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Travels into Several Remote Nations of the World. In Four Parts. By Lemuel Gulliver, First a Surgeon, and then a Captain of Several Ships

International Mortality and Longevity Symposium
15-17 September 2014

Xu Shi and Bridget Browne

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Adding stochastics to a deterministic forecasting method (such as the CMI method) – calibrated and exemplified with Australian data, or A "simple" stochastic model for longevity risk revisited through bootstrap

International Mortality and Longevity Symposium
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Or…
How to make a deterministic mortality forecast stochastic.

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Why does the issue arise?

- CMI method is deterministic
- How to determine percentiles for
  - Pricing
  - Reserving
  - Internal capital modelling
  - Etc
- How to incorporate prudence or conservatism with quantification?

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The Original Model (Koller 2011)

$$q_{x,t} = \hat{q}_{x,t} \times C_t + \varepsilon_{x,t}$$

$$C_t = e^{X_t} \times C_{t-1}$$

$$C_0 = 1$$

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The Original Model (Koller 2011)

$(X_t)_{t\in\mathbb{N}_0}$, iid N(μ, σ^2)

$(e^{X_t})_{t\in\mathbb{N}_0}$, iid lognormal

$\mathbb{E}[e^{X_t}] = 1$, is imposed, so that best estimate is followed

$\mathbb{E}[e^{X_t}] = e^{\mu+0.5\sigma^2}$, follows from above *

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Modifications

- What if X_t is not normal?
- Transformation - rejected
- Non-parametric approach – bootstrap
- Improved model selection
- Consider X rather than X_t

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A numerical example

- Australian Females
- Human Mortality Database
- Ages 55 to 89 inclusive
- 1954 to 2008
- Surface of 35 ages and 55 cal. years

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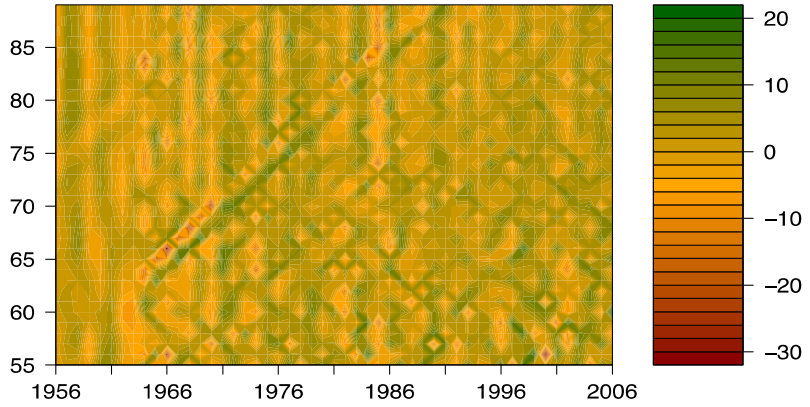
Crude Mortality Improvement Rates



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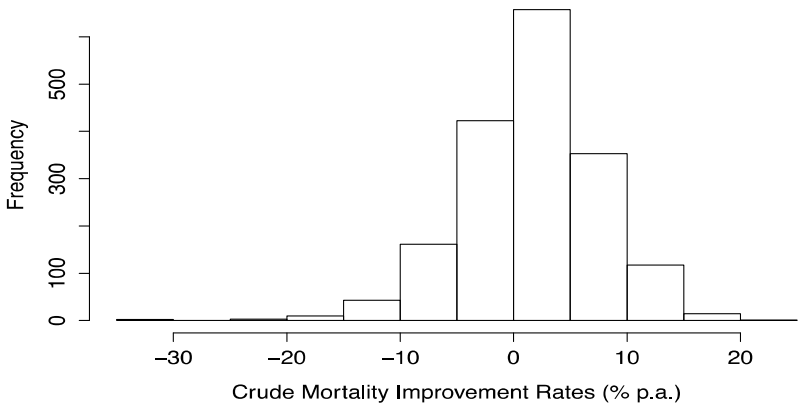
Histogram of Crude MI Rates



