The Actuarial Profession
making financial sense of the future

Statistical quality standards? Correlations? Help!
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Agenda

- Background
- Models
- Results

Background

Solvency II requirements

The current approach

The view from my ivory tower

Why we don't parameterise dependencies

Solvency II requirements

Statistical quality standards in the Solvency II directive

Article 121

Statistical quality standards

1. The methods used to calculate the probability distribution forecast shall be based on adequate, applicable and relevant actuarial and statistical techniques and shall be consistent with the methods used to calculate technical provisions.

   The methods used to calculate the probability distribution forecast shall be based upon current and credible information and realistic assumptions.

   Insurance and reinsurance undertakings shall be able to justify the assumptions underlying their internal model to the supervisory authorities.

5. As regards diversification effects, insurance and reinsurance undertakings may take account in their internal model of dependencies within and across risk categories, provided that supervisory authorities are satisfied that the system used for measuring those diversification effects is adequate.

Source: European Commission Solvency II Level 1 Directive

Statistical quality standards
The blue text – diversification effects

Supervisory authorities shall be satisfied that the system for measuring and recognising diversification effects is adequate if, as a minimum, the undertaking:

- identifies the key variables driving dependencies;
- provides support for the existence of diversification effects;
- justifies the assumptions underlying the modelling of dependencies;
- takes into particular consideration extreme scenarios and tail dependence;
- tests the robustness of this system on a regular basis, e.g. as part of the model validation process;
- takes diversification effects actively into account in business decisions.

Source: CEIOPS Level 2 Advice on Tests and Standards for Internal Model Approval (former CP 56)

Statistical quality standards
The white text

CEIOPS is aware that in implementing an aggregation mechanism for an internal model undertakings face a number of challenges:

a) Dependencies are very hard to estimate and validate
Dependencies are harder to estimate or calibrate than marginal distributions (or the quantification of individual risks). In many cases, there may be no conclusive evidence regarding the theoretically correct dependency or aggregation mechanism. The required parameters may be based on expert judgement which will require extra efforts in the validation approach.

b) In addition, aggregation mechanisms can be inherently sensitive to parameter changes. Seemingly small changes in parameterisation may result in large changes in overall capital.

c) Methods to account for dependency are not necessarily stationary across confidence levels, i.e. dependency measured at a central point may become inoperative at the confidence level required for capital calculations.

Combining the points above, CEIOPS concludes that modelling of dependencies and the aggregation mechanism requires special attention by the supervisory authority.

Source: CEIOPS Level 2 Advice on Tests and Standards for Internal Model Approval (former CP 56)

Statistical quality standards
The blue text

Expert judgment may be used to complement or substitute data. When data is available, expert judgement shall be reconciled with the data.

Where expert judgement as complement to or substitute for data has a material impact, its use must be well-founded and is admissible only if its derivation and usage follows a scientific method, i.e:

a) The expert judgement must be falsifiable, i.e. circumstances under which the expert judgement would be considered false can be clearly defined even though they may only be realised at a point in time far in the future.

b)The expert must be able to make transparent the uncertainty surrounding the judgement, e.g. by providing the context of the judgement, its scope, basis and limitations.

c)Standards concerning the operation of the methodology used must exist and be maintained.

d)The expert judgement must be documented. In particular, a track record of the expert judgements used must be available.

e)The expert judgement must be validated. Validation may include assessing the track record of expert judgements to assess reliability; challenging the expert judgement using scrutiny from other experts; comparing the expert judgement with existing and emerging data.

•Source: CEIOPS Level 2 Advice on Tests and Standards for Internal Model Approval (former CP 56)

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The current approach

The current approach

- The current approach to reviewing dependencies can best be called judgemental
- Correlations are set based on discussions with senior business experts, attempting to identify:
  - Any common exposures
  - Any historical incidents of joint behaviour (usually downside risk)
  - Any common risk drivers
- Following a discussion, a judgmental band (often low/medium/high) is set for all pairs, and for each level a judgmental correlation value is set
- This was often done when ICAs were first introduced (2002) and may not have been updated since.

Current approach: strengths and weaknesses

- The value of the current approach is in the discussions which feed it
  - Identification of risk drivers
  - Increased awareness of common (and hence significant to the firm) risk drivers
  - Drive awareness of common risk drivers through firm

- But there are significant weaknesses
  - All very judgemental
  - Not clear how broadly people are thinking through this process
  - Likely to focus only on extreme effects
  - Books of business change – how often is this updated
  - Large numbers of correlations – does focus slip?
  - Little help in identify dependency type

Benchmarks

- Of course, these subjective views are often backed up and validated with benchmarks
- In particular there are some published market studies, e.g. Aon Benfield's "Insurance Risk Study", (5th edition published last month)

- However use of benchmarks in risk modelling is fraught with difficulty
  - What allowance to make for differences between books?
  - How does the size of the book affect the correlation?
  - What correlation, specifically, is being referred to?

- These appear to predominantly focus on underwriting risk
  - This is often the most crucial driver…
  - but significant drivers of this correlation may be covered directly elsewhere within models (e.g. through underwriting cycle & cat models)

- We will focus predominantly on reserve risk (which is expected to be lower than underwriting risk)

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The view from my ivory tower

Dependency is all about understanding the drivers of risk

What might the effects be on the Profit & Loss Account and the Balance Sheet?

Oil Price Increase

- Increase in interest rates
- Change in discounted liabilities (all lines)
- Increased cost of shipping – increase in marine business interruption
- Change in Ogden values
- Decreased profits for cargo - Risk governance cut back?
- Recession - staff less happy - Increase in EL
- More profits for energy sector – increased supply – more chance for human error

An ivory tower approach to justifying the dependency assumptions

- The firm could:
  - Identify the risk drivers affecting each class
  - This needs to be at a highly granular level
    - not just attritional claims higher than expected
    - but attritional claims driver upwards by high unemployment leading to increased crime

    (This information should be known as part of the risk management anyway)
  - As well as identifying the risk drivers, their relative importance to the class of business' volatility should be estimated
  - Following this, a cross comparison can be performed between any two classes – common risk drivers identified and that risk drivers influence on each class observed
  - This can be combined over all risk drivers to provide a (qualitative) view of the dependency

- This is still judgemental, but is arguably easier to validate and more falsifiable, since each risk driver could, in theory, be observed, and its impact on each book could be measured
- Even better (ivory tower, remember) – firms could model each risk driver individually, and apply its impact on their books directly

Modelling key drivers

Advantages

- Linked to business influences
- Easier discussions with stakeholders
- Enhanced buy-in
- Complex interactions can be analysed
- Clarity of definitions

Limitations

- Data availability
- Statistical quality standards?
- Falsifiable?
- Do we need this detail?
- Increasing assumptions
- Need to as-if data
- Increasing number of parameters
- Increasing complexity of model
- Run-times
- Transparency
- Reconciliation to high level views
- Have we captured all drivers?

Why don’t we parameterise dependencies?

Back to the real world…

- It is unreasonable to expect that firms will be able to model all risk drivers with sufficient credibility to fully capture dependencies
- While it is best practice to strip out major risk drivers (e.g. cats, inflation) and model these separately, there will always be residual dependency
- While you may think that what is left is immaterial, this assumption should be validated

- The rest of this section discusses the main arguments we have heard used to justify not parameterising this residual dependency from data
  – that capital is very sensitive to the dependencies
  – that in order to produce credible results you need a lot of data
  – that there is no theoretical best basis to start with
  – And that dependency may not be stationary at different confidence levels

Argument 1:
Capital is sensitive to the dependencies

- This does not justify not trying to use historical data at all!
- Surely for a critical parameter we should be using all available methods, including both data led, benchmarks, and expert judgement.
- Expert judgement will remain fundamental, but it should be informed expert judgement
- Are we applying consistent stress tests?
  - Large claim CVs up 10%
  - Dependencies moved from 30% to 60%
- Are we over-stating the sensitivities due to a lack of understanding of acceptable range?

Argument 2:
Parameterising dependencies needs a lot of data

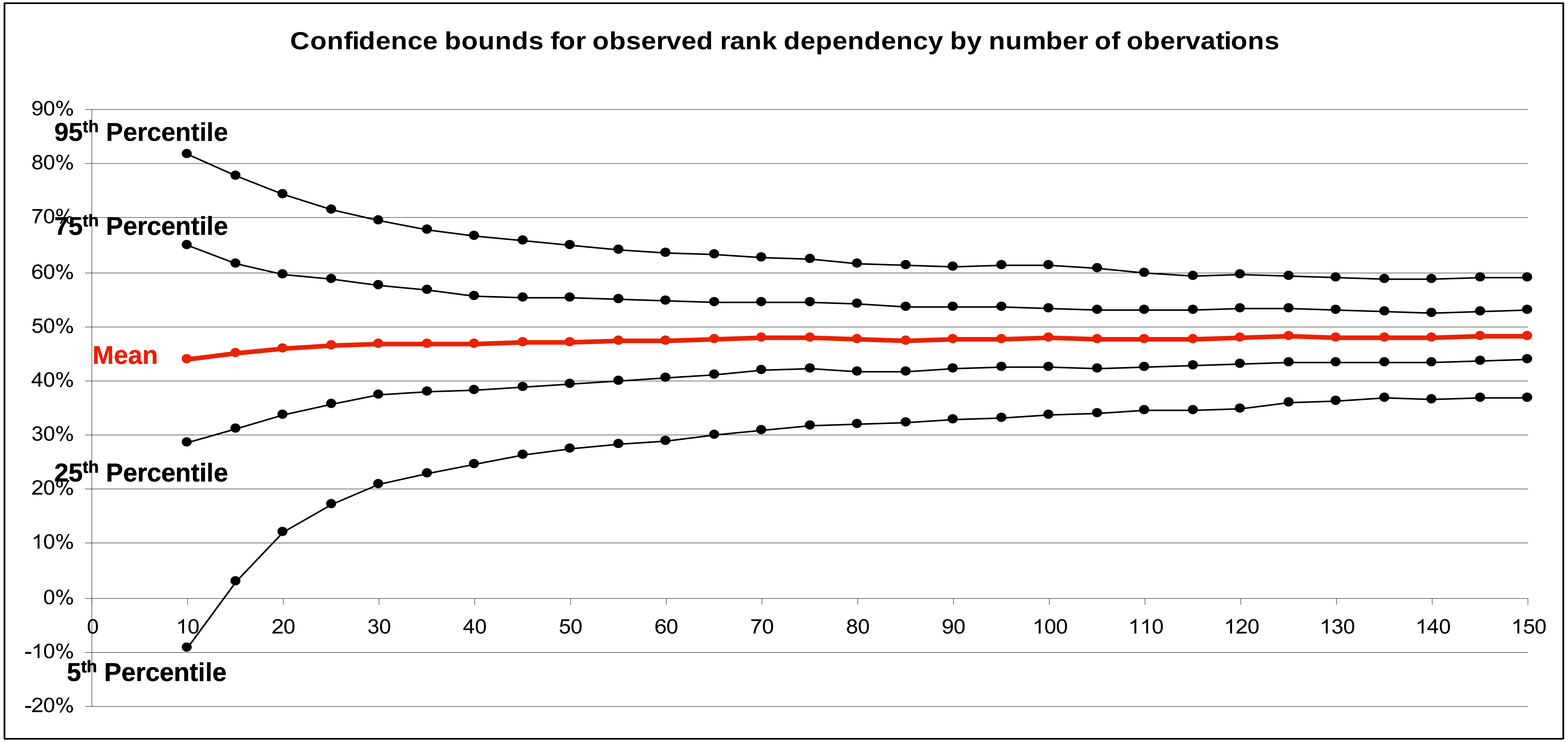
- To demonstrate this, we generated n aggregate claim amounts from a lognormal distribution (with mean 100, and CV 15%), for each of two classes.
- We applied a Gaussian (rank) dependency of 50% between each pair of data (there was no dependency between different draws for the same class)

