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The Design of Pension Contracts, on the perspective of customers

Zhaoxun Mei
Heriot-Watt University, Edinburgh

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What contract is attractive?

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- Is the largest expected value optimal to the customers?

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- Is the largest expected value optimal to the customers?
- Expectation = $\sum x_i p_i$

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What contract is attractive?

St. Petersburg Paradox

- Tossing a fair coin until the first head appears. What you will receive is £2^n if the first head appears at n_{th} coin tossing.
- How much would you pay to play this game?

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$$E = \frac{1}{2} \times 2 + \frac{1}{2^2} \times 4 + \frac{1}{2^3} \times 8 + \ \dots + \frac{1}{2^n} \times 2^n$$

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$$= \ 1 \ + \ 1 \ + \ 1 \ + \dots + \ 1 \ = \infty$$

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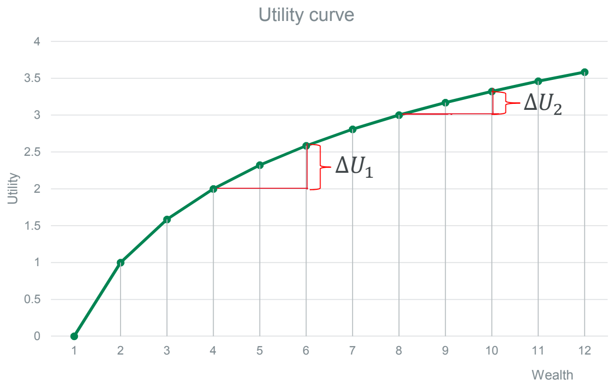
Expected Utility Theory

St. Petersburg Paradox

- Daniel Bernoulli (1738) proposed a solution using utility function.



Characteristics:

- prefer more to less

$$U'(x) > 0.$$

- diminishing marginal utility

$$U''(x) < 0.$$

Expected Utility Theory

- Von Neumann and Morgenstern (1947) show that rational people will always prefer actions that maximize expected utility.

- Mathematically, the expected utility is calculated as

$$E = \sum U(x_i)p_i$$

Expected Utility Theory

- $U(x_i) = \boldsymbol{log_2(x)}$

x_i	2	4	8	…	2^n
p_i	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	…	$\frac{1}{2^n}$
$U(x_i)$	1	2	3	…	n

$$E = \frac{1}{2} \times log_2 2 + \frac{1}{2^2} \times log_2 4 + \frac{1}{2^3} \times log_2 8 + \ldots + \frac{1}{2^n} \times log_2 2^n$$

$$= \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \ldots + \frac{n}{2^n}$$

$$= 2$$

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Product design

- Determine the optimal investment strategy:
  - how much to invest in equities to maximize the expected utility
- Test which product design gives the highest expected utility.

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Optimal investment strategies

- Merton's portfolio
  - Assume simple financial market model throughout that there is only an equity (risky asset) and a bond (risk-free asset) available for investment.
  - Results based on Black-Scholes model.
- Aim: find an investment strategy which maximizes the expected utility of wealth at a fixed time,

$$E(U(X_T^\pi)).$$

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Optimal investment strategies under EUT

- Merton's (1969) solution based on $U(x) = \frac{x^{1-\gamma}}{1-\gamma}$.
- Calculated as $\frac{\mu - r}{\gamma\sigma^2}$,
- Optimal to put a fixed proportion in equities at all times, which is not dependent on time or wealth.

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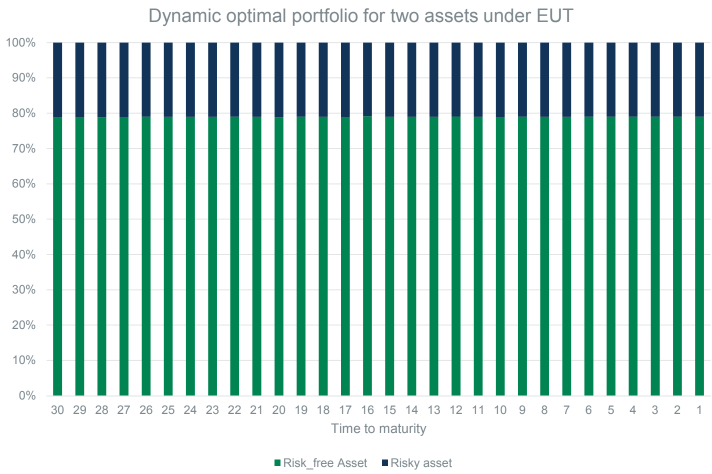
Optimal investment strategies under EUT



