Institute
and Faculty
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Towards an industry standard to assess Longevity Basis Risk

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This presentation has been prepared for attendees at the Institute and Faculty of Actuaries Life Conference 2014.

It covers work produced by a joint team from Cass Business School[1] and Hymans Robertson LLP[2] in response to research commissioned by the Longevity Basis Risk Working Group of the Institute & Faculty of Actuaries and the Life & Longevity Markets Association.

The work presented here is subject to peer review; the final version will be published at a Sessional Meeting on 8^{th} December 2014.

1 Prof Steven Haberman FIA, Prof Vladimir Kaishev, Dr Pietro Millossovich & Andres Villegas MACA
2 Steven Baxter FIA, Andrew Gaches FIA, Sveinn Gunnlaugsson GradStat, Mario Sison

Aims of today's session

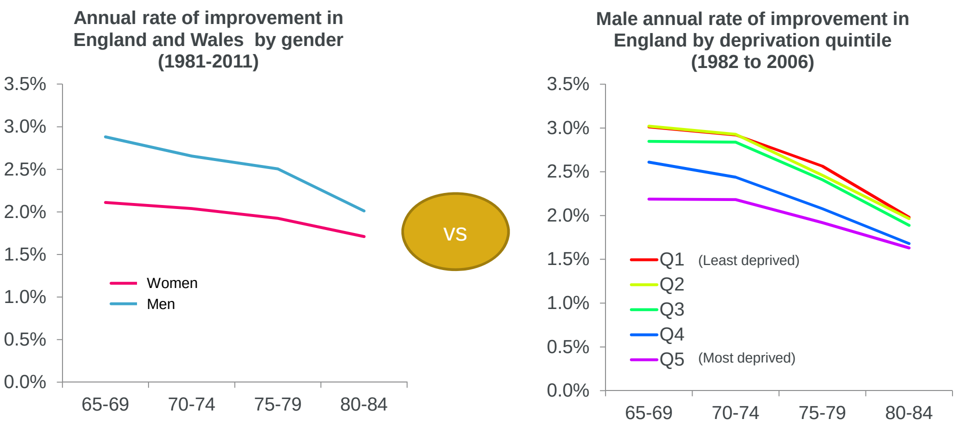
1. Introduce you to the Basis Risk problem
2. Give you a feel for the framework we have developed
3. Provide confidence in the framework
4. Encourage you to attend our sessional meeting on 8^{th} December 2014

One day at work…

I am keen to ensure we have the agility to adjust our longevity risk exposure up or down.

How effective would index-based longevity swaps be on our back-book?

Should we be concerned about hedge effectiveness?

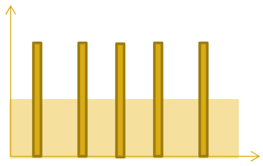


Source: Own calculations based on HMD data

Source: Based on Table 1 in Lu et al (2013)

Structuring, Sampling & Demographic Risk

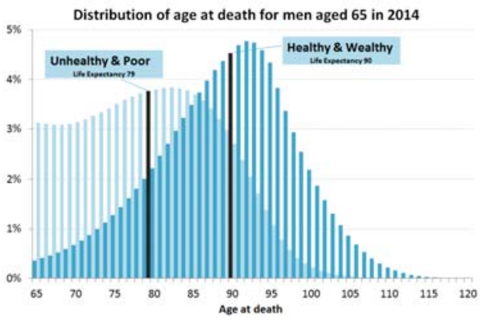
Structuring risk



✓ Risk that payoffs from hedging differs to that of portfolio

Sampling risk

Demographic risk



✓ The random outcomes of the individual lives within the portfolio and the index population

Demographic differences in the composition of the portfolio ?

Self-credible?...

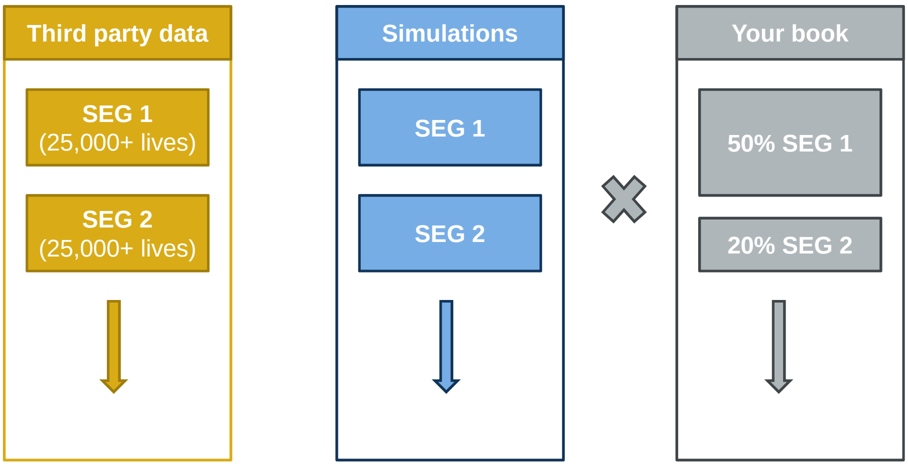
Do I have more than 25,000 lives in my book?

For a few of us….

If $\mu_R(x, t)$ is the force of mortality for E&W, we need to generate $\mu_B(x,t)$ (mortality for the book).

What form should $\mu_B(x,t)$ take?

For the most of us…



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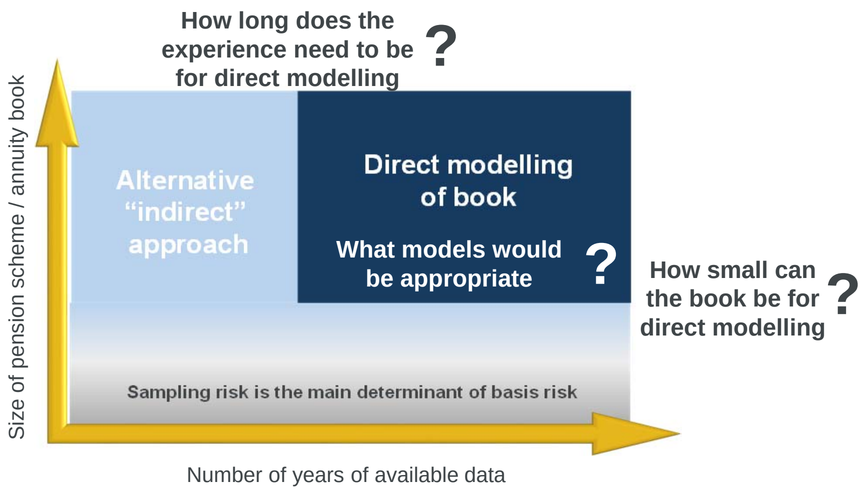
How to choose between different models?

Choice of two-population model



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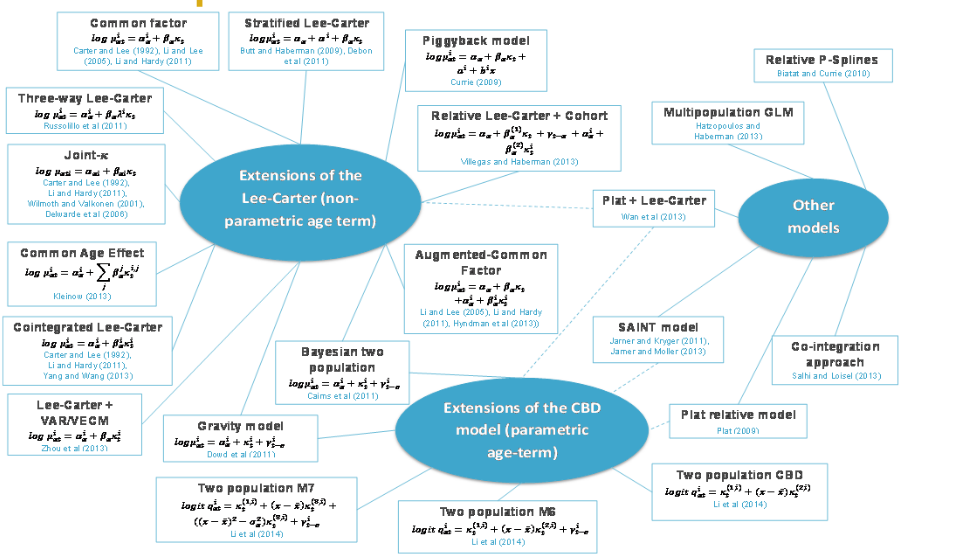
Landscape of available models



Selecting an appropriate two-population model for basis risk assessment

		Models		
		Extensions of the Lee-Carter (non-parametric age term)	Extensions of the CBD model (parametric age-term)	Other models
Criteria	Stage 1	No data required		
	Stage 2	Goodness-of-fit and reasonableness		
	Stage 3	Robustness		

- Detailed assessment of the models against criteria for a good model
- Focus today on the main highlights driving our conclusions

