

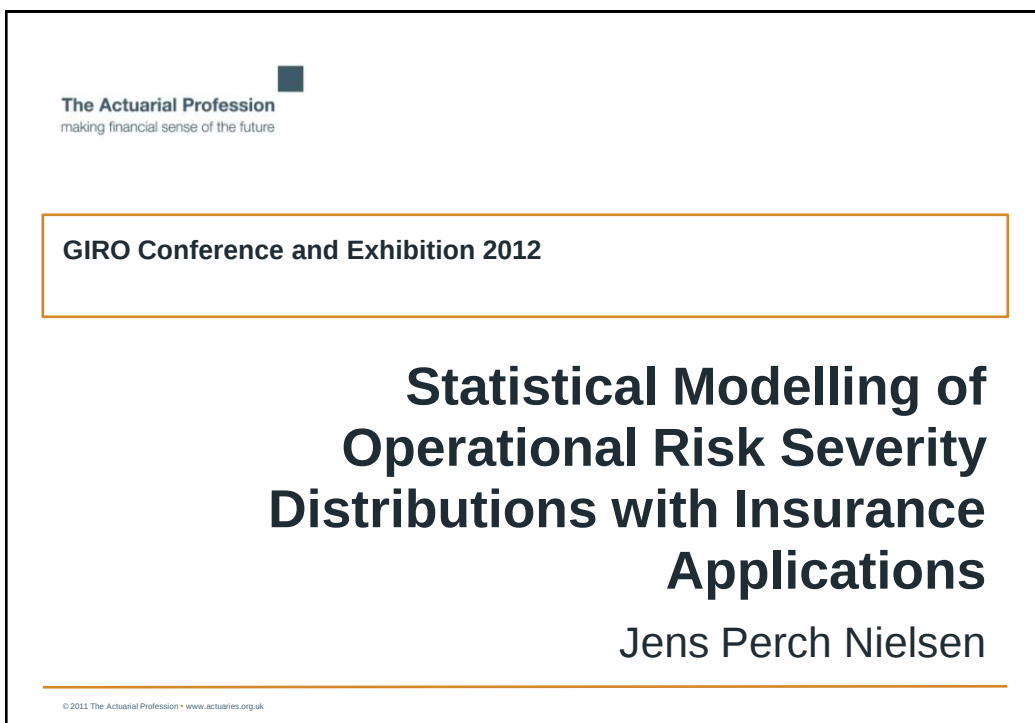
A person is silhouetted against a vibrant sunset sky, juggling several red balls. The scene is set on a beach with waves in the background. The sky is filled with orange and yellow clouds, and a bright sun is visible on the horizon.

The Actuarial Profession
making financial sense of the future

GIRO Conference and Exhibition 2012
Juggling uncertainty the actuary's part to play

20 September 2012

© 2012 The Actuarial Profession • www.actuaries.org.uk

A person is silhouetted against a vibrant sunset sky, juggling several red balls. The scene is set on a beach with waves in the background. The sky is filled with orange and yellow clouds, and a bright sun is visible on the horizon.

The Actuarial Profession
making financial sense of the future

GIRO Conference and Exhibition 2012

**Statistical Modelling of
Operational Risk Severity
Distributions with Insurance
Applications**

Jens Perch Nielsen

© 2011 The Actuarial Profession • www.actuaries.org.uk

Four published papers on operational risk

- A. Guillen, Gustafsson, Nielsen and Pritchard (2007). Using external data in operational risk. Geneva papers, 178-189.
- B. Buch-Kromann, Englund, Gustafsson, Nielsen, Thuring (2007). Non-Parametric estimation of operational risk losses adjusted for under-reporting. Scandinavian Actuarial Journal, 293-304.
- C. Gustafsson and Nielsen (2008). A mixing model for operational risk. Journal of operational risk, 3, 25-37.
- D. Guillen, Gustafsson and Nielsen (2008). Combining under-reported internal and external data for operational risk measurement. Journal of operational risk, 4, 1-22.

Festina Lente

AGENDA

Festina Lente

AGENDA



➤ **Painting the picture**

Festina Lente

AGENDA



➤ **Painting the picture**



➤ **Collecting the materials**

Festina Lente

AGENDA



➤ **Painting the picture**



➤ **Collecting the materials**



➤ **Behind the scenes**

Festina Lente

AGENDA



➤ **Painting the picture**



➤ **Collecting the materials**



➤ **Behind the scenes**



➤ **Mixing the colours**

Festina Lente

AGENDA



➤ **Painting the picture**



➤ **Collecting the materials**



➤ **Behind the scenes**



➤ **Mixing the colours**



➤ **Painting by numbers**

Festina Lente

AGENDA



➤ **Painting the picture**



➤ **Collecting the materials**



➤ **Behind the scenes**



➤ **Mixing the colours**



➤ **Painting by numbers**



➤ **Framing the canvas**

Festina Lente



➤Painting the picture

Festina Lente

Painting the picture

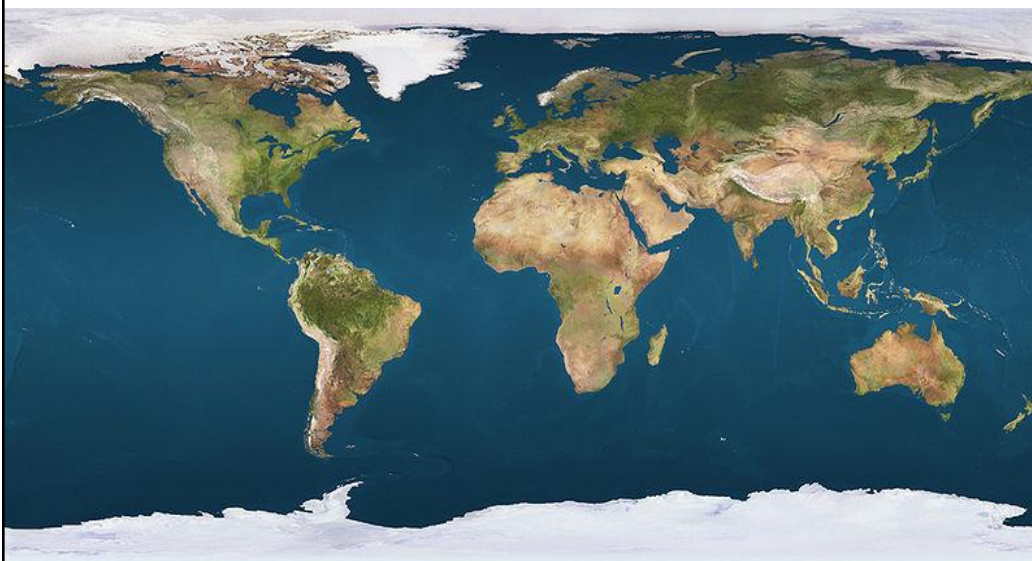
Collecting the material

Behind the scenes

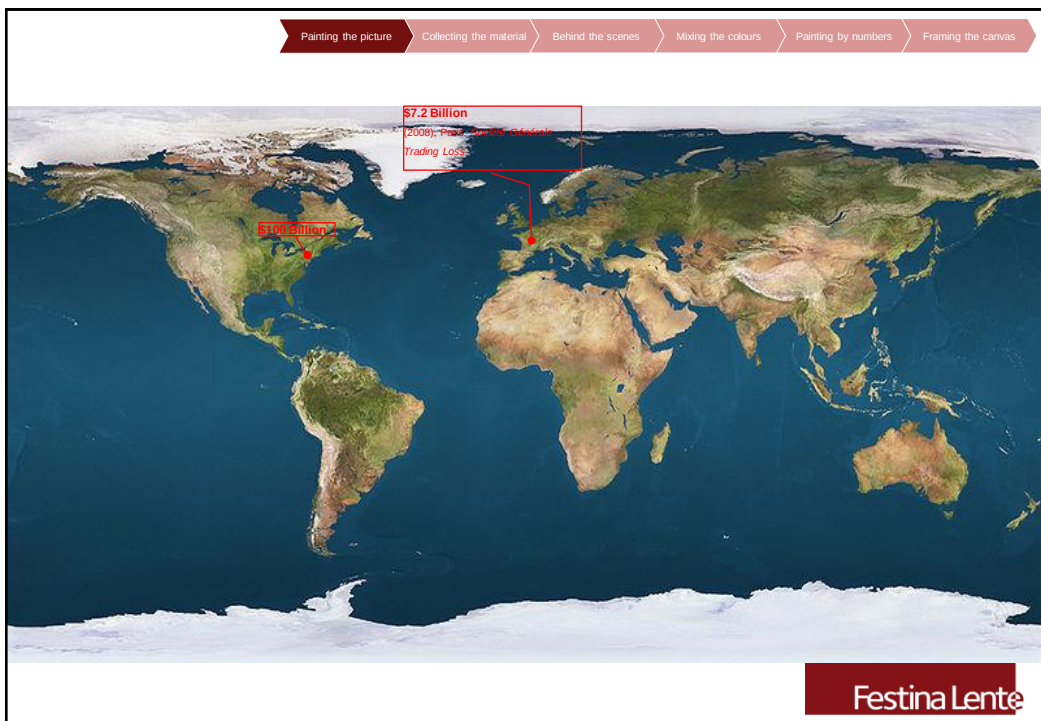
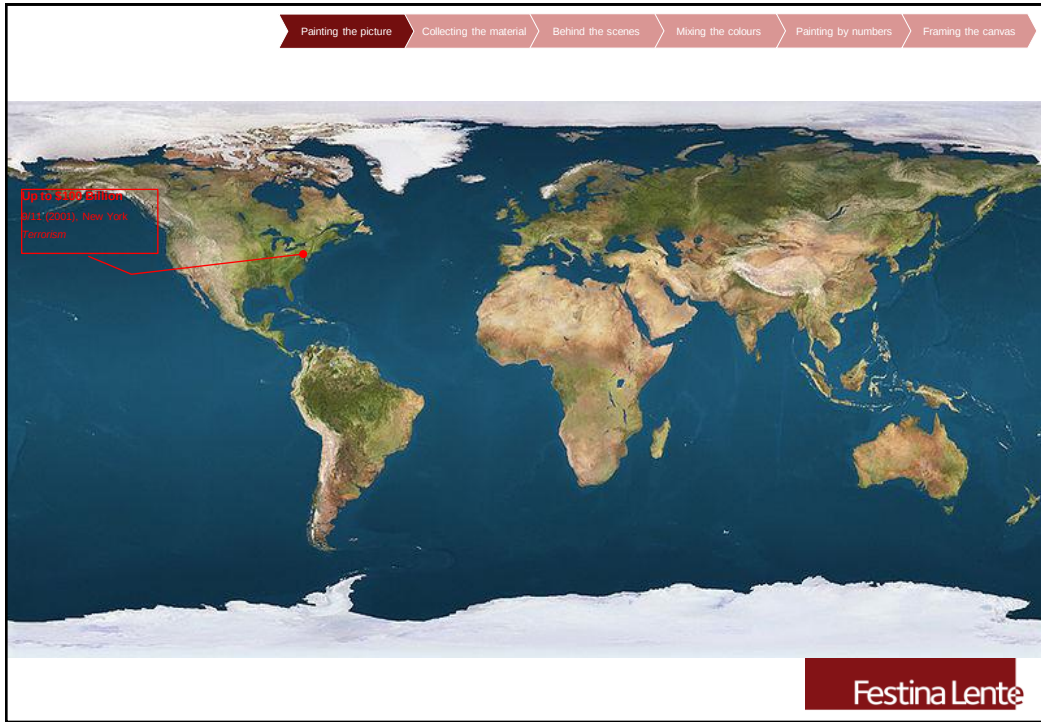
Mixing the colours

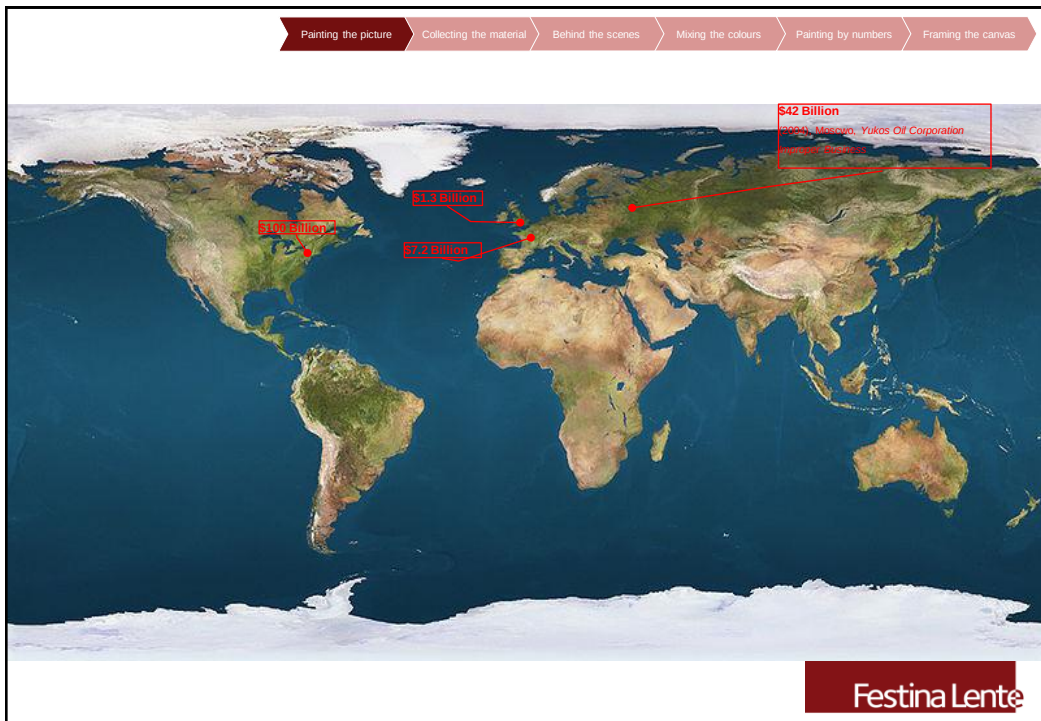
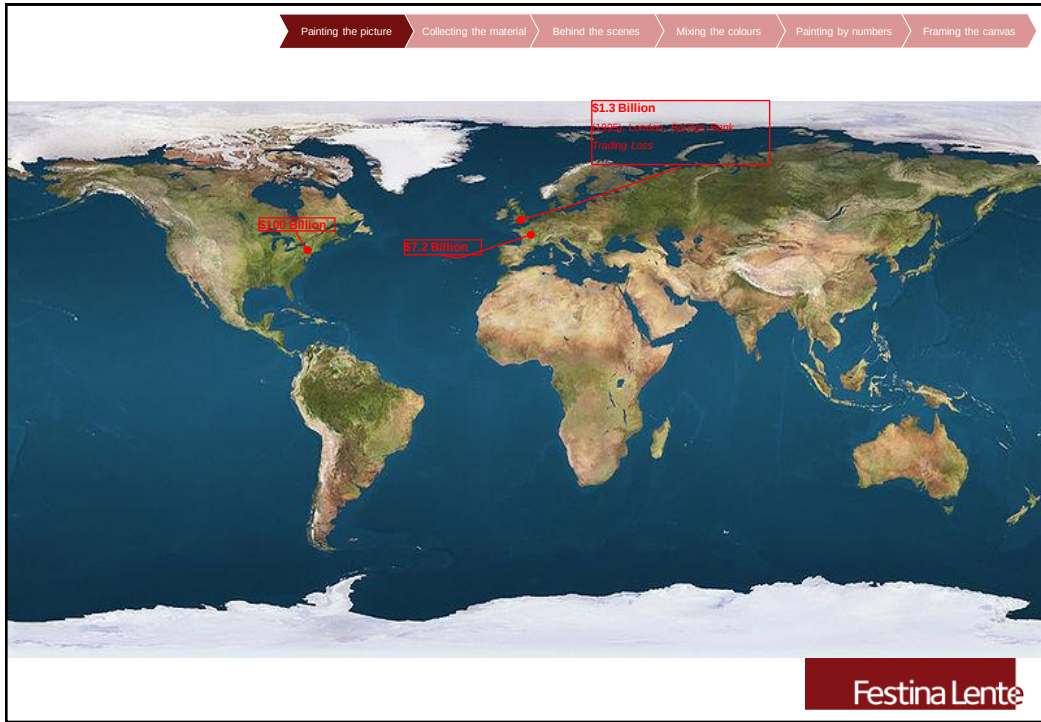
Painting by numbers