- We then calculated the rank correlation of our simulations

- This was repeated for 5000 simulations

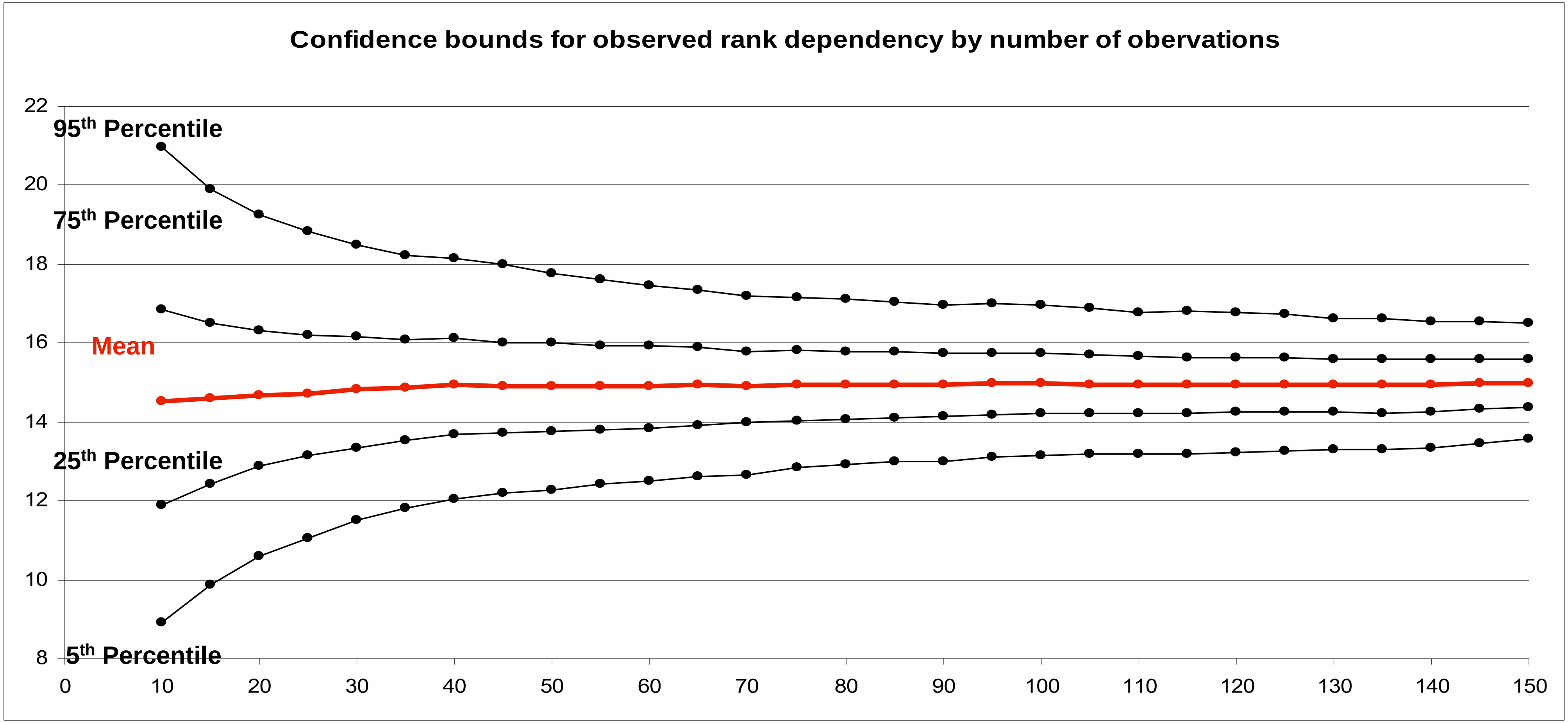
- We looked at the distribution of the parameterised rank correlation, as we increased the number of observations, n
- The graph has been slightly smoothed to minimise simulation error

How many data points do we need to fit a dependency?



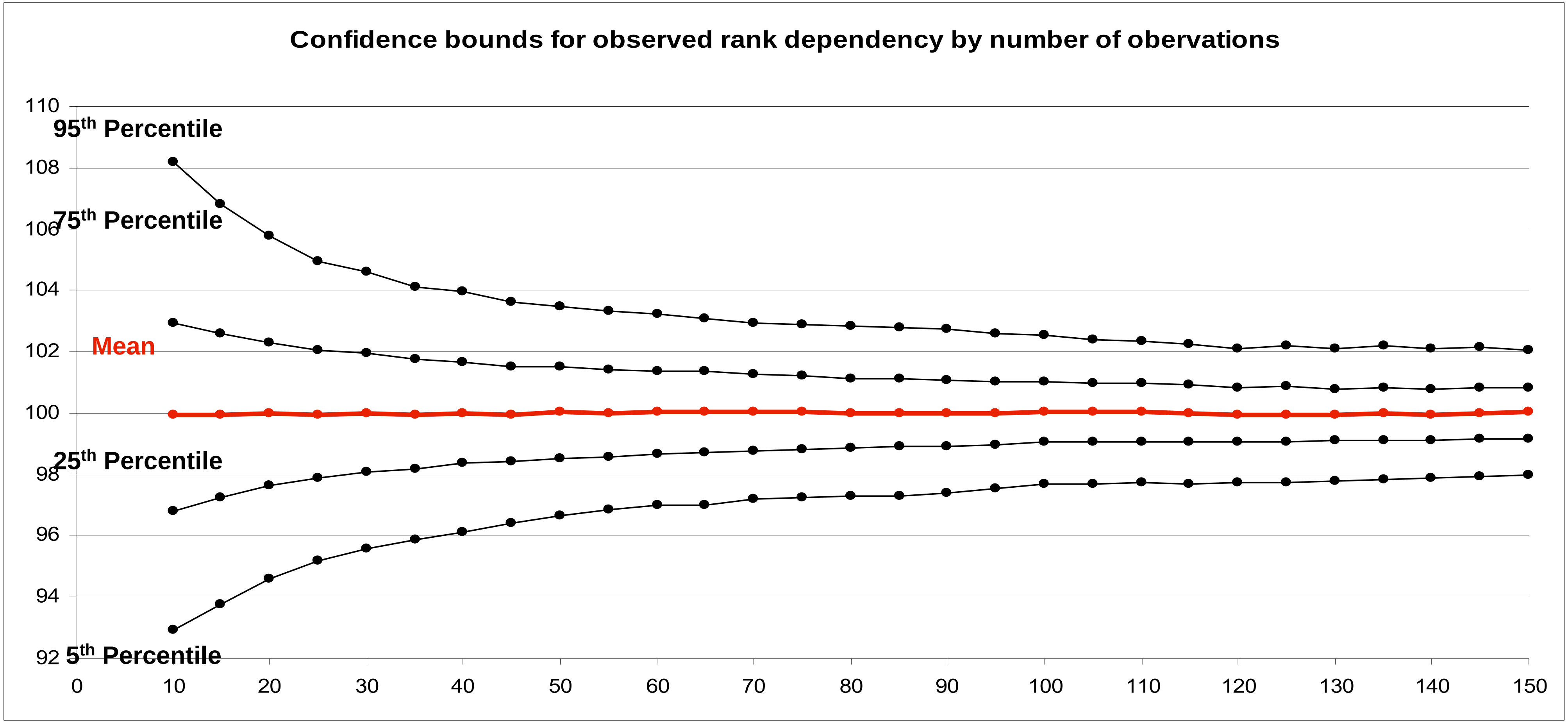
- The 90% confidence bound with 10 observations is +/- 100%
- The 90% confidence bound with 150 observations is +/- 20%

How many data points do we need to fit a standard deviation?



- The 90% confidence bound with 10 observations is +/- 40%
- The 90% confidence bound with 150 observations is +/- 10%

How many data points do we need to fit a mean?