Selecting an appropriate two-population model for basis risk assessment

		Models		
		Extensions of the Lee-Carter (non-parametric age term)	**Extensions of the CBD model (parametric age-term)**	**Other models**
Criteria	**Stage 1**	No data required		
	Stage 2	Goodness-of-fit and reasonableness		
	Stage 3	Robustness		

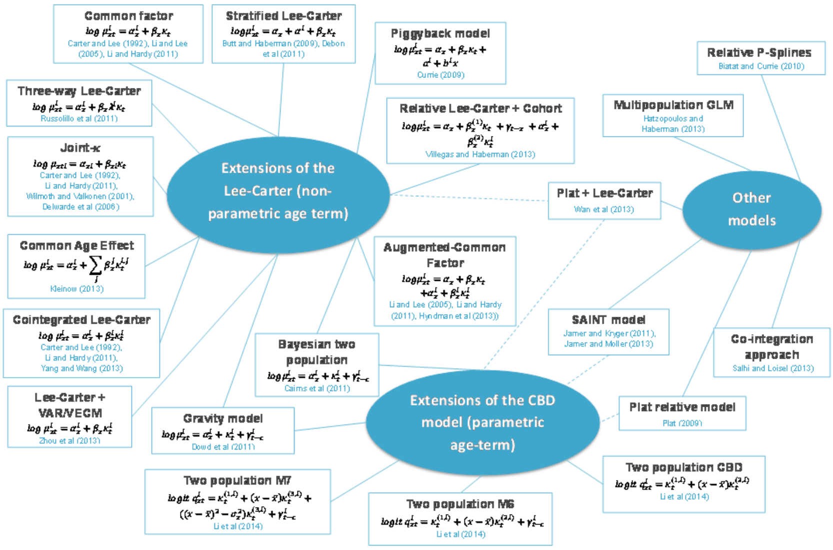
Theoretical characteristics of the model include:

- Correlation structure between book and reference
- Compatibility with data
- Transparency and ease of implementation

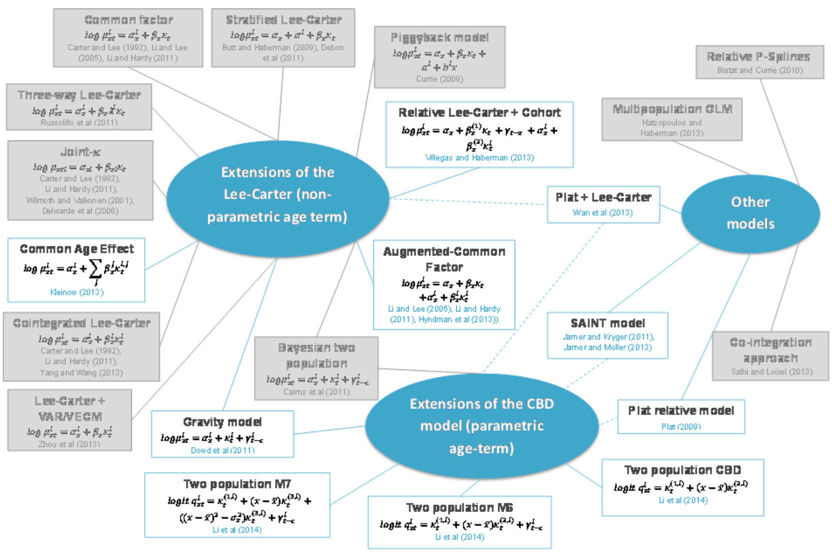
Stage 1 filtering: Criteria requiring no data to assess

- Models with perfect correlation between the book and the reference **imply no or very low basis risk**
  - e.g $\log \mu^i_{xt} = \alpha^i_x + \beta_x \kappa_t$
- Prefer models which **comply with data characteristics**
  - e.g. Models with co-integrated time indices between the book and the reference require longer history
- Transparency on model:
  - assumptions
  - meaning of parameters
- Ease of implementation:
  - reasonably simple mathematical structure
  - availability of software platform

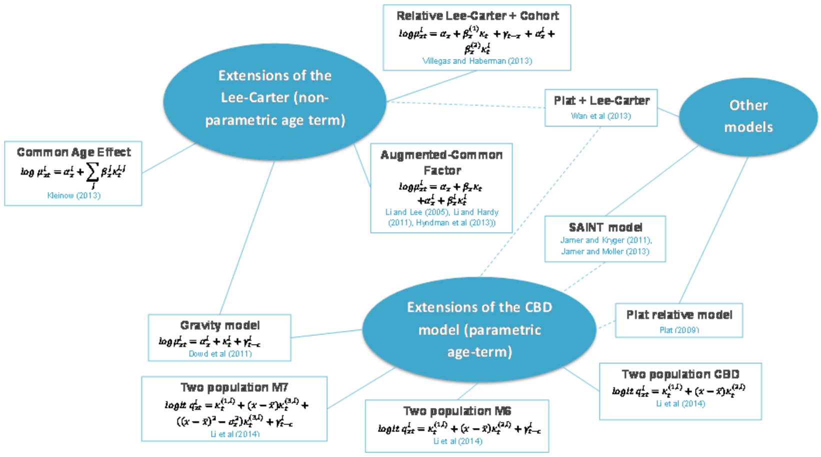
Stage 1 filtering: Criteria requiring no data to assess



Stage 1 filtering: Criteria requiring no data to assess



Stage 1 filtering: Criteria requiring no data to assess



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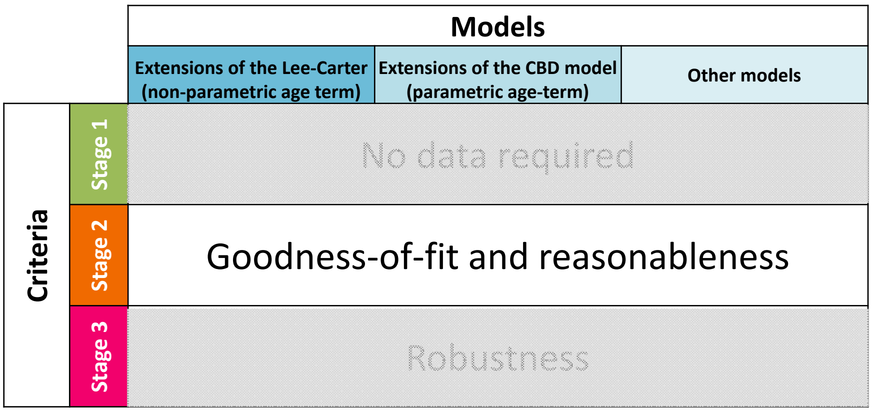
Selecting an appropriate two-population model for basis risk assessment



Assessing these criteria requires:

1. Test data for the reference and book population
2. A framework for fitting the two population mortality models

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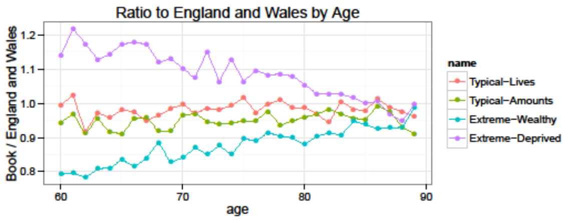
Testing datasets

Reference

- England and Wales male population for 1961-2010 and age 60-89

Book

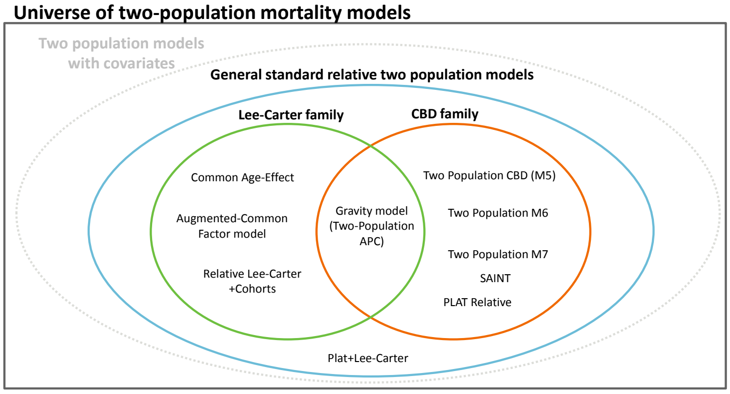
- **Synthetic datasets** for 1981-2010 and age 60-89 constructed by randomly sampling from the national data broadly in line with the **splits between deprivation groups** seen within **real Club Vita pension schemes**
  - Allows for controlling some characteristics of the book while changing others
  - Allow for backtesting
- Four different distributions of members by deprivation groups



Modelling framework

Families of models

Universe of two-population mortality models



Lee-Carter family	General standard relative two population models	CBD family
Non-Parametric β_x^R and β_x^B	$\text{logit}\, q_{xt}^R = \alpha_x^R + \sum_{j=1}^{N} \beta_x^{(j,R)} \kappa_t^{(j,R)} + \gamma_{t-c}^R$ $\text{logit}\, q_{xt}^B - \text{logit}\, q_{xt}^R = \alpha_x^B + \sum_{j=1}^{M} \beta_x^{(j,B)} \kappa_t^{(j,B)} + \gamma_{t-c}^B$	Parametric β_x^R and β_x^B