- $r_f = 4\%$
- $\mu = 6.5\%$
- $\sigma = 15\%$
- $\gamma = 5$

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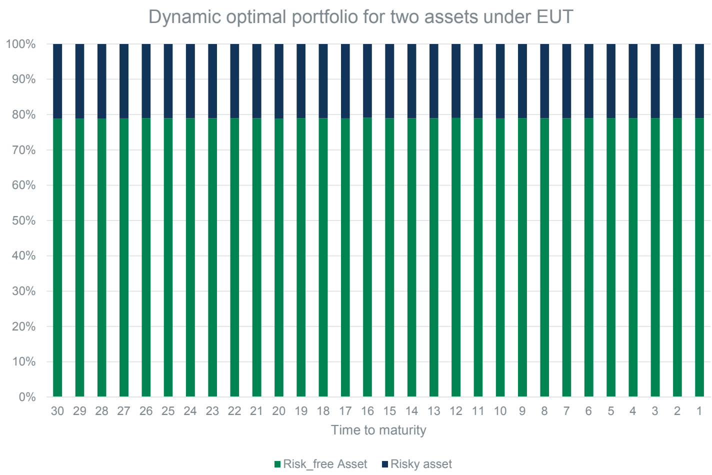
Optimal investment strategies under EUT



- $r_f = 4\%$
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Constant Proportion

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Limitations of EUT

- Framing effect
- Non-linear preference (Allais's paradox)
- Source dependence (Ellsberg's paradox)
- Risk seeking
- Loss aversion

Limitations of EUT

- Framing effect

Imagine that the we are preparing for the outbreak of an unusual disease, which is expected to kill 600 people.

Treatment A	Treatment B
"Saves 200 lives"	"A 33% chance of saving all 600 people, 66% possibility of saving no one."

Treatment C	Treatment D
"400 people will die"	"A 33% chance that no people will die, 66% probability that all 600 will die."

Limitations of EUT

- Allais's paradox

Investment A		Investment B		Investment C		Investment D	
Win	Chance	Win	Chance	Win	Chance	Win	Chance
£1 million	100%	£1 million	89%	Nothing	89%	Nothing	90%
		Nothing	1%	£1 million	11%		
		£5 million	10%			£5 million	10%

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Limitations of EUT

- Ellsberg's paradox

Consider an urn containing 30 red balls and 60 other balls that are either black or yellow. It is unknown how many black or yellow balls there are, but the total number of black balls and yellow balls equals 60.

Gamble A	Gamble B
You receive £100 if you draw a red ball	You receive £100 if you draw a black ball

Gamble C	Gamble D
You receive £100 if you draw a red or yellow ball	You receive £100 if you draw a black or yellow ball

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Cumulative Prospect Theory

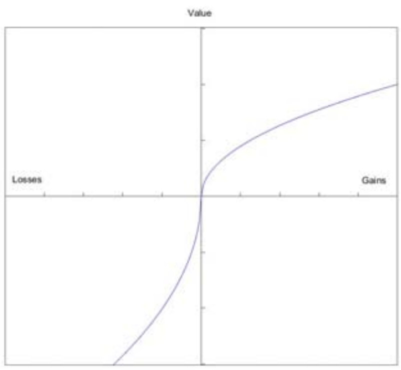
- Tversky and Kahneman (1992) proposed the Cumulative Prospect Theory (CPT). It is viewed as a better model in explaining people's behaviour in decision making under uncertainty.