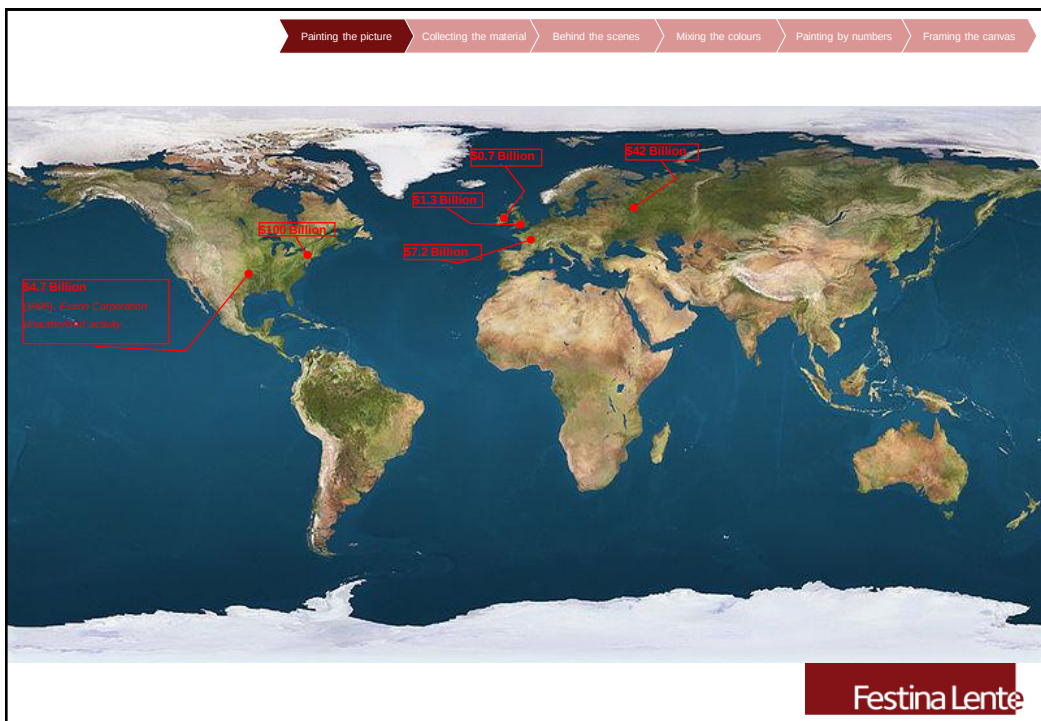
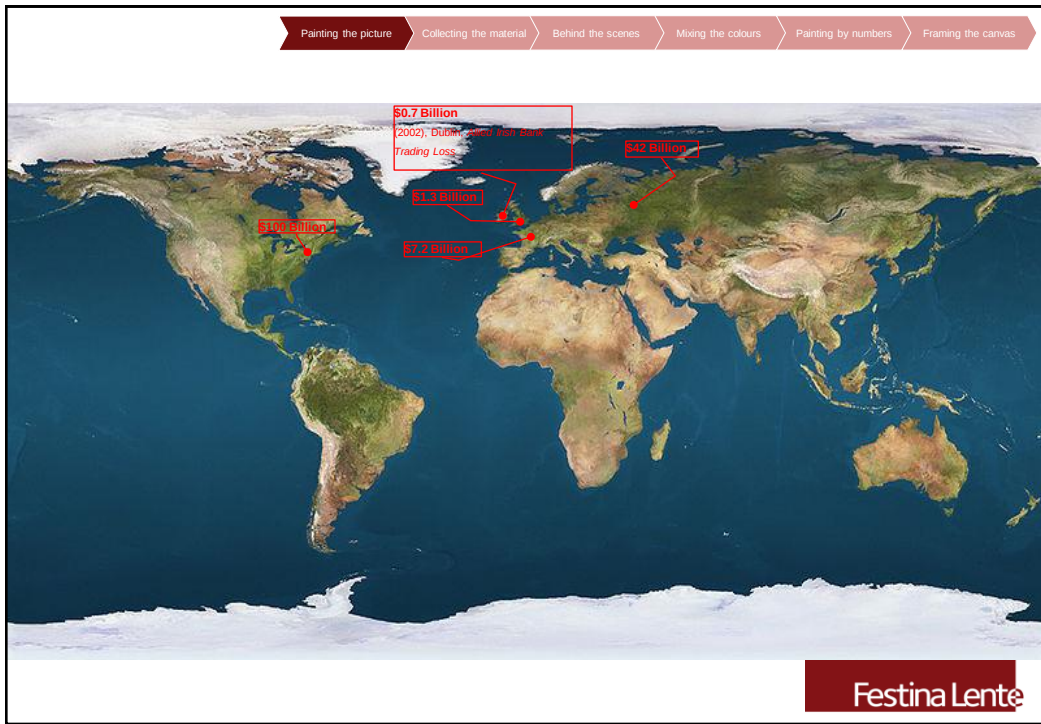
Framing the canvas

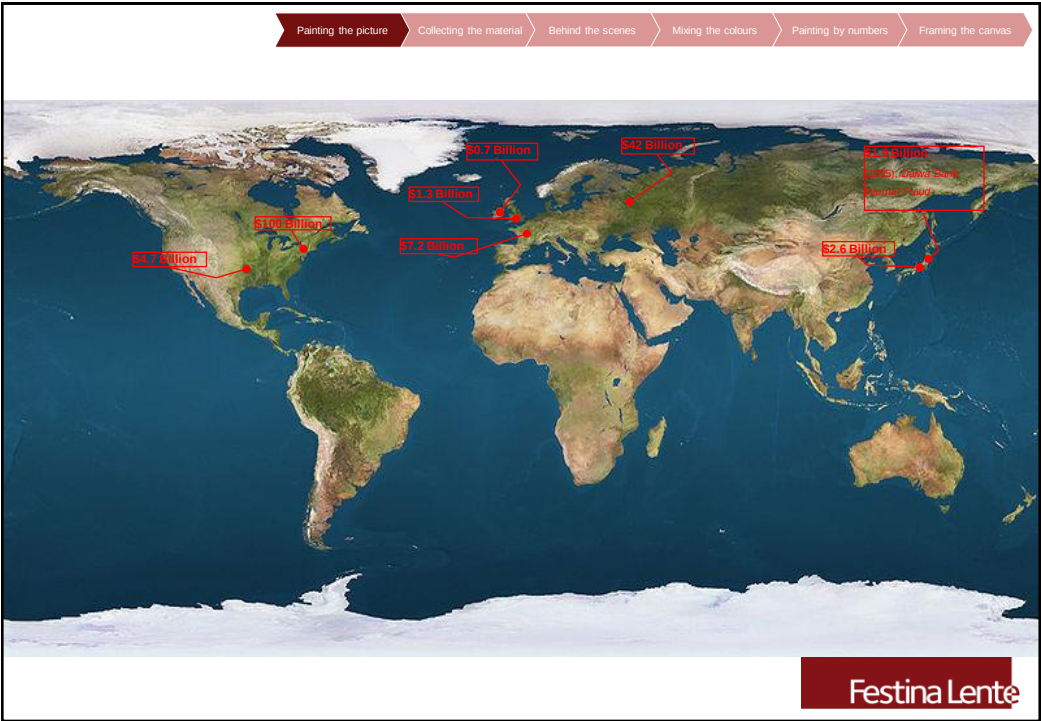
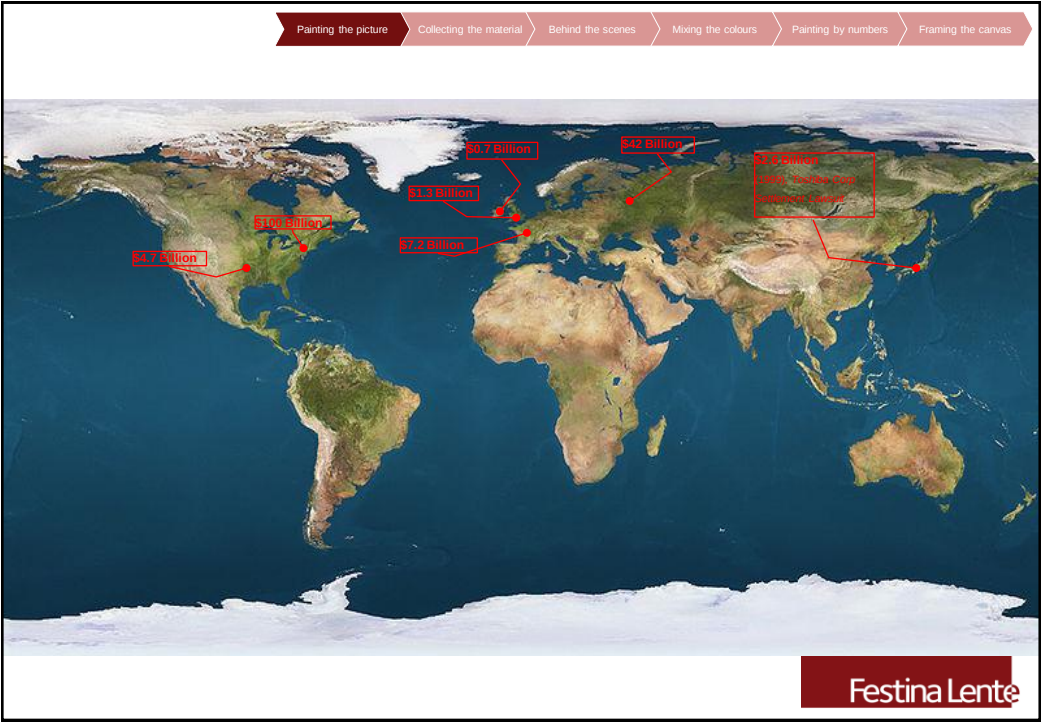


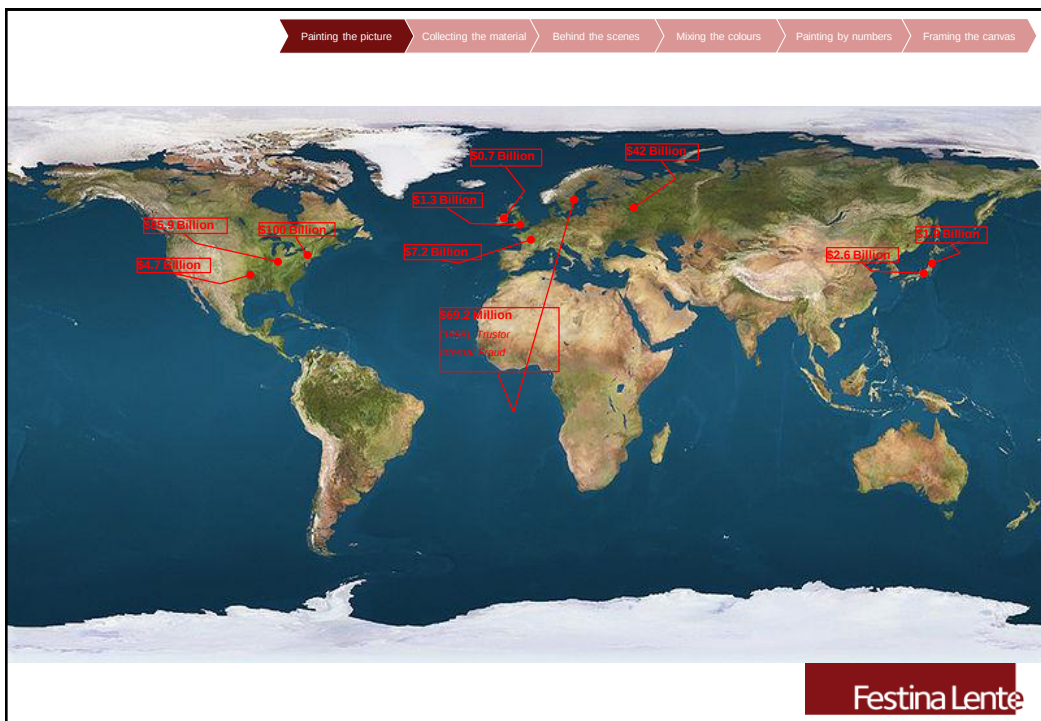
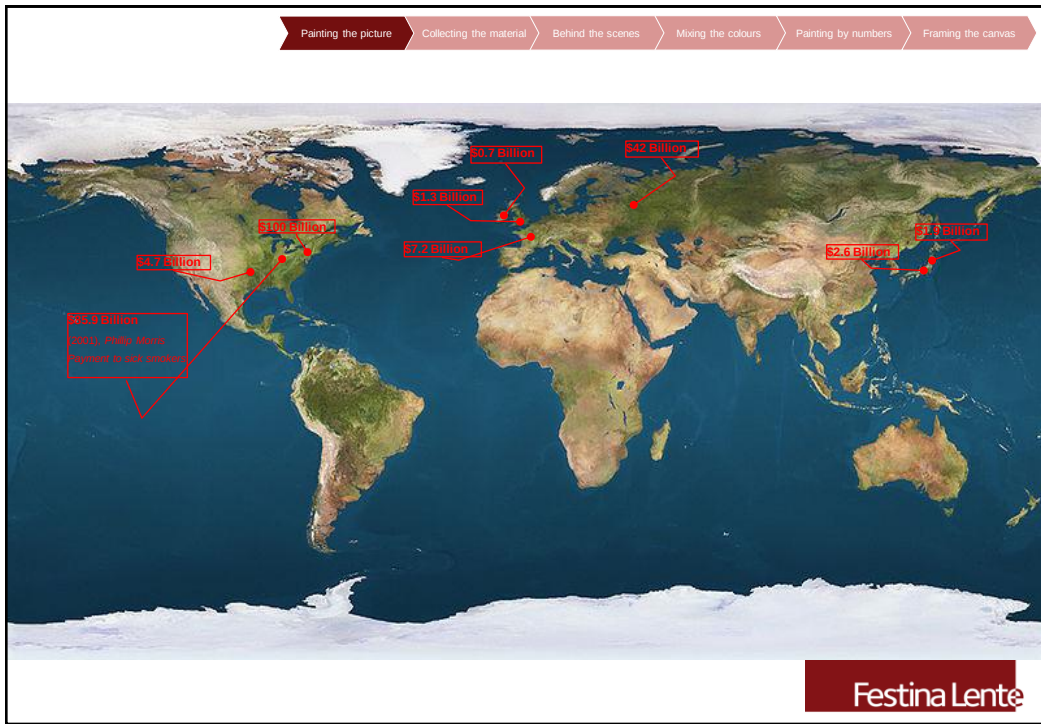
Festina Lente

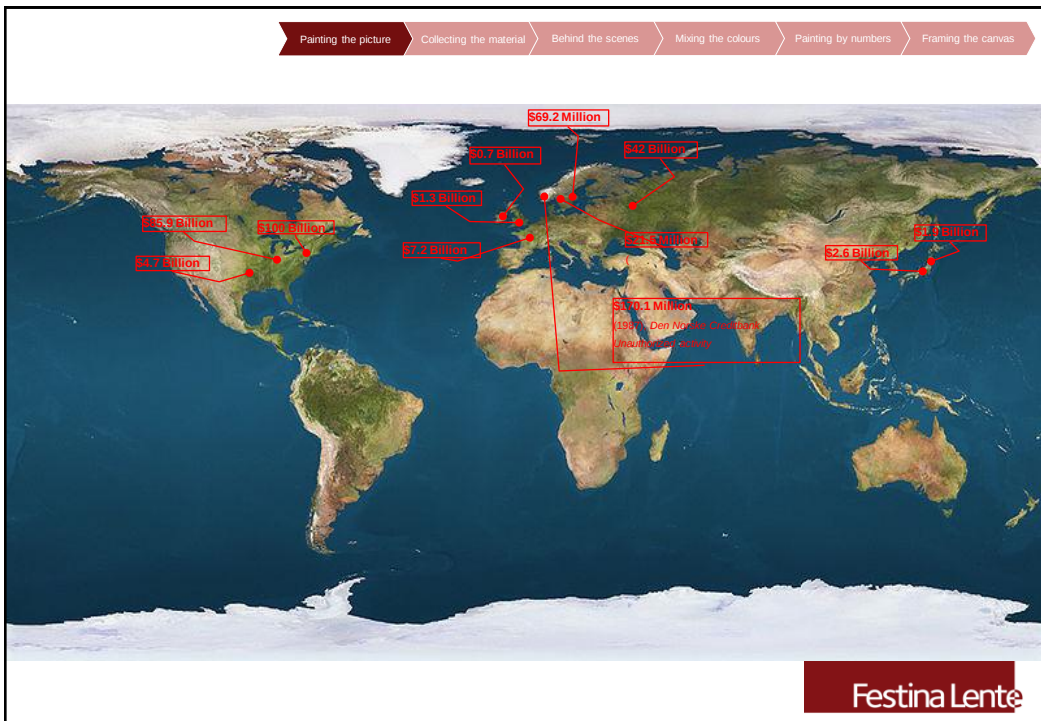
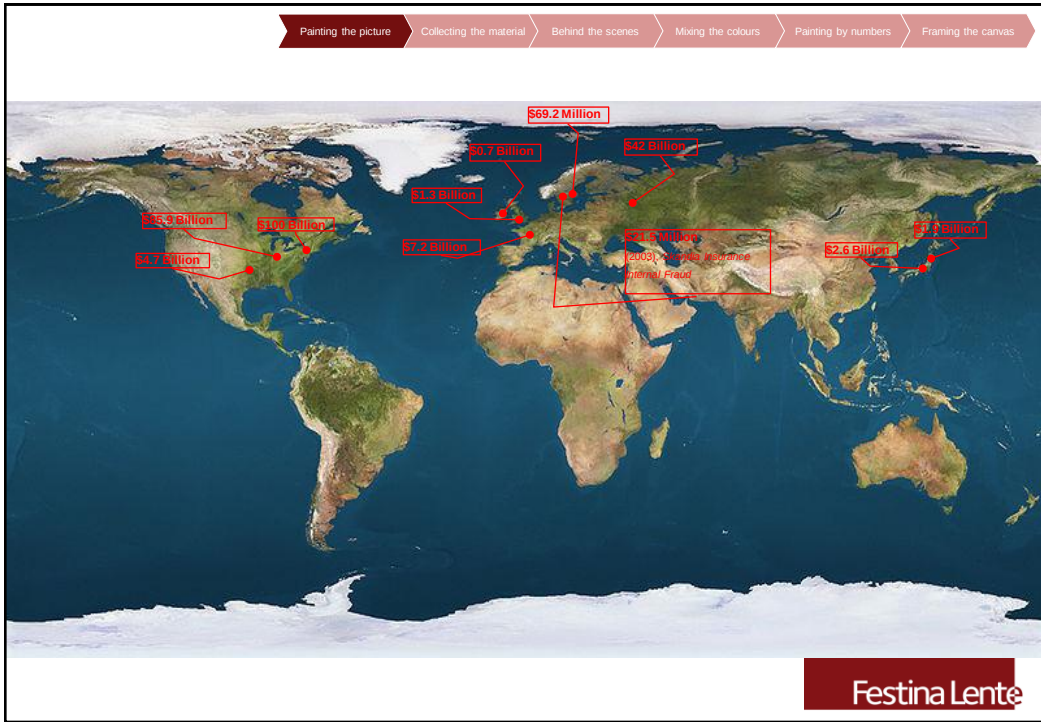