Modelling framework
General considerations

- We choose a **relative** approach (Jarner and Kryger, 2011)
- Advantages:
  - Allows data mismatch between reference and book
  - Reference population considerably larger than book population
  - Reference population models readily available so inform decisions on book model
- Assume non-divergence between book and reference in the long term
- Target death rates q_{xt} using a binomial distribution and a logit link function

Modelling framework
General mathematical formulation

Reference population

$$D_{xt}^{R} \sim Bin\,(E_{xt}^{R}, q_{xt}^{R})$$

$$\operatorname{logit} q_{xt}^{R} = \alpha_x^{R} + \sum_{j=1}^{N} \beta_x^{(j,R)} \kappa_t^{(j,R)} + \gamma_{t-x}^{R}$$

MRWD: $\boldsymbol{\kappa}_t^{\mathbf{R}} = \mathbf{d} + \boldsymbol{\kappa}_{t-1}^{\mathbf{R}} + \boldsymbol{\xi}_t^{\mathbf{R}}, \quad \boldsymbol{\xi}_t^{\mathbf{R}} \sim \boldsymbol{N}(\mathbf{0}, \boldsymbol{\Sigma}^{\mathbf{R}})$

ARIMA(1,1,0): $\Delta\gamma_c^{R} = d + \phi_1 \Delta\gamma_{c-1}^{R} + \varepsilon_c^{R}, \quad \varepsilon_c^{R} \sim N(0, \sigma_R^2)$

Book population

$$D_{xt}^{B} \sim Bin\,(E_{xt}^{B}, q_{xt}^{B})$$

$$\operatorname{logit} q_{xt}^{B} - \operatorname{logit} q_{xt}^{R} = \alpha_x^{B} + \sum_{j=1}^{M} \beta_x^{(j,B)} \kappa_t^{(j,B)} + \gamma_{t-x}^{B}$$

VAR(1): $\boldsymbol{\kappa}_t^{\mathrm{B}} = \boldsymbol{\Phi}_{\mathbf{0}} + \boldsymbol{\Phi}_{\mathbf{1}} \boldsymbol{\kappa}_{t-1}^{\mathrm{B}} + \boldsymbol{\xi}_t^{\mathrm{B}}, \quad \boldsymbol{\xi}_t^{\mathrm{B}} \sim N(\mathbf{0}, \Sigma^{B})$

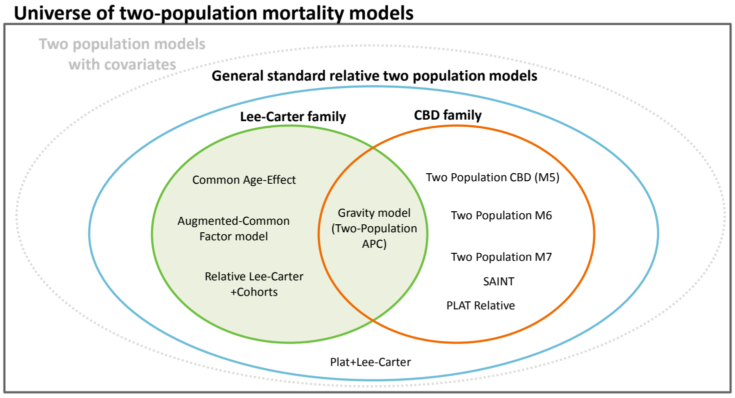
AR(1): $\gamma_c^{\mathrm{B}} = \phi_0 + \phi_1 \gamma_{c-1}^{\mathrm{B}} + \varepsilon_c^{B}, \varepsilon_c^{B} \sim N(0, \sigma_B^2)$

Enables a flexible R implementation

Lee-Carter family

Candidate models

Universe of two-population mortality models



Lee-Carter family

Non-Parametric β_x^R and β_x^B

General standard relative two population models

$$\operatorname{logit} q_{xt}^R = \alpha_x^R + \sum_{j=1}^{N} \beta_x^{(j,R)} \kappa_t^{(j,R)} + \gamma_{t-c}^R$$

$$\operatorname{logit} q_{xt}^B - \operatorname{logit} q_{xt}^R = \alpha_x^B + \sum_{j=1}^{M} \beta_x^{(j,B)} \kappa_t^{(j,B)} + \gamma_{t-c}^B$$

CBD family

Parametric β_x^R and β_x^B

Lee-Carter family

Candidate models

- We consider the following models from the Lee-Carter family

Reference: $\log\left(\frac{q_{xt}^R}{1-q_{xt}^R}\right)$	**Book:** $\log\left(\frac{q_{xt}^B}{1-q_{xt}^B}\right) - \log\left(\frac{q_{xt}^R}{1-q_{xt}^R}\right)$	**Name**
$\alpha_x^R + \beta_x^R \kappa_t^R + \gamma_{t-x}^R$	α_x^B	CF+Cohort
	$\alpha_x^B + \beta_x^R \kappa_t^B$	CAE+Cohort
	$\alpha_x^B + \beta_x^B \kappa_t^B$	RelLC+Cohort
$\alpha_x^R + \kappa_t^R + \gamma_{t-x}^R$	$\alpha_x^B + \kappa_t^B + \gamma_{t-x}^B$	APC(Gravity)

Lee-Carter family
Candidate models

- We consider the following models from the Lee-Carter family

Reference: $\log\left(\frac{q_{xt}^R}{1-q_{xt}^R}\right)$	Book: $\log\left(\frac{q_{xt}^B}{1-q_{xt}^B}\right) - \log\left(\frac{q_{xt}^R}{1-q_{xt}^R}\right)$	Name
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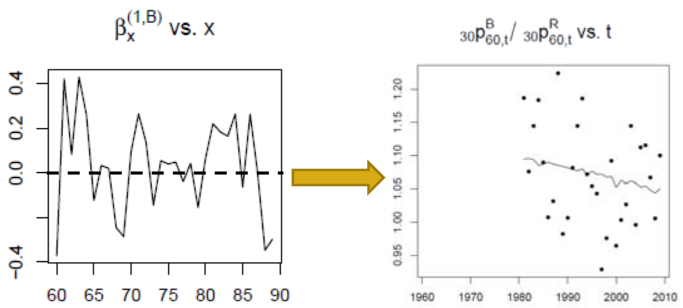
$\alpha_x^R + \kappa_t^R + \gamma_{t-x}^R$	$\alpha_x^B + \kappa_t^B + \gamma_{t-x}^B$	APC(Gravity)

Lee-Carter family
Avoid models with book-specific age-modulating parameter

Reference	Book	Model
$\alpha_x^R + \beta_x^R \kappa_t^R + \gamma_{t-x}^R$	$\alpha_x^B + \beta_x^B \kappa_t^B$	RelLC+Cohort



Not enough data in the book to estimate β_x^B

May produce over-smoothed aggregate demographic metrics so the model behaves as if it implied perfect correlation

Lee-Carter family
Candidate models

- We consider the following models from the Lee-Carter family

Reference: $\log\left(\frac{q^R_{xt}}{1-q^R_{xt}}\right)$	**Book:** $\log\left(\frac{q^B_{xt}}{1-q^B_{xt}}\right) - \log\left(\frac{q^R_{xt}}{1-q^R_{xt}}\right)$	**Name**
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	$\alpha^B_x + \beta^R_x \kappa^B_t$	CAE+Cohort
	$\alpha^B_x + \beta^B_x \kappa^B_t$	RelLC+Cohort
$\alpha^R_x + \kappa^R_t + \gamma^R_{t-x}$	$\alpha^B_x + \kappa^B_t + \gamma^B_{t-x}$	APC(Gravity)

Lee-Carter family
Candidate models

- We consider the following models from the Lee-Carter family

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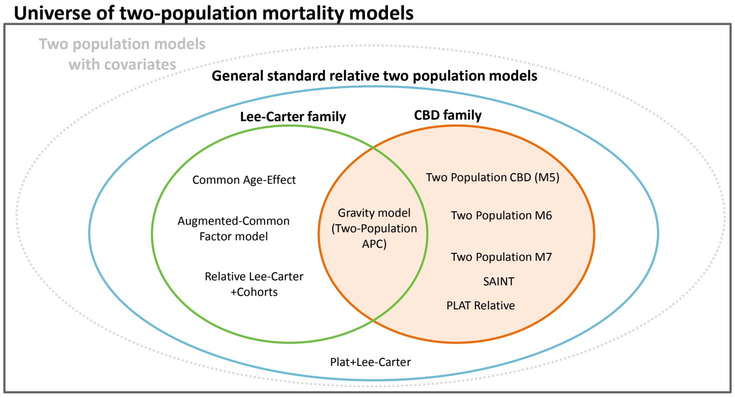
$\alpha^R_x + \beta^R_x \kappa^R_t + \gamma^R_{t-x}$	α^B_x	CF+Cohort
	$\alpha^B_x + \beta^R_x \kappa^B_t$	CAE+Cohort
	$\alpha^B_x + \beta^B_x \kappa^B_t$	RelLC+Cohort
$\alpha^R_x + \kappa^R_t + \gamma^R_{t-x}$	$\alpha^B_x + \kappa^B_t + \gamma^B_{t-x}$	APC(Gravity)