- The 90% confidence bound with 10 observations is +/- 8%
- The 90% confidence bound with 150 observations is +/- 2%

Parameterising dependencies needs a lot of data

- It looks like much more data is needed to tie down dependencies (compared to the mean and the standard deviation)

Number of Observations	10	150
Dependency – 90% confidence bound	+/- 100%	+/- 20%
Standard deviation – 90% confidence bound	+/- 40%	+/- 10%
Mean – 90% confidence bound	+/- 8%	+/- 2%

- But why is this? - Parameterising a dependency is equivalent to parameterising the variance of the sum (once we have locked down each class' own distribution)
- Arguably, we are not interested in the standard deviation or the correlation, per se
- (We may well be interested in the mean as a stand-alone result though)
- They are a means to an end - capital. How influential are they on the 99.5th VaR

Impact of uncertainty – 10 observations

True 99.5th VaR	278

Mean	SD	Correlation	90% Confidence Interval
True	True	Observed	+/- 14
Observed	True	True	+/- 7
True	Observed	True	+/- 17

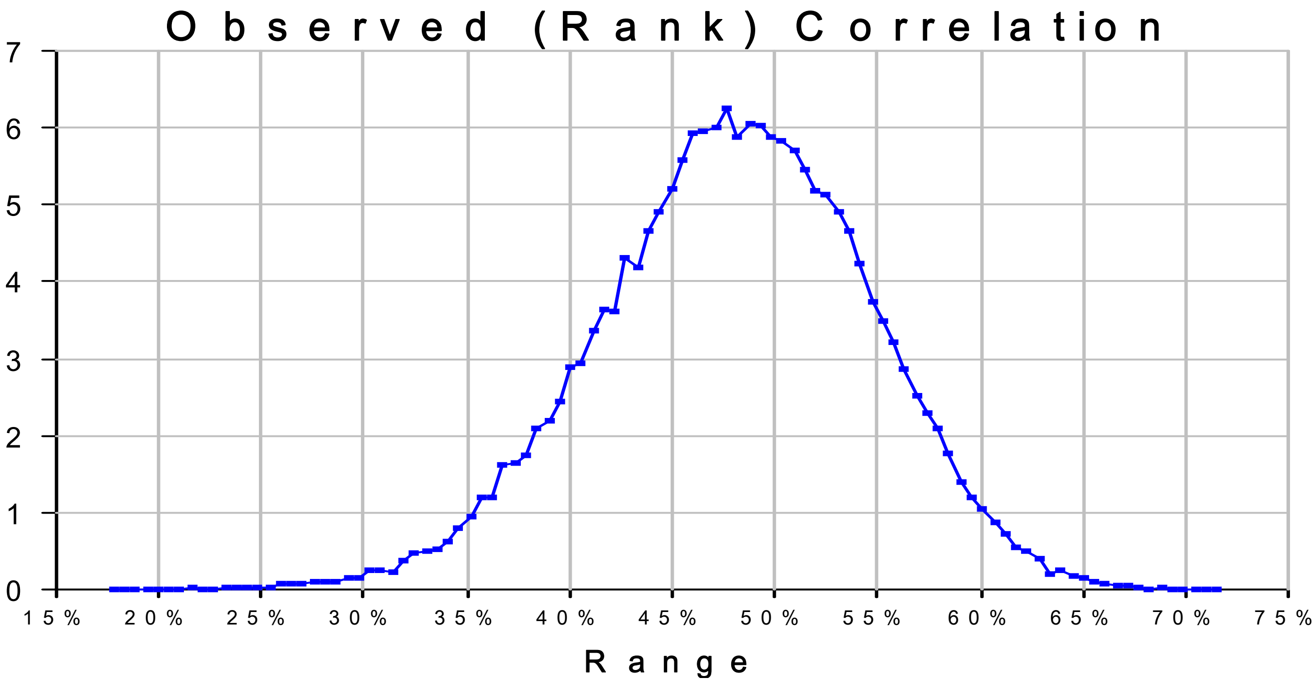
Impact of uncertainty – 150 observations

True 99.5th VaR	278		

Mean	SD	Correlation	Interval
True	True	Observed	+/- 3
Observed	True	True	+/- 2
True	Observed	True	+/- 4

Warning

- One other point we noticed in this simple test
- This is a distribution of the observed correlation (with 150 observations, and 50k sims)



- The result appears to be negatively skewed
- So beware! Observing low correlations may not mean that the correlations are not there?

Argument 3: No theoretical “best” dependency structure

- We can rely on extreme value theory for the large claims to propose the theoretical best distributions

- But how strong a recommendation is that?
  - Relies on assumptions
  - Only a limiting case

- What theory do we rely on for attritional claims?

- Nevertheless, more understanding of appropriate dependency structures would be useful, and perhaps new dependency structures which are more appropriate for General Insurance – further research is needed

Argument 4: Dependency is not stationary at different confidence levels

- In general insurance this is known to be true
  - E.g. catastrophe losses
- Nonetheless, there is value in understanding the “normal” dependency, and then building “extreme” variability on a solid base
- As data volumes increase we may ***start*** to see tail effects coming through, though we would need a long established stable book to view these credibly
- This problem may be mitigated be identifying and separately modelling tail drivers of dependency, which is often done
- But there are no guarantees, and the analysis is always open to additional tail dependencies being missed
- Qualitative validation will always remain important here!

Models

The current model

An alternative model

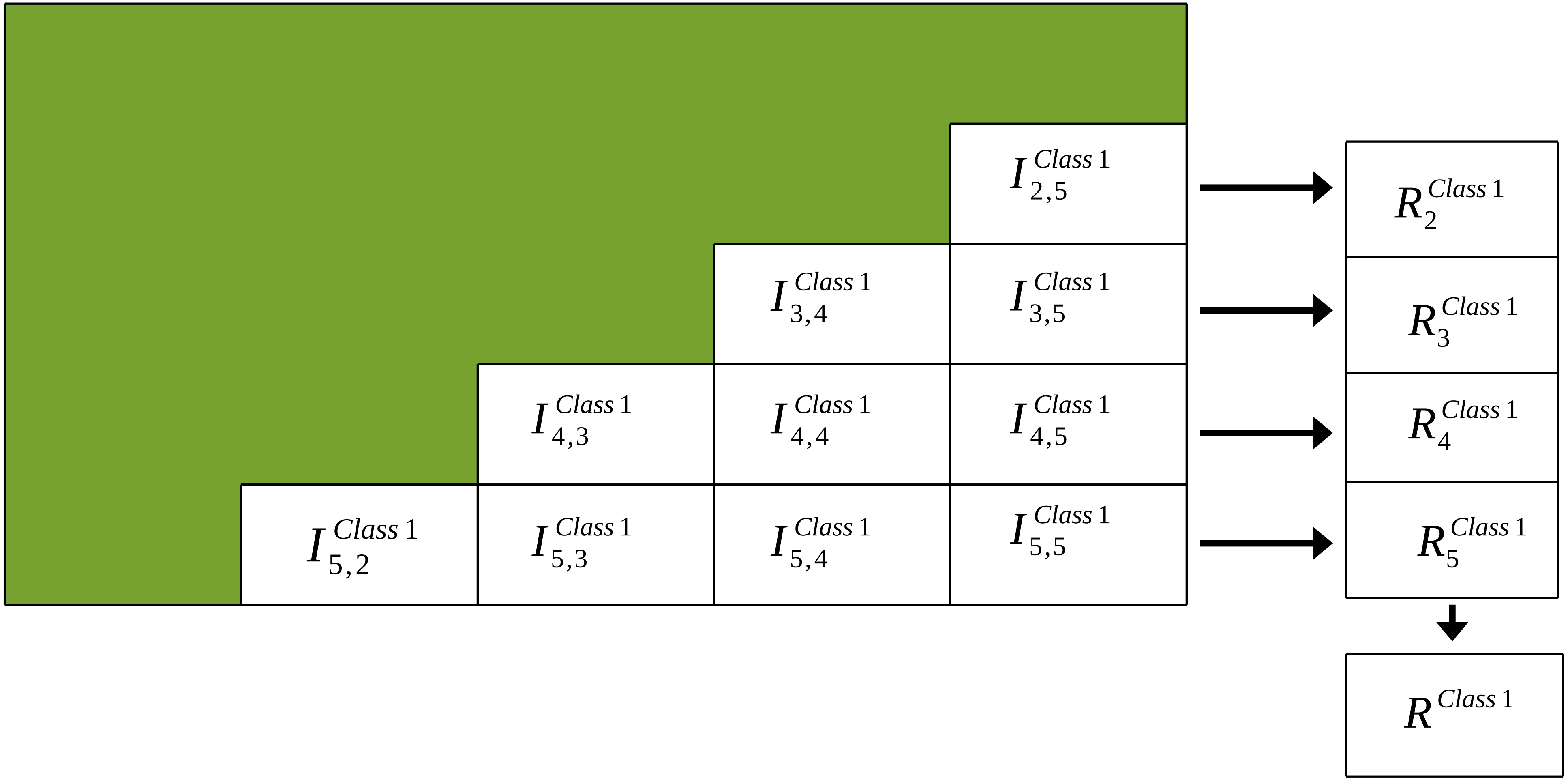
Parameterisation

The current model

Notation used

- Origin periods: op=1..n
- Development periods: dp=1..n (assume annual/annual model for clarity)
- Incremental Amounts: $I_{op,dp}^{Class}$, known for op+dp<=n+1, otherwise unknown
- Cumulative Amounts $C_{op,dp}^{Class}$, known for op+dp<=n+1, otherwise unknown
- Ultimate (by origin period) $U_{op}^{Class} = \sum_{dp} I_{op,dp}^{Class}$
- Reserve (by origin period) $R_{op}^{Class} = \sum_{dp > n+1-op} I_{op,dp}^{Class}$
- Total reserve $R^{Class} = \sum_{op} R_{op}^{Class}$
- While the terminology suggests that this example is based on paid, in practice I would expect to perform these calculations based on incurred data
- We use the convention $\overline{X}$ for the projected value of X

Reserve uncertainty: by class



Reserve uncertainty: aggregate model

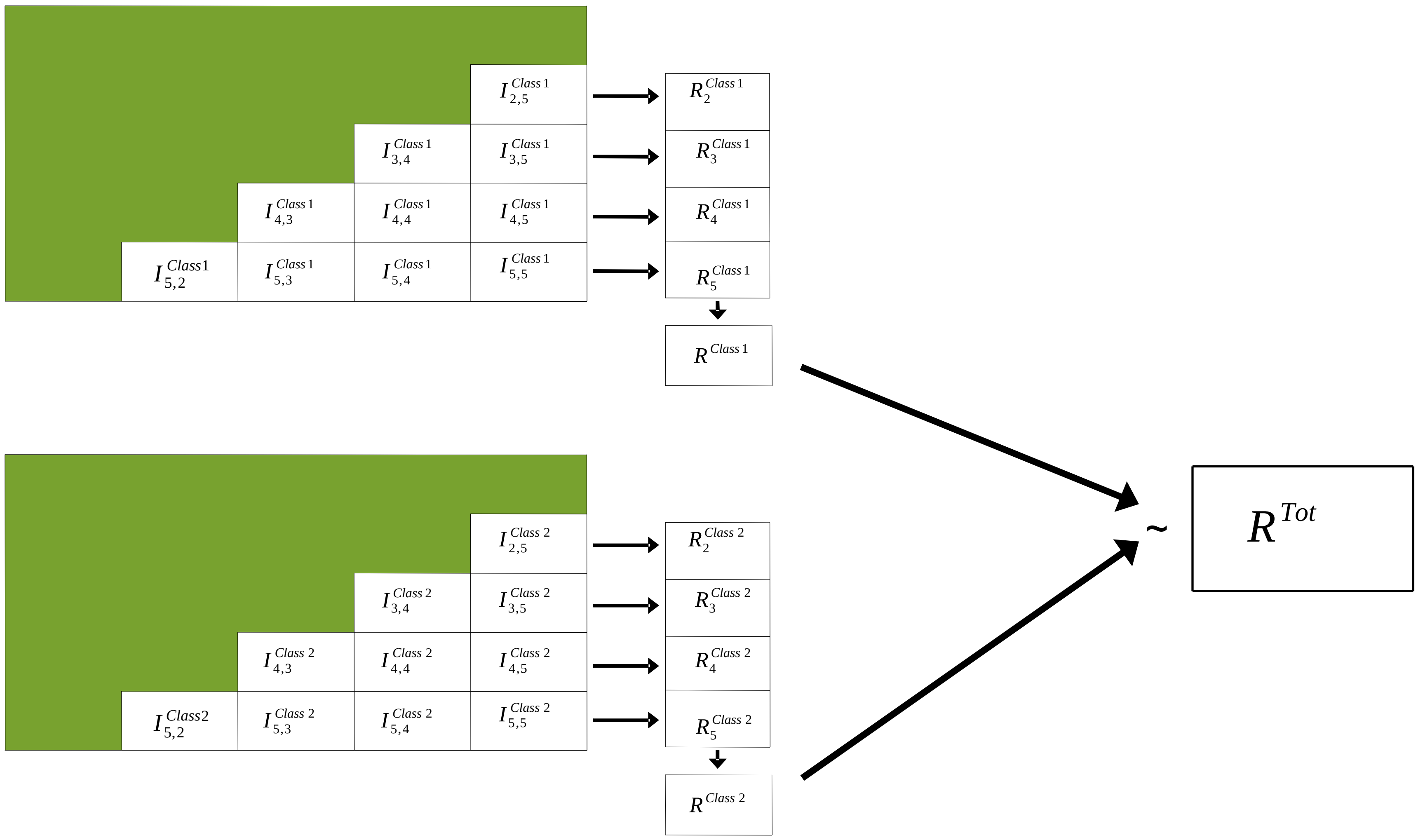
- We want the total reserve, $R^{Tot} = R^{Class\,1} + R^{Class\,2}$