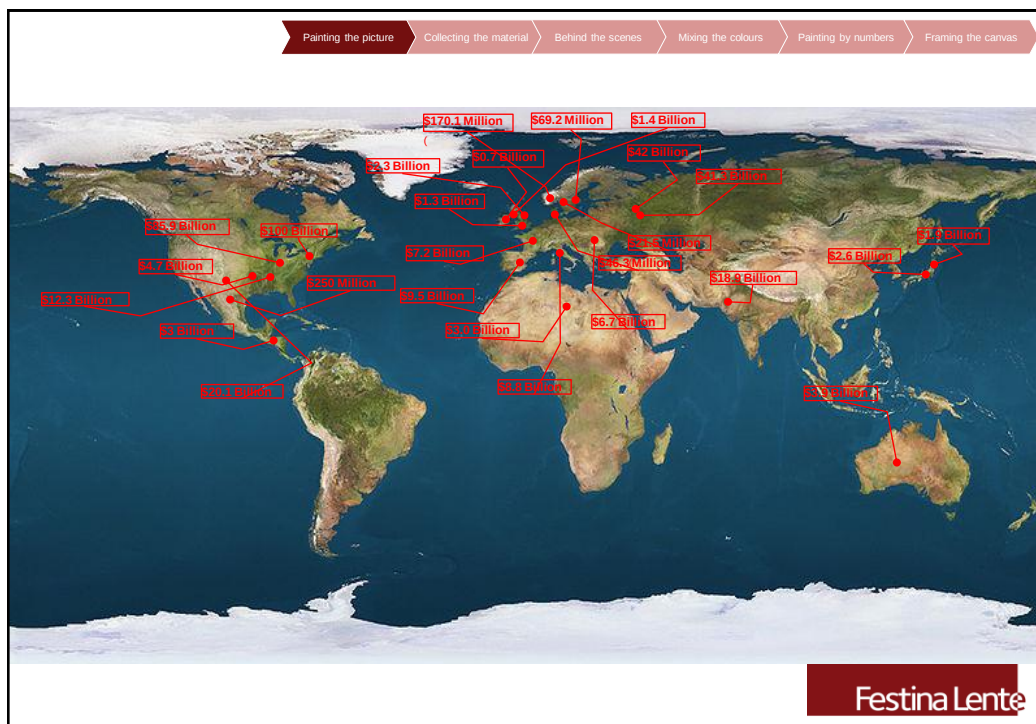








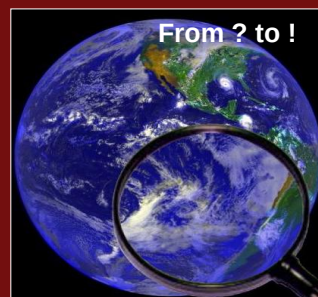




Painting the picture Collecting the material Behind the scenes Mixing the colours Painting by numbers Framing the canvas

Definition

Operational risk is the risk of economic loss resulting from inadequate or failed internal process and methodologies, people, systems, or from external events.



Festina Lente

Painting the picture Collecting the material Behind the scenes Mixing the colours Painting by numbers Framing the canvas

Operational Risk Universe



Festina Lente



➤ Collecting the materials

Festina Lente

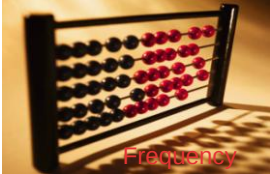
Painting the picture > Collecting the material > Behind the scenes > Mixing the colours > Painting by numbers > Framing the canvas

Operational Risk Quantification

Festina Lente

Painting the picture Collecting the material Behind the scenes Mixing the colours Painting by numbers Framing the canvas

Operational Risk Quantification



Frequency



Severity

Festina Lente

Painting the picture Collecting the material Behind the scenes Mixing the colours Painting by numbers Framing the canvas

Operational Risk Quantification

Scenario 1



Frequency



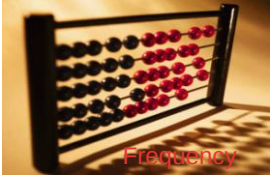
Severity




Festina Lente

Painting the picture Collecting the material Behind the scenes Mixing the colours Painting by numbers Framing the canvas

Operational Risk Quantification



Frequency



Severity

Scenario 1



Scenario 2






Festina Lente

Painting the picture Collecting the material Behind the scenes Mixing the colours Painting by numbers Framing the canvas

Operational Risk Quantification



Frequency



Severity

Scenario 1



Scenario 2



Scenario 3



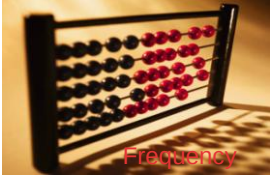




Festina Lente

Painting the picture Collecting the material Behind the scenes Mixing the colours Painting by numbers Framing the canvas

Operational Risk Quantification



Frequency



Severity

Scenario 1




Scenario 2




Scenario 3




...

Scenario n




Festina Lente

Painting the picture Collecting the material Behind the scenes Mixing the colours Painting by numbers Framing the canvas

Operational Risk Quantification



Frequency



Severity

Scenario 1





Scenario 2




Scenario 3




...

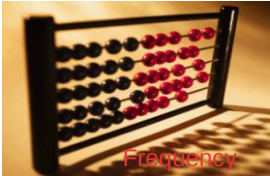
Scenario n




Festina Lente

Painting the picture Collecting the material Behind the scenes Mixing the colours Painting by numbers Framing the canvas

Operational Risk Quantification




Scenario 1



Scenario 2



Scenario 3



Scenario n



Festina Lente

Painting the picture Collecting the material Behind the scenes Mixing the colours Painting by numbers Framing the canvas

Operational Risk Quantification




Scenario 1



Scenario 2



Scenario 3



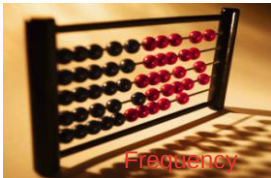
Scenario n



Festina Lente

Painting the picture Collecting the material Behind the scenes Mixing the colours Painting by numbers Framing the canvas

Operational Risk Quantification



Frequency



Severity

Scenario 1




Scenario 2




Scenario 3




...

Scenario n




Festina Lente

Painting the picture Collecting the material Behind the scenes Mixing the colours Painting by numbers Framing the canvas

Operational Risk Quantification



Frequency



Severity

Scenario 1




Scenario 2




Scenario 3




...

Scenario n




What type of frequency and severity model should a company use to predict a correct operational risk exposure?

Festina Lente

Painting the picture

Collecting the material

Behind the scenes

Mixing the colours

Painting by numbers

Framing the canvas

A Comprehensive overview

Festina Lente

Painting the picture

Collecting the material

Behind the scenes

Mixing the colours

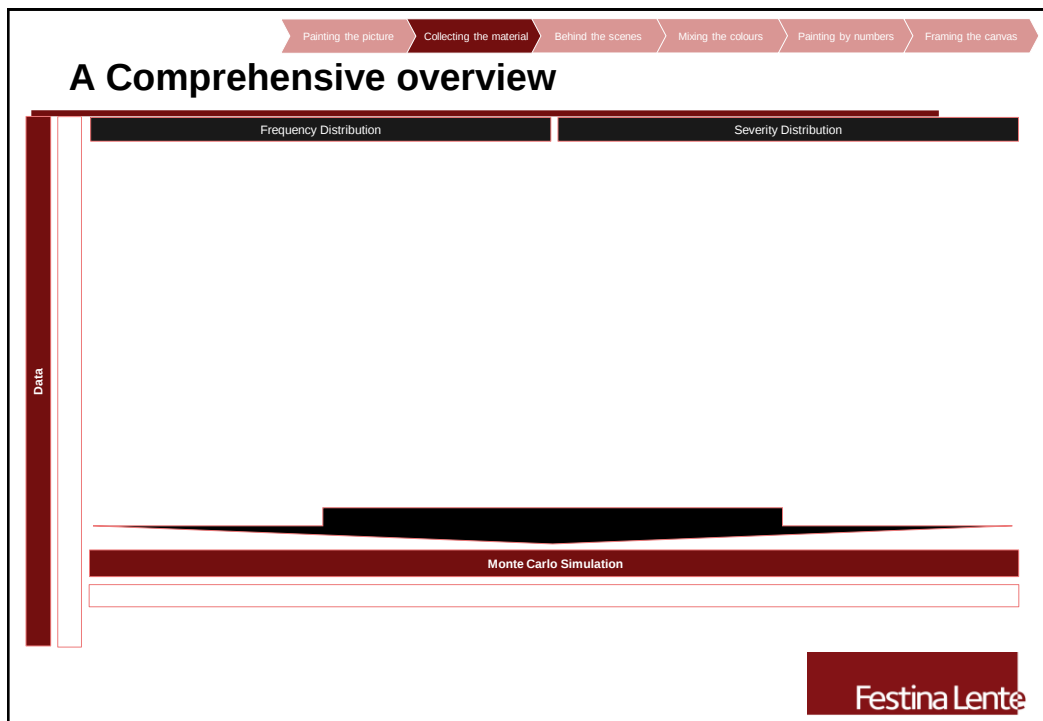
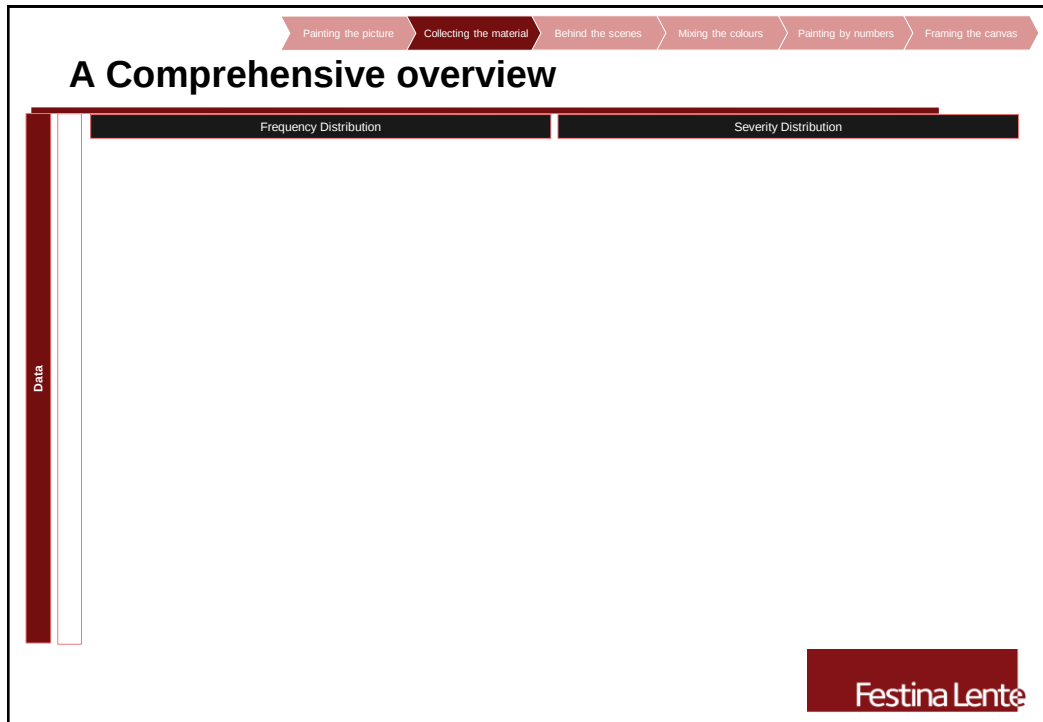
Painting by numbers

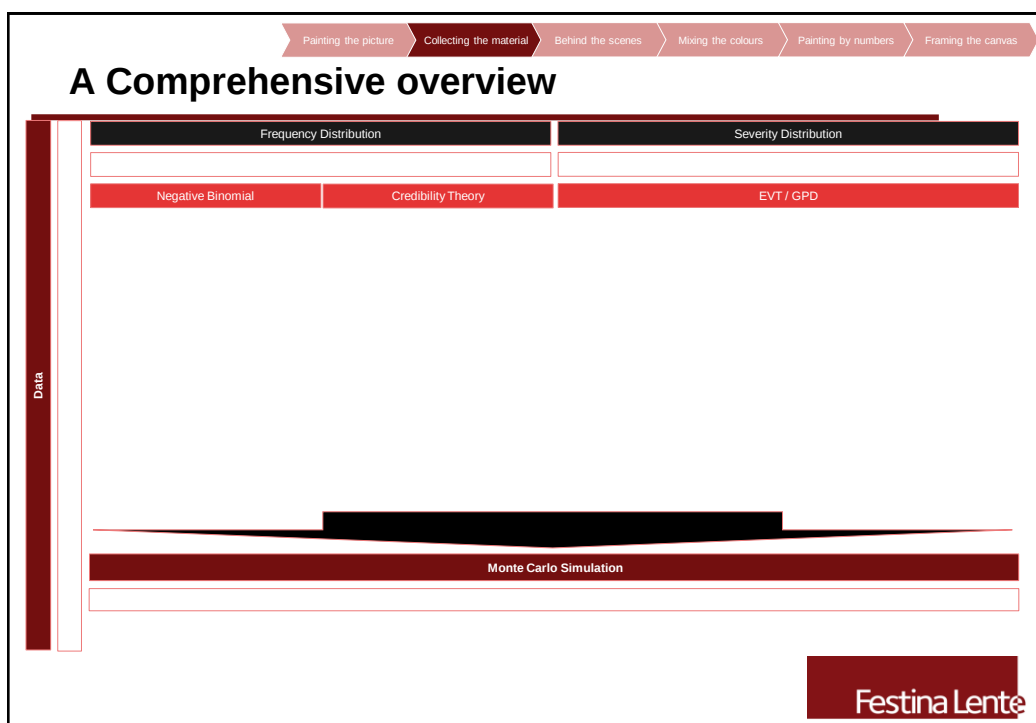
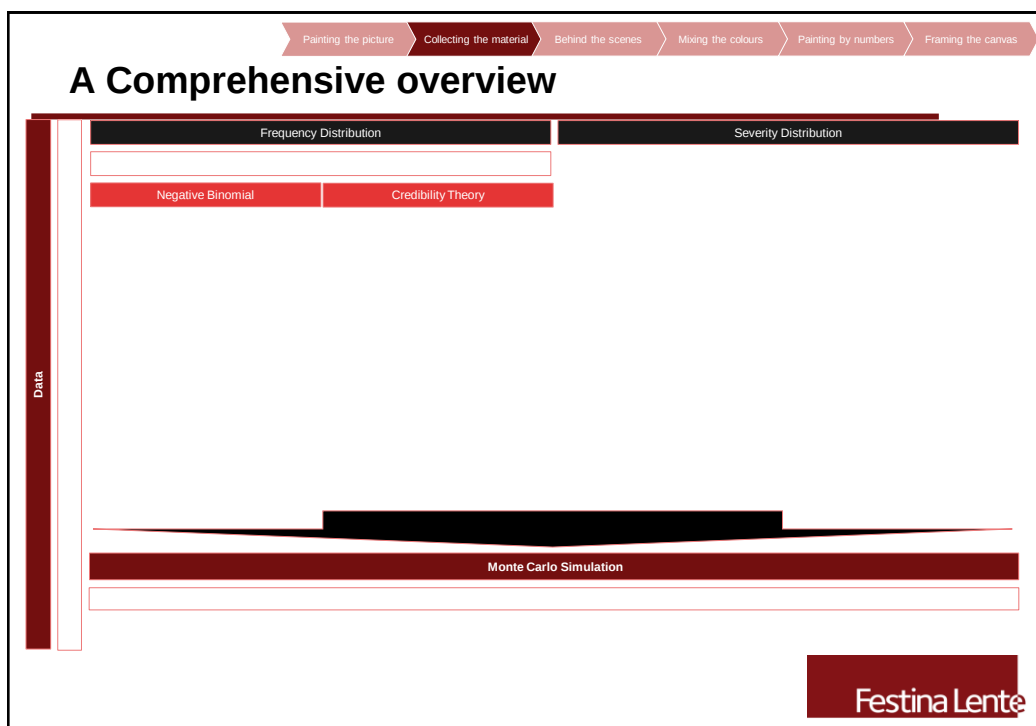
Framing the canvas

