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How to Net Down an Aggregate Gross Distribution

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Agenda

- **What is the Challenge?**
- **Conditional Ceded Approach**
 - Round 1 - Simple Solution
 - Round 2 - Concept of Ceded Curve
 - Graphical representation
 - Mathematical derivation
 - Round 3 – Simulation
 - Insight to the dynamics between Excess of Loss structure and Ceded Curve
- **Ceded Ratio Approach**
 - Beta distribution and dependency assumption
 - Ceded Ratio graphs
 - Ceded Ratio vs Gross Aggregate Copula
- **Comparison of Results**
- **Questions / Comments**



What is the Challenge

- **How to derive a distribution of reserves on a net basis (net of reinsurance)**
 - Direct use of net data
 - Use gross data then “net it down”
- **Issues with using net data directly**
 - Net data contains reinsurance impact
 - What if a line hasn’t had any reinsurance recoveries?
 - Does the history have an above average or below average reinsurance recoveries?
 - Is a low residual in a particular cell due to low noise OR large noise but dampened by RI?
- **Gross analysis then “Netting it down”**
 - Unless individual claim analysis is performed, the result will be an aggregate claim distribution
 - Even if Large Losses were treated separately, Attritional claims might developed to Large!
 - The challenge is to incorporate RI program to convert the distribution from a Gross to a Net basis



What is the Challenge

- Given a Gross empirical aggregate reserve distribution (10,000 points), derive a Net empirical aggregate reserve distribution (10,000 points)
- Reserving actuaries provided Booked Reserves
- Objective: Produce Net reserve distribution on a trial consistent basis by incorporating reinsurance program (XoL) by line by year

AY 2012 Reserve			
Sim#	GROSS \$		NET \$
1	2,001		?
2	7,119		?
3	7,368		?
4	7,748		?
5	7,956		?
6	8,353		?
7	8,692		?
.	.		.
.	.		.
.	.		.
9999	55,311		?
10000	63,825		?

Mean	49,820	44,838
	= Gross Booked Ultimate	= Net Booked Ultimate

Conditional Ceded Approach



Conditional Ceded Approach - Round 1a

- Look at Ultimate, not Reserve
- Look at Ceded, not Net
- Ceded Aggregate \$ using constant percentage (like Quota Share)

AY 2012 Ultimate				
#	GROSS \$000	CEDED \$000		
1	25,715	2,572		
2	30,833	3,083		
3	31,082	3,108		
4	31,462	3,146		
5	31,671	3,167		
6	32,067	3,207		
7	32,406	3,241		
.	.	.		
.	.	.		
.	.	.		
9999	79,025	7,903		
10000	87,540	8,754		

		NET \$000
		23,144
		27,750
		27,974
		28,316
		28,504
		28,860
		29,166
		.
		.
		.
		71,123
		78,786

Mean	49,820	4,982
	= Gross Booked Ultimate	= Ceded Booked Ultimate

Conditional Ceded Approach - Round 1b

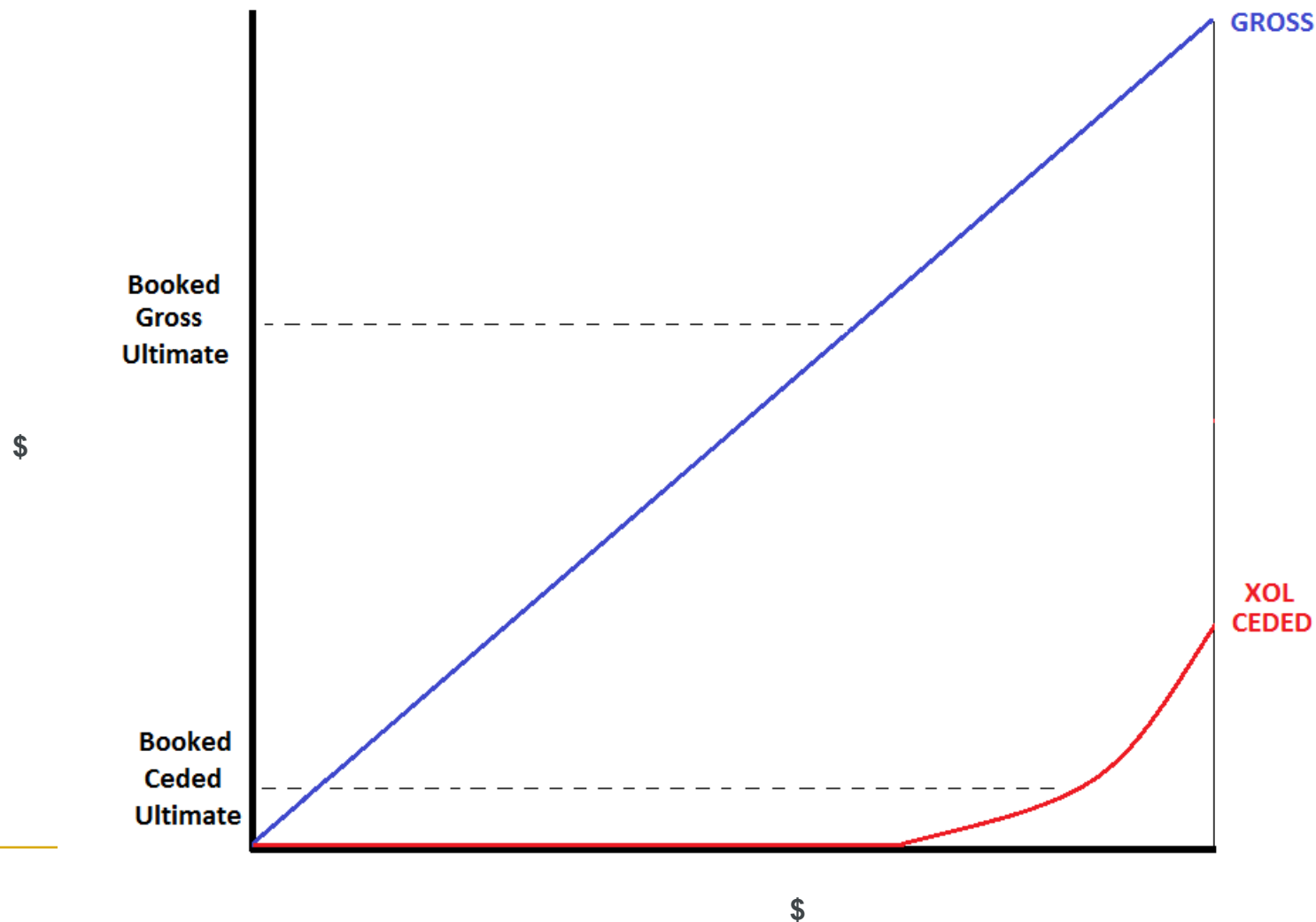
- Increasing Ceded% but start after a threshold as a Starting Point
- Linear interpolate
 - 0% if Gross Aggregation Ultimate < **StartingPoint**
 - **MaxCeded** is a free parameter
 - Constrain: Average Ceded \$ must match Ceded Booked Ultimate