- We generally assume a model such that:

R^{Class1} and R^{Class2} each have some distribution (possibly complex, but obtainable by simulation)

and $R^{Class1} \sim R^{Class2}$ via some copula (e.g. Gaussian (RankNormal))

Aggregate model correlations



Aggregate model - strengths and weaknesses

Advantages

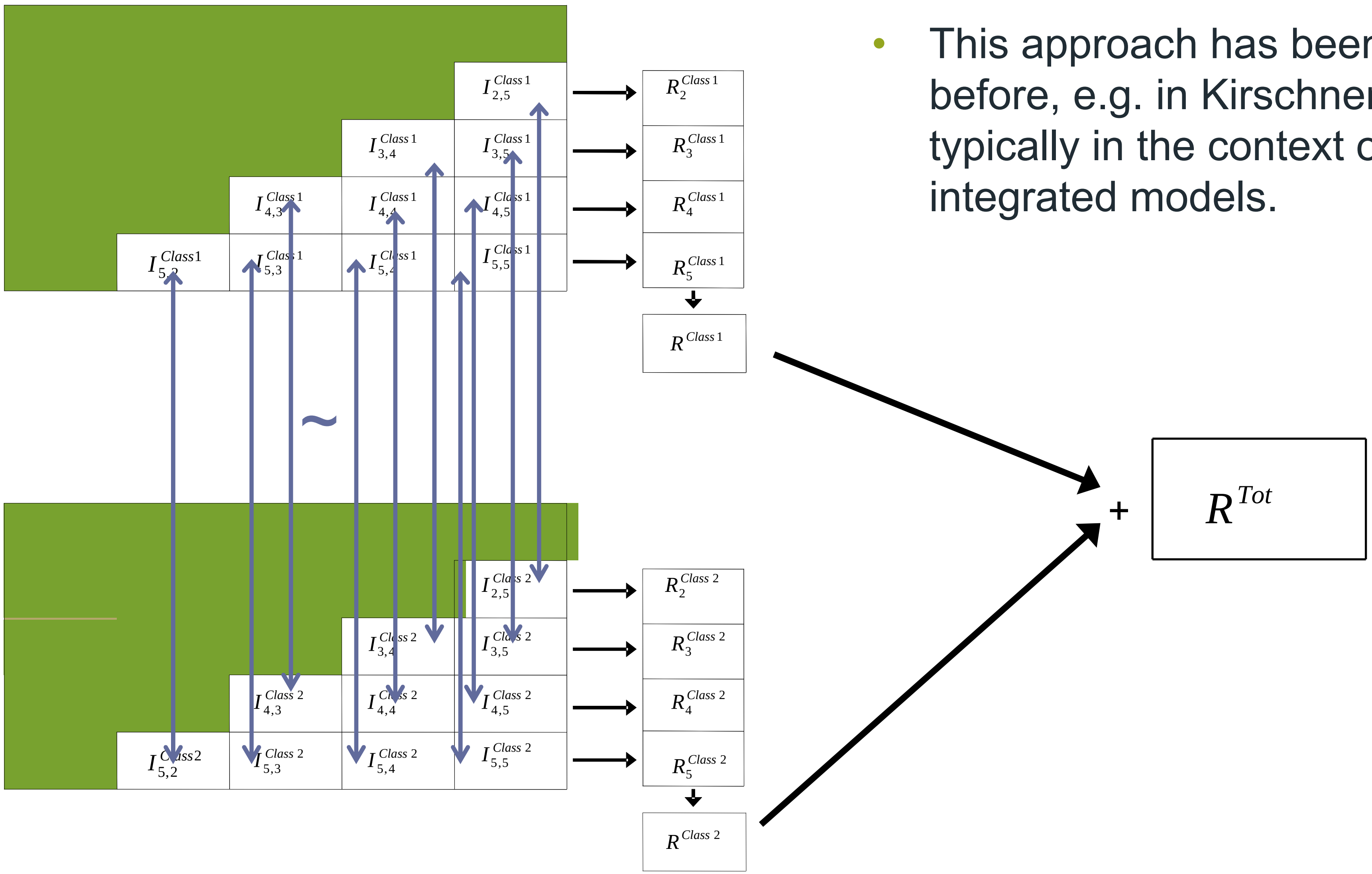
- Simple
- (Relatively) easy to explain
- Can be consistently applied
- (Relatively) easy to review

Limitations

- Results may not be consistent
- Impossible to parameterise from history
- Does not help to explain why classes move together
- Correlation may be driven by many factors, acting in different directions
- Is it “falsifiable”?

An alternative model

Aggregate model - alternative



- This approach has been seen before, e.g. in Kirschner et al, typically in the context of full integrated models.

What impact does this have?

- We tested the correlation on the total reserve, for various correlations at the cell level
- There is an assumption that the (rank) correlation at cell level is independent of position of the cell, or the distribution of those cells
- We assumed a Rank Gaussian dependency structure

Cell correlation (input)	Total reserve correlation (output)
0%	0%
15%	14%
30%	29%
50%	49%
75%	74%

Complications

- There are a number of practical issues
- There will be dependencies within a triangle (we assumed independence):
  - within an origin period
  - within a development period
  - within a calendar period

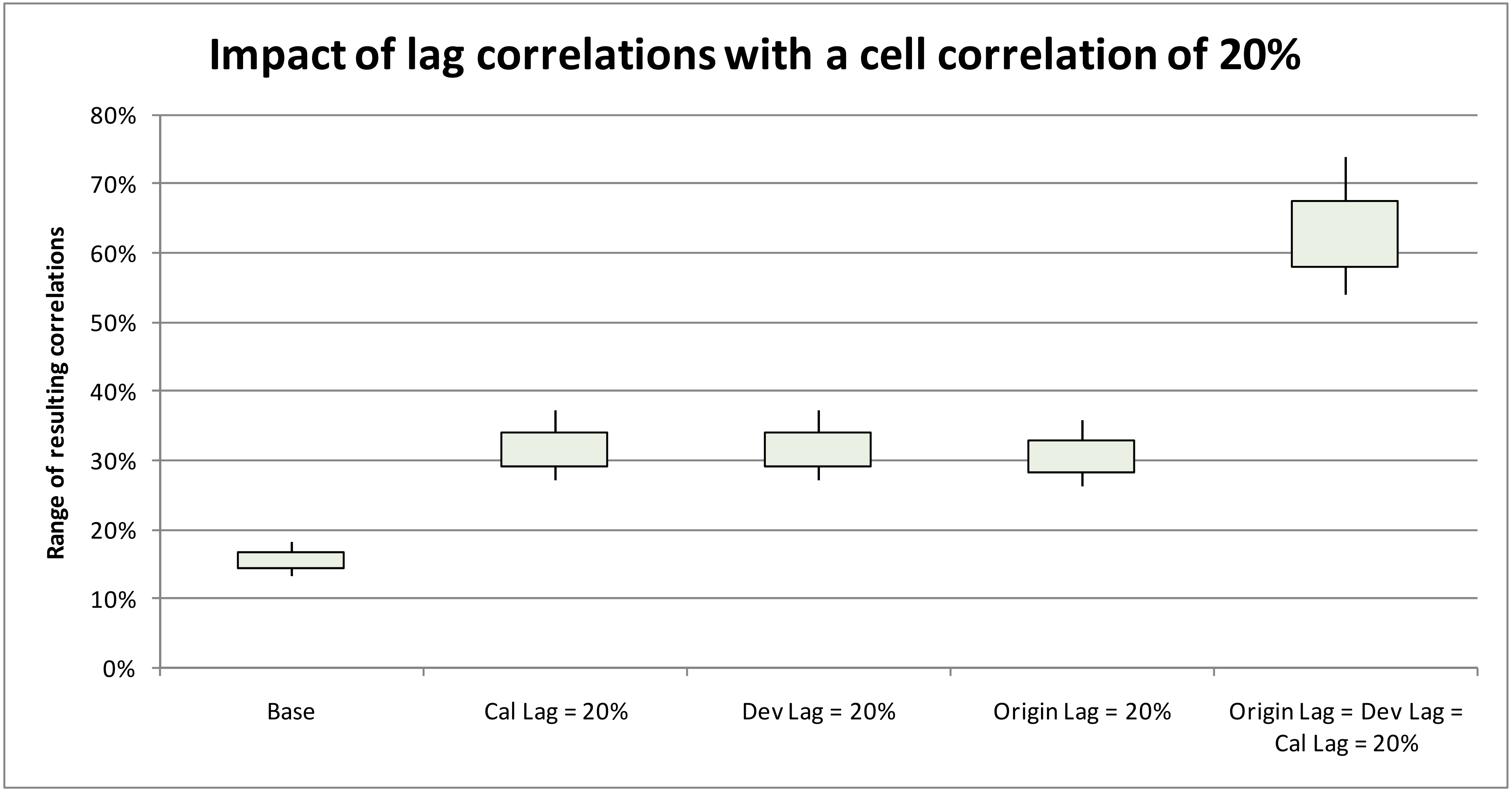
    (not all of these are currently modelled)

- Cross-correlations may also not affect the same cell, e.g.:
  - there could be a reporting lag in one class, which means the same event affects different development periods
  - there could be different exposure lengths on contracts, which means the same event could affect different origin periods (but at the same calendar period)
  - both could happen, so the same event affects different origin and calendar periods!