CBD family
Candidate models

Universe of two-population mortality models



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CBD family
Candidate models

- We consider the following models from the parametric (CBD) family

Reference: $\log\left(\frac{q_{xt}^R}{1-q_{xt}^R}\right)$	Book: $\log\left(\frac{q_{xt}^B}{1-q_{xt}^B}\right) - \log\left(\frac{q_{xt}^R}{1-q_{xt}^R}\right)$	Name
$\alpha_x^R + \beta_x^R \kappa_t^R + \gamma_{t-x}^R$	$\alpha_x^B + \beta_x^R \kappa_t^B$	CAE+Cohort
$\alpha_x^R + \kappa_t^R + \gamma_{t-x}^R$	$\alpha_x^B + \kappa_t^B + \gamma_{t-x}^B$	APC
	$\kappa_t^{(1,B)} + (x-\bar{x})\kappa_t^{(2,B)}$	M7-M5
	$\kappa_t^{(1,B)} + (x-\bar{x})\kappa_t^{(2,B)} + \gamma_{t-x}^B$	M7-M6
$\kappa_t^{(1,R)} + (x-\bar{x})\kappa_t^{(2,R)} + \left((x-\bar{x})^2 - \sigma_x^2\right)\kappa_t^{(3,R)} + \gamma_{t-x}^R$	$\kappa_t^{(1,B)} + (x-\bar{x})\kappa_t^{(2,B)} + \left((x-\bar{x})^2 - \sigma_x^2\right)\kappa_t^{(3,B)}$	M7-SAINT
	$\kappa_t^{(1,B)} + (x-\bar{x})\kappa_t^{(2,B)} + \left((x-\bar{x})^2 - \sigma_x^2\right)\kappa_t^{(3,B)} + \gamma_{t-x}^B$	M7-M7
	$\frac{100-x}{100-\bar{x}}\kappa_t^{(1,B)}$	M7-PLAT

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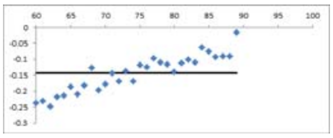
CBD family

Differences (vs reference population) captured by the CBD book models

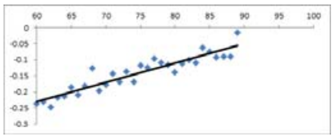
	Level	Slope	Curvature	Cohort
M5	Yes	Yes	No	No
M6	Yes	Yes	No	Yes
SAINT	Yes	Yes	Yes	No
M7	Yes	Yes	Yes	Yes
PLAT	?	?	No	No

Looking at $\log\left(\frac{q_{xt}^{B}}{1-q_{xt}^{B}}\right) - \log\left(\frac{q_{xt}^{R}}{1-q_{xt}^{R}}\right)$ for a **fixed** calendar year

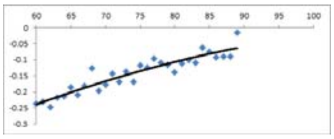
Only Level



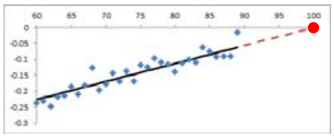
Level + Slope (M5,M6)



Level + Slope + Curvature(SAINT, M7)

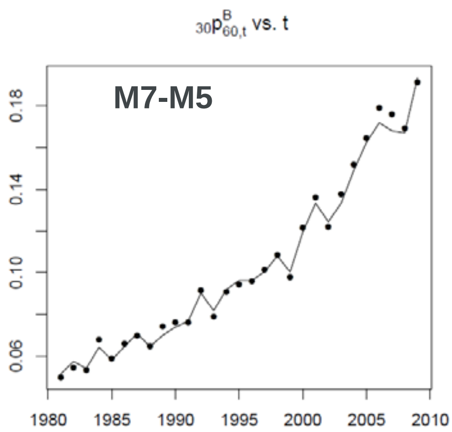


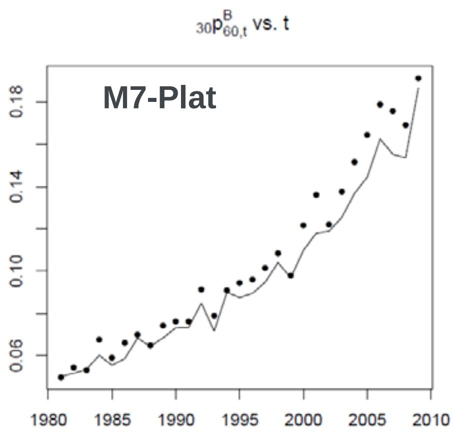
Constrained Level + Slope (PLAT)



CBD family

Some model showed poor goodness-of-fit





CBD family
Candidate models

- We consider the following models from the parametric (CBD) family

Reference: $\log\left(\frac{q^R_{xt}}{1-q^R_{xt}}\right)$	Book: $\log\left(\frac{q^B_{xt}}{1-q^B_{xt}}\right) - \log\left(\frac{q^R_{xt}}{1-q^R_{xt}}\right)$	Name
$\alpha^R_x + \beta^R_x \kappa^R_t + \gamma^R_{t-x}$	$\alpha^B_x + \beta^B_x \kappa^B_t$	CAE+Cohort
$\alpha^R_x + \kappa^R_t + \gamma^R_{t-x}$	$\alpha^B_x + \kappa^B_t + \gamma^B_{t-x}$	APC
$\kappa^{(1,R)}_t + (x-\bar{x})\kappa^{(2,R)}_t + \left((x-\bar{x})^2 - \sigma^2_x\right)\kappa^{(3,R)}_t + \gamma^R_{t-x}$	$\kappa^{(1,B)}_t + (x-\bar{x})\kappa^{(2,B)}_t$	M7-M5
	$\kappa^{(1,B)}_t + (x-\bar{x})\kappa^{(2,B)}_t + \gamma^B_{t-x}$	M7-M6
	$\kappa^{(1,B)}_t + (x-\bar{x})\kappa^{(2,B)}_t + \left((x-\bar{x})^2 - \sigma^2_x\right)\kappa^{(3,B)}_t$	M7-SAINT
	$\kappa^{(1,B)}_t + (x-\bar{x})\kappa^{(2,B)}_t + \left((x-\bar{x})^2 - \sigma^2_x\right)\kappa^{(3,B)}_t + \gamma^B_{t-x}$	M7-M7
	$\frac{100-x}{100-\bar{x}}\kappa^{(1,B)}_t$	M7-PLAT

CBD family
Candidate models

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	$\frac{100-x}{100-\bar{x}}\kappa^{(1,B)}_t$	M7-PLAT

Book population specification

Goodness of fit vs. Parsimony

Model	Number of book parameters	BIC Ranking (Book part of the model) Typical-Lives	Typical-Amounts	Extreme-Wealthy	Extreme-Deprived
CAE+Cohort	58	2	1	2	1
M7-M5	58	1	2	1	2
M7-SAINT	87	3	3	3	3
M7-M6	114	4	4	4	5
M7-M7	142	6	6	5	6
APC (Gravity)	114	5	5	6	4

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Book population specification

Goodness of fit vs. Parsimony

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- CAE+Cohort and M7-M5 have a good compromise between goodness-of-fit and parsimony

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Book population specification
Goodness of fit vs. Parsimony

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M7-M6	114	4	4	4	5
M7-M7	142	6	6	5	6
APC (Gravity)	114	5	5	6	4

- CAE+Cohort and M7-M5 have a good compromise between goodness-of-fit and parsimony
- Models with a book-specific cohort effect have the worst trade-off between goodness-of-fit and parsimony.

Book population specification
Goodness of fit vs. Parsimony

Model	Number of book parameters	BIC Ranking (Book part of the model) Typical-Lives	Typical-Amounts	Extreme-Wealthy	Extreme-Deprived
CAE+Cohort	58	2	1	2	1
M7-M5	58	1	2	1	2
M7-SAINT	87	3	3	3	3
M7-M6	114	4	4	4	5
M7-M7	142	6	6	5	6
APC (Gravity)	114	5	5	6	4

- CAE+Cohort and M7-M5 have a good compromise between goodness-of-fit and parsimony
- Models with a book-specific cohort effect have the worst trade-off between goodness-of-fit and parsimony.
- M7-SAINT and M7-M7 have a poor trade-off between goodness-of-fit and parsimony, indicating that it suffices to capture level and slope differences, and **there is no need to capture curvature differences**.

