Quantifying and allocating mortality risk in defined-benefit pension schemes

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Motivation

- Small defined-benefit pension schemes
- Mortality risk components:
  - Systematic risk - distribution of deaths.
  - Idiosyncratic risk - random fluctuations.

Outline

Homogeneous pension scheme

- N members all age 40.
- $Y_n :=$ P.V. r.v. of £1 p.a. from age 65, for member n.
- P.V. r.v. of total liability

$$L_N = \sum_{n=1}^{N} Y_n.$$

- For example,

$$\mathbb{E}(L_N) = N\, v^{65-40}\, {}_{25}p_{40}\, \bar{a}_{65}.$$

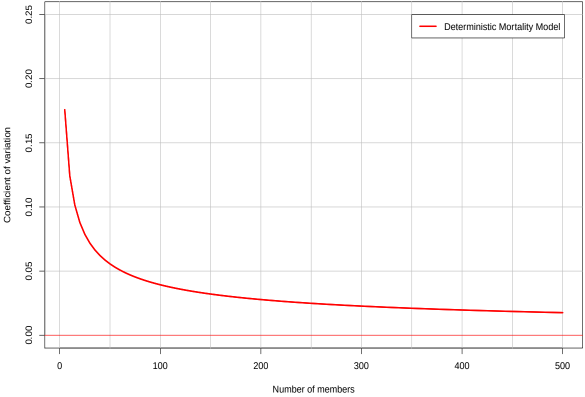
Risk measure

$$\boxed{\text{Coefficient of variation} = \frac{\text{standard deviation of total liability}}{\text{expectation of total liability}}}$$

If $Y_1, Y_2, \ldots, Y_N$ are independent, then

$$\mathrm{VCo}(L_N) \quad = \quad \frac{\mathrm{sd}(L_N)}{\mathbb{E}(L_N)} \quad \to \quad 0 \quad \text{as } N \to \infty.$$

Numerical results: deterministic mortality model (PMA92C10) and $\delta = 4\%$ p.a.



Stochastic mortality model

Stochastic mortality model r

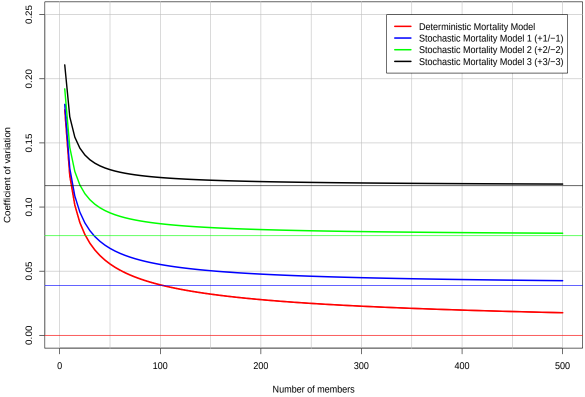
$$\text{age rating} = \begin{cases} +r & \text{with probability } 0.5, \\ -r & \text{with probability } 0.5. \end{cases}$$

based on PMA92C10.

e.g. either *everyone* is PMA92C10 $+1$,

or *everyone* is PMA92C10 -1.

Numerical results: $\delta = 4\%$ p.a.



Stochastic mortality model r

Here $Y_1, Y_2, \ldots, Y_N$ are not independent, and

$$\mathrm{VCo}_r(L_N) \quad = \quad \frac{\mathrm{sd}_r(L_N)}{\mathbb{E}_r(L_N)} \quad \to \quad \frac{\sqrt{\mathrm{Cov}_r(Y_1, Y_2)}}{\mathbb{E}_r(Y_1)} \quad \text{as } N \to \infty.$$

Interpretation:

- Systematic risk measure:

$$\frac{\sqrt{\mathrm{Cov}_r(Y_1, Y_2)}}{\mathbb{E}_r(Y_1)}.$$

- Idiosyncratic risk measure:

$$\mathrm{VCo}_r(L_N) - \frac{\sqrt{\mathrm{Cov}_r(Y_1, Y_2)}}{\mathbb{E}_r(Y_1)}.$$

Summary of homogeneous pension scheme

1. Uncertainty about distribution of deaths $\uparrow$, then $\mathrm{VCo}(L_N) \uparrow$.

Systematic risk: cannot eliminate by increasing number of members N.

2. Number of members $\downarrow$, then $\mathrm{VCo}(L_N) \uparrow$.

Idiosyncratic risk: reduce by increasing number of members N.