What impact do the complications have?

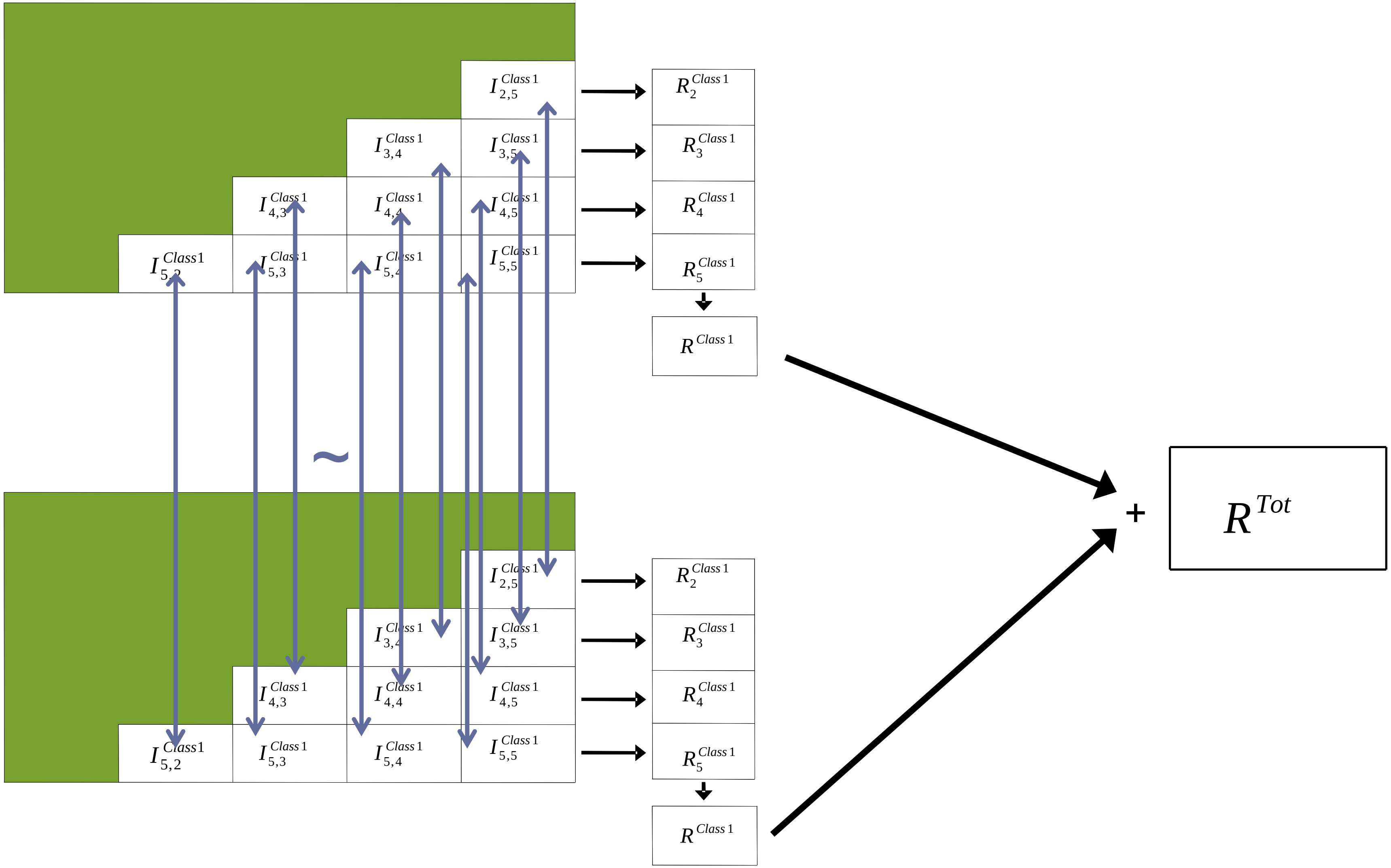
- We've tested a more complex model with:
  - intra triangle dependencies
    - origin period
    - development period
    - calendar period
  - cross correlation dependencies
    - by cell
    - origin period
    - development period
    - calendar period
- If the each dependency can take five values, this gives 5^7 = 78,125 options to test – more than we can show in this presentation!

Total reserve correlation results

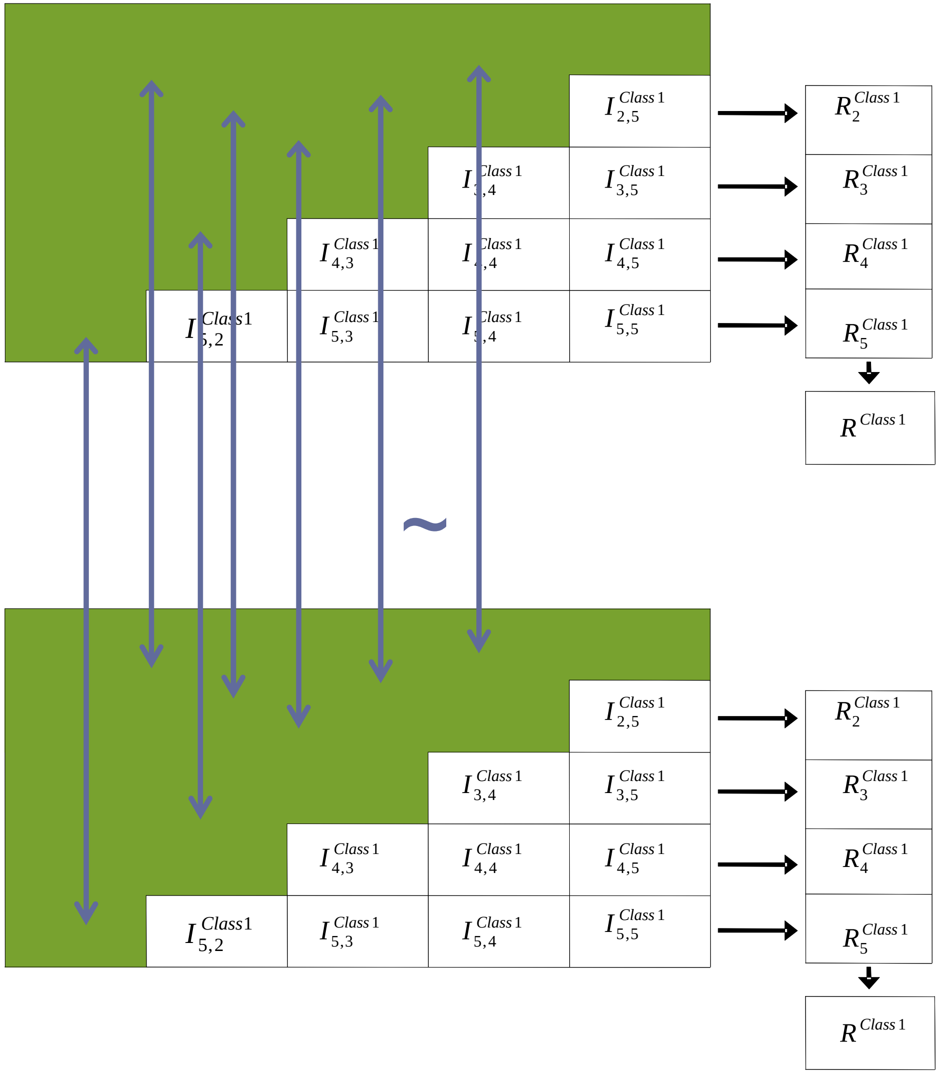


Parameterisation

Aggregate model: alternative



But then…



- So in a 10 x 10 triangle, we have 55 observations
- In a quarterly triangle we have 210 observations

Issues 1

Issue:	Volumes may be very different – not a linear effect?
Solution:	Fit a rank dependency

Issue:	Dependency may not be constant (e.g. normally no dependency, but shock events could impact both classes)
Solution:	Investigate non-Gaussian dependency structures

Issues 2

Issue:

My business volumes have been growing in both classes
This will add a dependency to the historical observations, since in both classes we would expect that claim amounts from the most recent origin periods were larger than observations from earlier origin periods. Since the business volumes are now fixed and known, this dependency is not relevant for future reserve deterioration risk.

Issue:

My patterns are not simple linear
This will add a dependency to the historical observations, since in both classes we would expect that incremental claim amounts in the first few development years are larger than incremental claim amounts after ten years. Since the mean projections are know, this dependency is not relevant for future reserve deterioration risk.

Solution:

Standardise the historical data, so only deviations from the expected claim amounts are considered

Standardising the data

- We require a method which will standardise the data for the expected claims amounts, and leave only the surprise value of the observation
- In fact, we should also recognise that the volatility (in cash terms) is higher when the mean is higher, so we need to also standardise the surprise value.
- Fortunately such a standardisation is already in common use: bootstrapping

Bootstrapping process

1. Fit a development model
2. Generate expected historical data from model
3. Compare actual to expected to assess volatility
4. Calculate historical residuals
5. Re-sample from residuals, and re-build pseudo-data from sampled residuals
6. Re-apply model, to derive new mean run-off projections
7. Simulate run-off using new mean run-off projections, and adding process variance from the volatility in step 3

For further details see, for example, England & Verall

Bootstrapping process

1. Fit a development model
2. Generate expected historical data from model
3. Compare actual to expected to assess volatility
4. **Calculate historical residuals**

- The residuals generated in this way are, given the model, an empirical distribution of the surprise value. They are standardised so they are entirely comparable
- This is the justification for sampling from any observed residual to rebuild the pseudo data
- Hence if we calculate these residuals we have all of the surprise information from the triangle, without bias for the volume effects which were not relevant for future reserve deterioration risk modelling.

