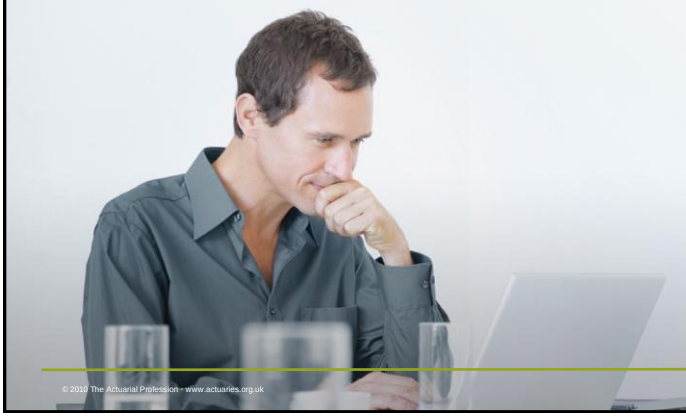
The Actuarial Profession
making financial sense of the future

How to Calibrate Risk Distributions

Andrew D Smith (andrewdsmith8@deloitte.co.uk)



2 December 2011

Agenda

Modelling Extreme Events

Standard Formula, Internal Models and Scope of Validation		
Example IM Calibrations	Interest Rates	Equities
"Living memory" Test	Fit to Overlapping One-year Changes	
Fit to Past Data	Histogram, P-P plot, moments, KS Test	
Stability / Contra-Cyclicality	Rolling estimates, Through-Cycle Methodology	
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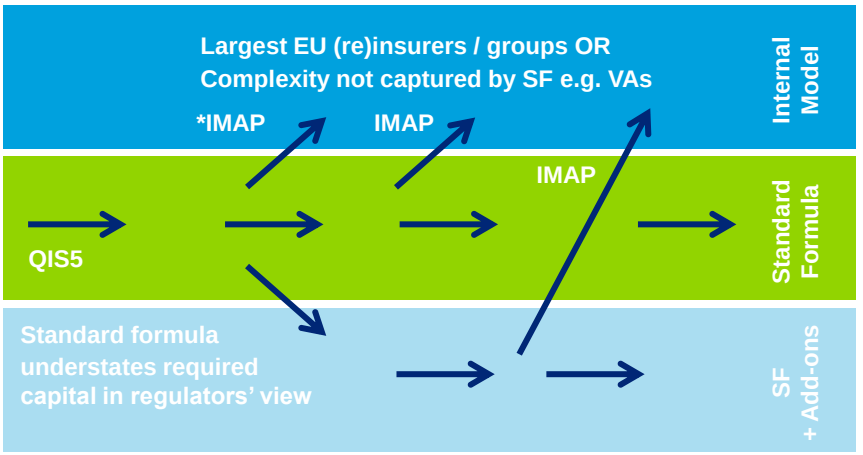
Practical challenges, Conclusions and Questions		

SCR Calculation: Standard Formula vs Internal Model

How firms and the regulator calculate the Solvency Capital Requirement



*IMAP = internal model application process

Internal Model validation

Regulatory requirements & purpose

Insurers seeking approval to use an internal model to calculate their capital requirements will have to demonstrate, as part of their IMAP, that they have had their internal model "independently validated" (Article 112 and Article 124).

Regulatory requirement

The specific "validation standards" requirements are outlined in Article 124 of the Directive and in the detailed text that supports the Level 2 and emerging Level 3 texts. The requirements include:

- having a regular cycle of model validation;
- monitoring the performance of the internal model;
- reviewing the on-going appropriateness of its specification;
- testing its results against experience;
- analysing the stability of the internal model;
- reviewing the sensitivity of the results to changes in key underlying assumptions;
- demonstrating the model's use – "use test"; and
- assessing the accuracy, completeness and appropriateness of the data used.