AY 2012 Ultimate				NET \$000	
#	GROSS \$000	CEDED \$000			
1	25,715	0		25,715	
2	30,833	0		30,833	
3	31,082	0		31,082	
4	31,462	0		31,462	
5	31,671	0		31,671	
6	32,067	32		32,035	
7	32,406	65		32,341	
.	.	.		.	
.	.	.		.	
.	.	.		.	
9999	79,025	21,416		57,609	
10000	87,540	23,811		63,729	

Mean	49,820	4,982
	= Gross Booked Ultimate	= Ceded Booked Ultimate

Conditional Ceded Approach - Round 2

- Concept of Ceded Curve
 - Visual representation allows us to generalize the approach
 - Need a mathematical curve to formulate and implement the approach



Conditional Ceded Approach - Round 2a

- Derivate the Ceded Curve mathematically using Frequency & Severity

$$X_i \sim \text{Severity e.g. LogNorm}(\mu, \sigma)$$

$$N \sim \text{Frequency e.g. Poisson}(\lambda)$$

$$G = \sum_{i=1}^N X_i \quad C = \sum_{i=1}^N \min(\max(X_i - \text{Attachment}, 0), \text{Limit})$$

- Ceded Curve is a conditional expectation on Gross

$$E[C \mid G = g] = \int_c c f(c \mid G = g), \text{ where } f(c \mid G = g) = \frac{f(c, g)}{\int_c f(c, g)}$$

- Unlikely the integral has a close-form solution but it must be a transformation of Exponential