Calculating a rank correlation

- We have two datasets of residuals, A & B
- First we calculate the ranks of those results – R & S (normalised to lie between 0 and 1 – i.e. percentiles)

$$Corr\,(R,S) = \frac{Cov\,(R,S)}{SD\,(R) \times SD\,(S)}$$

$$Cov\,(R,S) = E\left[R \times S\right] - E\left[R\right]E\left[S\right]$$

- r, s are percentiles within a rank, so follow a (discrete approximation to a) uniform distribution

E[R]=E[S]=0.5
SD[R]=SD[S]=0.29 (this varies slightly depending on the number of data points)
Var[R]=Var[S]=0.0833

- So problem reduces to calculating the expected value of R x S, this is obtained empirically

$$Corr\,(R,S) = \frac{E\left[R \times S\right] - 0.25}{0.0833}$$

Fitting a Rank Gaussian dependency via MLE

- Gaussian copula over two datasets (given a correlation parameter, ρ) is a distribution from [0,1] x [0,1]->[0,1]

$$C(u_1, u_2) = \Phi_R\left(\Phi^{-1}(u_1), \Phi^{-1}(u_2)\right)$$

$$= \int_{-\infty}^{\Phi^{-1}(u_1)} \int_{-\infty}^{\Phi^{-1}(u_2)} \frac{1}{(2\pi)^{\frac{n}{2}} |R|^{\frac{1}{2}}} \exp\left(-\frac{1}{2} x^T R^{-1} x\right) dx_1 dx_2$$

Where u_1 and u_2 are the percentiles of the data
Φ is the standard normal distribution function
Φ^{-1} is the inverse of the standard normal distribution function
And R is the (positive definite) correlation matrix (with 100% on the diagonal, and ρ off the diagonal)

Maximise likelihood proceeds by finding the numerical correlation parameter, ρ, which maximises the likelihood of this joint distribution, given the data

For further details see, for example, Zhang

Proposed dependency parameterisation process

- Obtain historical triangulations (we will use Incurred)
- Fit a development model – we will use Mack's model
- Standardise triangles – obtain residuals
- Flatten residuals triangles
- Apply dependency modelling to the two columns of data – any dependency method can be used, but we will only consider Rank Gaussian dependency
  - Calculate the rank/percentile of each result within its data set
  - Calculate the simple (rank) correlation
  - Fit a Gaussian dependency structure via MLE

Results

Base results

Confidence intervals

Results

Application of the parameterisation method

- To demonstrate the method we have analysed the dependency between a variety of classes

- The data is based on FSA returns
- We have extracted data for paid and incurred triangles per reporting class for several firms, and aggregated these to market triangles for each class
- We selected only companies with 10 years worth of returns, and an average earned premium of at least 10,000
- Several companies data exhibited significant calendar effects (e.g. due to M&A activity) – these have been excluded
- As such, in some classes we have many companies aggregated, while in others we have only one company

- Our analysis has been based on a Mack model, using Annual development Incurred data

A note on what dependency we have parameterised

- The dependency we parameterised related to the “surprise value” between two classes of business.
- It covered the element of uncertainty remaining given our ***current*** estimates for the incremental amounts in each cell, which were projected based on the data in the triangle
- In essence, it is the correlation that would occur between classes in the “actual versus expected” analysis
- In particular, the expected ultimate claims level is an input into this process (at least, if parameterised according to chain ladder assumptions) – correlation between row parameters is not considered
- For example, this method will not include dependency caused by;
  - the underwriting cycle (this would already have impacted the mean claims level for each underwriting year),
  - or any dependency caused by uncertain levels of exposures, e.g. poor underwriting controls in one year due to staffing shortages (but it ***would*** catch dependency caused by joint exposures)
- As such, the parameters may be appropriate for reserve risk, but we would expect underwriting risk correlations to be higher

Gaussian correlations

	Private motor - comprehensive	Private motor - non-comprehensive	Total Household & domestic all risks	Employers liability	Public & products liability
Private motor - comprehensive	**100%**	-17%	7%	49%	24%
Private motor - non-comprehensive	*-17%*	**100%**	20%	-20%	-23%
Total Household & domestic all risks	*7%*	*20%*	**100%**	-8%	19%
Employers liability	*49%*	*-20%*	*-8%*	**100%**	26%
Public & products liability	*24%*	*-23%*	*19%*	*26%*	**100%**

What went wrong?

What went wrong?

- There are three possibilities:
  - The method doesn't work (hopefully not, or we've all been wasting our time!)
  - The data didn't work
  - Our intuition didn't work (i.e. that is the right result)

Example: Private motor comprehensive versus private motor – non comprehensive

Private motor - comprehensive

424,604	475,300	477,437	497,284	488,112	489,068	478,930	483,255	477,804	474,036
386,573	422,434	424,472	428,739	430,460	406,742	406,232	405,622	402,031	
466,106	540,771	546,563	551,308	536,284	527,700	521,837	510,991		
547,890	627,256	634,207	623,295	622,597	620,358	605,471			
578,340	658,768	666,497	656,296	655,576	652,343				
566,997	619,141	601,882	598,157	609,111					
595,038	682,867	687,266	682,013						
596,796	702,800	719,095							
577,579	697,067								
588,516									

Private motor - non-comprehensive

54,835	69,558	68,712	67,389	68,824	69,262	70,337	70,124	71,798	71,355
56,008	77,515	79,961	81,017	81,841	84,526	85,375	84,822	84,872	
64,783	90,915	94,121	92,812	93,569	97,950	97,652	97,302		
59,418	82,407	75,974	82,683	84,595	87,978	93,091			
61,750	78,765	82,111	83,835	84,803	86,285				
57,812	69,459	70,905	74,507	74,759					
79,464	90,499	122,648	103,449						
67,943	89,887	94,856							
56,111	72,278								
45,193									

Example: Private motor comprehensive versus private motor – non comprehensive

Private motor - comprehensive

425	475	477	497	488	489	479	483	478	474
387	422	424	429	430	407	406	406	402	
466	541	547	551	536	528	522	511		
548	627	634	623	623	620	605			
578	659	666	656	656	652				
567	619	602	598	609					
595	683	687	682						
597	703	719							
578	697								
589									

Link Ratio

1.145	1.006	1.000	0.996	0.987	0.985	0.995	0.990	0.992

Link ratio Triangle

1.119	1.004	1.042	0.982	1.002	0.979	1.009	0.989	0.992
1.093	1.005	1.010	1.004	0.945	0.999	0.998	0.991	
1.160	1.011	1.009	0.973	0.984	0.989	0.979		
1.145	1.011	0.983	0.999	0.996	0.976			
1.139	1.012	0.985	0.999	0.995				
1.092	0.972	0.994	1.018					
1.148	1.006	0.992						
1.178	1.023							
1.207								

Private motor - non-comprehensive

55	70	69	67	69	69	70	70	72	71
56	78	80	81	82	85	85	85	85	
65	91	94	93	94	98	98	97		
59	82	76	83	85	88	93			
62	79	82	84	85	86				
58	69	71	75	75					
79	90	123	103						
68	90	95							
56	72								
45									

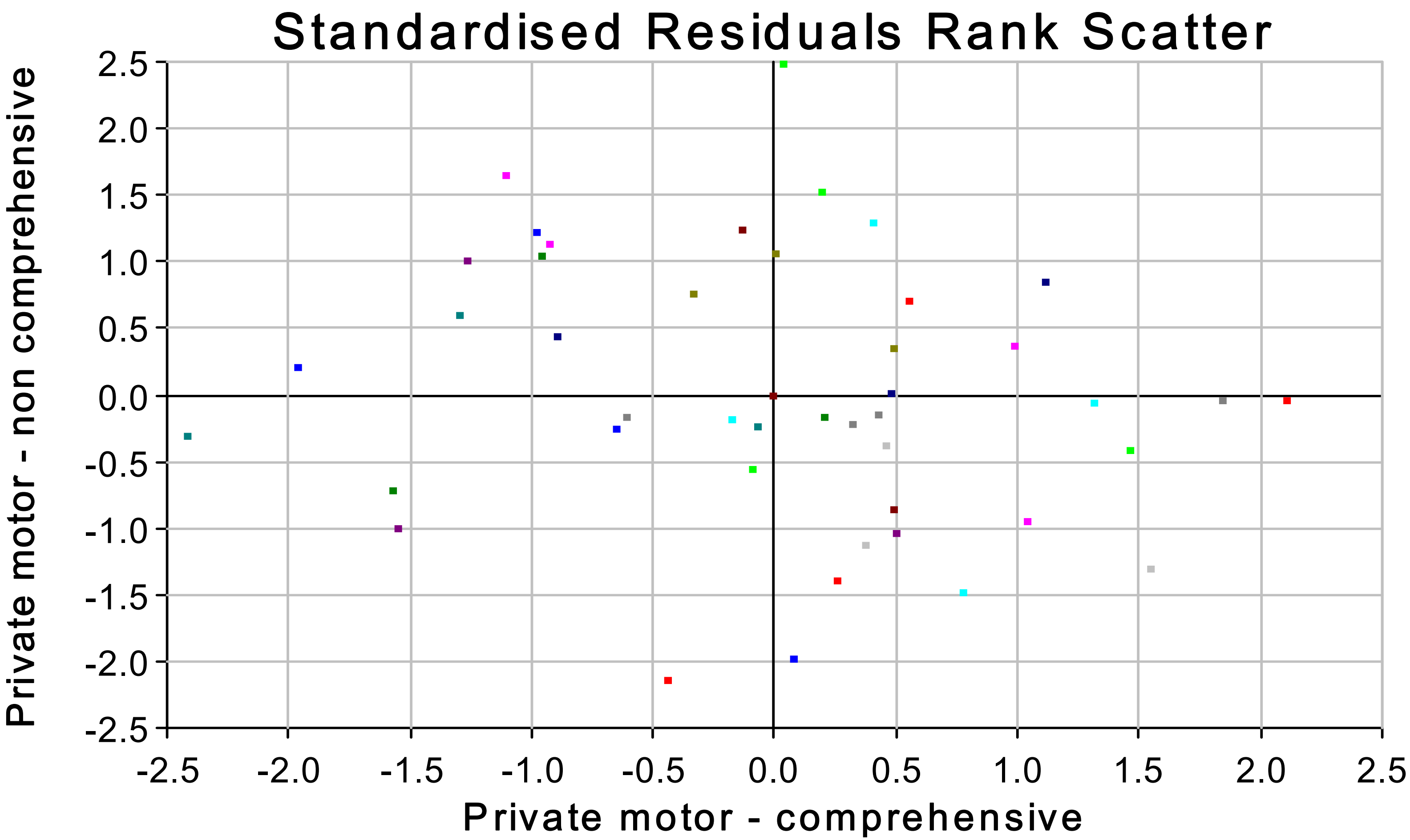
Link Ratio

1.292	1.062	0.985	1.013	1.030	1.020	0.996	1.011	0.994

Link ratio Triangle

1.268	0.988	0.981	1.021	1.006	1.016	0.997	1.024	0.994
1.384	1.032	1.013	1.010	1.033	1.010	0.994	1.001	
1.403	1.035	0.986	1.008	1.047	0.997	0.996		
1.387	0.922	1.088	1.023	1.040	1.058			
1.276	1.042	1.021	1.012	1.017				
1.201	1.021	1.051	1.003					
1.139	1.355	0.843						
1.323	1.055							
1.288								

Example: Private motor comprehensive versus private motor – non comprehensive



What did go wrong?