- CPT utility is calculated by value function and weighting function.

$$E = \sum V(x_i)W(p_i)$$

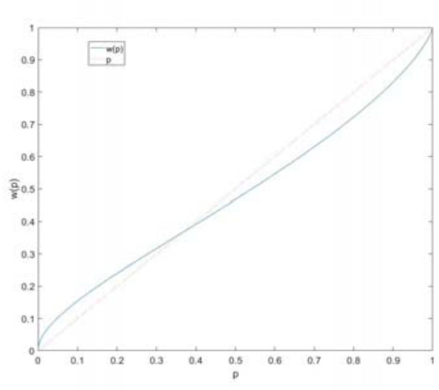
Cumulative Prospect Theory

- Value Function



Cumulative Prospect Theory

- Weighting function



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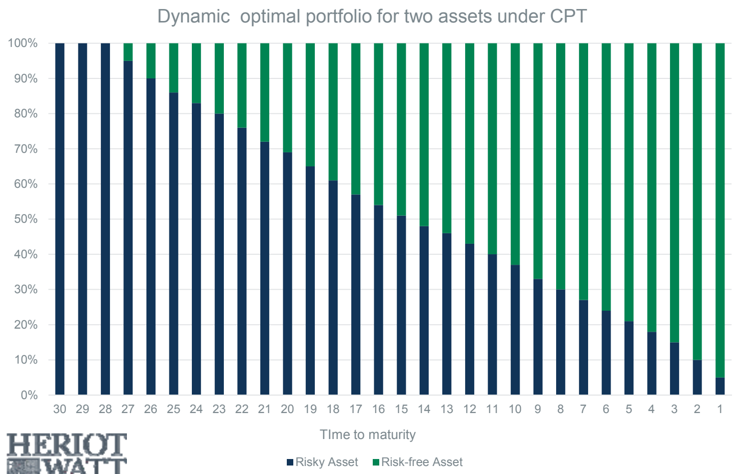
Optimal investment strategies under CPT



- $r_f = 4\%$
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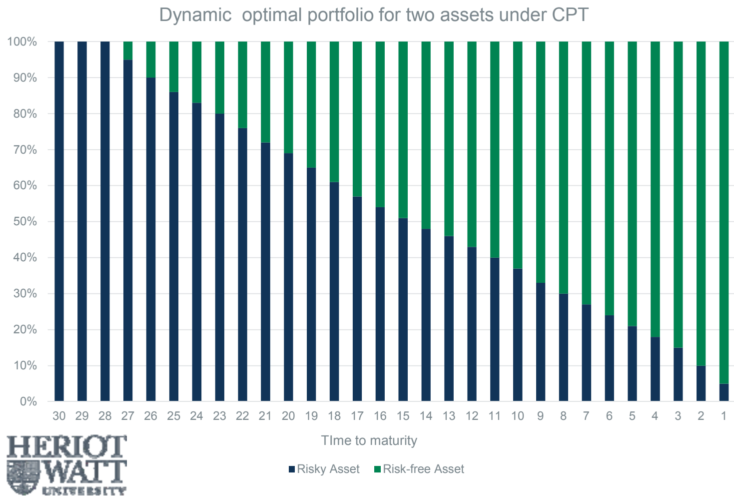
Optimal investment strategies under CPT



- $r_f = 4\%$
- $\mu = 6.5\%$
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Life Cycle

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Danish Time Pension

- Guillen et al (2006) analyse a popular pension contract from Denmark.
- Danish "Time Pension".
- Annual customer value calculated as

Underlying value

$$D'_n = \begin{cases} P, & n = 0 \\ (1+g')D'_{n-1} + \alpha'[A_n - (1+g')D'_{n-1}], & n \in \{1, 2, \ldots, n\} \end{cases}$$

Guarantee rate

Participation ratio

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The new contract

- Danish “Time Pension” . Annual customer value calculated as

$$D'_n = \begin{cases} P, & n = 0 \\ (1+g')D'_{n-1} + \alpha'[A_n - (1+g')D'_{n-1}], & n \in \{1, 2, \dots, n\} \end{cases}$$

- The new contract. Annual customer value calculated as

$$D_n = \begin{cases} P, & n = 0 \\ (1+g)D_{n-1} + \alpha \max[A_n - (1+g)D_{n-1}, 0], & n \in \{1, 2, \dots, n\} \end{cases}$$