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Model assessment

- Use smoothing models only to help determine our best estimate of future variability
- M1-M8 from LifeMetrics (JP Morgan)
  - Standardised residuals and BIC
  - Forecasting properties

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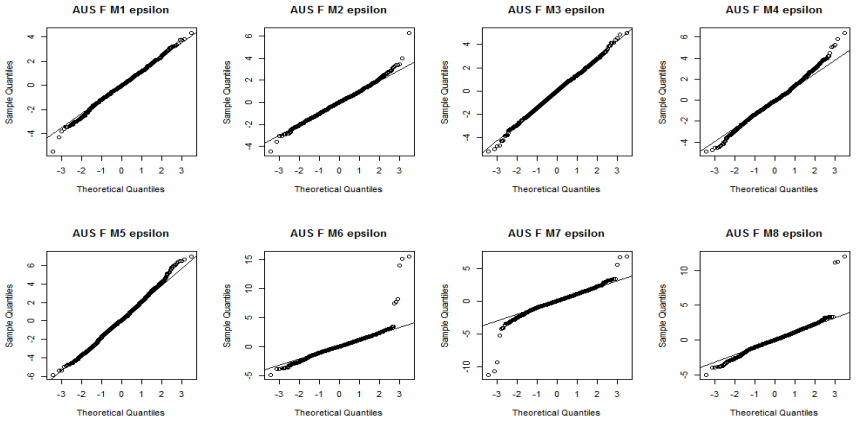
QQ Plots



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Summary statistics for epsilon and BIC

Model	Mean	Variance	Skewness	BIC
1	-0.000606	1.489723	-0.087587	-10 152
2	0.007536	1.088392	0.134849	-10 192
3	-0.003997	2.028962	-0.085894	-10 917
4	-0.019290	2.078517	0.172425	2 136
5	0.166069	3.876925	0.139640	-12 399
6	0.034808	1.842268	2.350169	-10 309
7	-0.005507	1.496240	-1.015056	-10 397
8	0.020308	1.582045	1.035231	-10 297

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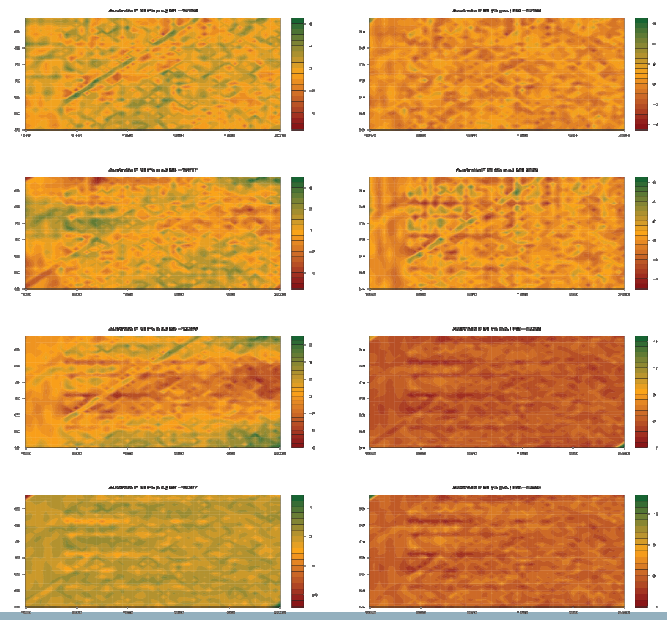