Modelling Extreme Events

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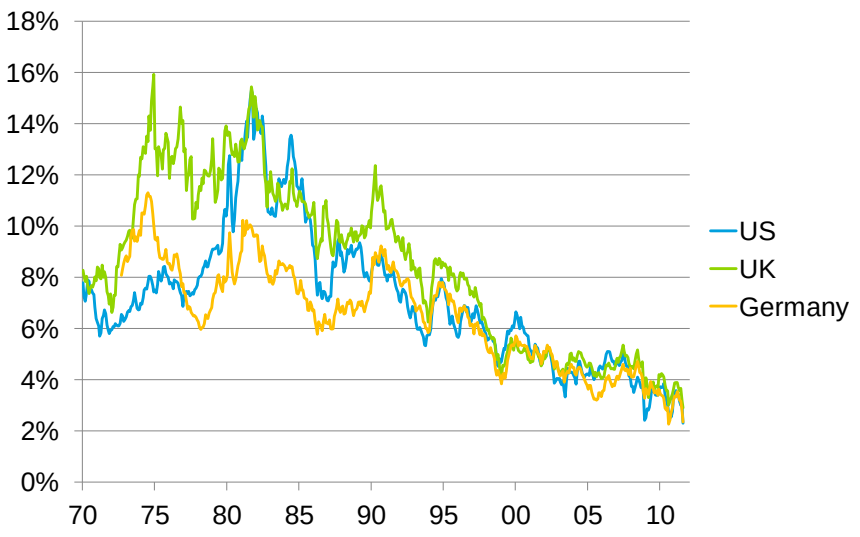
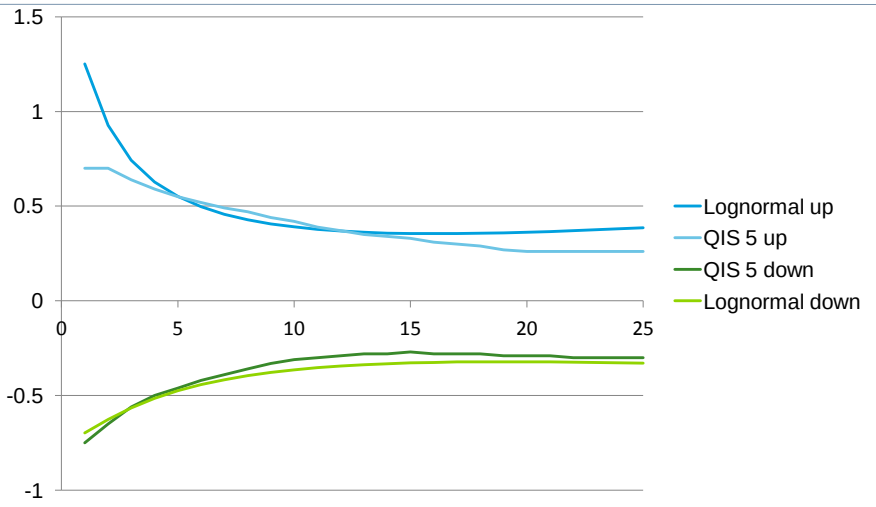
10-Year Spot Rates by Term, 1970-2011

Source: Federal Reserve, Bank of England, Bundesbank



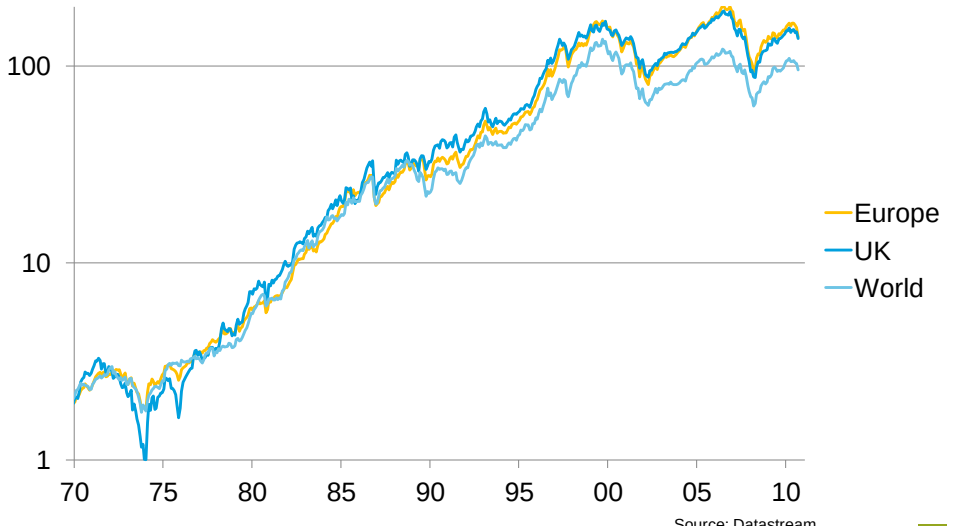
Predictive Distribution for Δ Bund Spot Curve