- So we can understand why the result is being produced.
- It is still possible that the dependency doesn't apply to the surprise information in the way we have assumed (i.e. the "Actual vs Expected" way of thinking about reserve dependencies is wrong)
  - Lags?
  - The standardisation method?
- But is it also possible that the issue was with the data?
- We were aggregating Motor – Comp and Motor – Non Comp triangles for different firms to get a "market" dependency
- But do different firms have different reserving practices?
  - Could this be dominating our results?

Focusing on the data

- After extracting the data, and excluding any spurious triangles, how many triangles did we have left?
  - Motor Comprehensive: 19
  - Motor Non-Comprehensive: 10
- Not all of the market!
  - Firms participating in both Motor Comprehensive and Motor Non-Comprehensive: 10
- Ah ha! We have 19 companies in our sample writing Motor Comprehensive only! Could dependency between companies be swamping dependency between lines?

Results – Motor Comprehensive versus Motor Non-Comprehensive (revised)

- Dependencies between Motor – Comprehensive and Motor – Non Comprehensive for the 10 firms who wrote both

11%	19%	25%	30%	34%
41%	53%	56%	56%	62%

- Average is 39%

- That's better!

Gaussian correlations

	Private motor - comprehensive	Private motor - non-comprehensive	Total Household & domestic all risks	Employers liability	Public & products liability
Private motor - comprehensive	**100%**	39%	8%	34%**	18%
Private motor - non-comprehensive	*39%*	**100%**	4%	NA	29%*
Total Household & domestic all risks	*8%*	*4%*	**100%**	8%	-1%**
Employers liability	*34%***	*NA*	*8%*	**100%**	25%
Public & products liability	*18%*	*29%**	*-1%***	*25%*	**100%**

* 1 observation only

**high uncertainty compared to other values

Confidence intervals

Difference between firm's dependencies

- Dependencies between Motor – Comprehensive and Motor – Non Comprehensive for the 10 firms who wrote both:

11%	19%	25%	30%	34%	41%	53%	56%	56%	62%

- A range of around 50% between lowest and highest
- Differences could be the result of:
  - Firm specific factors
    - Proportion of claims reserve representing BI in each class
    - Reserving philosophy for each class
    - Different people in reserving process
  - Parameter uncertainty
- How can we tell how much of the variation comes from each source?
- We will investigate parameter uncertainty – and see whether the "market average" of 39% is consistent with the data
- We will focus on the firms which generated the values of 11%, 41%, and 62%

Bayesian analysis

- Bayesian analysis is used in general insurance as an approach to add parameter uncertainty to projections
- It requires two components:
  - An a priori view of the distribution which the parameters could take
    - Often taken to be uninformative, i.e. no a priori view
  - Deriving the likelihood of the data for each possible (set of) parameters
    - View this as a function of the parameters
- These are combined to give a distribution of the parameters.
  - Essentially how likely is it that that is the true parameter and we got that data, times how likely did we think it was beforehand that that was the true parameter
- We simulate a parameter from this, then use this in the simulation of the underlying process
- For more details see Borrowicz & Norman

Using Bayesian analysis to derive confidence intervals of correlations

It requires two components:

- An a priori view of the distribution which the parameters could take
  - Often taken to be uninformative, i.e. no a priori view
- Deriving the likelihood of the data for each possible (set of) parameters
  - View this as a function of the parameters

These are combined to give a distribution of the parameters.

- Essentially how likely is it that that is the true parameter and we got that data, times how likely did we think it was beforehand that that was the true parameter

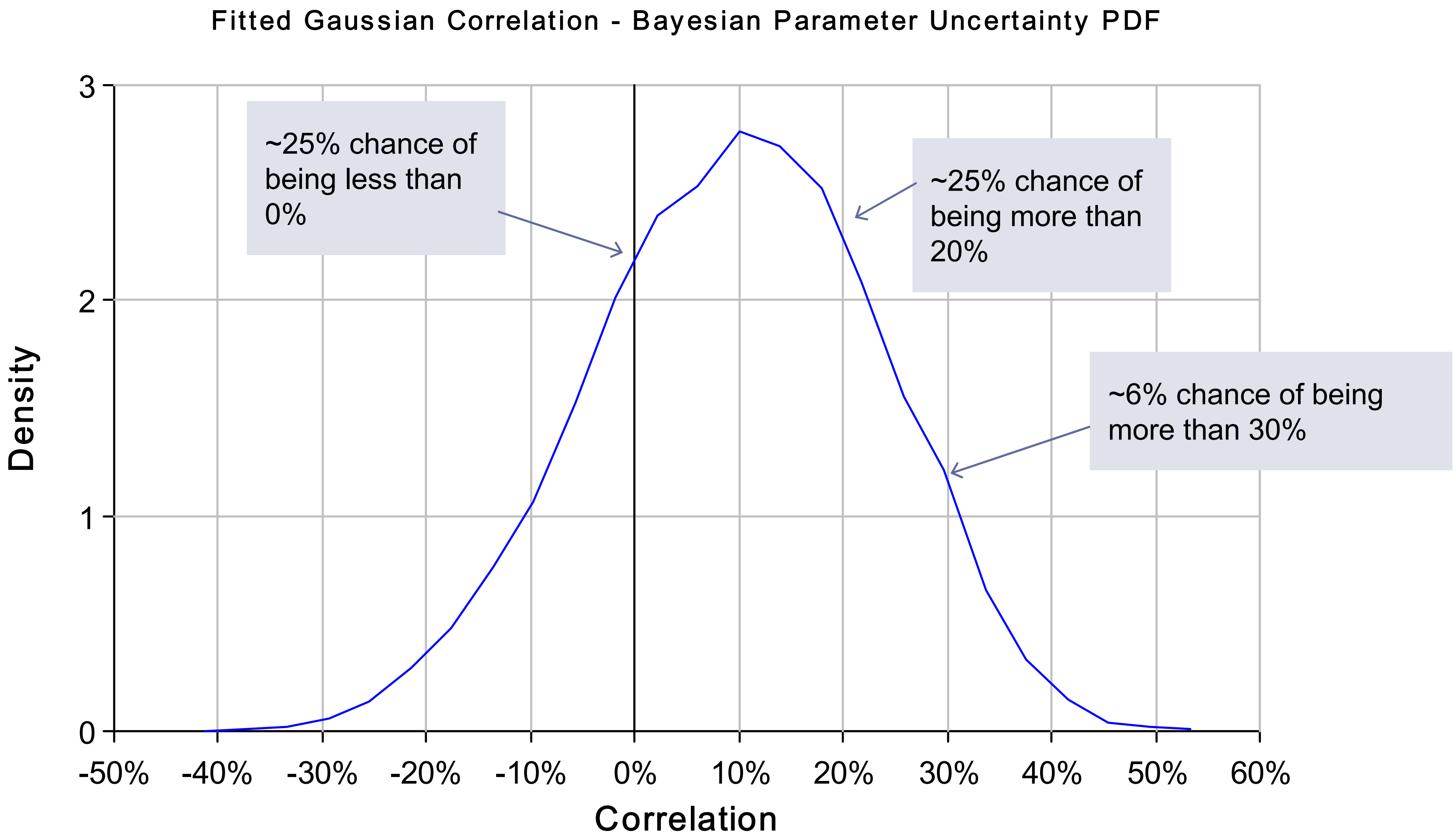
We simulate a parameter from this, then use this in the simulation of the underlying process

We will use an uninformative prior – U[-1,1]

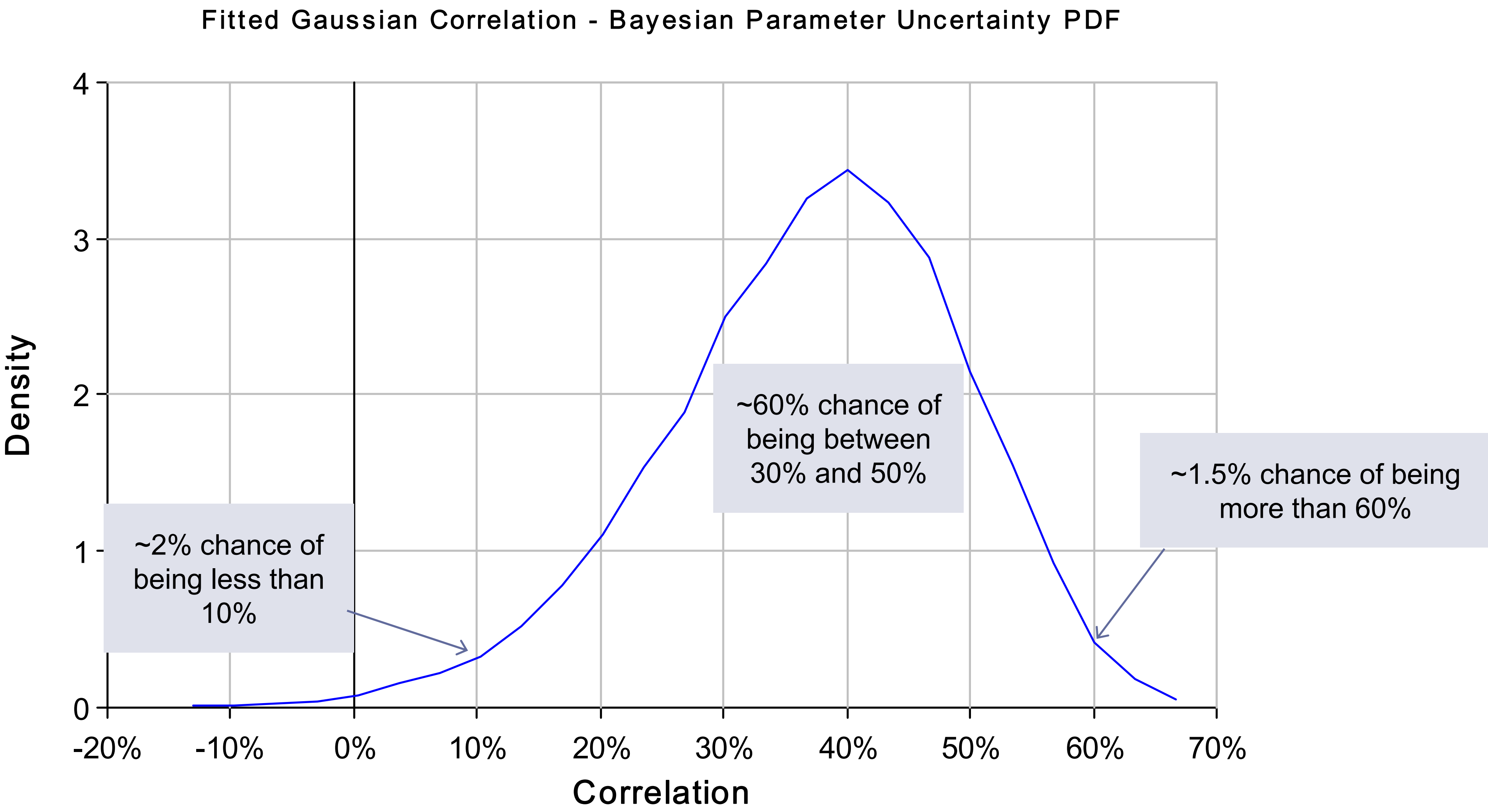
For each possible correlation calculate the likelihood as before, but we will just record these (i.e. we want all of the likelihoods for each possible correlation – not just the maximum)

We will stop here, at a distribution of the true correlations.