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Forecasting properties

- Now choosing between M2 and M3
- M2 unstable
- Backtesting for M3

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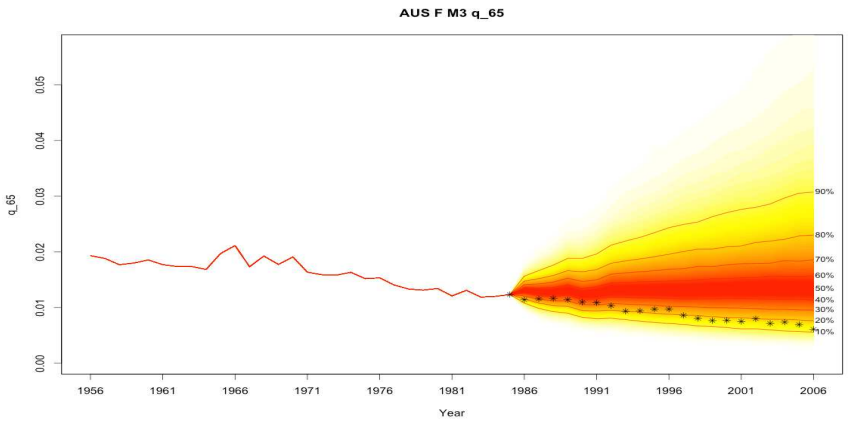
Prediction Intervals I



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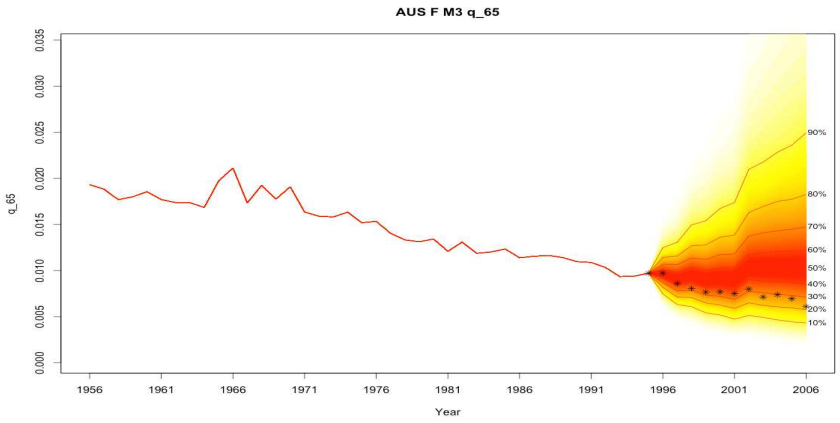
Prediction Intervals II



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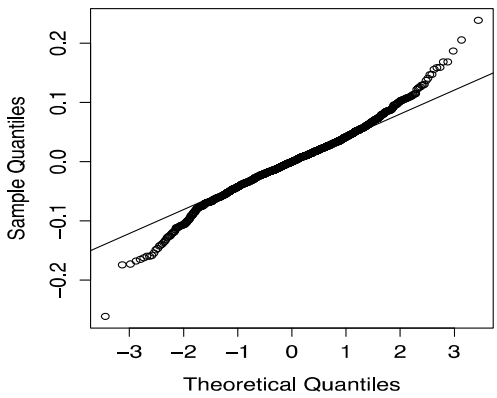
QQ Plot of X_t for M3 Australian Females



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Age dependence or independence?

- P-values of two-variable regression of X against age (x) and year (t)

Model Type	Variable	ANOVA p-value
Continuous Case	Year	0.1079
	Age	0.8350
Discrete Case	Year	0.9965
	Age	0.9999

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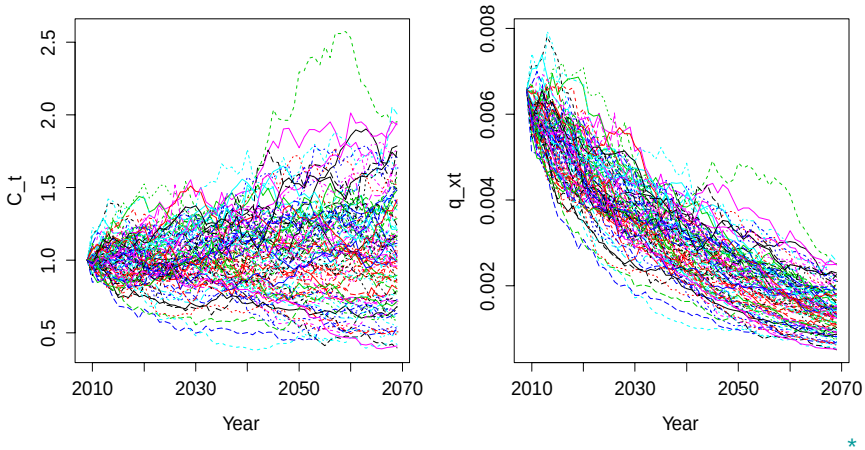
Paths of Ct, q65,t for Australian Females