Comparison of Lognormal Model with QIS 5



Historic Equity Returns

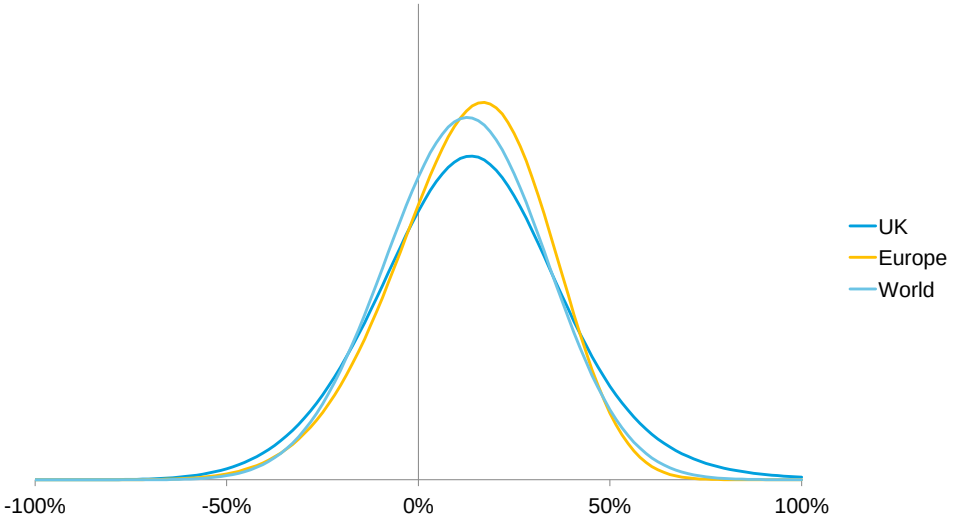
Total return MSCI in Euro (reconstructed basket prior to 1999)



Source: Datastream

Fitted Return Distributions

Based on EGB2 distribution family



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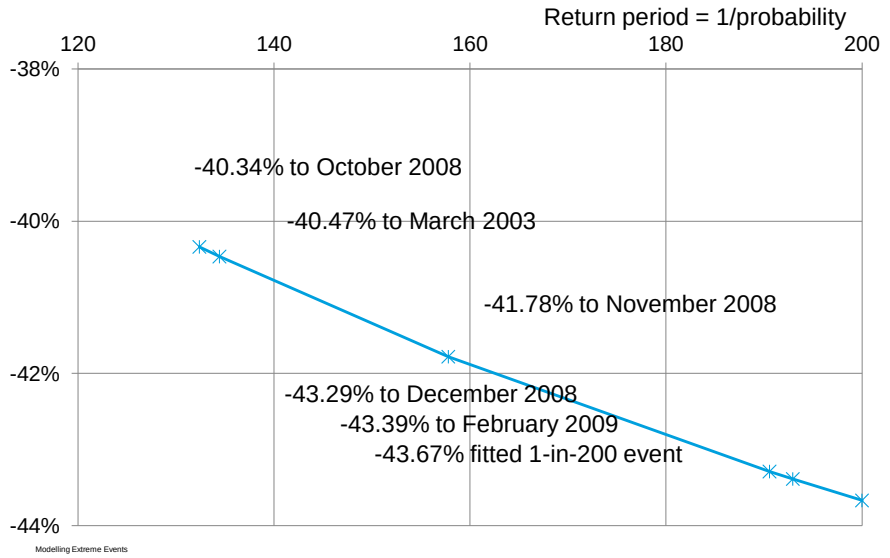
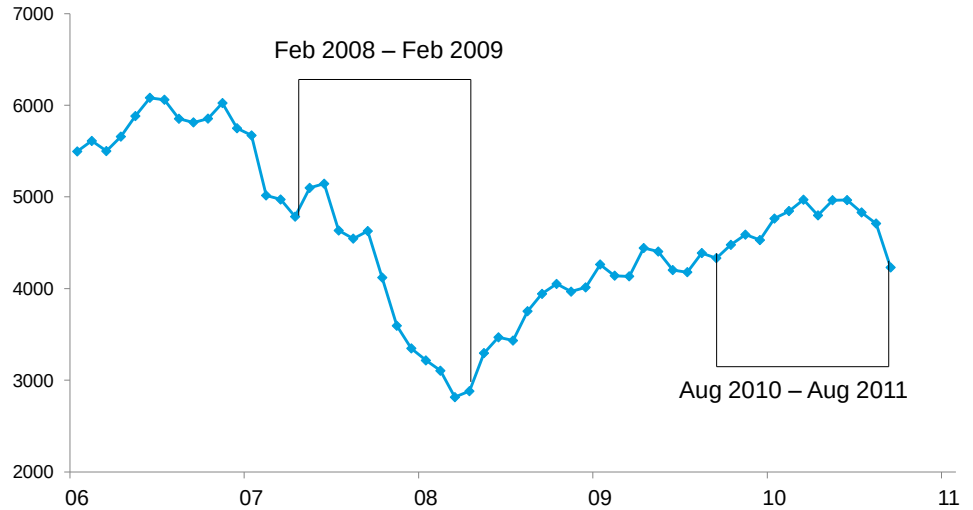
Extreme Histories: The Living Memory Test