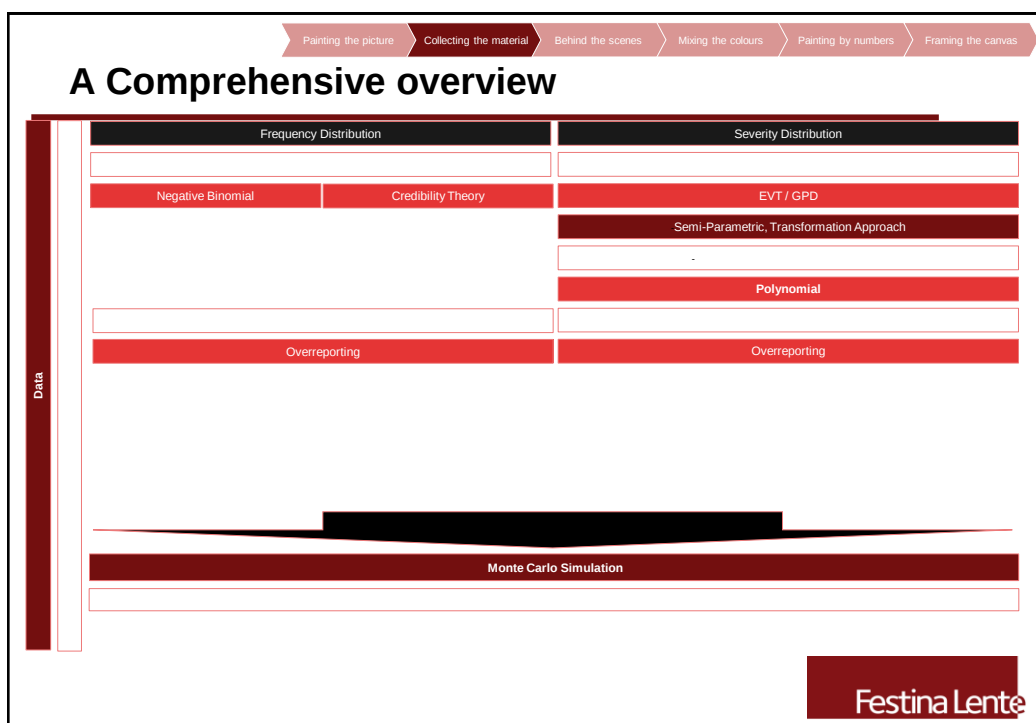
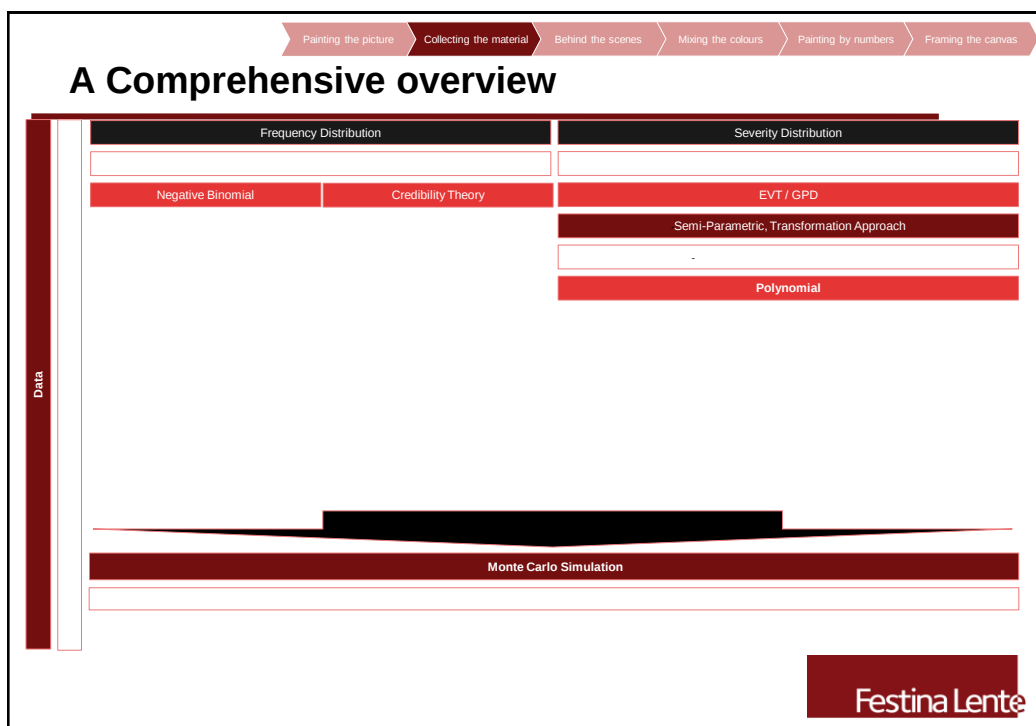
A Comprehensive overview

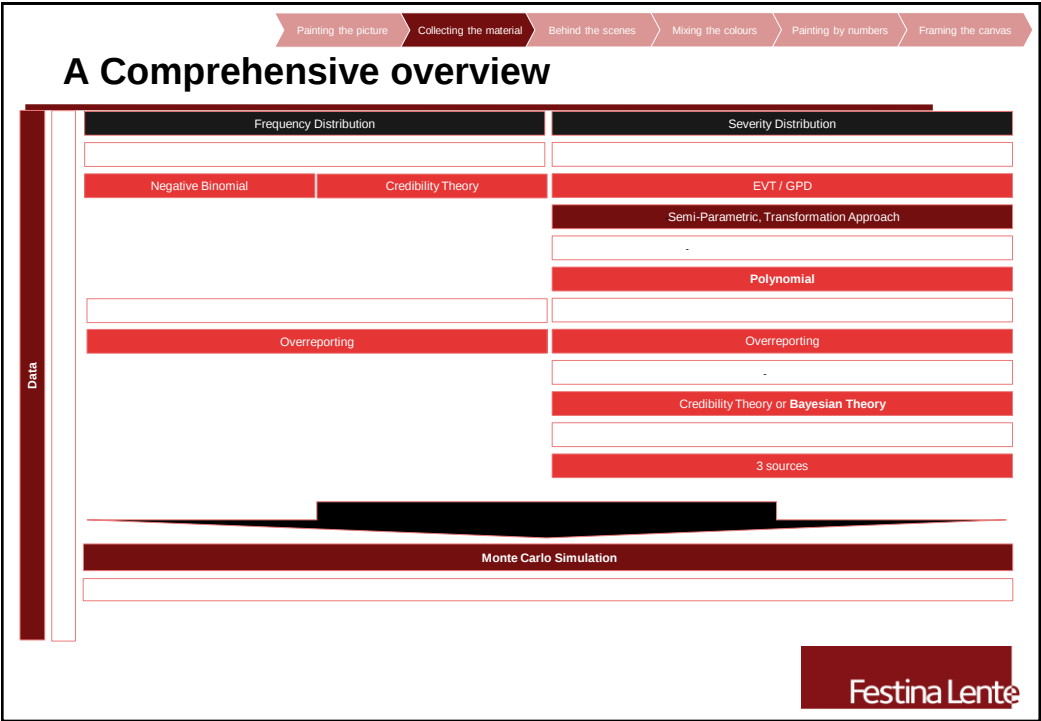
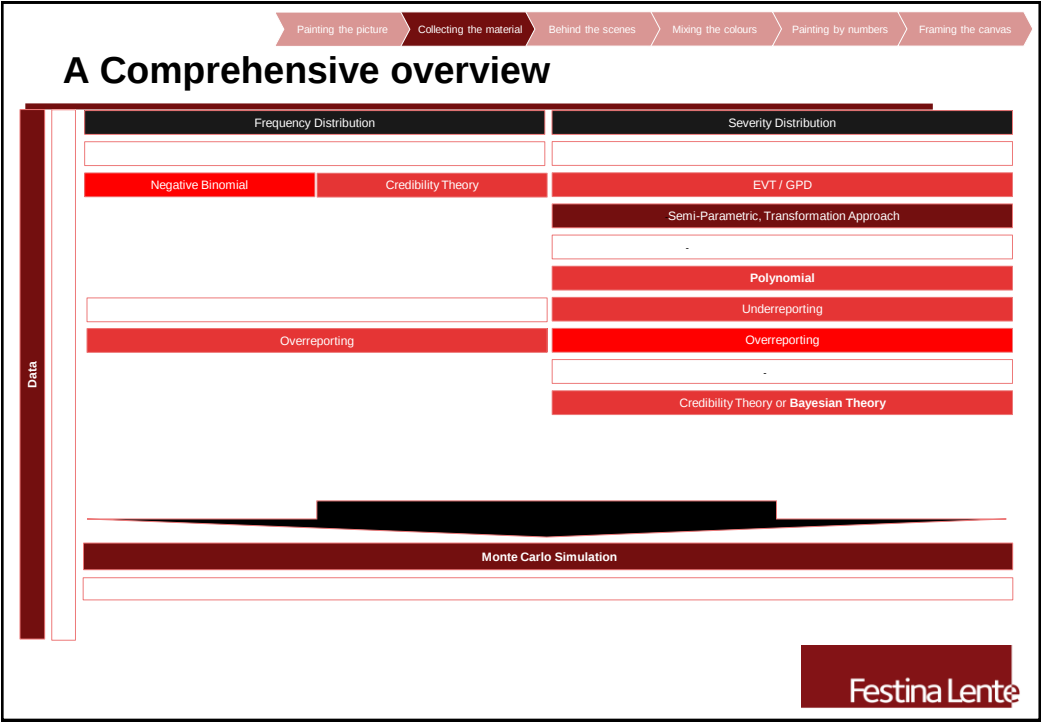
Data

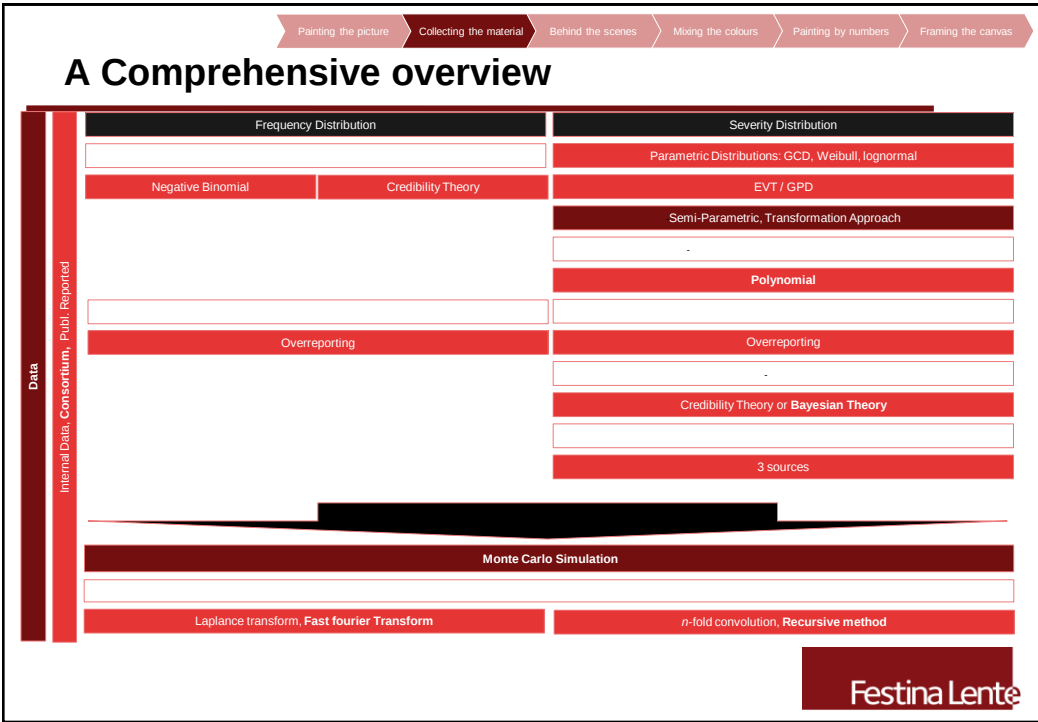
Festina Lente



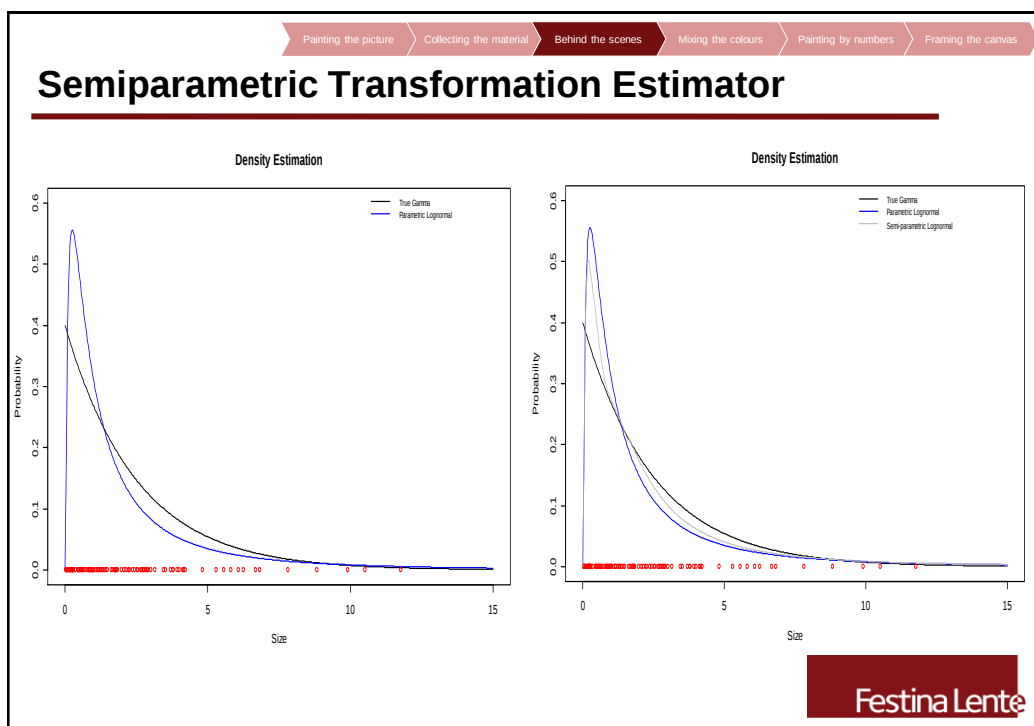








➤ Behind the scenes



Painting the picture Collecting the material Behind the scenes Mixing the colours Painting by numbers Framing the canvas

Semiparametric Transformation Estimator

- When data are limited, the model is close to a parametric model.
- As the number of losses increases, the model becomes more non-parametric.

Distribution Assumption. Estimate the distribution parameters on the data $(X_i)_{1 \leq i \leq 100}$

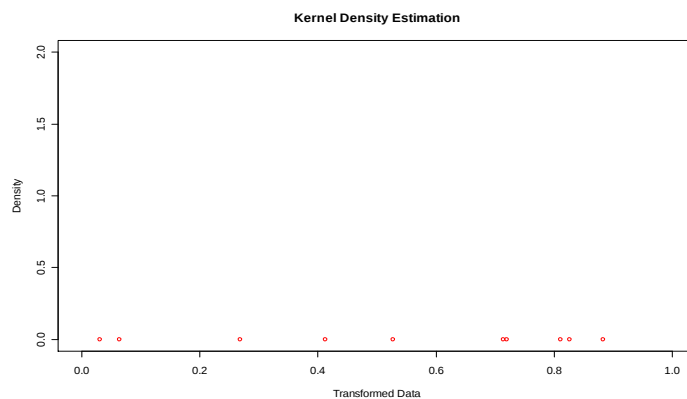
Transform losses to the bounded support $[0,1]$ with $Z_i = F_\theta(X_i)$

KDE: $\hat{g}(z) = \frac{1}{100} \sum_{i=1}^{100} K_h(z - z_i)$ with $K(z) = \frac{3}{4} \begin{cases} -z^2 & |z| < 1 \\ 0 & |z| \geq 1 \end{cases}$ and $K_h(\cdot) = \frac{1}{h} K\left(\frac{\cdot}{h}\right)$

Semi-parametric density, $\hat{f}(x) = f_\theta(x) \hat{g}(F_\theta(x))$

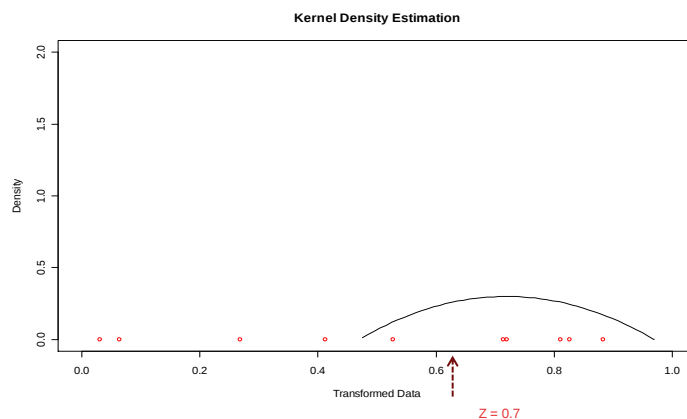
Festina Lente

Semiparametric Transformation Estimator



Festina Lente

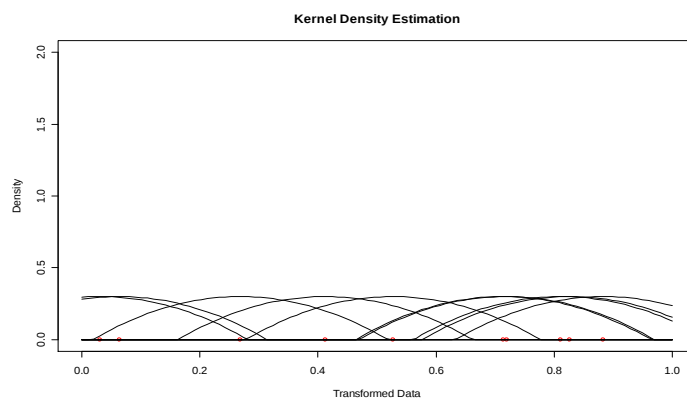
Semiparametric Transformation Estimator



Festina Lente

Painting the picture > Collecting the material > Behind the scenes > Mixing the colours > Painting by numbers > Framing the canvas

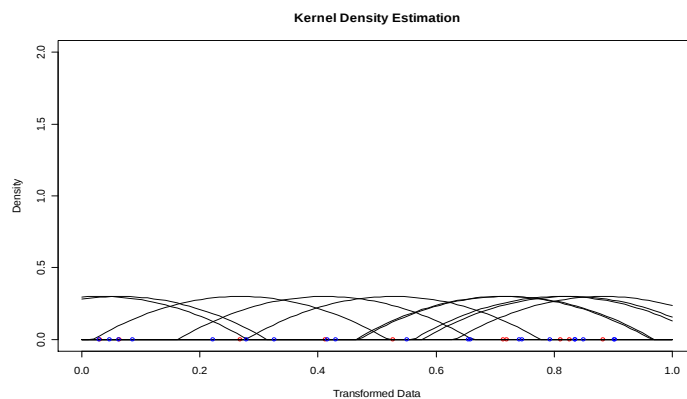
Semiparametric Transformation Estimator



Festina Lente

Painting the picture > Collecting the material > Behind the scenes > Mixing the colours > Painting by numbers > Framing the canvas