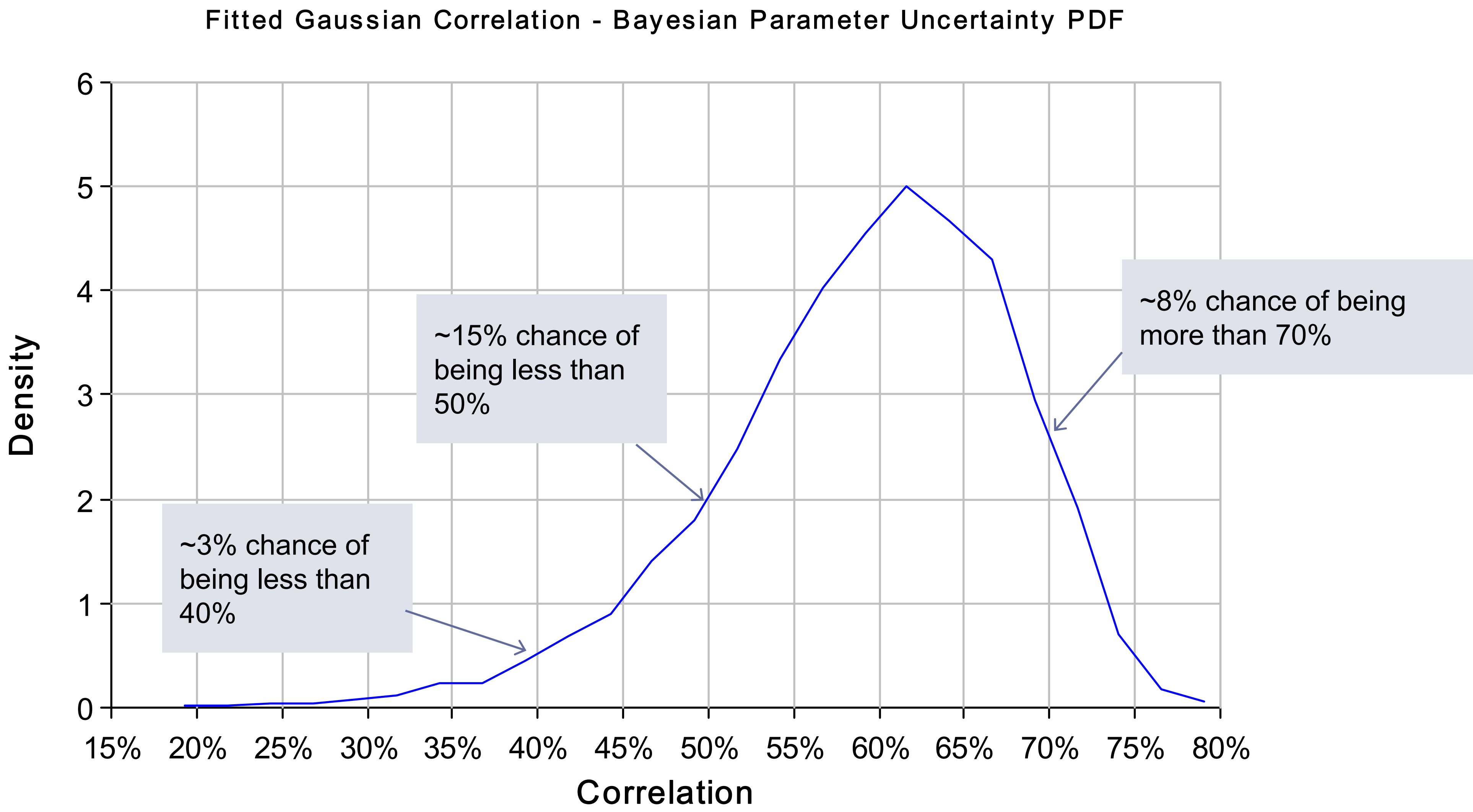
Example: The firm with a correlation of 11%



Example: The firm with a correlation of 41%



Example: The firm with a correlation of 62%



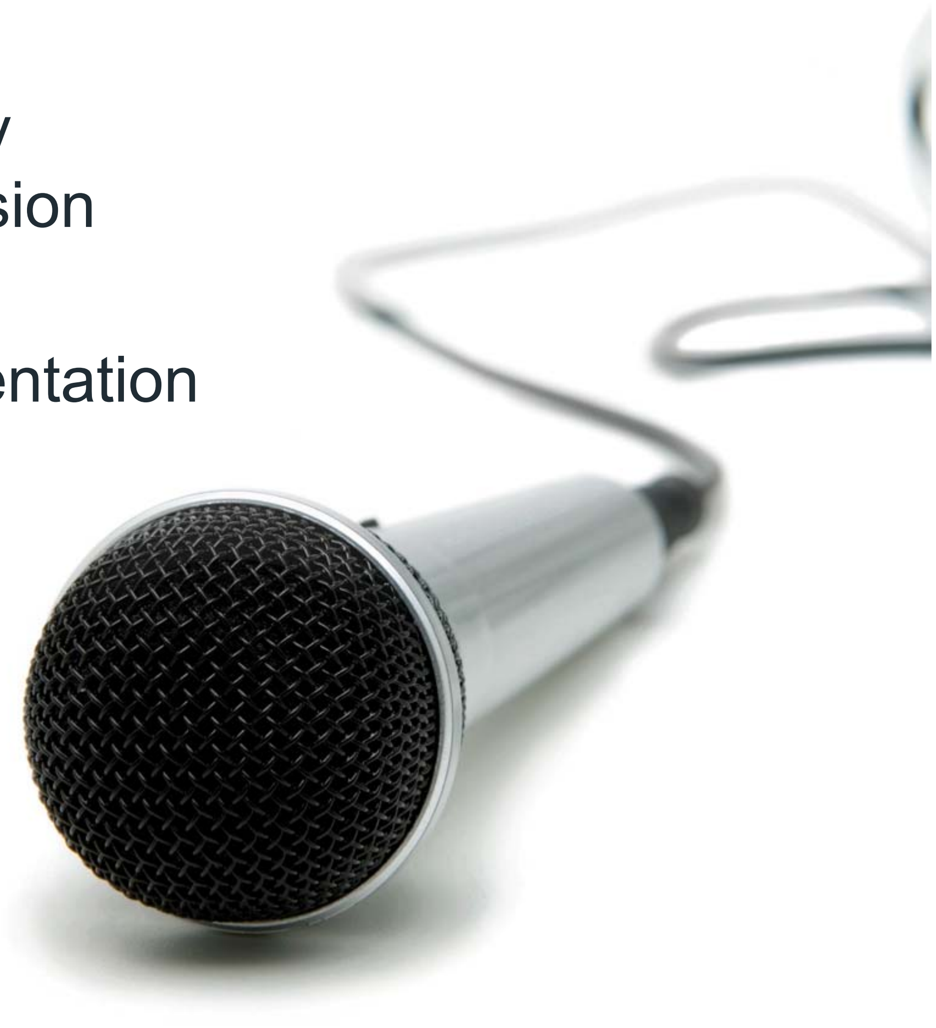
Summary

- Dependencies are a critical, and very difficult area of financial modelling
- Special importance has been given to them within Solvency II requirements
- However our understanding of appropriate values remains weaker than for other important parameters
  - This may lead to stress tests which are more extreme than for other areas
- Best practice would be to document all (material) drivers of volatility per pair of classes which introduce dependency
- Where possible these should be individually modelled
  - E.g. catastrophe losses, inflation, underwriting cycle etc.
- Historically we have tended to assess (at least residual) dependencies primarily via qualitative techniques and expert judgement
  - Mainly due to lack of data
- This workshop has presented an alternative dependency model, which is more obviously “falsifiable”
- This leads to a more data rich parameterisation methodology, at least for reserving risk
- However there remains significant debate to be had on appropriate industry benchmarks, and how much credibility to attach to own data
- And further work would be required to assess dependencies for underwriting risk

Questions or comments?

Expressions of individual views by members of The Actuarial Profession and its staff are encouraged.

The views expressed in this presentation are those of the presenter.



References

- Solvency II Level 1 Directive - DIRECTIVE 2009/138/EC OF THE EUROPEAN PARLIAMENT AND OF THE COUNCIL of 25 November 2009 on the taking-up and pursuit of the business of Insurance and Reinsurance (Solvency II) (recast)
- CP 56 - CEIOPS' Advice for Level 2 Implementing Measures on Solvency II: Articles 120 to 126 Tests and Standards for Internal Model Approval (former Consultation Paper 56), October 2009
- QIS 3 Calibration - QIS3 Calibration of the underwriting risk, market risk and MCR, April 2007
- Aon Benfield - Insurance Risk Study Fifth Edition, 2010
- Zhang - Estimating bivariate Gaussian Copula by the means of Maximization by Parts in Likelihood Inference - Seminar *Copula: Theory and Applications,* Ran Zhang, 2006
- Borrowicz & Norman – The effects of parameter uncertainty in dependency structures, Jakub Borowicz and James Normal, International Congress of Actuaries, 2006
- England & Verall - Stochastic Claims Reserving in General Insurance (with discussion), British Actuarial Journal 8, III (2002) – England, P, and Verall, RJ
- Kirschner et al – Two Approaches to Calculating Correlated Reserve Indications Across Multiple Lines of Business – Variance, Vol 2., Gerald S. Kirschner, Colin Kerley, and Belinda Isaacs