Largest Falls in MSCI Europe € Index since 1970



Fitting to Overlapping Intervals

Monthly time series analysis may fail the "Living Memory" test



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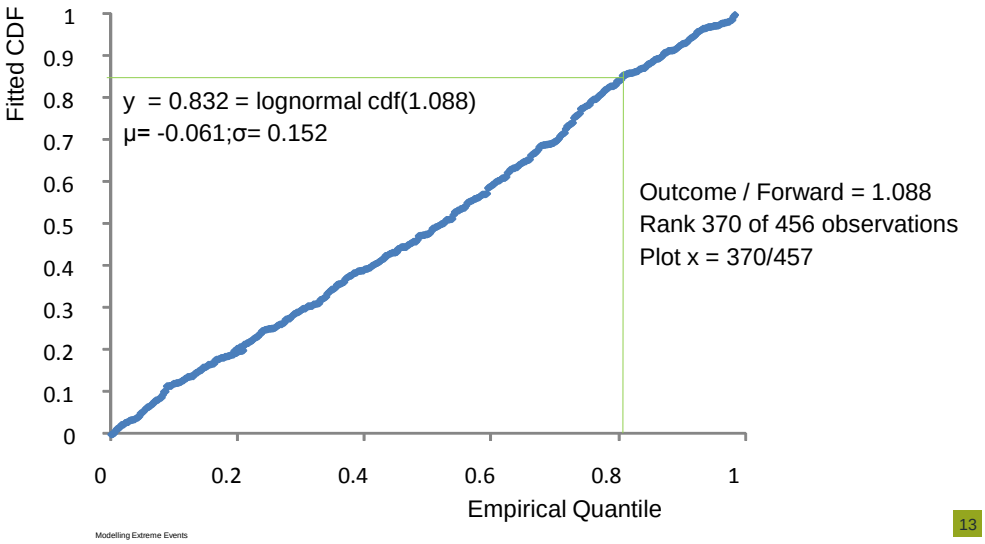
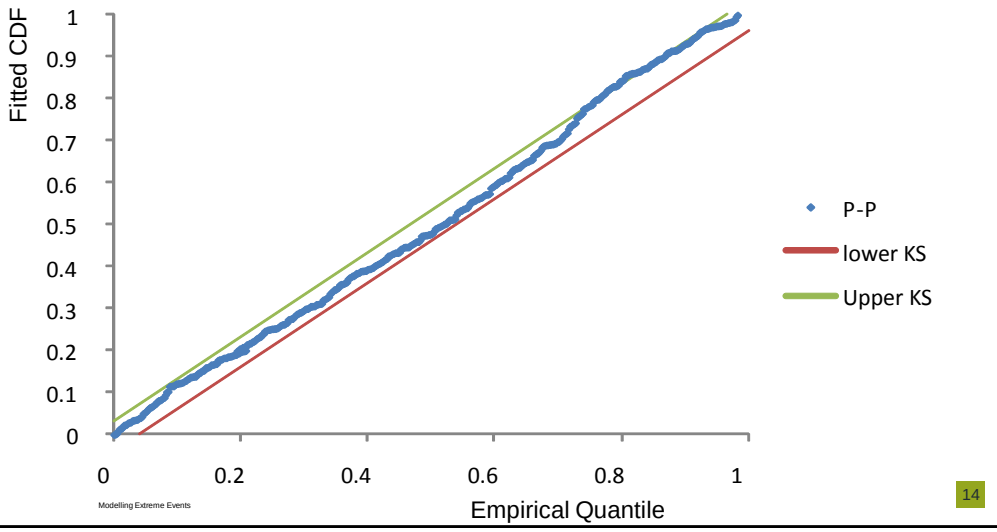
P-P Plot (10 year interest rate)