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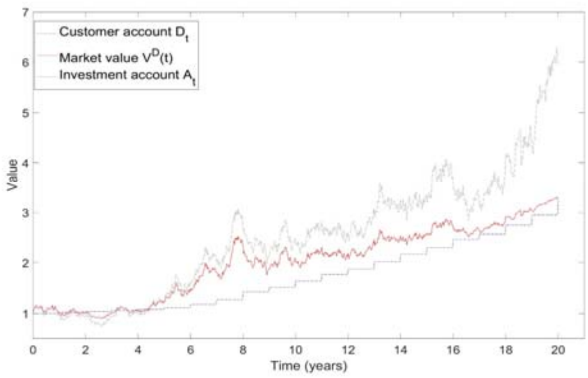
Sample path of new contract

- Bull Market



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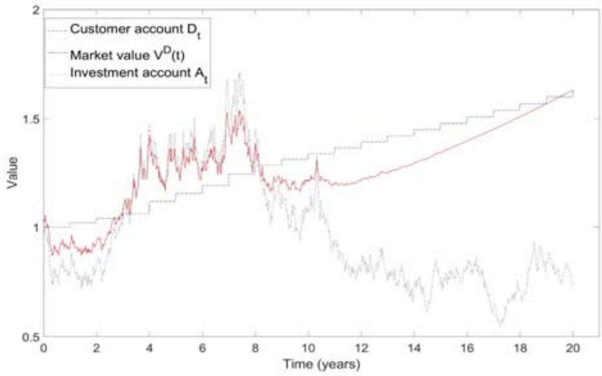
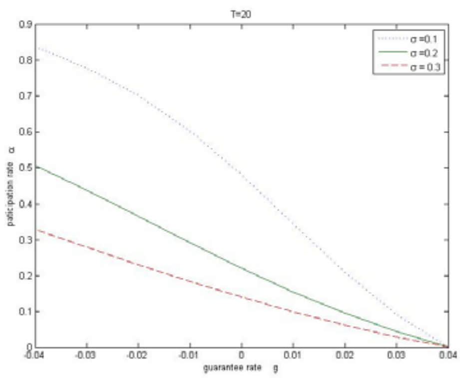
Sample path of new contract

- Bear Market



Parameter sensitivity



- $r_f = 4\%$
- $\mu = 6.5\%$
- $\sigma = 15\%$
- $\mathrm{T} = 20$

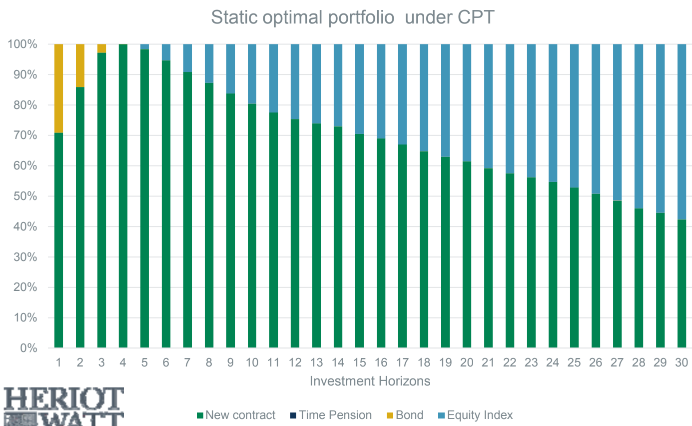
Results

	Our new contract	Danish Time Pension	100% Bond	100% Equity
Expected TV (terminal value)	2.8793	3.0612	2.1911	3.5243
Standard deviation	1.4889	1.6829	0	2.6340
Mean CPT utility	1.3330	1.2816	1.0914	1.2785

	Our new contract	Danish Time Pension	100% Bond	100% Equity
Proportion of Optimal portfolio	61%	0	0	39%

- $P = 1$
- $\alpha = 13\%$
- $g' = 4\%$
- $g = 2\%$
- $r_f = 4\%$
- $\mu = 6.5\%$
- $\sigma = 15\%$
- $T = 20$

Results



- $r_f = 4\%$
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Conclusion

- In this paper, we introduce a new pension contract with the features of guarantees and bonuses. It has transparent structure and clear distribution rule.
- Under cumulative prospect theory, the contract generates higher utility than the Time Pension. The result provides the evidence why the guarantees should be included in the pension contract.

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The views expressed in this presentation are those of the presenter.

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