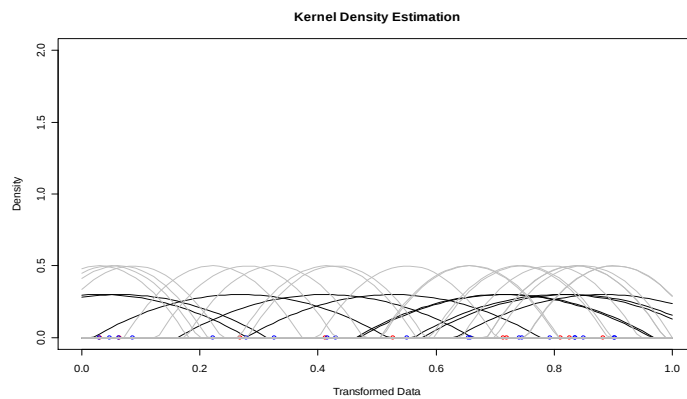
Semiparametric Transformation Estimator



Festina Lente

Painting the picture Collecting the material Behind the scenes Mixing the colours Painting by numbers Framing the canvas

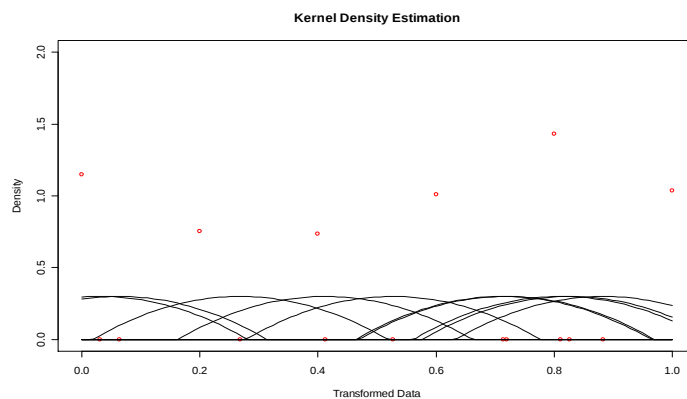
Semiparametric Transformation Estimator



Festina Lente

Painting the picture Collecting the material Behind the scenes Mixing the colours Painting by numbers Framing the canvas

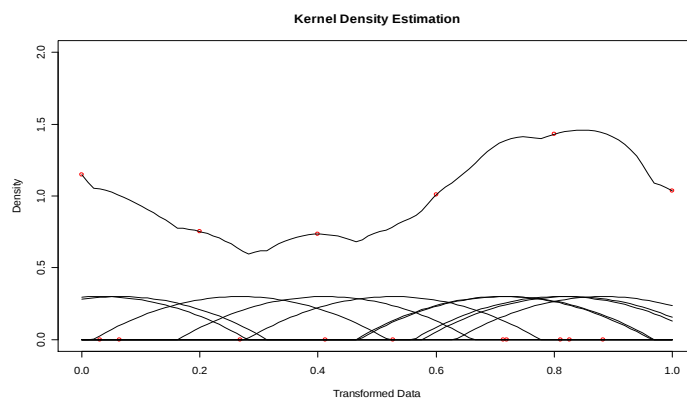
Semiparametric Transformation Estimator



Festina Lente

Painting the picture Collecting the material Behind the scenes Mixing the colours Painting by numbers Framing the canvas

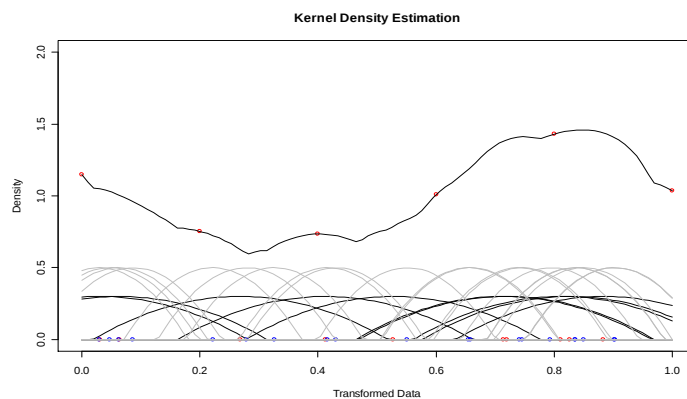
Semiparametric Transformation Estimator



Festina Lente

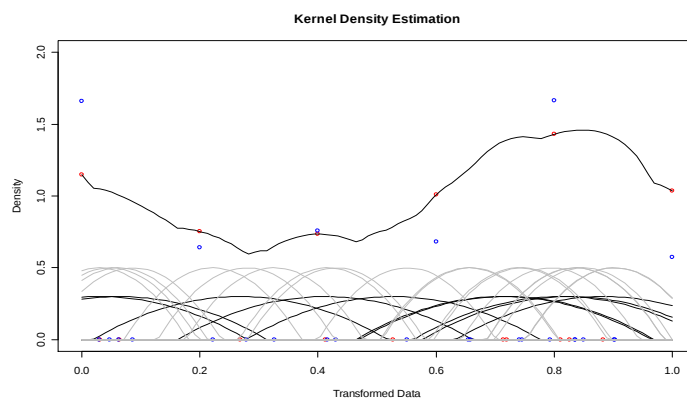
Painting the picture Collecting the material Behind the scenes Mixing the colours Painting by numbers Framing the canvas

Semiparametric Transformation Estimator



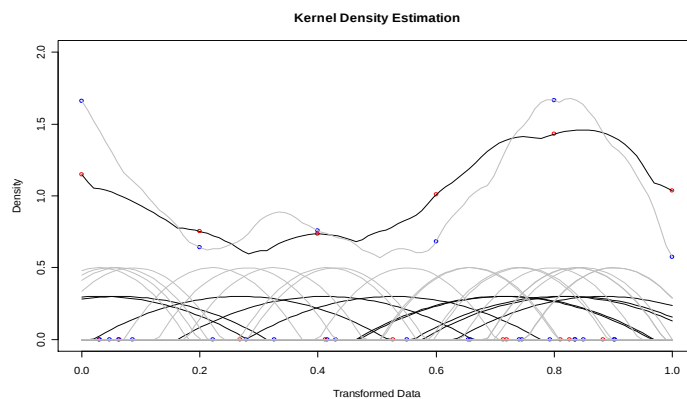
Festina Lente

Semiparametric Transformation Estimator



Festina Lente

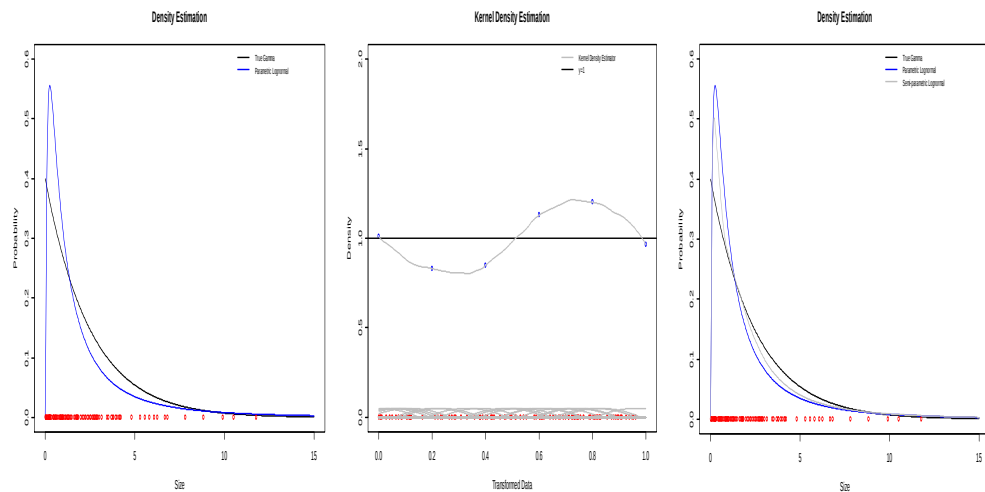
Semiparametric Transformation Estimator



Festina Lente

Painting the picture Collecting the material Behind the scenes **Mixing the colours** Painting by numbers Framing the canvas

Semiparametric Transformation Estimator



Festina Lente



➤ **Mixing the colours**

Festina Lente

Painting the picture Collecting the material Behind the scenes Mixing the colours Painting by numbers Framing the canvas

Underreporting Model

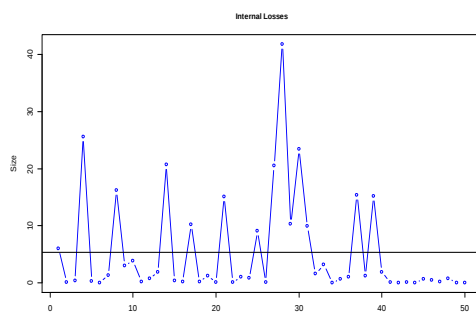
Underreporting means that not all losses in the company are reported.

Festina Lente

Painting the picture Collecting the material Behind the scenes Mixing the colours Painting by numbers Framing the canvas

Underreporting Model

Underreporting means that not all losses in the company are reported.



Occurred losses $(X_i)_{1 \leq i \leq g}$

Indicator function

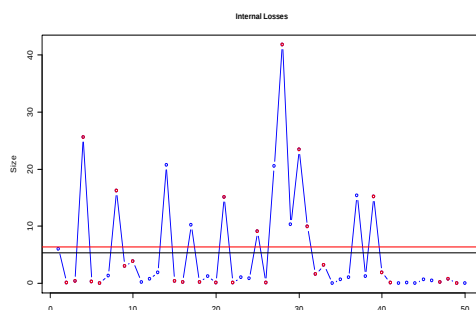
$$I(i) = \begin{cases} 1 & \text{if } X_i \text{ is reported} \\ 0 & \text{otherwise} \end{cases}$$

Festina Lente

Painting the picture Collecting the material Behind the scenes Mixing the colours Painting by numbers Framing the canvas

Underreporting Model

Losses could be hidden, ignored, unrecorded or reported elsewhere



Reported losses $(Y_j)_{1 \leq j \leq 25} \approx f$

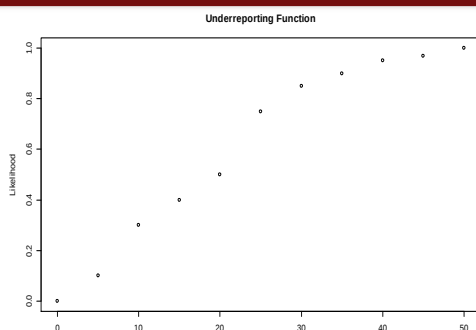
Total number of reported: $N = \sum_{i=1}^{50} I(i)$

Festina Lente

Painting the picture Collecting the material Behind the scenes Mixing the colours Painting by numbers Framing the canvas

Underreporting Model

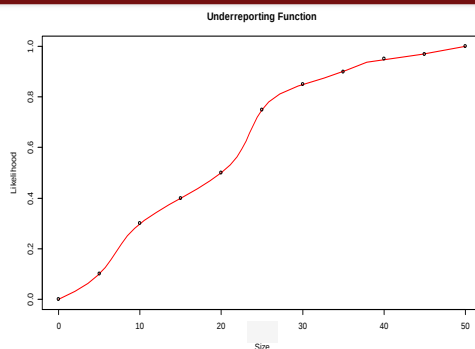
An underreporting function encodes the likelihood that a loss of a particular size is being reported



Festina Lente

Underreporting Model

An underreporting function encodes the likelihood that a loss of a particular size is being reported



Define g as a function of f and underreporting function u

$$g(x) = \frac{f(x)\{u(x)\}^{-1}}{\int f(w)\{u(w)\}^{-1}dw}$$

The probability of observing an Operational risk loss

$$P = \int_0^{\infty} g(w)u(w)dw$$

Assume Poisson distribution, then occurred / reported losses

$$M \in Po(\lambda), \quad N \in Po(\lambda \cdot P)$$

Festina Lente

Mixing Model

- Include prior knowledge from external data.
- Correct the external global start with internal observed data.

Festina Lente

Painting the picture Collecting the material Behind the scenes Mixing the colours Painting by numbers Framing the canvas

Mixing Model

- Include prior knowledge from external data.
- Correct the external global start with internal observed data.

Distribution Assumption. Estimate the distribution parameters on the data $(Y_j)_{1 \leq j \leq M}$

Transform losses to the bounded support $[0,1]$ with $Z_i = F_{\hat{\theta}_y}(X_i)$

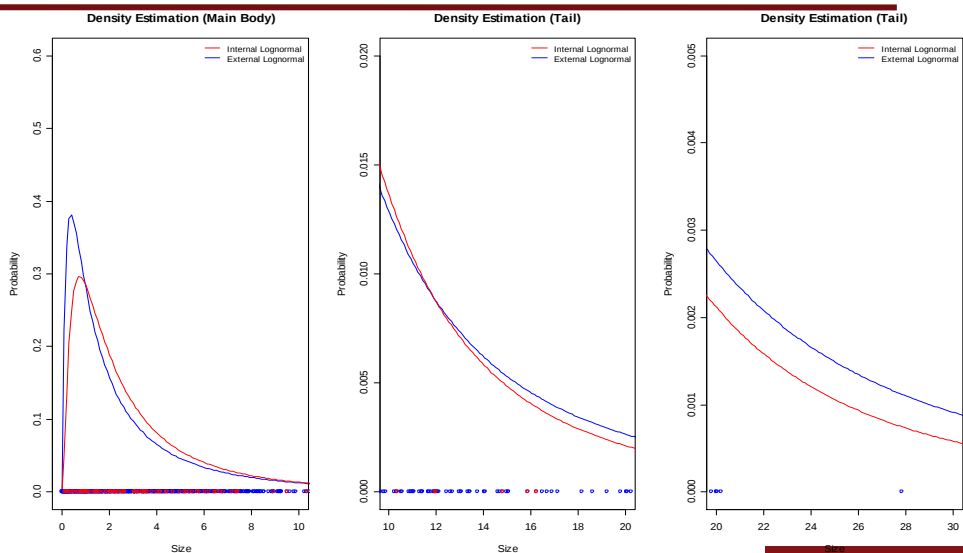
KDE: $\hat{g}(z) = \frac{1}{100} \sum_{i=1}^{100} K_h(z - z_i)$ with $K(z) = \frac{3}{4} \left(1 - z^2 \right) \mathbb{I}_{|z| < 1}$ and $K_h(\cdot) = \frac{1}{h} K\left(\frac{\cdot}{h}\right)$

Semi-parametric density, $\hat{f}_M(x) = f_{\hat{\theta}_y}(x) \hat{g}(F_{\hat{\theta}_y}(x))$

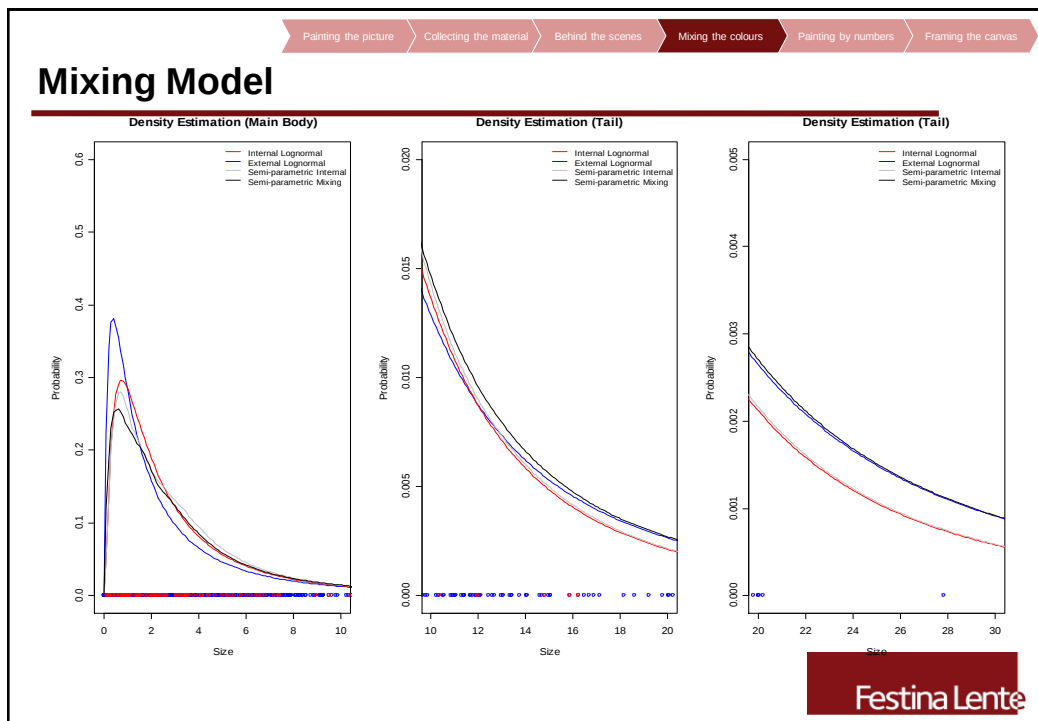
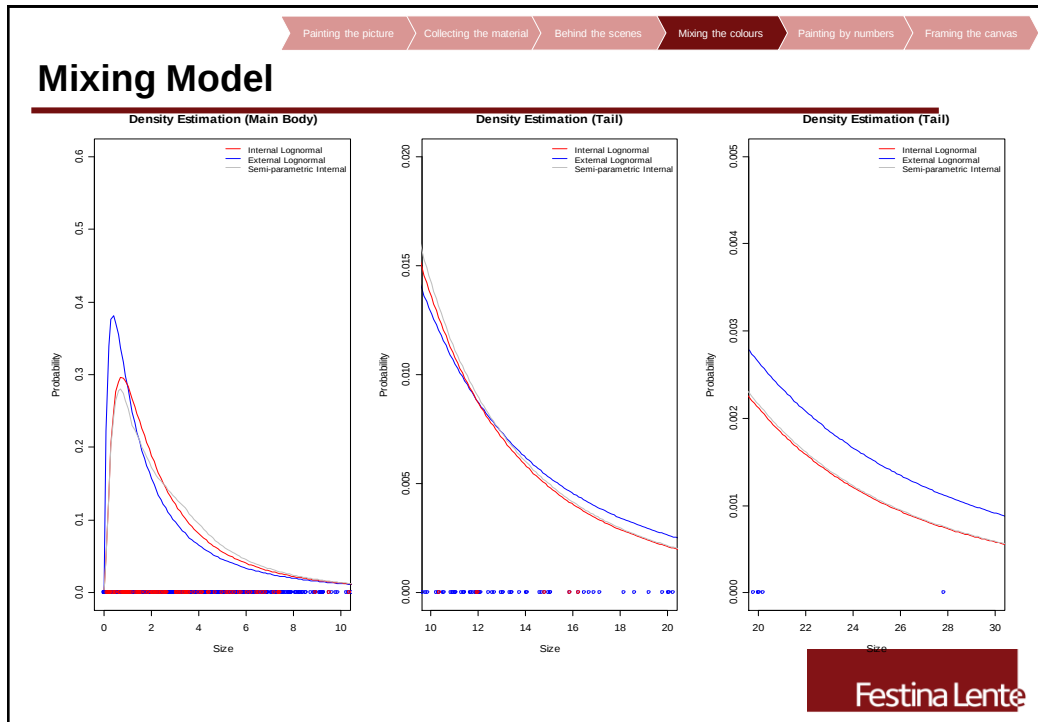
Festina Lente