Pension scheme with executive section

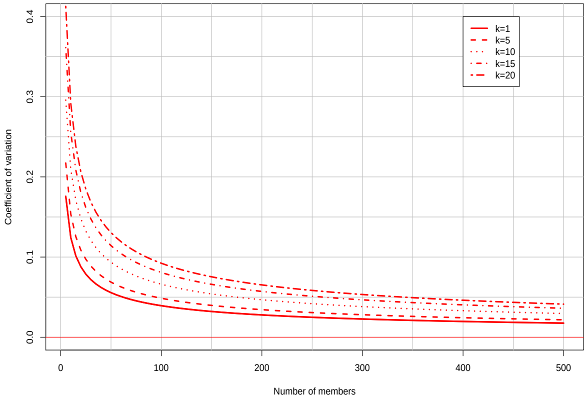
As before except

- αN are executives.
- Executive benefit: £k p.a. from age 65.

e.g. $N = 100$, $\alpha = 5\%$, $k = 20$. Then

- 95 members receive £1 p.a. from age 65 (non-execs).
- 5 members receive £20 p.a. from age 65 (execs).

Numerical results: deterministic mortality model (PMA92C10), $\delta = 4\%$ p.a. and $\alpha = 5\%$.



Pension scheme with executive section

- Setting

$$f(\alpha, k) = \frac{\alpha k^2 + 1 - \alpha}{(\alpha k + 1 - \alpha)^2},$$

we find

$$\begin{aligned}
&\mathrm{VCo}_r(L_N) \\
=&\frac{1}{\mathbb{E}_r(Y_1)} \\
&\cdot \left(\frac{1}{N} f(\alpha, k) \left(\mathrm{Var}_r(Y_1) - \mathrm{Cov}_r(Y_1, Y_2)\right) + \mathrm{Cov}_r(Y_1, Y_2) \right)^{1/2}.
\end{aligned}$$

Summary of pension scheme with executive section

1. Executive benefits $\uparrow$, then $\mathrm{VCo}(L_N) \uparrow$.

Idiosyncratic risk: reduce by

- increasing number of members N, and
- decreasing executive benefit levels k.

Risk capital

- Risk capital: cushion against unexpected losses.
- "unexpected losses"... define as:
  - extreme quantile-type, e.g. $\mathrm{VaR}_{99\%}(L_N)$, $\mathrm{ES}_{99\%}(L_N)$?
  - interval around the mean, e.g. standard deviation, semi-standard deviation of L_N?
- We focus on standard deviation.

Risk capital definition

$$\boxed{\text{Risk capital} := \mathrm{sd}_r(L_N)}$$

Scheme 1: All members currently age 40.

- 195 members receive £1 000 p.a. from age 65 (non-execs).
- No executive members.

Basis: PMA92C10, interest rate 4% p.a.

- Expected liability is £869 500.
- Standard deviation of liability is £24 500.
- Risk capital is £24 500.

Risk capital definition

$$\boxed{\text{Risk capital} := \text{sd}_r(L_N)}$$

Scheme 2: All members currently age 40.

- 95 members receive £1 000 p.a. from age 65 (non-execs).
- 5 members receive £20 000 p.a. from age 65 (execs).

Basis: PMA92C10, interest rate 4% p.a.

- Expected liability is £869 500 (same as Example 1),
- BUT standard deviation of liability is £80 300.
- Risk capital is £80 300.

Risk capital allocation

Idea: find concentrations of risk by allocating risk capital.

Risk capital allocation problem:

For member n, find an amount π_n such that

$$\sum_{n=1}^{N} \pi_n = \text{Risk capital} = \mathrm{sd}_r(L_N).$$

Ideas?

- Benefit-weighted allocation.
- Covariance principle.

Benefit-weighted allocation

Benefit-weighted allocation: allocate risk capital among members in proportion to benefit.

Scheme 1: All members currently age 40.

- 195 members receive £1 000 p.a. from age 65 (non-execs).
- No executive members.
- Risk capital is £24 500.

Benefit-weighted allocation: all members have identical allocation $\pi^{\text{non-exec}}$.

Find: $\pi^{\text{non-exec}} =$£126.

Benefit-weighted allocation

Benefit-weighted allocation: allocate risk capital among members in proportion to benefit.

Scheme 2: All members currently age 40.

- 95 members receive £1 000 p.a. from age 65 (non-execs).
- 5 members receive £20 000 p.a. from age 65 (execs).
- Risk capital is £80 300.

Benefit-weighted allocation: $\pi^{\text{exec}} = 20\pi^{\text{non-exec}}$.

Find: π^{exec} =£8 236 and $\pi^{\text{non-exec}}$ =£412.