CBD family
Candidate models

- We consider the following models from the parametric (CBD) family

Reference: $\log\left(\frac{q_{xt}^R}{1-q_{xt}^R}\right)$	**Book:** $\log\left(\frac{q_{xt}^B}{1-q_{xt}^B}\right) - \log\left(\frac{q_{xt}^R}{1-q_{xt}^R}\right)$	**Name**
$\alpha_x^R + \beta_x^R \kappa_t^R + \gamma_{t-x}^R$	$\alpha_x^B + \beta_x^B \kappa_t^B$	CAE+Cohort
$\alpha_x^R + \kappa_t^R + \gamma_{t-x}^R$	$\alpha_x^B + \kappa_t^B + \gamma_{t-x}^B$	APC
$\kappa_t^{(1,R)} + (x-\bar{x})\kappa_t^{(2,R)} + \left((x-\bar{x})^2 - \sigma_x^2\right)\kappa_t^{(3,R)} + \gamma_{t-x}^R$	$\kappa_t^{(1,B)} + (x-\bar{x})\kappa_t^{(2,B)}$	M7-M5
	$\kappa_t^{(1,B)} + (x-\bar{x})\kappa_t^{(2,B)} + \gamma_{t-x}^B$	M7-M6
	$\kappa_t^{(1,B)} + (x-\bar{x})\kappa_t^{(2,B)} + \left((x-\bar{x})^2 - \sigma_x^2\right)\kappa_t^{(3,B)}$	M7-SAINT
	$\kappa_t^{(1,B)} + (x-\bar{x})\kappa_t^{(2,B)} + \left((x-\bar{x})^2 - \sigma_x^2\right)\kappa_t^{(3,B)} + \gamma_{t-x}^B$	M7-M7
	$\frac{100-x}{100-\bar{x}}\kappa_t^{(1,B)}$	M7-PLAT

CBD family
Candidate models

- We consider the following models from the parametric (CBD) family

Reference: $\log\left(\frac{q_{xt}^R}{1-q_{xt}^R}\right)$	**Book:** $\log\left(\frac{q_{xt}^B}{1-q_{xt}^B}\right) - \log\left(\frac{q_{xt}^R}{1-q_{xt}^R}\right)$	**Name**
$\alpha_x^R + \beta_x^R \kappa_t^R + \gamma_{t-x}^R$	$\alpha_x^B + \beta_x^B \kappa_t^B$	CAE+Cohort
$\alpha_x^R + \kappa_t^R + \gamma_{t-x}^R$	$\alpha_x^B + \kappa_t^B + \gamma_{t-x}^B$	APC
$\kappa_t^{(1,R)} + (x-\bar{x})\kappa_t^{(2,R)} + \left((x-\bar{x})^2 - \sigma_x^2\right)\kappa_t^{(3,R)} + \gamma_{t-x}^R$	$\kappa_t^{(1,B)} + (x-\bar{x})\kappa_t^{(2,B)}$	M7-M5
	$\kappa_t^{(1,B)} + (x-\bar{x})\kappa_t^{(2,B)} + \gamma_{t-x}^B$	M7-M6
	$\kappa_t^{(1,B)} + (x-\bar{x})\kappa_t^{(2,B)} + \left((x-\bar{x})^2 - \sigma_x^2\right)\kappa_t^{(3,B)}$	M7-SAINT
	$\kappa_t^{(1,B)} + (x-\bar{x})\kappa_t^{(2,B)} + \left((x-\bar{x})^2 - \sigma_x^2\right)\kappa_t^{(3,B)} + \gamma_{t-x}^B$	M7-M7
	$\frac{100-x}{100-\bar{x}}\kappa_t^{(1,B)}$	M7-PLAT

Shortlisted two population models

Reference: $\log\left(\frac{q_{xt}^R}{1-q_{xt}^R}\right)$	**Book:** $\log\left(\frac{q_{xt}^B}{1-q_{xt}^B}\right) - \log\left(\frac{q_{xt}^R}{1-q_{xt}^R}\right)$	**Name**
$\alpha_x^R + \beta_x^R \kappa_t^R + \gamma_{t-x}^R$	$\alpha_x^B + \beta_x^B \kappa_t^B$	CAE+Cohort
$\alpha_x^R + \kappa_t^R + \gamma_{t-x}^R$	$\alpha_x^B + \kappa_t^B + \gamma_{t-x}^B$	APC
$\kappa_t^{(1,R)} + (x-\bar{x})\kappa_t^{(2,R)} + \left((x-\bar{x})^2 - \sigma_x^2\right)\kappa_t^{(3,R)} + \gamma_{t-x}^R$	$\kappa_t^{(1,B)} + (x-\bar{x})\kappa_t^{(2,B)}$	M7-M5
	$\kappa_t^{(1,B)} + (x-\bar{x})\kappa_t^{(2,B)} + \gamma_{t-x}^B$	M7-M6

- So far we have shortlisted four models (preference for CAE+Cohorts and M7-M5) based on theoretical properties, practicality and goodness-of-fit performance
- For basis risk purposes it is crucial to check for both single and two population metrics that these models produce **reasonable forecast levels of uncertainty** that are in line with historical variability
- Several sources of risk which determine uncertainty levels
  - **Process risk (PR)** from the possible future trajectories of the time series of the period and cohort indexes.
  - **Parameter uncertainty (PU)** from the estimation of the parameters of the model (Bootstrapping)
  - **Sampling risk (SR)** due to the volatility of the actual mortality experience depending on the size of the population

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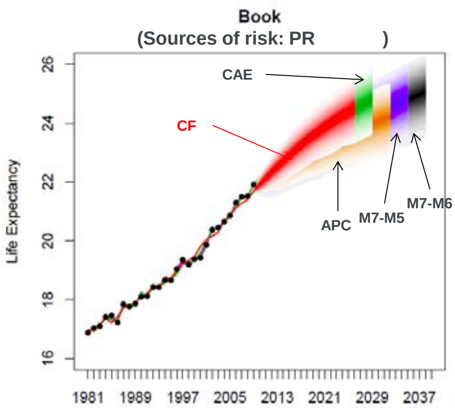
Reasonable forecast level of uncertainty

Period 30 year curtailed life expectancy from age 60 (*Extreme amounts-75000 lives*)

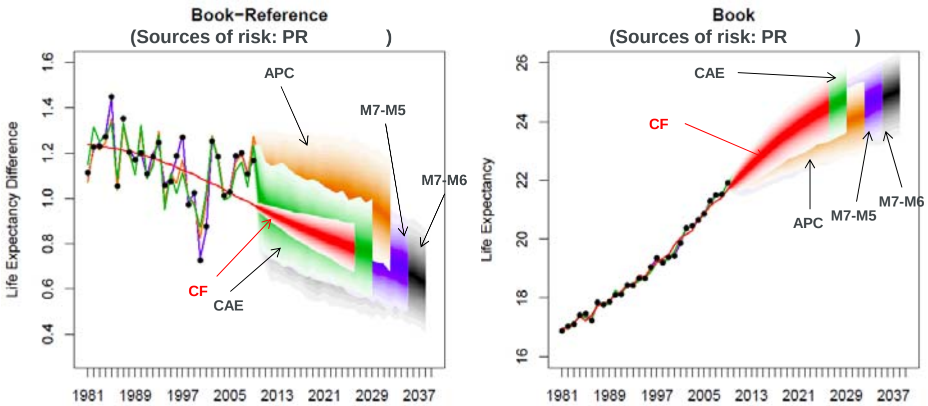


- The levels of uncertainty in **single population metrics** are reasonable and consistent in the reference and the book.

42

Reasonable forecast level of uncertainty

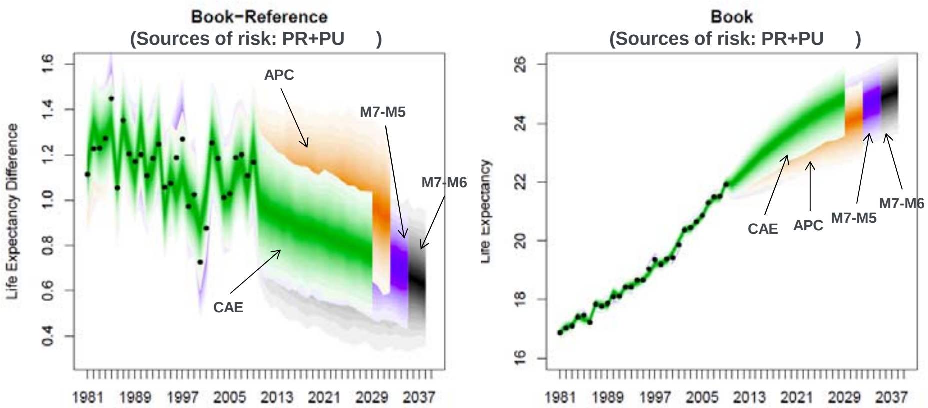
Period 30 year curtailed life expectancy from age 60 (*Extreme amounts-75000 lives*)



- The levels of uncertainty in **single population metrics** are reasonable and consistent in the reference and the book.
- The levels of uncertainty in the **differences** are on the tight side and vary considerably across models.
- The **issues** with models assuming a **perfect correlation** (e.g CF) become evident.