Painting the picture Collecting the material Behind the scenes Mixing the colours Painting by numbers Framing the canvas

Mixing Model



Festina Lente



Painting the picture > Collecting the material > Behind the scenes > Mixing the colours > Painting by numbers > Framing the canvas

Asymptotic Theory: Semiparametric Estimator

Let the transformation function T be two times differentiable known function. Assume that f is also two times differentiable. Then the bias of $\hat{f}$ is given by

$$\begin{aligned} E\hat{f}(x) - f(x) &= \mu_2(K)B_x b^2 + o(b^2) \\ &= \frac{1}{2} \int w^2 K(w) dw \left[\left(\frac{f(x)}{T'(x)} \right)' \frac{1}{T'(x)} \right] b^2 + o(b^2) \end{aligned}$$

and the variance is given by

$$\begin{aligned} V\hat{f}(x) &= (nb)^{-1} R(K)T'(x) f(x) + o((nb)^{-1}) \\ &= (nb)^{-1} \int K^2(w) dw T'(x) f(x) + o((nb)^{-1}) \end{aligned}$$

where the asymptotics is given for $n \rightarrow \infty$.



Festina Lente

Painting the picture > Collecting the material > Behind the scenes > Mixing the colours > Painting by numbers > Framing the canvas

Asymptotic Theory: Semiparametric Estimator corrected for UR

Let the transformation function T and the underreporting function u be two times differentiable known function. Assume that g is also two times differentiable. Then the bias of $\hat{g}$ is given by

$$E\hat{g}(x) - g(x) = \mu_2(K) \left[\frac{B_x}{u(x)B} - \frac{A}{B^2} \int \frac{B_w}{u(w)} dw \right] b^2 + o(b^2)$$

where

$$A = \frac{f(x)}{u(x)}, B = \int \frac{f(w)}{u(w)} dw \text{ and } B_x = \left[\left(\frac{f(x)}{T'(x)} \right)' \frac{1}{T'(x)} \right]$$

and the variance is given by

$$V\hat{g}(x) = \frac{R(K)T'(x)g(x)}{u(x)Bmb} + o((mb)^{-1})$$

where the asymptotics is given for

$$m \rightarrow \infty, b \rightarrow 0 \text{ and } n = P \cdot m.$$



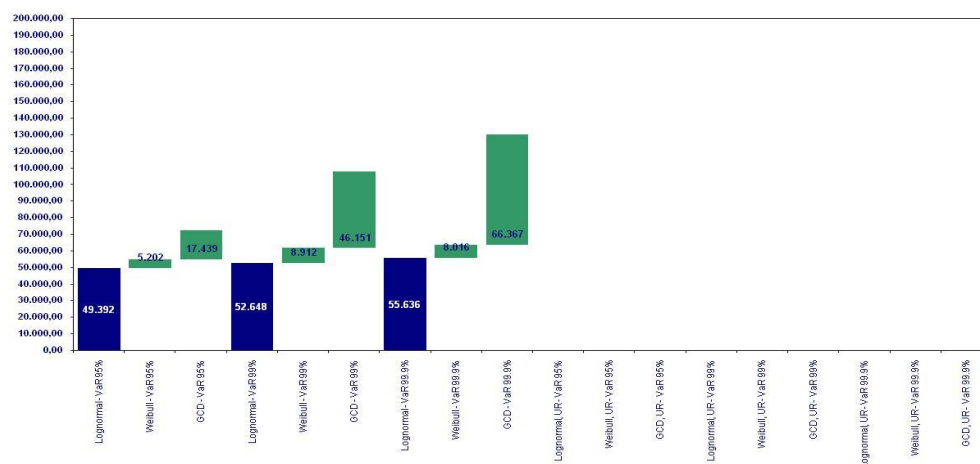
Festina Lente



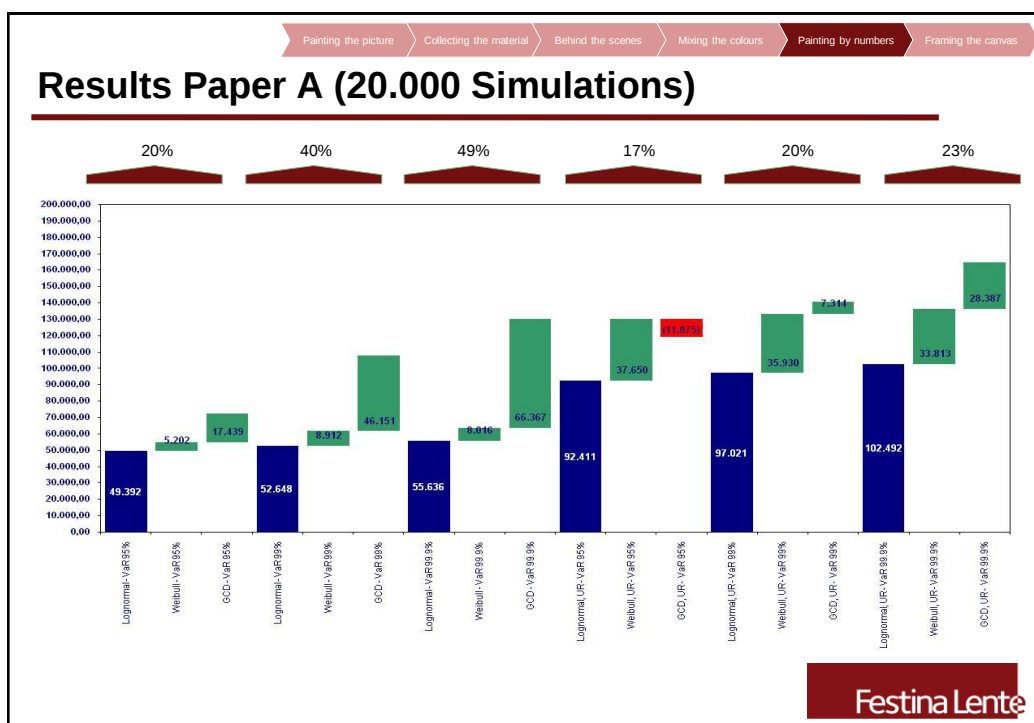
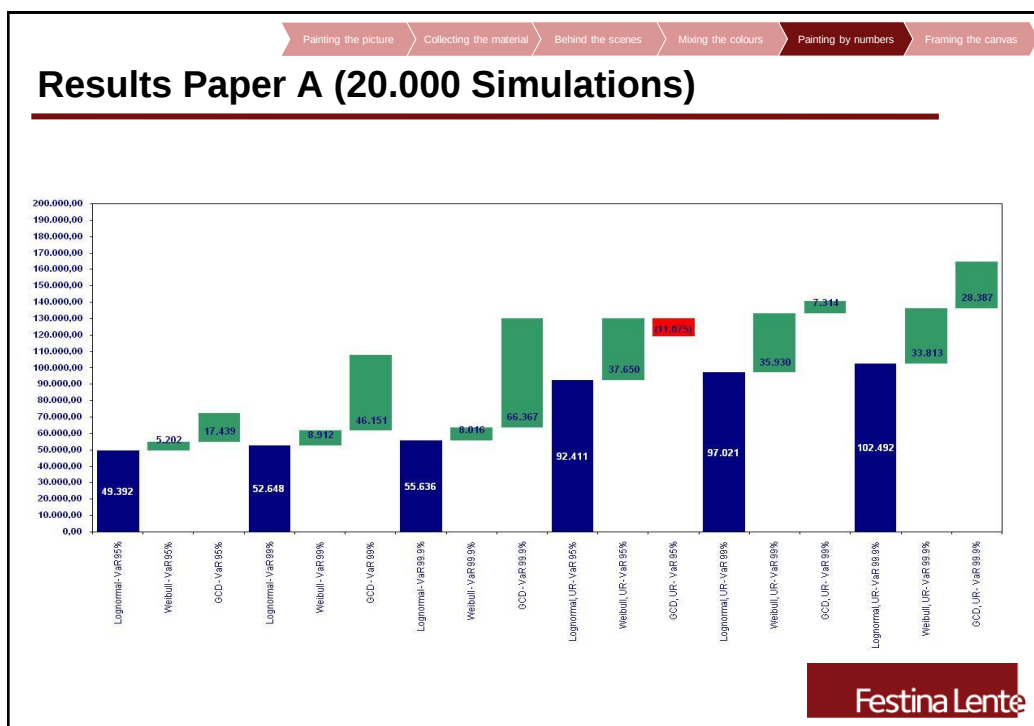
➤ Painting by numbers

Festina Lente

Results Paper A (20.000 Simulations)



Festina Lente



Painting the picture Collecting the material Behind the scenes Mixing the colours Painting by numbers Framing the canvas

Key Findings Paper A

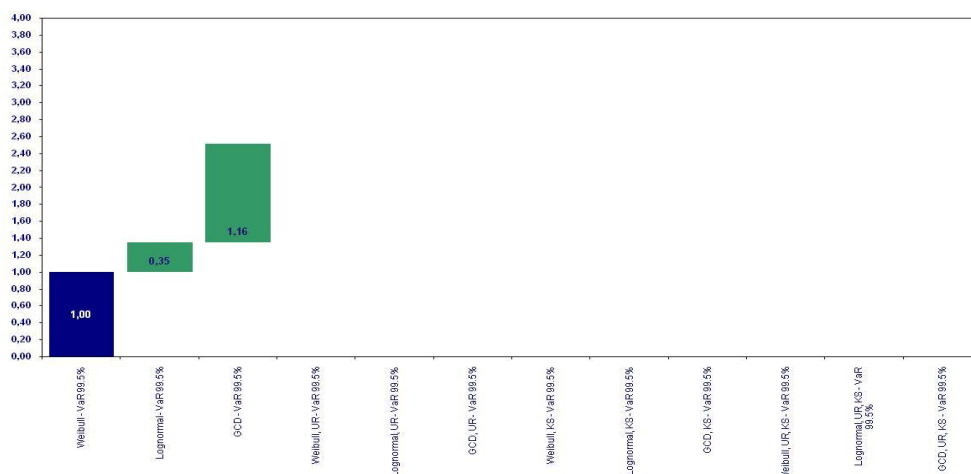
- Distribution dependent results
- Correction for underreporting increases the capital
- A small stabilizing effect by introducing a correction for underreporting



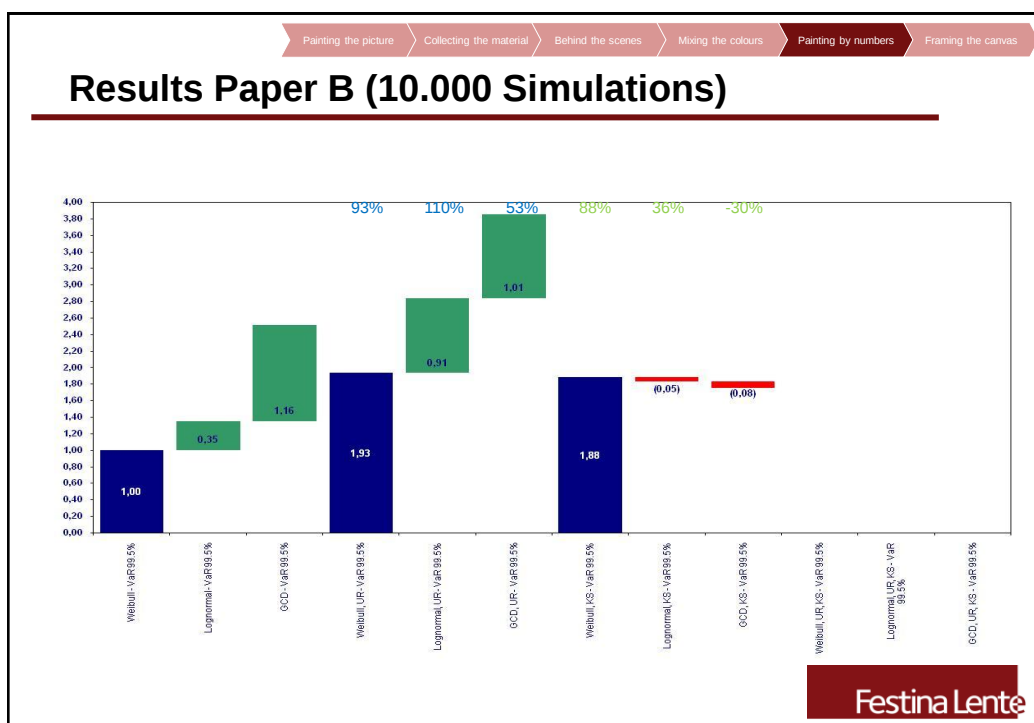
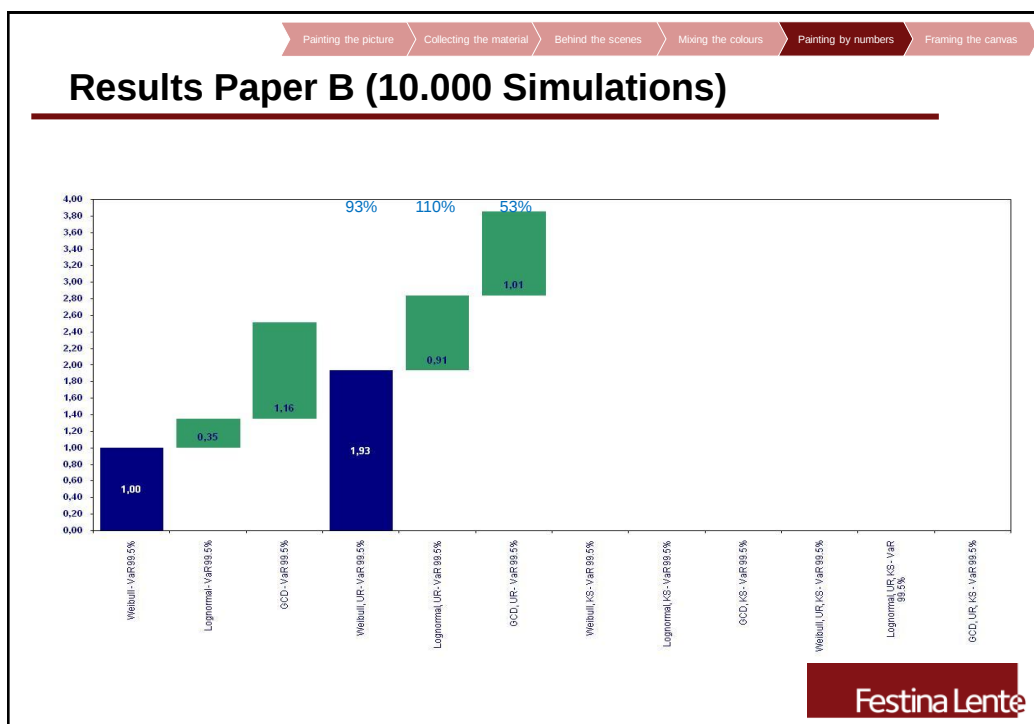
Festina Lente