Visualising Goodness of Fit



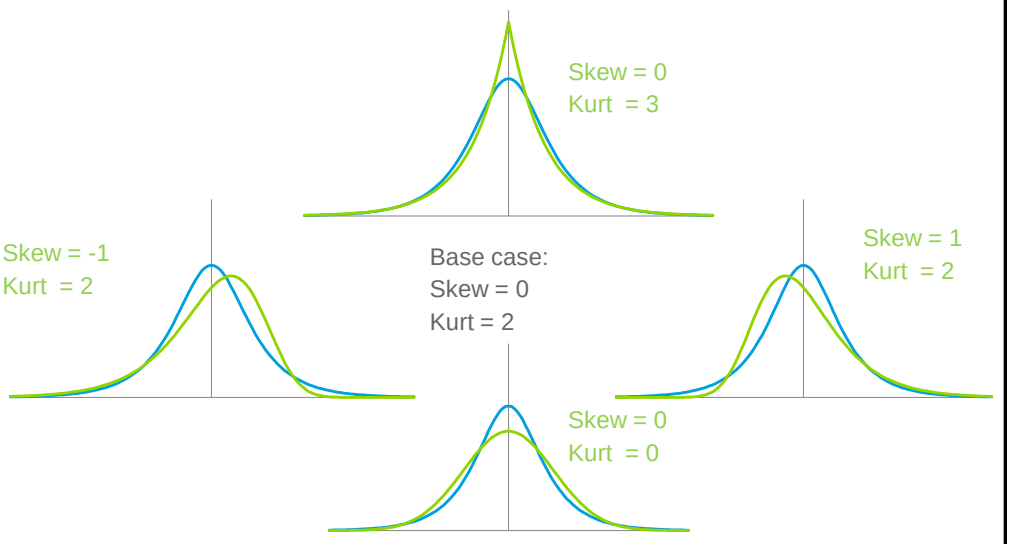
Kolmogorov-Smirnov Goodness of Fit Test

Look at largest deviation in P-P Plot



Skew & Kurtosis to fit Equity Returns

All distributions have mean = 0; standard deviation = 1



References

Some textbooks to read in your spare time!

- Continuous Univariate Distributions by Johnson, Kotz and Balakrishnan (JKB), published by Wiley in two volumes (Norman Johnson was an actuary).
- Volume 1 of Kendall Advanced Theory of Statistics (revised by Stuart & Ord).
- Quantitative Risk Management by McNeil, Frey and Embrechts (Princeton).

- Pearson Type IV (covered in JKB, vol 1 p15 et seq and in Kendall p221). Also look at http://www-cdf.fnal.gov/physics/statistics/notes/cdf6820_pearson4.pdf
- Johnson's SU distributions. Covered in JKB vol 1 p33 and Kendall p240.
- EGB2 distributions. Covered in JKB, vol 2 p141
- Generalised hyperbolic distributions. Treated by McNeil et al p78. See also http://cran.r-project.org/web/packages/ghyp/vignettes/Generalized_Hyperbolic_Distribution.pdf
- MULE distributions. You won't find these in the literature because they are my invention. Unlike the other classes, the MULE (mixed exponential uniform logistic) permits distributions with negative kurtosis, with uniform logistic and exponential distributions as special cases. The inverse CDF is a linear combination of {1,x, ln(x), ln(1-x)}.

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Pro and Contra-Cyclical Tests

Arguments to avoid pro-cyclicality

- Unconditional estimates = "Through the cycle"
  - Average over states of the world
  - Estimate through historic distributions, as in this presentation
  - Can satisfy "1-in-200" test
  - Capital requirements can increase following large market moves