Reasonable forecast level of uncertainty

Period 30 year curtailed life expectancy from age 60 (*Extreme amounts-75000 lives*)



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- **Parameter uncertainty** has little impact on single population metrics but makes the confidence intervals in the **difference look reasonable**.

Reasonable forecast level of uncertainty

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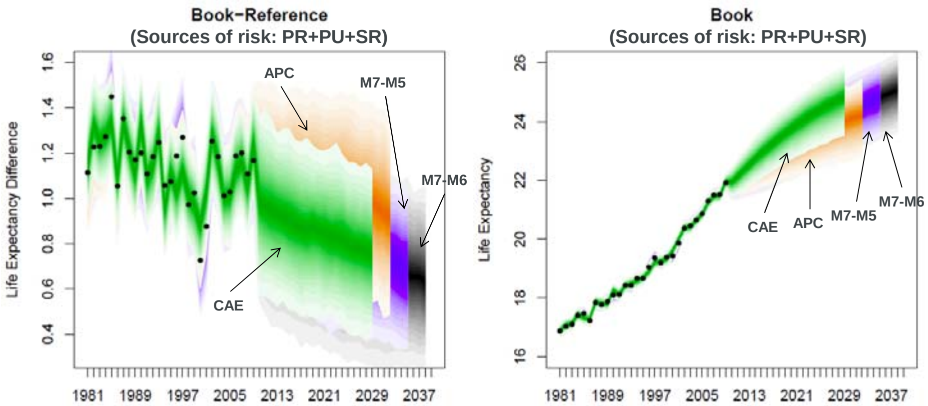
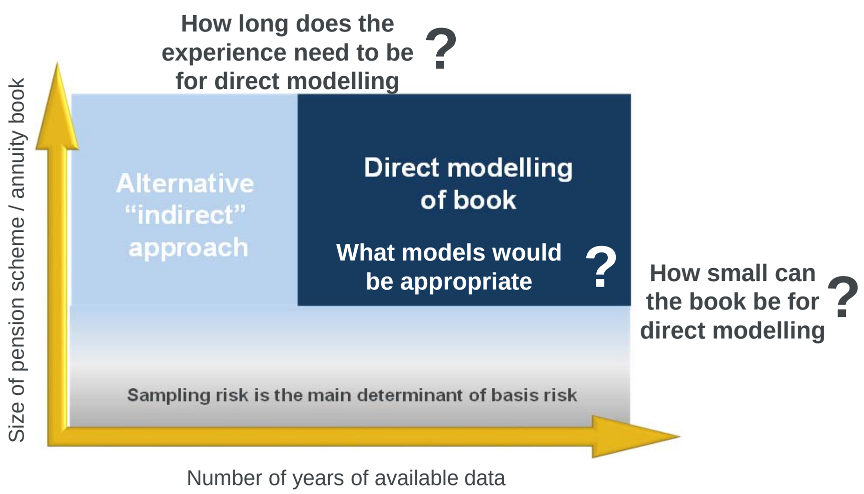
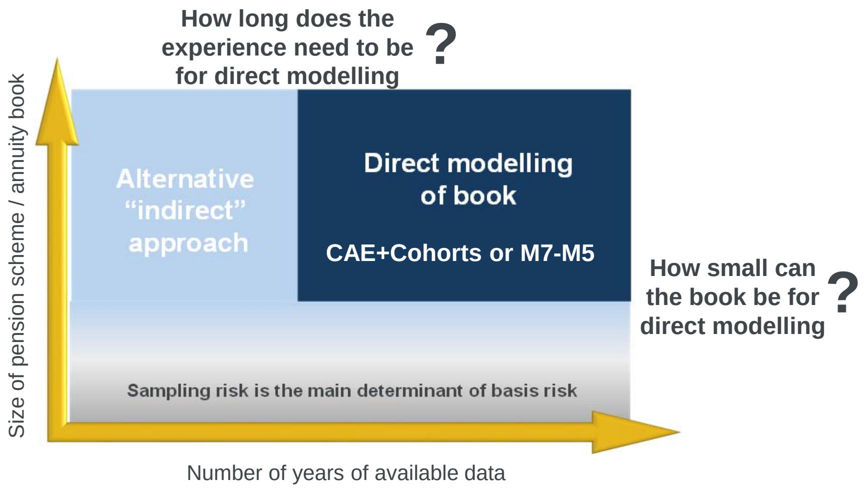


- The levels of uncertainty in **single population metrics** are reasonable and consistent in the reference and the book.
- The levels of uncertainty in the **differences** are on the tight side and vary considerably across models.
- The **issues** with models assuming a **perfect correlation** (e.g CF) become evident.
- **Parameter uncertainty** has little impact on single population metrics but makes the confidence intervals in the **difference look reasonable**.
- Once **sampling risk** is added the levels of uncertainty still **look reasonable** but on the wide side for some models (e.g. M7-M6).

Choice of two-population model



Choice of two-population model



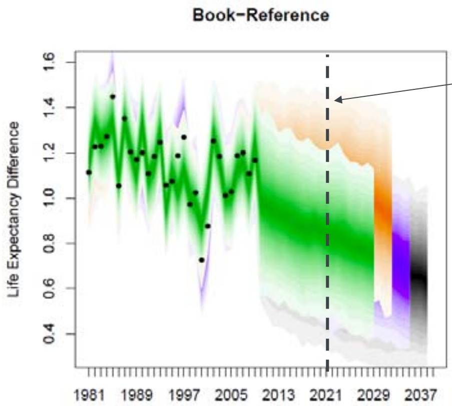
Selecting an appropriate two-population model for basis risk assessment

		Models		
		Extensions of the Lee-Carter (non-parametric age term)	Extensions of the CBD model (parametric age-term)	Other models
Criteria	Stage 1	No data required		
	Stage 2	Goodness-of-fit and reasonableness		
	Stage 3	Robustness		

Robustness of the models with respect to:

- Book size
- History length

How small can the book be for direct modelling?



Look at variance decomposition by risk and population size

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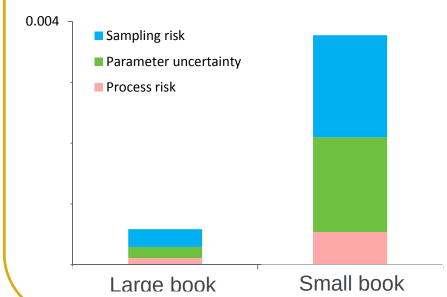
How small can the book be for direct modelling?

Variance decomposition



Much smaller book leads to a lot more sampling risk in the projection. ✓

Smaller book leads to greater parameter uncertainty.

Is this a fair assessment of basis risk? ✗

The sampling noise in the back history gives a larger fitted volatility to the time series.

This potentially **overestimates** basis risk. ✗

Sampling noise a problem for books with less than 25K lives

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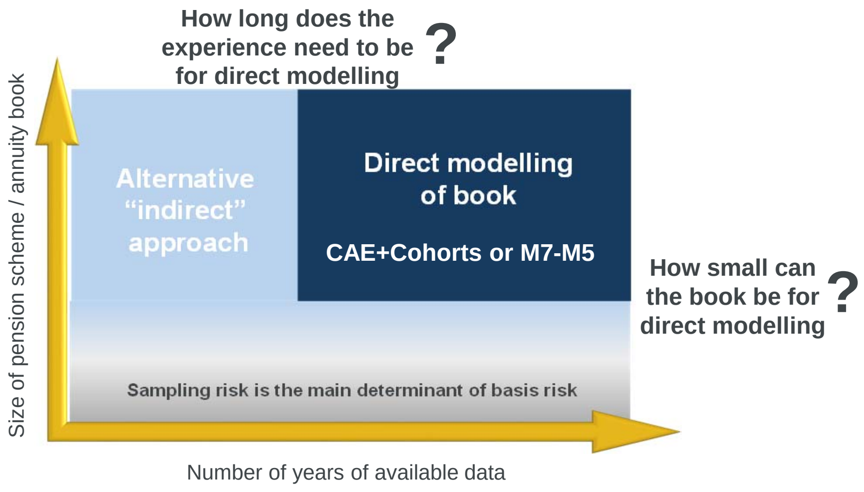
Choice of two-population model