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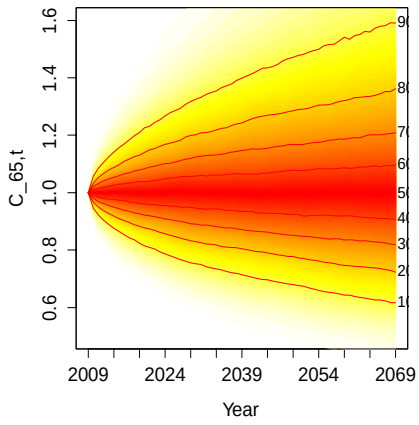
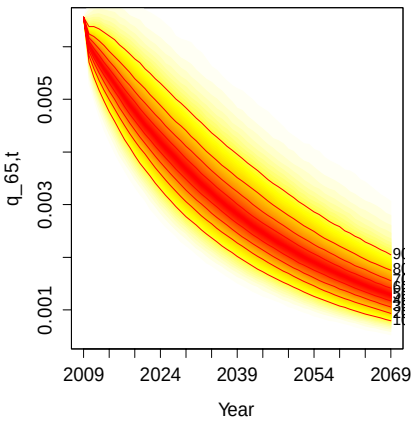
Fan plots of Ct, q65,t for Australian Females





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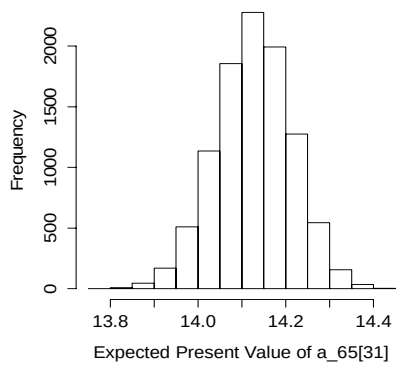
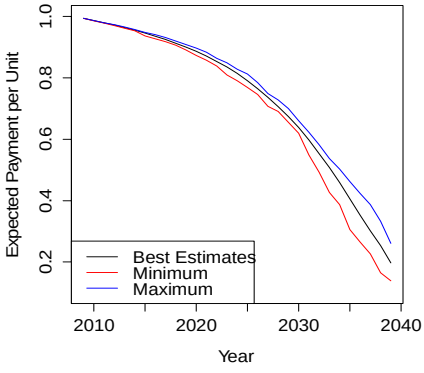
Aust Females (Age Independent Case, X)





Histogram (LHS) of $a_{65:[31]}$ (at 4% pa) and cohort survival curves(RHS)

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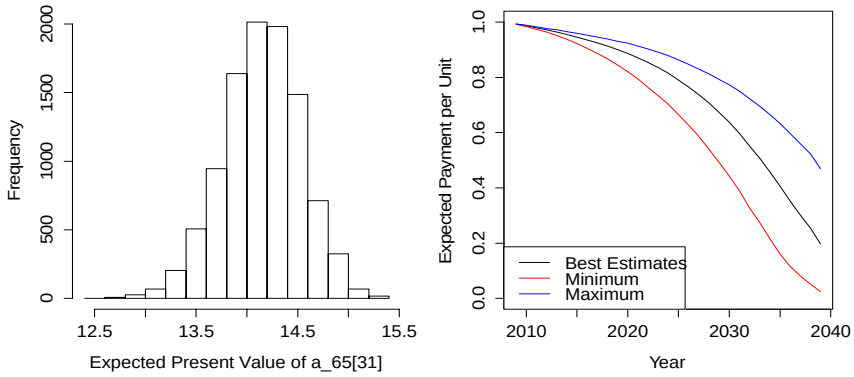
Aust Females (Age Dependent Case, X_t)



Histogram (LHS) of $a_{65:[31]}$ (at 4% pa) and cohort survival curves(RHS)