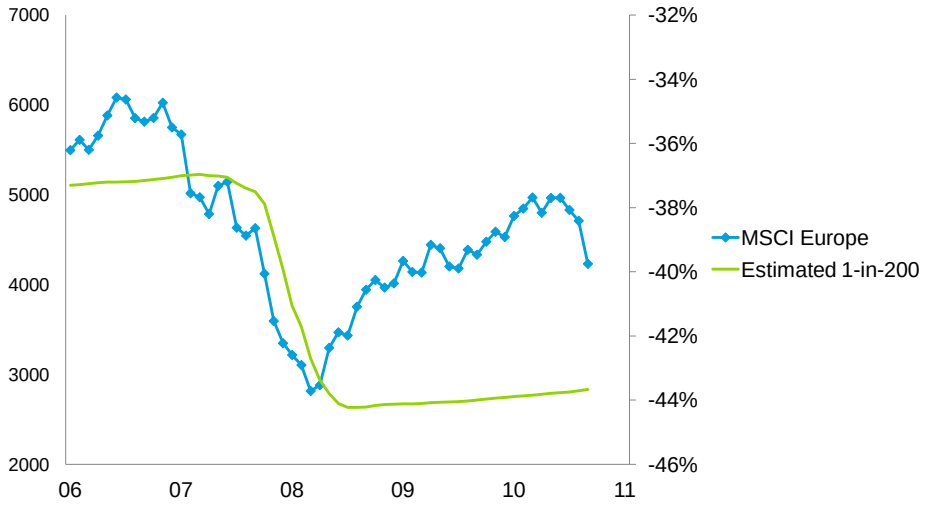
- Conditional estimate = "Point in Time"
  - Given current state of the world
  - Empirical validation by comparing to historic periods with the same starting point
  - Can satisfy "1-in-200" test
  - Sensitive to time series model formulation
  - Capital requirements may rise suddenly from small market moves, making this approach commercially unattractive
  - Arguments against this approach based on fear of "pro-cyclicality"

Fitting Methods: In Search of Stability

Method	Feature Replication	Fit Optimisation
Examples	Method of moments Modal fit	Maximum likelihood Minimax cdf difference (minimise Kolmogorov-Smirnov)
Pros	Can prove it has worked	Most powerful for large n Parameter standard error known for large n
Cons	Need Plan B outside feasible set	Solution may not exist May not converge Difficult to demonstrate method has worked

Rolling Estimates

Effect of 2008 Crisis on Estimated Stress Tests (fitted to data since 1970)



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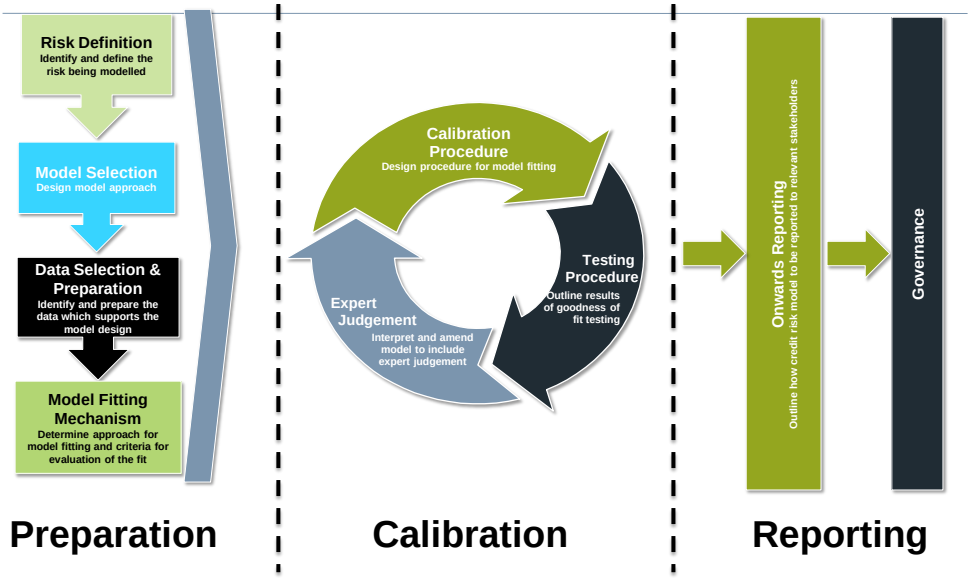
Integrated Modelling Framework