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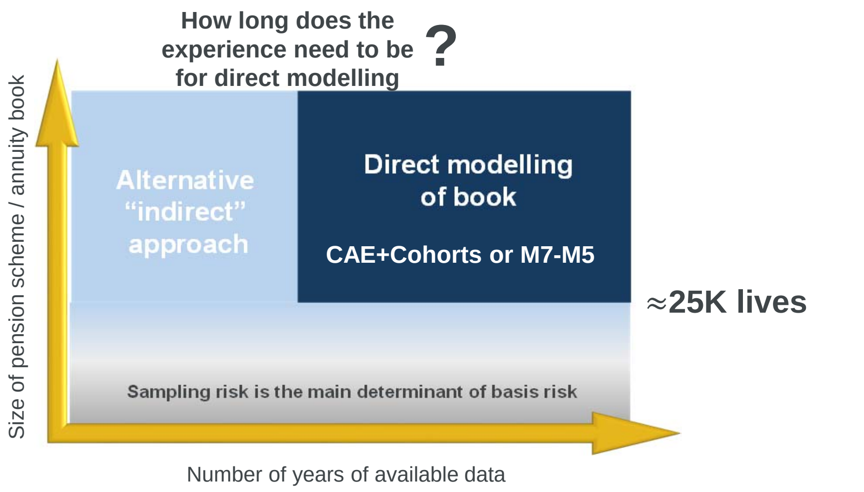
Choice of two-population model



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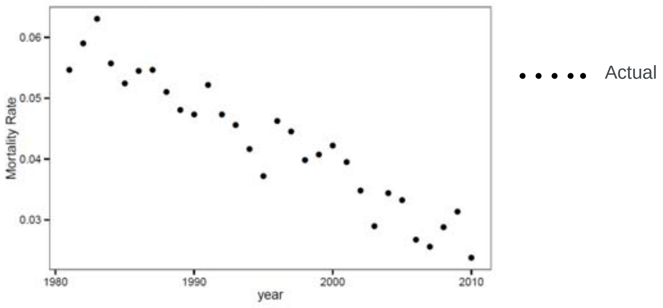
How long does the experience need to be for direct modelling?



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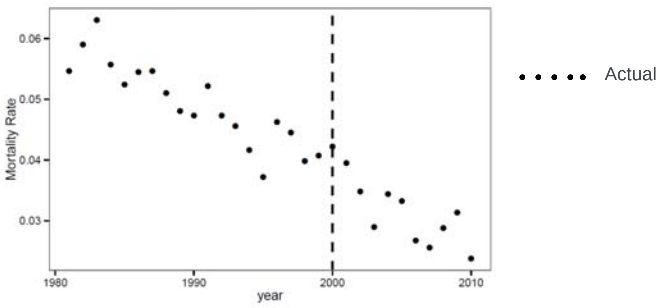

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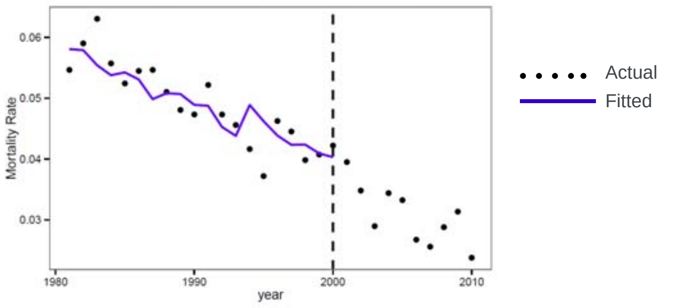

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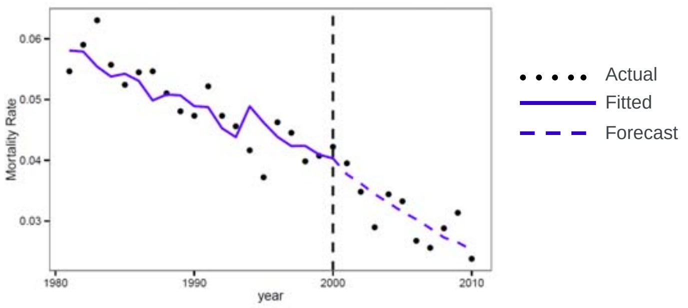
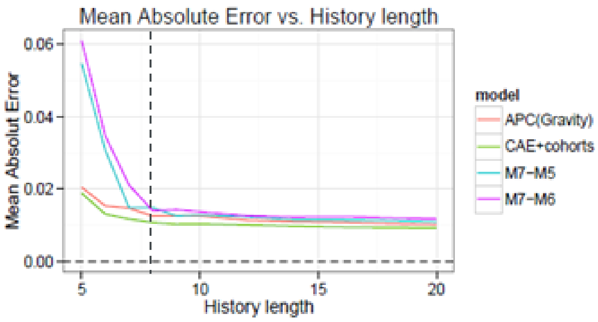


Look at performance of the models for different history lengths

Mean absolute error = |Actual – Forecast|

How long does the experience need to be for direct modelling?

Forecasting performance by history length (Back testing methodology)



*Average 5 to 15 year forecast horizon

For history lengths shorter than 8 years forecasting performance is poor

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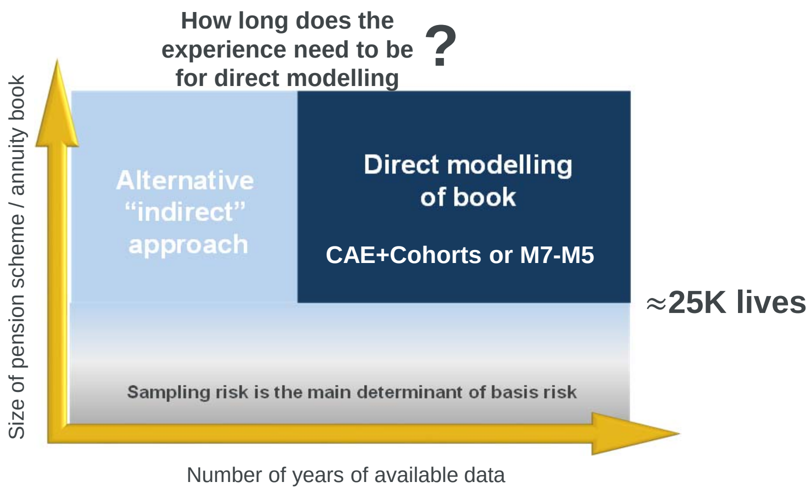
Choice of two-population model



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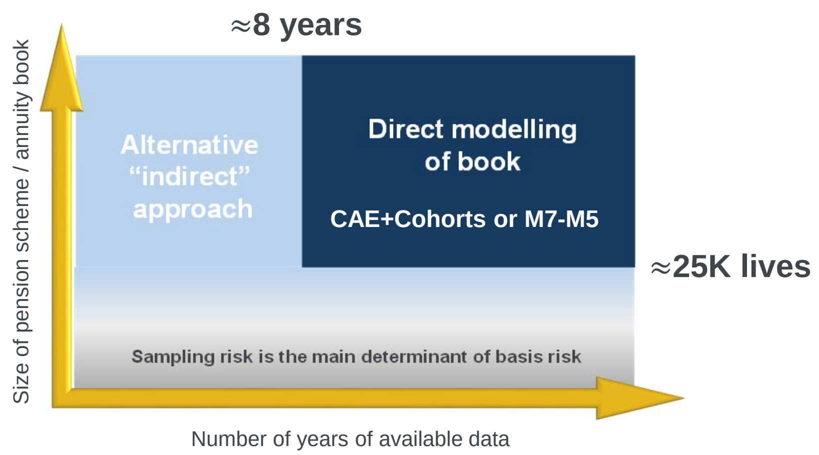
Choice of two-population model



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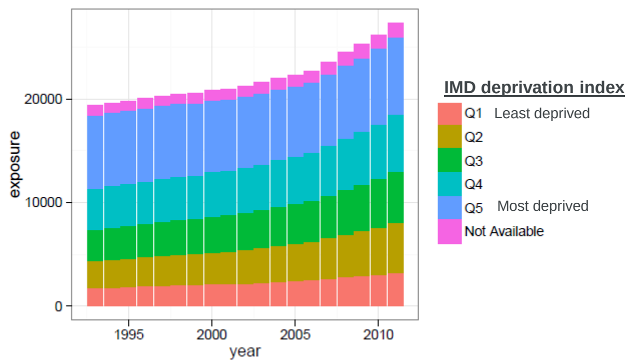
Two models in action

A case study of hedge effectiveness insights

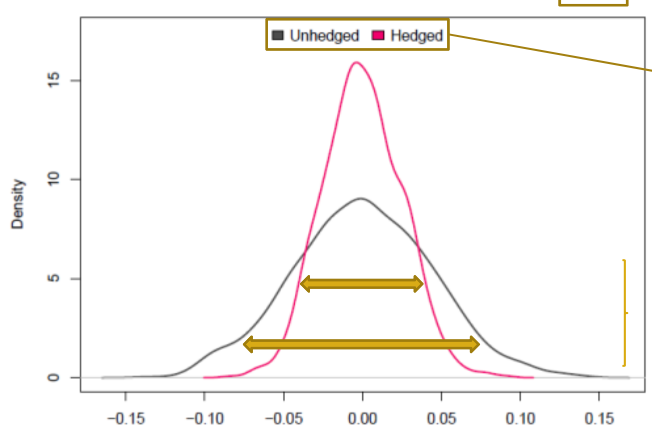
Our example portfolio

- ✓ **Real** book
- ✓ **Large**
- ✓ Long back **history**
- ✓ **Different** SEC mix to population
- ✓ **Stable** SEC mix over time