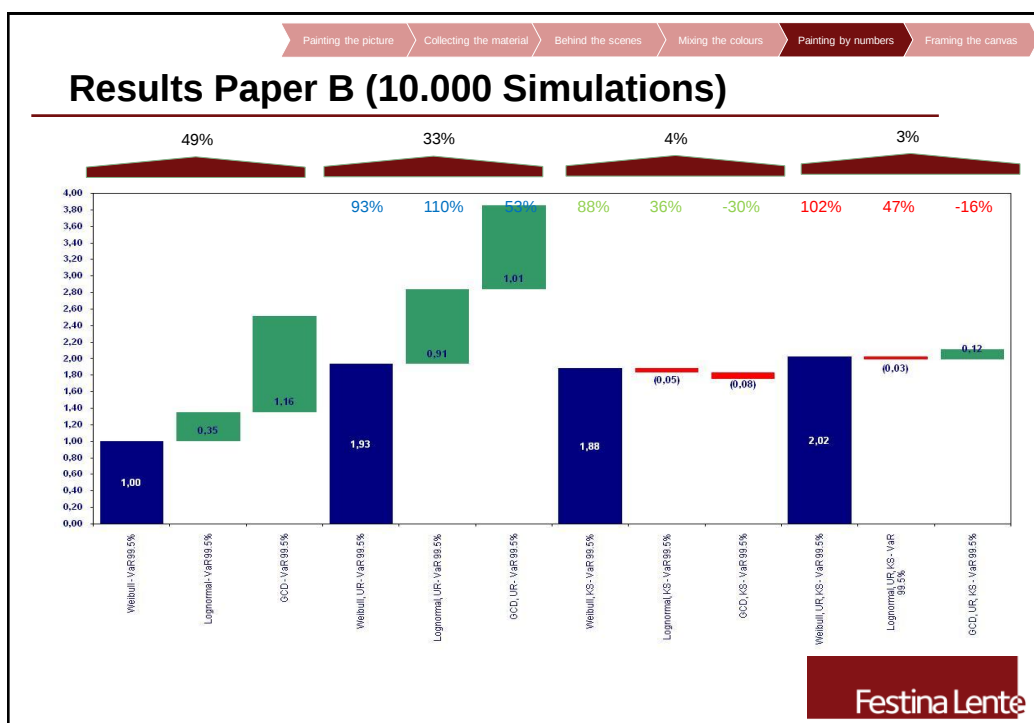
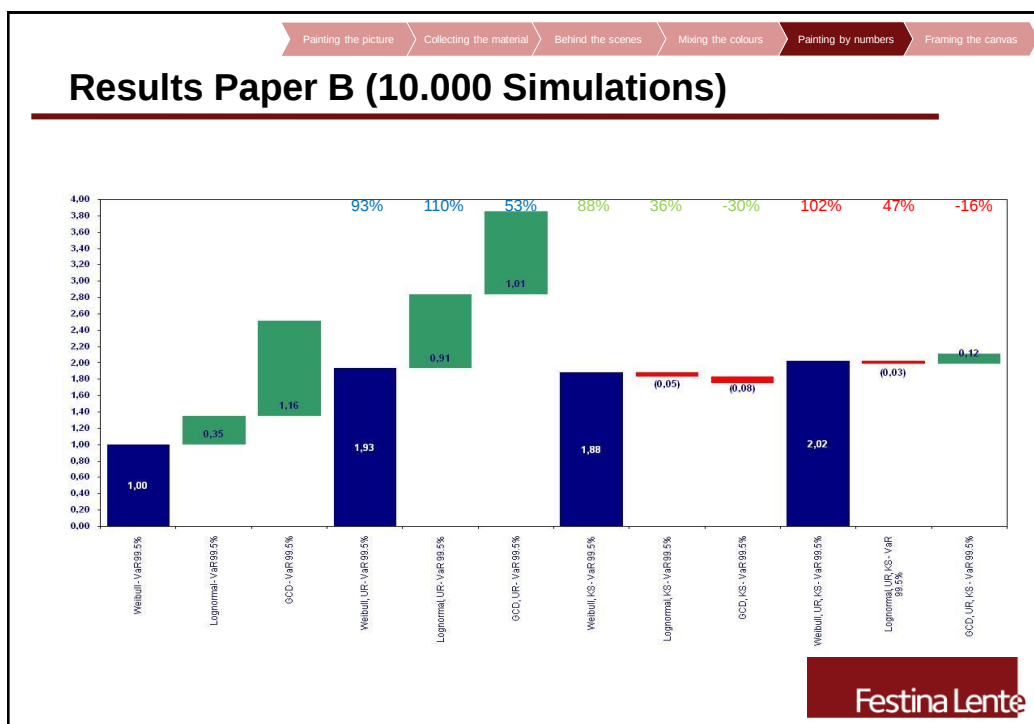
Painting the picture Collecting the material Behind the scenes Mixing the colours Painting by numbers Framing the canvas

Results Paper B (10.000 Simulations)



Festina Lente





Painting the picture > Collecting the material > Behind the scenes > Mixing the colours > Painting by numbers > Framing the canvas

Key Findings Paper B

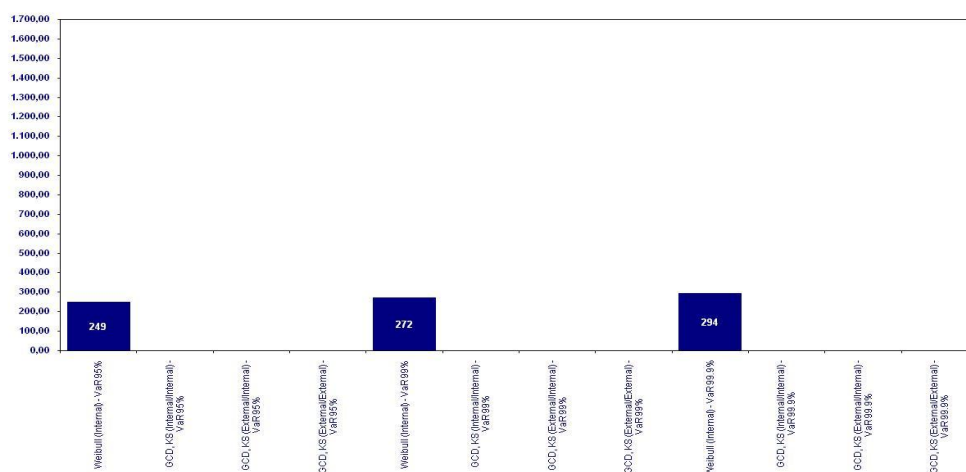
- Consistency with the findings in Paper A
- A stabilizing effect by introducing kernel smoothing
- Even more stabilizing effect by combining correction for UR and incorporate KS



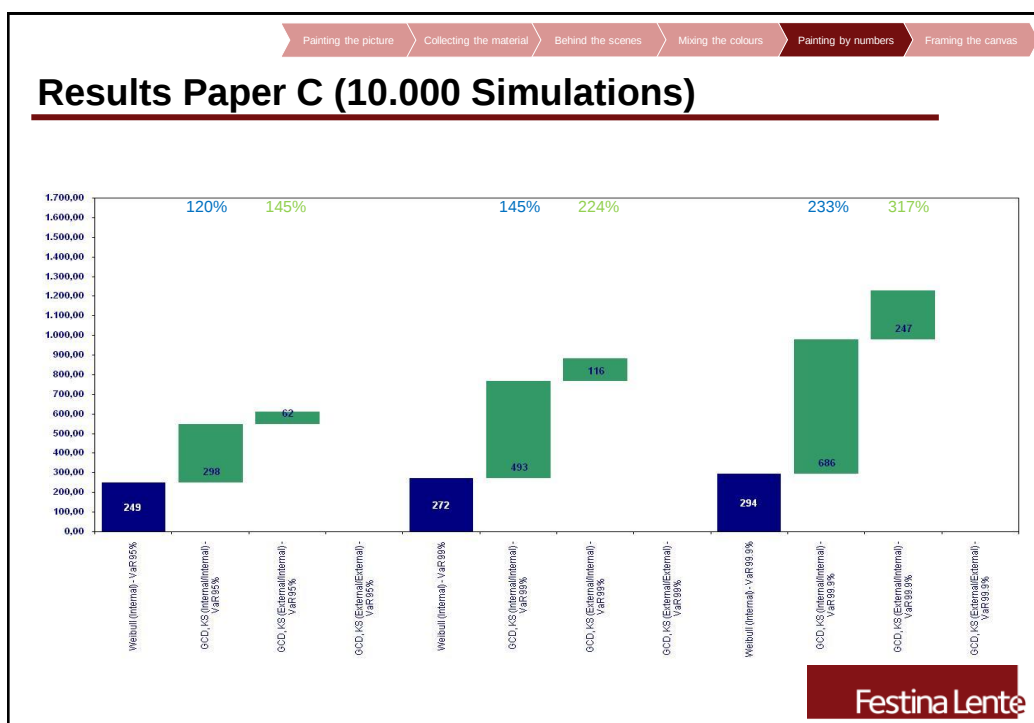
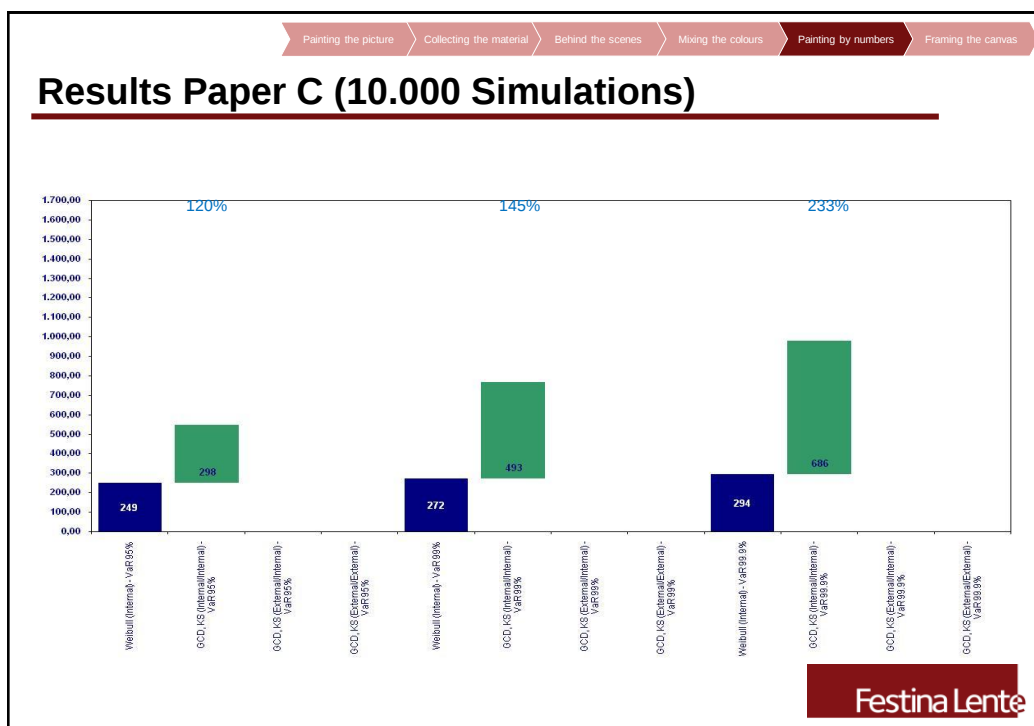
Festina Lente

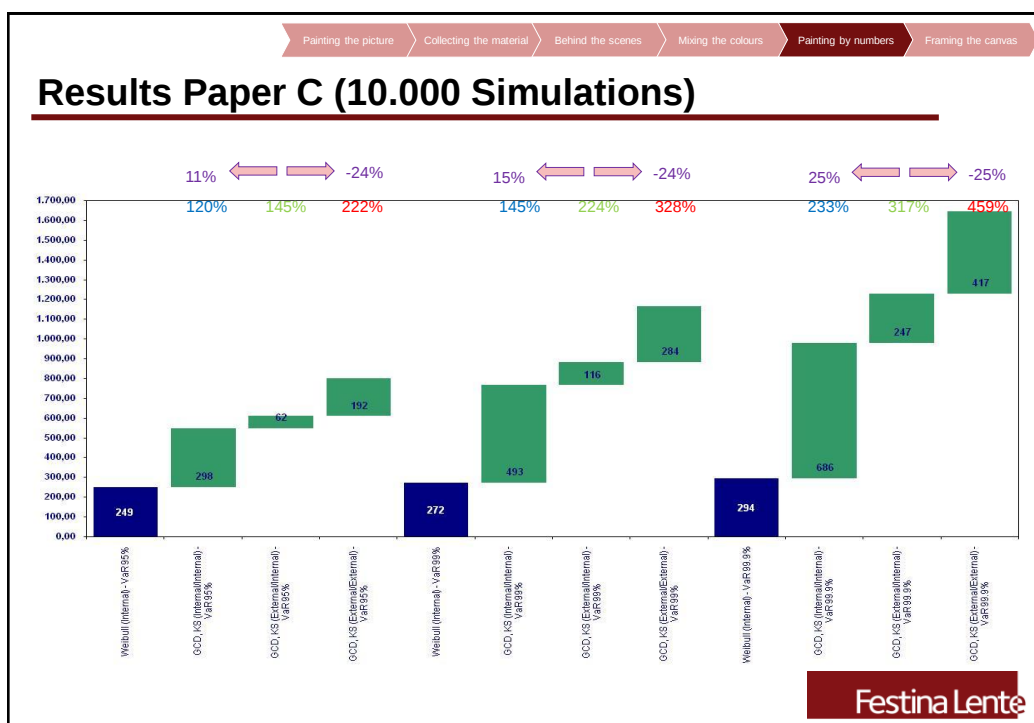
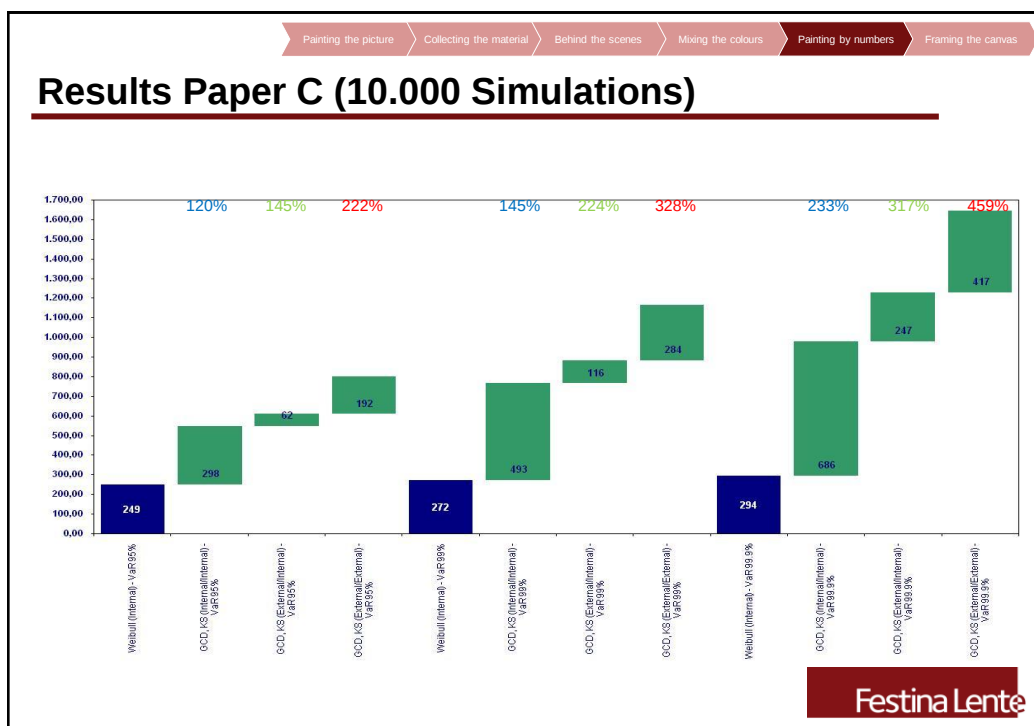
Painting the picture > Collecting the material > Behind the scenes > Mixing the colours > Painting by numbers > Framing the canvas

Results Paper C (10.000 Simulations)



Festina Lente





Painting the picture > Collecting the material > Behind the scenes > Mixing the colours > **Painting by numbers** > Framing the canvas

Key Findings Paper C

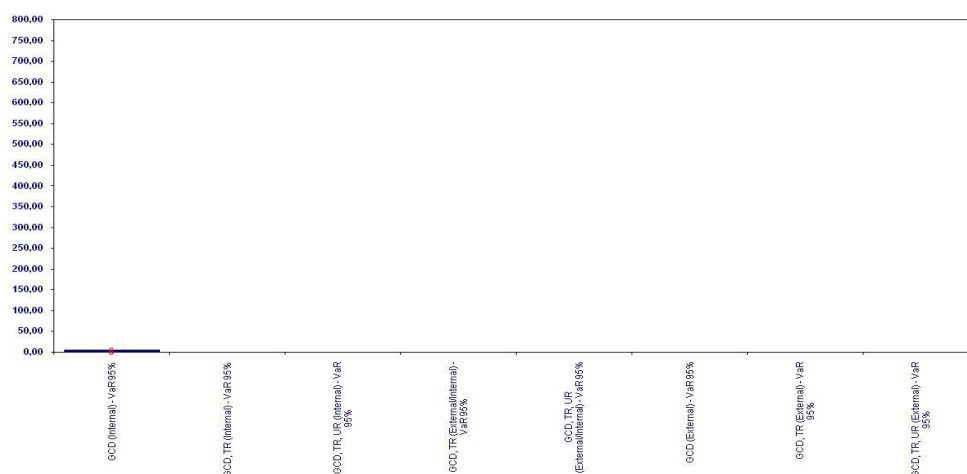
- For lower quantiles, the mixing model resembles the internal model
- For higher quantiles, more prior knowledge from the external source is present.