Benefit-weighted allocation

$$\boxed{\text{Is } 5 \times £20\,000 \text{ p.a. as risky as } 100 \times £1\,000 \text{ p.a?}}$$

Benefit-weighted allocation: **Yes**.

Scheme 1: 100 members allocated $£126 \times 100 = £12\,600$.

Percentage of risk capital $= \frac{12\,600}{24\,500} = 51\%$.

Scheme 2: 5 executives allocated $£8\,236 \times 5 = £41\,200$.

Percentage of risk capital $= \frac{41\,200}{80\,300} = 51\%$.

Covariance principle

Covariance principle: if X_n is P.V. r.v. of benefit due to member n then

$$\pi_n = \frac{\mathrm{Cov}_r(X_n, L_N)}{\mathrm{sd}_r(L_N)}$$

is the risk capital allocated to member n.

Also called the Euler capital allocation principle.

Covariance principle

Scheme 1: All members currently age 40.

- 195 members receive £1 000 p.a. from age 65 (non-execs).
- No executive members.
- Risk capital is £24 500.

All members in the scheme are identical.

Covariance principle: find: $\pi^{\text{non-exec}} =$£126.

Same as benefit-weighted allocation method.

Covariance principle

Scheme 2: All members currently age 40.

- 95 members receive £1 000 p.a. from age 65 (non-execs).
- 5 members receive £20 000 p.a. from age 65 (execs).
- Risk capital is £80 300.

Covariance principle, find $\pi^{\text{exec}} =$£15 332 and $\pi^{\text{non-exec}} =$£38.

Covariance principle

$$\boxed{\text{Is } 5 \times £20\,000 \text{ p.a. as risky as } 100 \times £1\,000 \text{ p.a?}}$$

Covariance principle: **No**.

Scheme 1: 100 members allocated $£126 \times 100 = £12\,600$.

Percentage of risk capital $= \frac{12\,600}{24\,500} = 51\%$.

Scheme 2: 5 executives allocated $£15\,332 \times 5 = £76\,700$.

Percentage of risk capital $= \frac{76\,700}{80\,300} = 95\%$.

Comparing capital allocation methods

Which method reflects risk best?

Scheme 2: All members currently age 40.

- 95 members receive £1 000 p.a. from age 65 (non-execs).
- 5 members receive £20 000 p.a. from age 65 (execs).
- Risk capital is £80 300.
- Benefit-weighted allocation: executives allocated 51% of total risk capital.
- Covariance principle: executives allocated 95% of total risk capital.

Connection between capital allocation methods

Fix α, k.

Let p^{exec} = proportion of risk capital allocated to execs.

- Under covariance principle and a deterministic mortality model,

$$p^{\text{exec}} = \frac{\alpha k^2}{\alpha k^2 + 1 - \alpha}.$$

- Under benefit-weighted allocation and any mortality model,

$$p^{\text{exec}} = \frac{\alpha k}{\alpha k + 1 - \alpha}.$$

Capital allocation methods

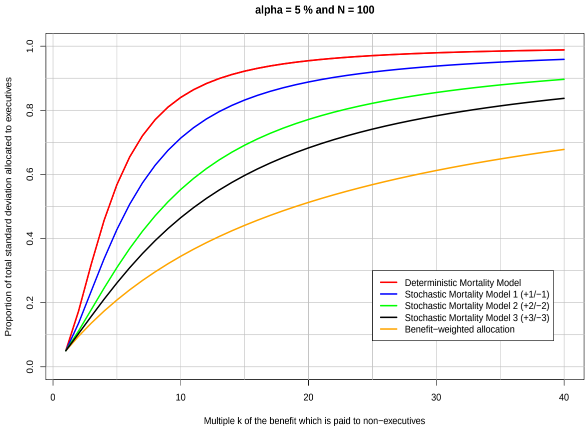
Under covariance principle and a stochastic mortality model,

- $$p^{\text{exec}} = \frac{\alpha k}{\alpha k + 1 - \alpha} \; + \; \mathrm{O}\left(\frac{1}{N}\right).$$
- $\{\text{exec systematic risk}\} = k \times \{\text{non-exec systematic risk}\}$.
- $\{\text{exec idiosyncratic risk}\} =$ $k \times \{\text{non-exec idiosyncratic risk}\} \; + \; k(k-1) \cdot \mathrm{O}\left(\frac{1}{N}\right)$.