Measuring hedge effectiveness

Simulated distribution of Hedged and Unhedged quantities $({}_{30}p^{B}_{60,t})$



1 30 year survival probability on a period basis – here t=10

2 Compare outcomes from book ('unhedged') and book net of reference population ('hedged')
Note: Both presented relative to average

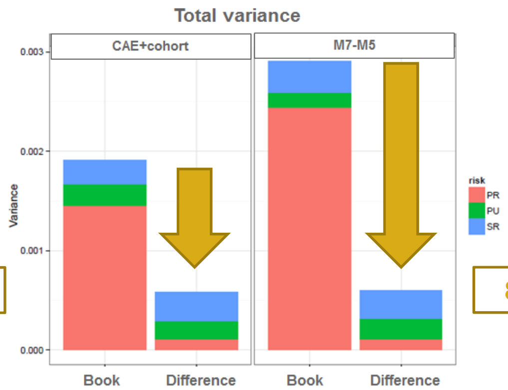
3 Compare spread of outcomes under 'hedged' to 'unhedged'. Reduction in spread is a measure of hedge effectiveness

$$1-\frac{variance\ of\ hedged}{variance\ of\ unhedged}$$

Model comparison – CAE+Cohort vs M7-M5

Survival probabilities from age 60 to age 90

Index based hedge for in-payment annuitants



68%

80%

Different variance reductions - Similar residual basis risk

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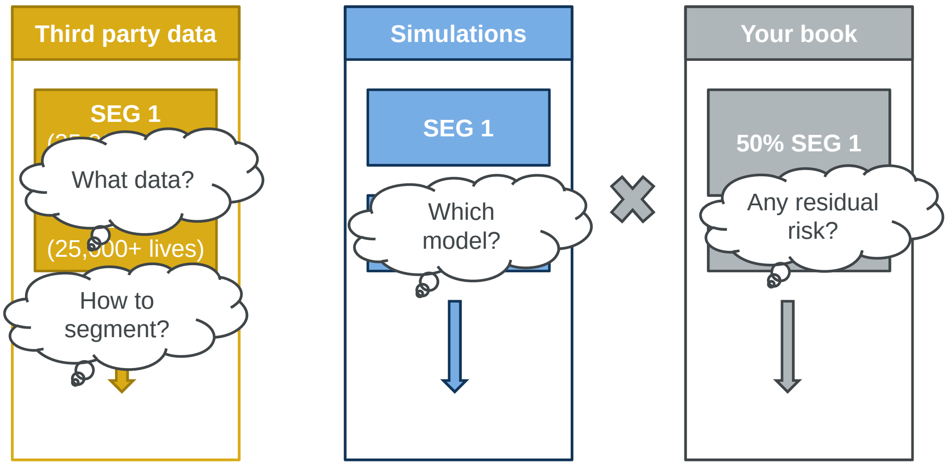
Characterisation Approach

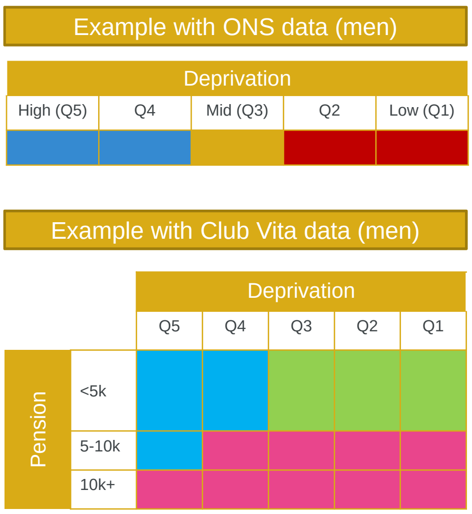
For annuity books below 25,000 lives

The characterisation approach



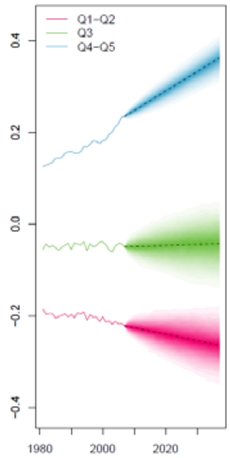
Data and segmenting

- Wide range of potential data sources:
  - ONS (segment by IMD)
  - CMI (segment by pension amount)
  - Club Vita (multiple potential factors)
- Principles for creating SEGs:
  - 25,000+ lives
  - Capture differences in trends
  - Keep groups with very different baseline apart
  - Widely usable
  - Parsimony

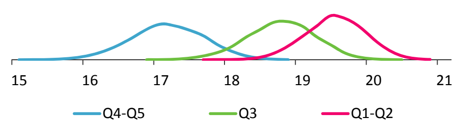


Modelling the SEGs

Time trend for mortality relative to average*



Life expectancy in 2020
(from age 65, curtate at age 90)



- Leverage the two population model but:
  - Allow for correlations between groups
  - Ensure adequate width to funnels
  - Do you want divergence or convergence?

* The κ_1 term under M7-M5

Testing the approach

Annuity book	Annual exposure[1]	Exposure period	IMD split Low Mid High Unknown	Club Vita Wealthy Middling Unhealthy Unknown	Commentary
Large A	28k	1993 2011 2013			• Single pension scheme • Large enough to do direct modelling • Long history
Large B	28k	1995 2007 1993 2013			• Combined scheme[2] • Large enough to do direct modelling • Medium history
Large C	28k	1997 2006 1993 2013			• Combined scheme[2] • Large enough to do direct modelling • Medium history
Medium	20k	1997 2006 1993 2013			• Single pension scheme • Borderline for direct modelling • Medium history • Wealthy
Small	12k	1993 2011 2013			• Single pension scheme • Too small for direct modelling • Long history • Very wealthy

Different (lives) splits

Notes:
1. Exposure in final year of data
2. Combined schemes are generated by pooling data from pensopn schemes in very similar industries to create a sufficiently large portfolio for direct modelling.

Example hedge effectiveness results

Annuity Book	Direct Modelling M7-M5 (VAR with Constant)	Different characterisation approaches: Club Vita Characterisation (VAR with Constant)	ONS Characterisation (MRWD)	ONS Characterisation (VAR around Trend)
Large A	78%	84%	77%	88%
Large B	80%	79%	73%	85%
Large C	65%	77%	73%	84%
Medium	77%	80%	75%	85%
Small		75%	70%	79%

Example hedge effectiveness results

Annuity Book	Direct Modelling M7-M5 (VAR with Constant)	Different characterisation approaches: Club Vita Characterisation (VAR with Constant)
Large A	78%	84%
Large B	80%	79%
Large C	65%	77%
Medium	77%	80%
Small		75%

Credible alternative to direct modelling

Example hedge effectiveness results

Annuity Book	Direct Modelling M7-M5 (VAR with Constant)	Club Vita Characterisation (VAR with Constant)
	Different characterisation approaches	
Large A	78%	84%
Large B	80%	79%
Large C		
Medium	77%	80%
Small		75%

Modest residual basis risk

Example hedge effectiveness results

Annuity Book	Direct Modelling M7-M5 (VAR with Constant)	Club Vita Characterisation (VAR with Constant)
	Different characterisation approaches	
Large A	78%	84%
Large B	80%	79%
Large C	65%	77%
Medium	77%	80%
Small		75%

Lots of ‘unknowns’

Need to be able to map most annuitants

Example hedge effectiveness results

Different characterisation approaches

Annuity Book	Club Vita Characterisation (VAR with Constant)	Different datasets	ONS Characterisation (VAR around Trend)
Large A	84%		88%
Large B	79%		85%
Large C	77%		84%
Medium	80%		85%
Small	75%		79%

Results depend on data used

But similar high-level conclusions

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Example hedge effectiveness results

Different characterisation approaches

Different types of time series

Annuity Book	ONS Characterisation (MRWD)	ONS Characterisation (VAR around Trend)
Large A	77%	88%
Large B	73%	85%
Large C	73%	84%
Medium	75%	85%
Small	70%	79%

Results depend on how interpret 'signals' in the data

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Summing up

Summing up

Today we have seen	On 8th December will also cover

Today we have seen

- Highlighted importance of demographic risk
- Illustrated a direct modelling approach
  - Including how we have narrowed down the wide range of possible models to 'best of breed'
- Introduced a method for smaller books
- Shown that it is possible to assess risk-reward trade-off of index-based swaps

On 8th December will also cover

- A decision framework:
  - When to use M7-M5 and when to use CAE+ Cohorts
  - Some other criteria we have glossed over today!
- Some key challenges faced in practice:
  - Men and women
  - Incorporating user (expert) judgement
  - The time series dilemma

We hope to see you at the sessional meeting on 8th December where we will launch the full framework.

Questions Comments

Expressions of individual views by members of the Institute and Faculty of Actuaries and its staff are encouraged.

The views expressed in this presentation are those of the presenter.

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