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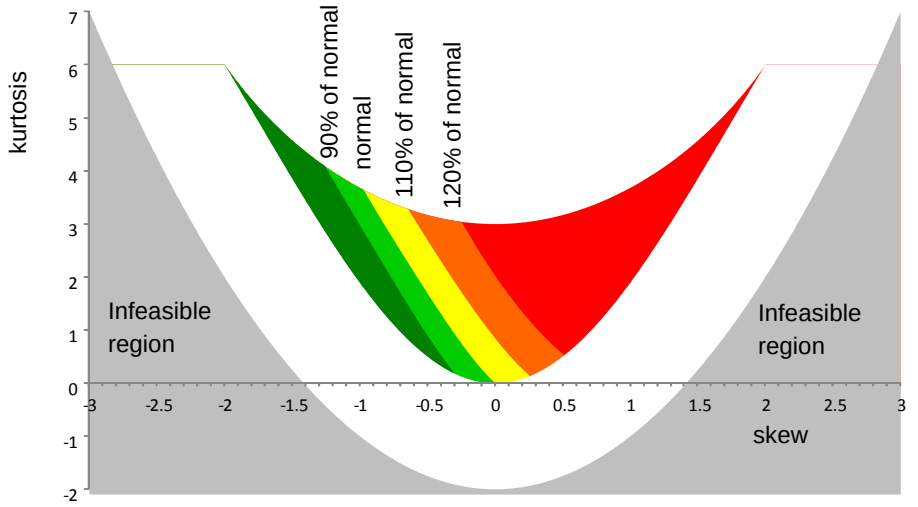
Effect of Skewness and Kurtosis on EGB2

Contours of 99.5%-ile expressed as number of standard deviations

To consider a 0.5%-ile (eg equity fall), reflect this diagram left to right.



Example Equity Stress Test Calculation

Fitted Gamma Power Distribution to MSCI Europe Returns

Parameter	Fitted Parameters
α	1.346608
β	1.999722
power	0.185368

Quantile q	Gamma inverse G = gammainv(q,α,β,true)	Return $R = G^{power}-1$
0.5%	0.045219	-43.67%
10.0%	0.455908	-13.55%
50.0%	2.063342	14.37%
90.0%	5.761522	38.35%
99.5%	12.17842	58.94%

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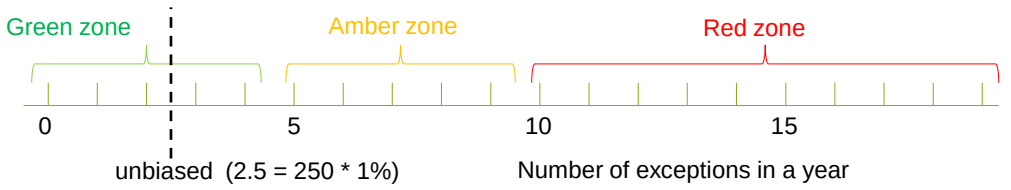
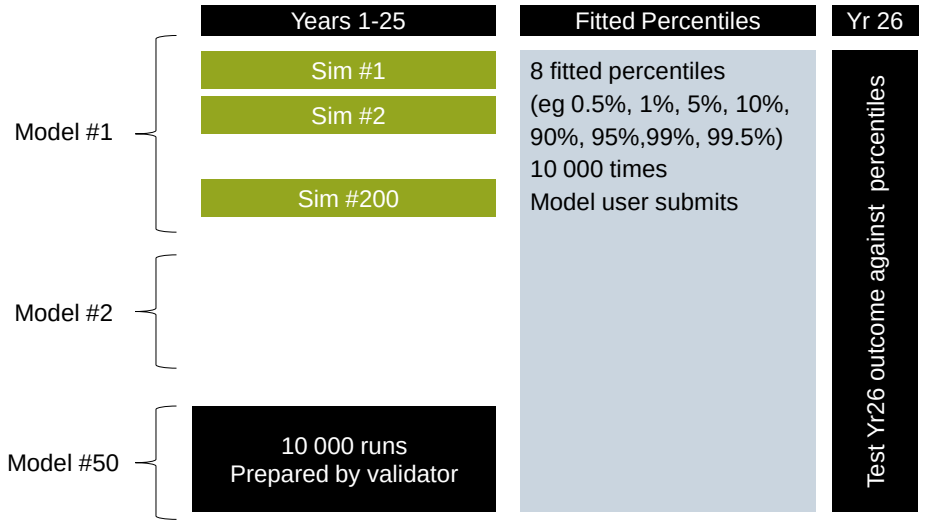
Validation under Basel - Banks

- Banks have different rules:10 day VaR at 99% Confidence
  - Look back over last year (250 trading days, overlapping periods each looking 10 days back) in which both VaR and profit are updated