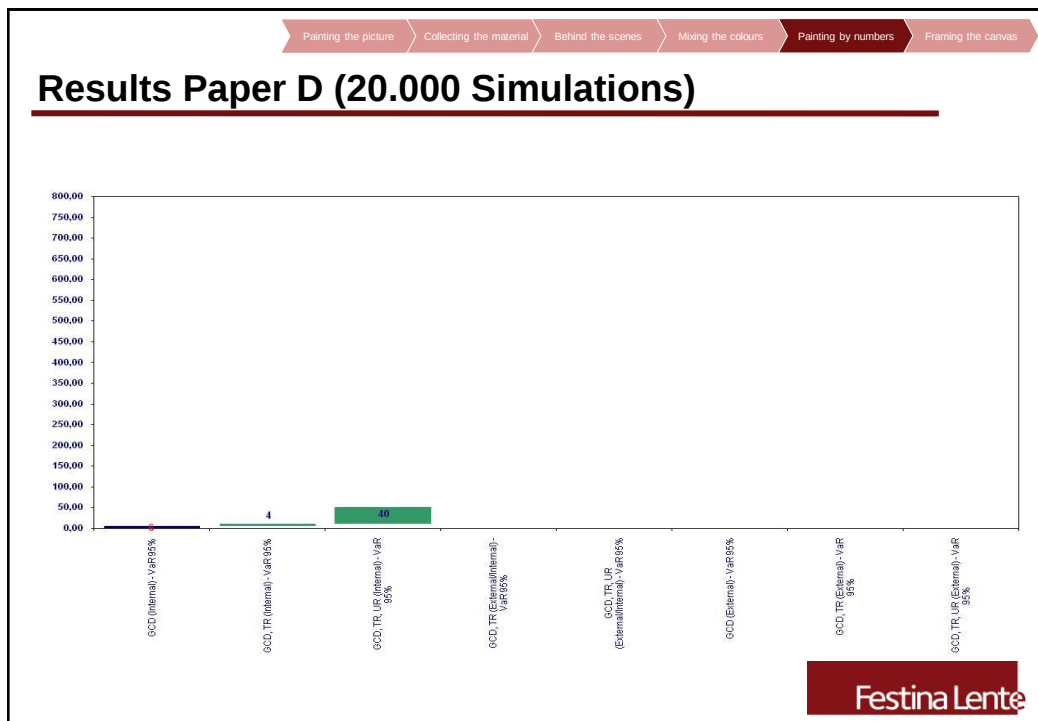
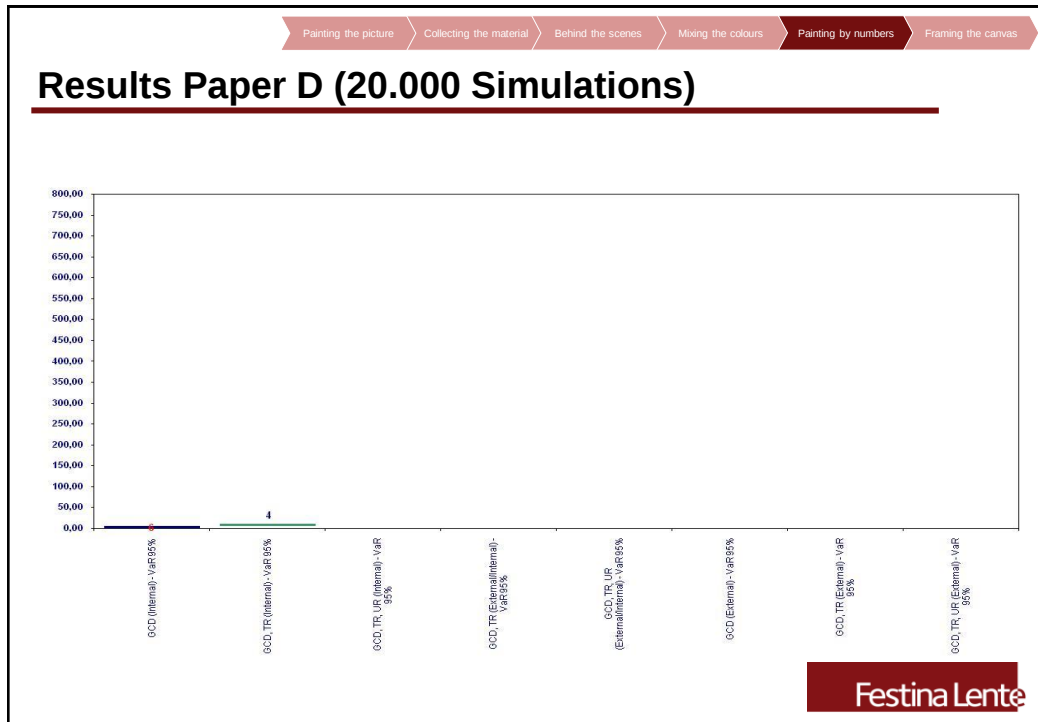
Festina Lente

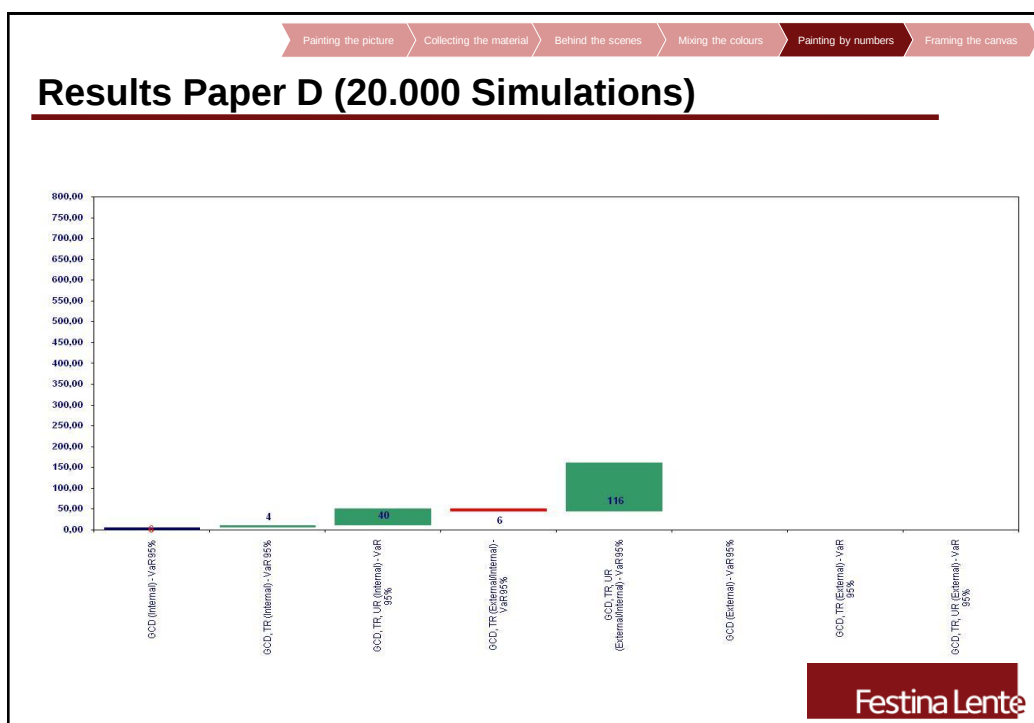
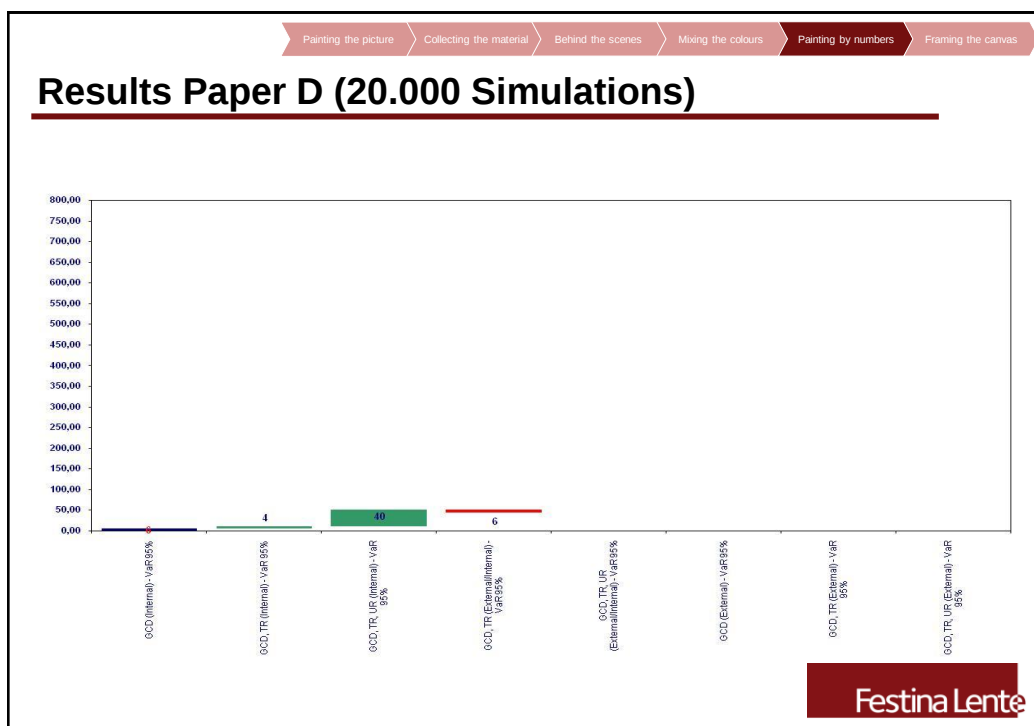
Painting the picture > Collecting the material > Behind the scenes > Mixing the colours > **Painting by numbers** > Framing the canvas

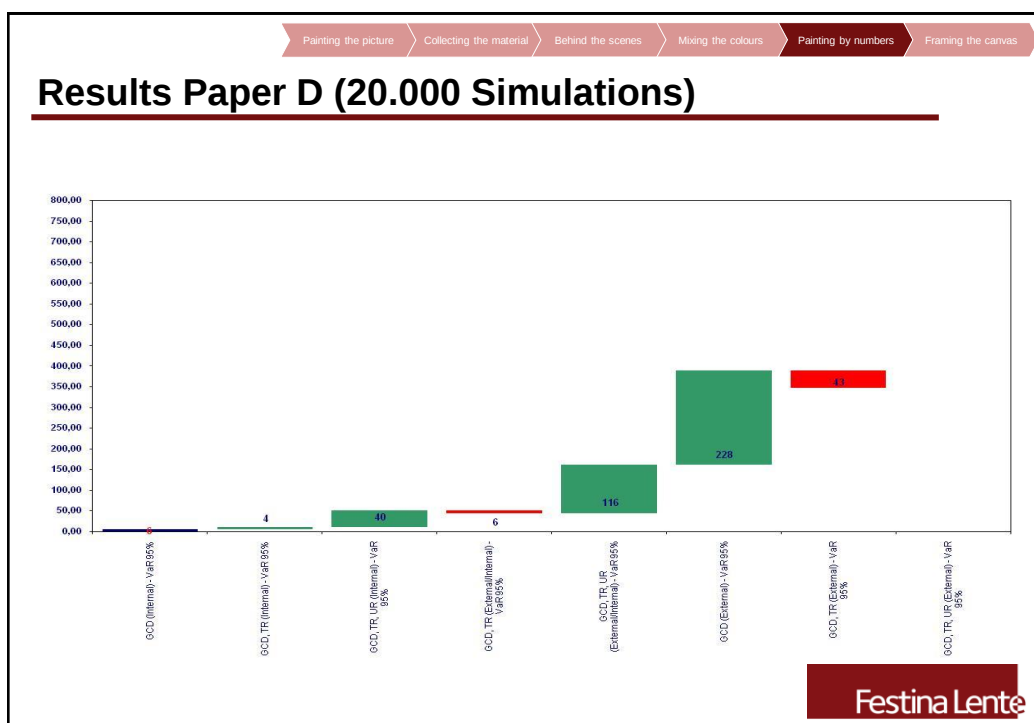
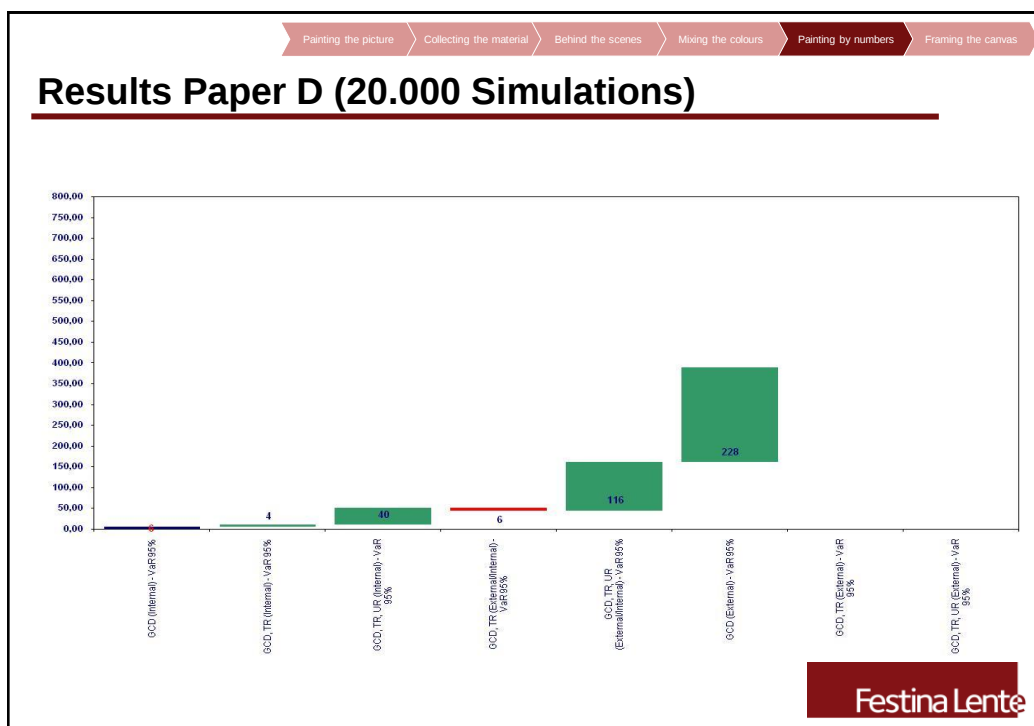
Results Paper D (20.000 Simulations)

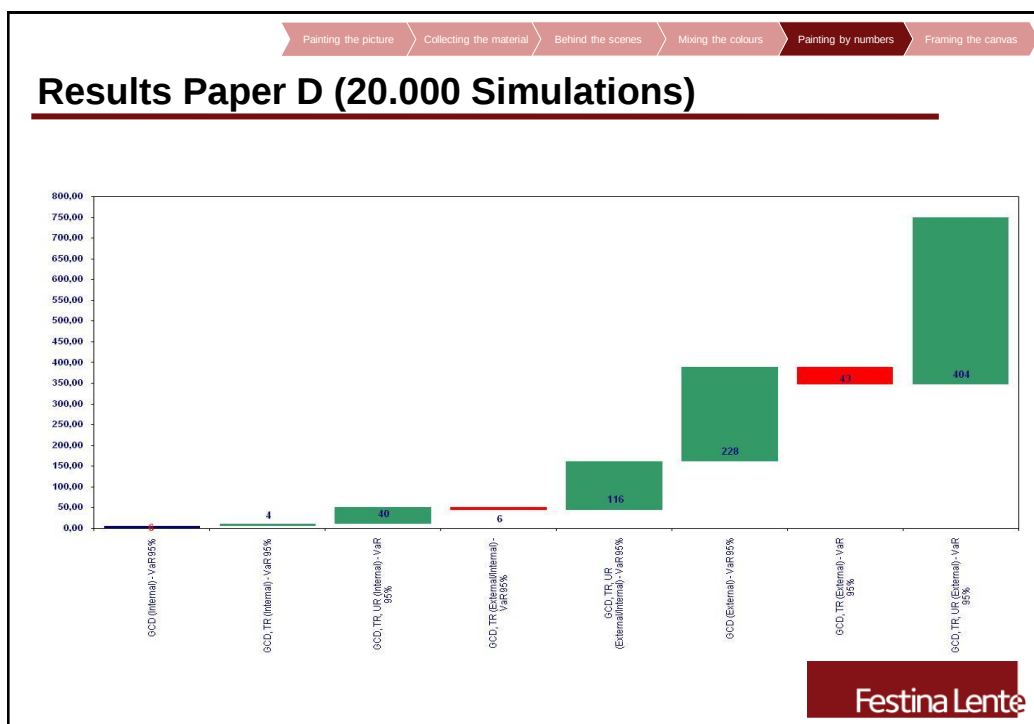


Festina Lente









Painting the picture Collecting the material Behind the scenes Mixing the colours **Painting by numbers** Framing the canvas

Key Findings Paper D

- Consistency with the findings in paper C:
 - Mixing model resembles the internal model for lower quantiles. For higher quantiles, more information from external data is present
- Consistency with findings in paper A and B:
 - Correction for underreporting increases the capital.

Festina Lente



➤ Framing the canvas

Festina Lente

Painting the picture Collecting the material Behind the scenes Mixing the colours Painting by numbers Framing the canvas

Main Findings and Conclusions

➤ The developed method can

- Pooling internal and external data to produce more credible capital estimates
- Incorporate correction for truncation (unrecorded losses) and random censoring (unreported losses)
- Robust capital method, not distribution dependent

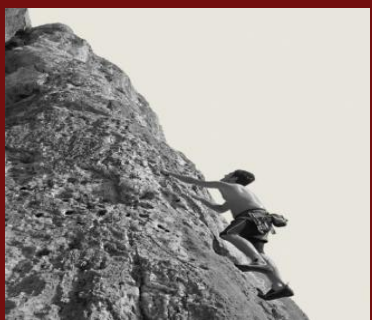


Festina Lente

Painting the picture > Collecting the material > Behind the scenes > Mixing the colours > Painting by numbers > Framing the canvas

Considerations and future research

- Improve boundary correction, bandwidth selection and correction function
- Investigate and implement possible correlation between event risk categories
- Scaling and filtration (external data)
- Take into account insurance cover
- Compare against other mixing models



Festina Lente

Sponsorship / Consultancy proposal

- We are looking for two sponsors of 50K£ each to sponsor our upcoming book on operational risk
- The money is used to buy off the four authors (academics as well as practitioners) from other duties
- The sponsors receive two VERY VALUABLE opportunities:
 1. Will play a key role when the book is launched at an event at Cass Business School
 2. One week exclusive course in the book in Copenhagen, copy of the book and the software. Accommodation paid for (up to four people).

Festina Lente

THANK YOU!

Festina Lente

Questions or comments?

Expressions of individual views by members of The Actuarial Profession and its staff are encouraged.

The views expressed in this presentation are those of the presenter.