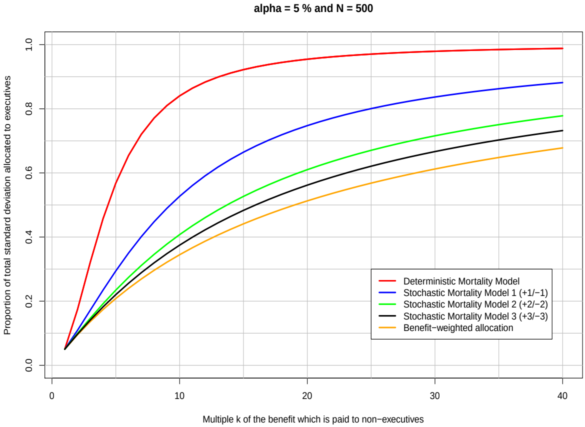
As $N \to \infty$,

$p^{\text{exec}}(\text{covariance, SMM}) \to p^{\text{exec}}(\text{benefit-weighted, any mortality})$.

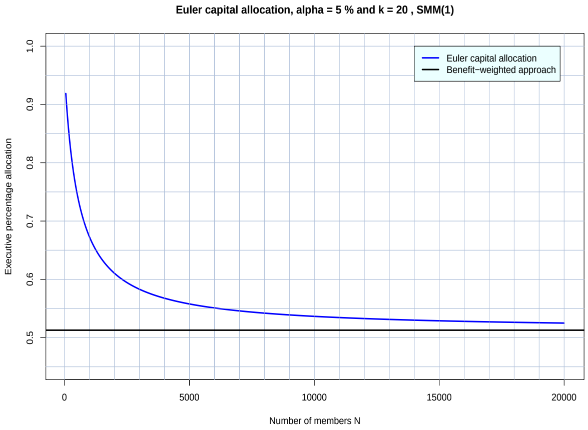
Numerical results: $\alpha = 5\%$, $N = 100$ and $\delta = 4\%$ p.a.



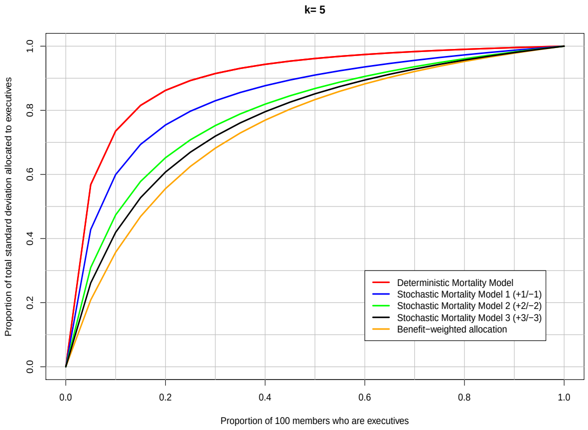
Numerical results: $\alpha = 5\%$, $N = 500$ and $\delta = 4\%$ p.a.



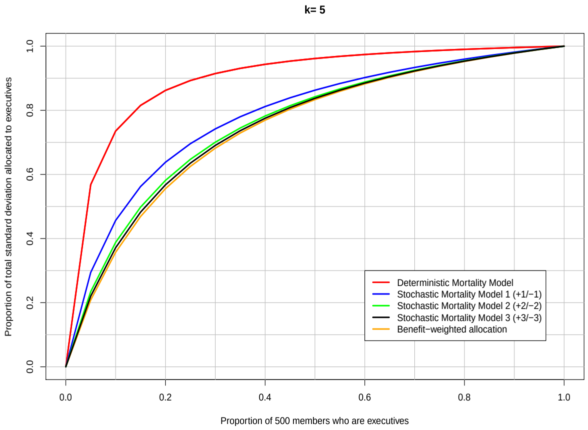
Numerical results: $\alpha = 5\%$, SMM(1) and $\delta = 4\%$ p.a.



Numerical results: $k = 5$, $N = 100$ and $\delta = 4\%$ p.a.



Numerical results: $k = 5$, $N = 500$ and $\delta = 4\%$ p.a.



Summary

- Quantify mortality risk.
- Can help us to understand risks better.
- Techniques exist to allocate risk.
- Measuring concentrations of risk can inform risk mitigation strategies.

Future work

- Study of different capital allocation techniques.
- Incorporate financial risks.
- Risk mitigation strategies.

Acknowledgements and references

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More details:

C. Donnelly. *Quantifying mortality risk in small defined-benefit pension schemes.* Scandinavian Actuarial Journal, to appear. DOI: 10.1080/03461238.2011.635803.

Website: `http://www.ma.hw.ac.uk/~cd134/`