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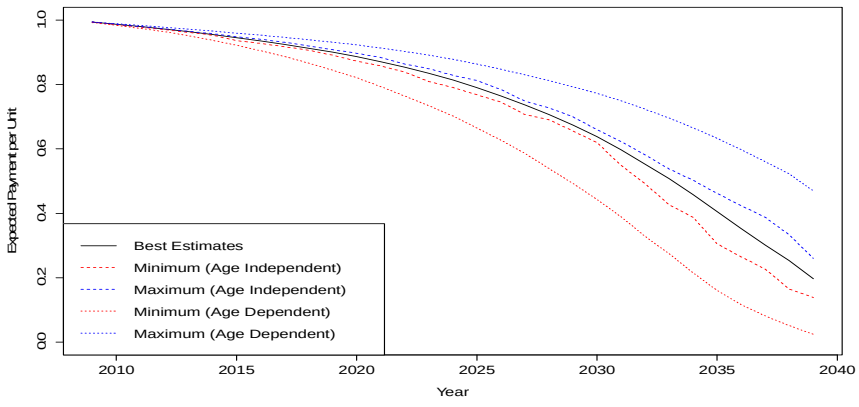
Paths of cohort survival curves for $a_{65:[31]}$ for Australian Females (Two Cases)



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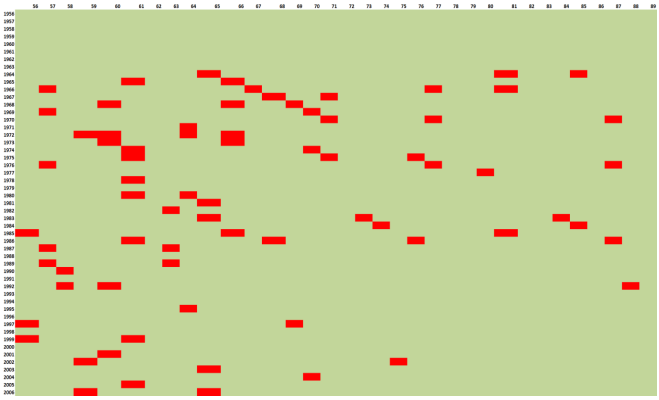
Age independence => coherence?



• Example of crossover in Crude Mortality Rates for Australian Females
(red cell indicates mortality in year t is greater for age x than for age x+1)

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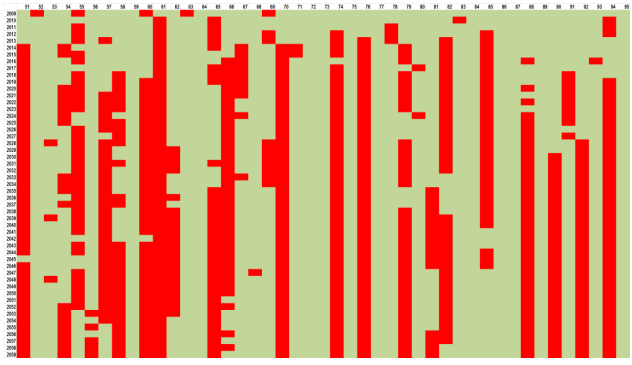
Age independence => coherence?



• Example of crossover in one simulation for Australian Females
(red cell indicates mortality in year t is greater for age x than for age x+1)

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Comparison with other methods

- Tickle and Booth (2014) evaluated and updated a forecast of mortality for Australian seniors using the Booth-Maindonald-Smith variant of Lee-Carter (generously shared underlying work)
  - Central estimate and 80% PI for qx,t
- LPS 115 Section 38 (APRA, 2013, p7)
  - 20% drop in qx,t
  - < 0.5% probability actual claims exceed