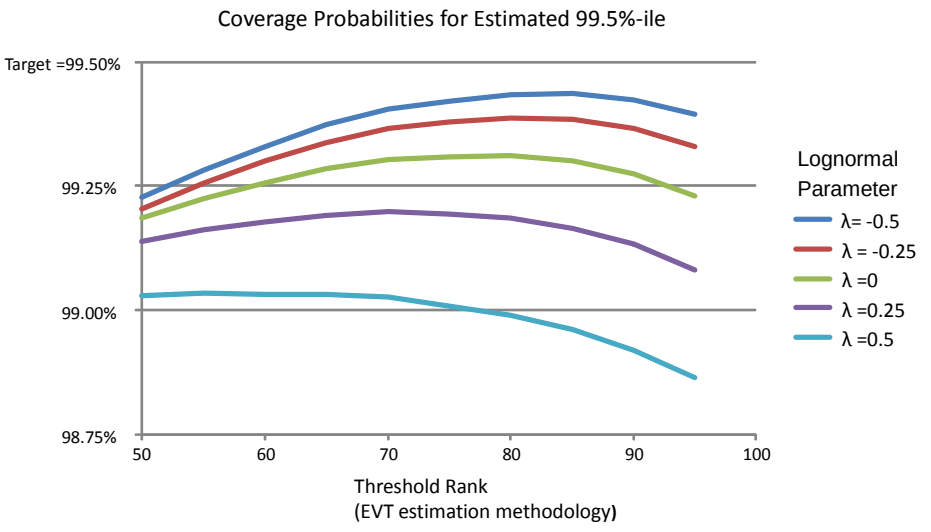
- What does this process test?
  - The "back test" includes implicit tests of model and parameter error as well as outcomes
  - Although it won't test risks that didn't materialise in the last year

Monte Carlo Calibration Test



Probability {next observation ≤ estimated percentile}

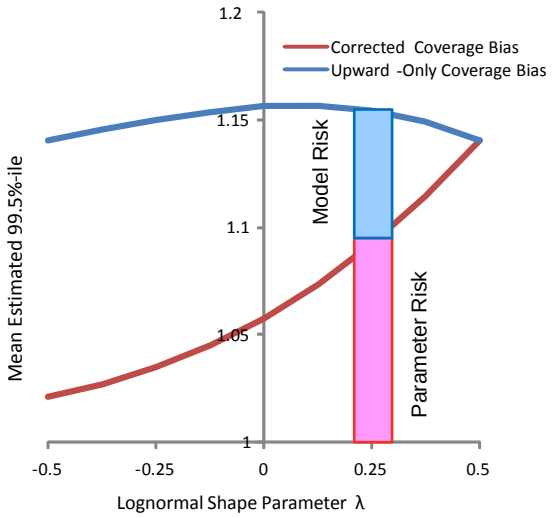
Coverage probability aka back-test probability



Modelling Extreme Events

Using Simulations for Model and Parameter Risk

Adjusting estimated 99.5%-ile for 99.5% coverage



The theoretical coverage bias correction depends on the underlying distribution which is (sadly) unknown.
We can quantify model risk using a robust estimate based on a bias correction for the "worst case" distribution.
This then overstates required capital (coverage probability > 99.5%) for other distributions.

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Internal Model validation
Practical challenges

Timing	• The Solvency II Directive requires an insurer's internal model to be independently validated at regular intervals once Solvency II is fully implemented. However, validation is also important before Solvency II implementation:
	  • Integrating validation modules into the process of developing, building and testing the model provides greater confidence in the model and reduces the risk that late stage validation identifies major re-working of the model.
	  • A complete independent validation must be provided to the Board as part of the evidence to support their approval of the model before it is submitted for review by the CBI.

Board involvement	• The validation policy and report will be used by the Board when reporting to the regulator. The validation report will need to be accessible to all members of the Board, taking into account their varying experience and familiarity with Solvency II. • The validation report should addresses the scope of the validation, the strengths and weaknesses of the model and the data and tools used in the validation process.
Documentation	• Detailed and complete validation documentation will help facilitate internal model approval. • Validation documentation should address model theory, model implementation and model governance.
Risk assessment	• "Expert judgement" and "data" are likely to be high risk areas given the subjectivity and regulatory scrutiny respectively around these inputs.

Internal Model validation
Practical challenges

Independence	• An effective internal model validation process requires independent and objective challenge. The use test should provide evidence that the model has been challenged.
	• Independence is a strict requirement but can be achieved in a number of ways, or through a combination of:
	  • Existing resources
	  • An internal audit team with specialist skills
	  • External resources / auditors
	• Care should be taken to ensure that model validators are independent from those who have been involved in designing and building the model . Reporting lines should also be independent.

Questions or comments?

Expressions of individual views by members of The Actuarial Profession and its staff are encouraged.

The views expressed in this presentation are those of the presenter.