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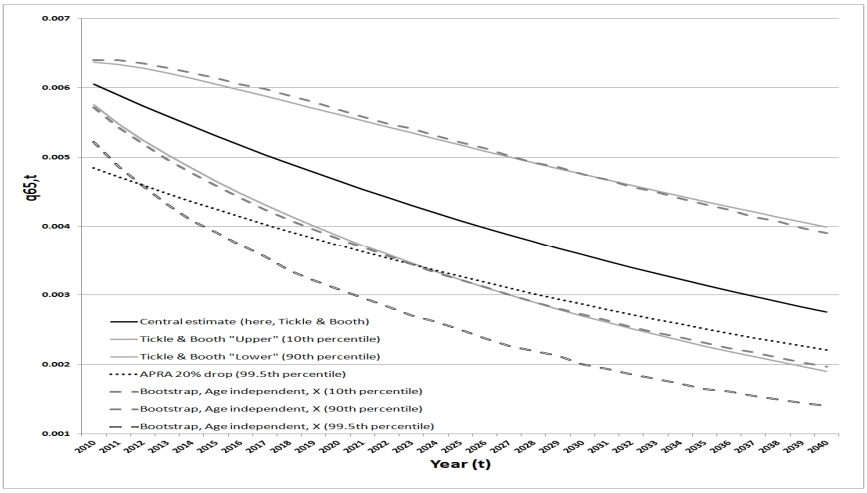
Forecast mortality rates for Australian Females aged 65, central estimate & selected percentiles



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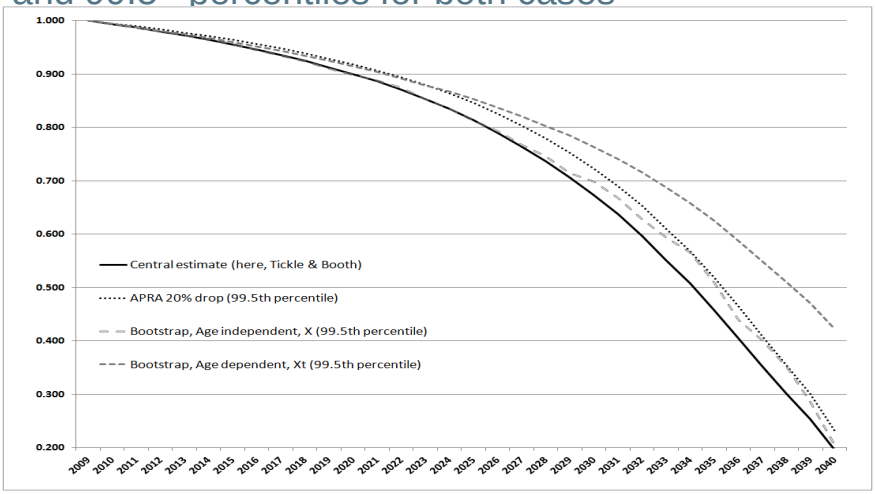
Paths of cohort survival curves (ie cashflows) for Australian Females, central estimate, APRA basis and 99.5th percentiles for both cases



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Comparison of Cohort Life Expectancy and Annuity Values (at 4% pa) for Australian Females aged 65 in 2010, central estimate and selected percentiles

	e65[31],2010	% diff	a65[31],2010	% diff
Central estimate (here, Tickle & Booth)	22.668		14.166	
APRA 20% drop (99.5th percentile)	23.655	4.35%	14.610	3.14%
Bootstrap, Age independent, X (10th percentile)	22.287	-1.68%	14.019	-1.03%
Bootstrap, Age independent, X (90th percentile)	22.845	0.78%	14.254	0.62%
Bootstrap, Age independent, X (99.5th percentile)	23.102	1.91%	14.325	1.12%
Bootstrap, Age dependent, Xt (10th percentile)	21.415	-5.53%	13.636	-3.74%
Bootstrap, Age dependent, Xt (90th percentile)	23.817	5.07%	14.645	3.39%
Bootstrap, Age dependent, Xt (99.5th percentile)	24.912	9.90%	15.038	6.16%

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Conclusion

- CMI method more widely used - UK, USA, Canada, Australia and China at least
- (SOA, 2014, Office of the Chief Actuary, 2014, Huang and Browne, 2014, CMI, 2009a, CMI, 2009b)
- Some national statistics bodies also deterministic (ABS, 2013, ONS, 2013)
- This method allows the user to add stochastic variation to a deterministic model
- Bootstrapping improves tail modelling
- Help to inform understanding of longevity risk

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