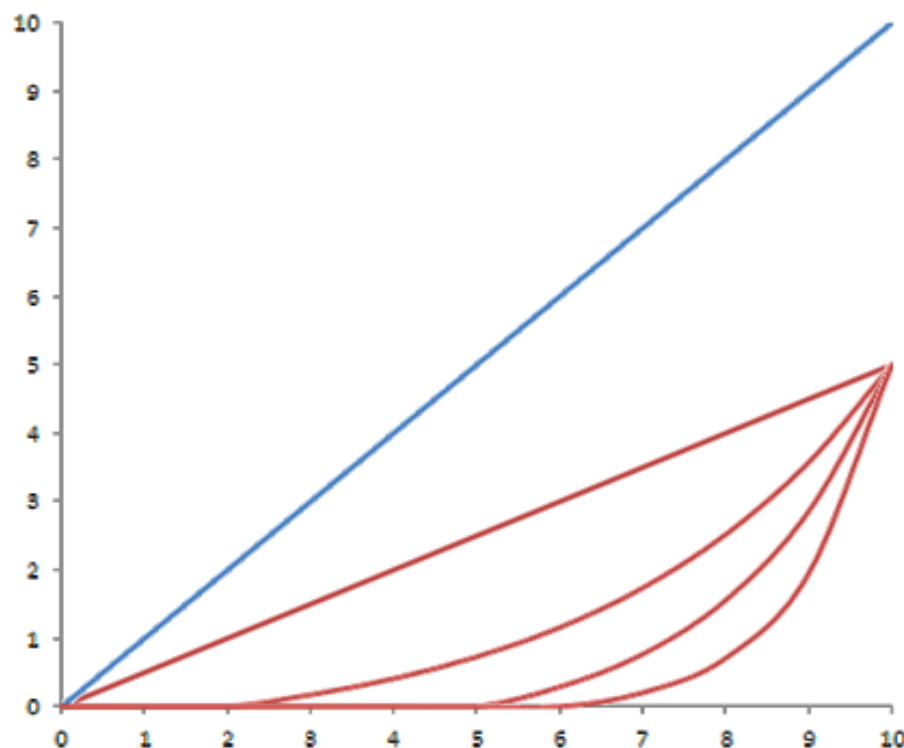
Round 2b



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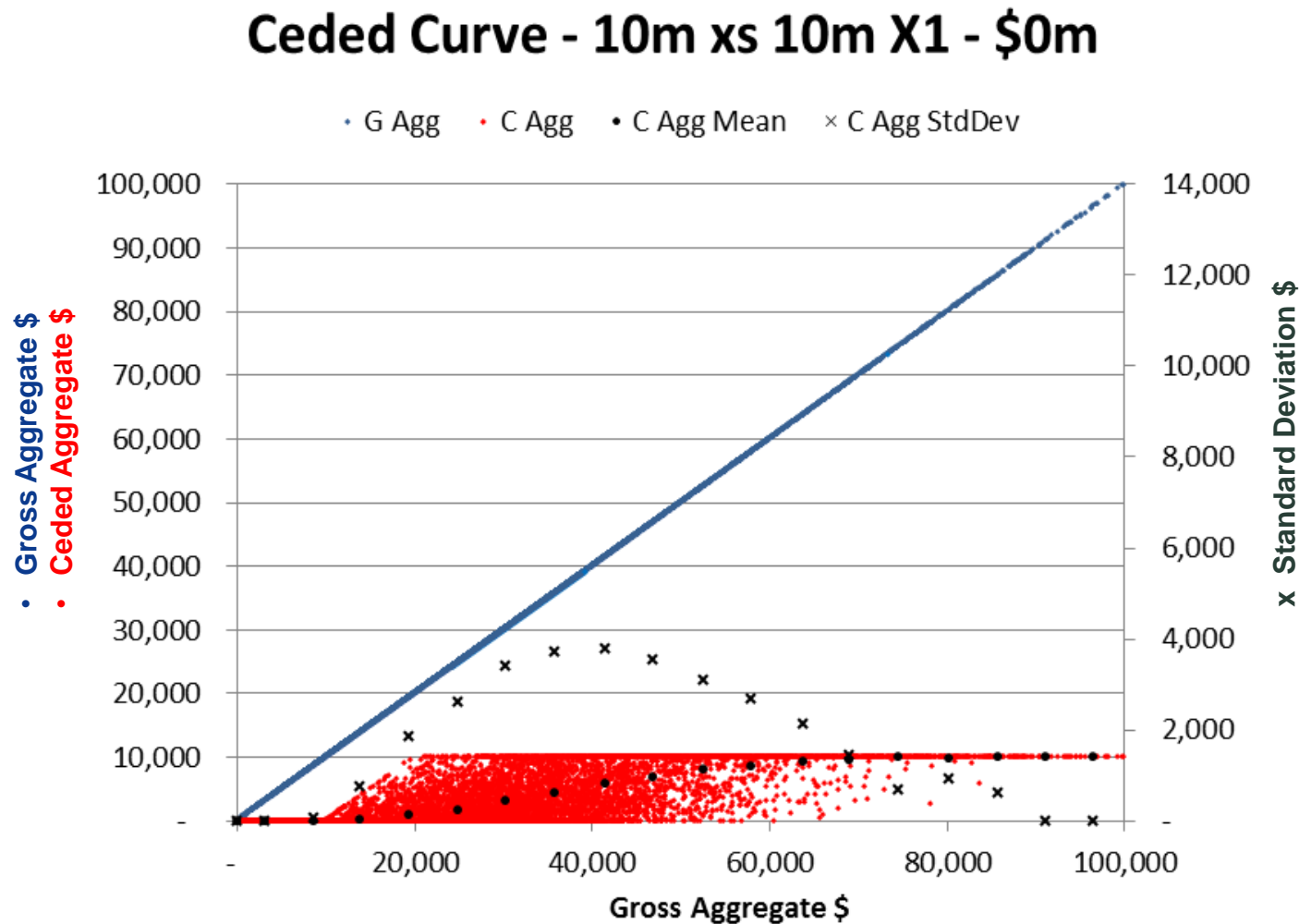
Conditional Ceded Approach - Round 2b

- *StartingPoint*: At what Gross aggregate \$ likely to start benefiting from RI?
- *EndingPoint*: At the very extreme worst Gross, what is the max benefit from RI?
- *Convexity*: Freedom parameter to control the steepness
- Even included **Quote Share!** ($StartingPoint = \$0$, $Convexity = 0$)



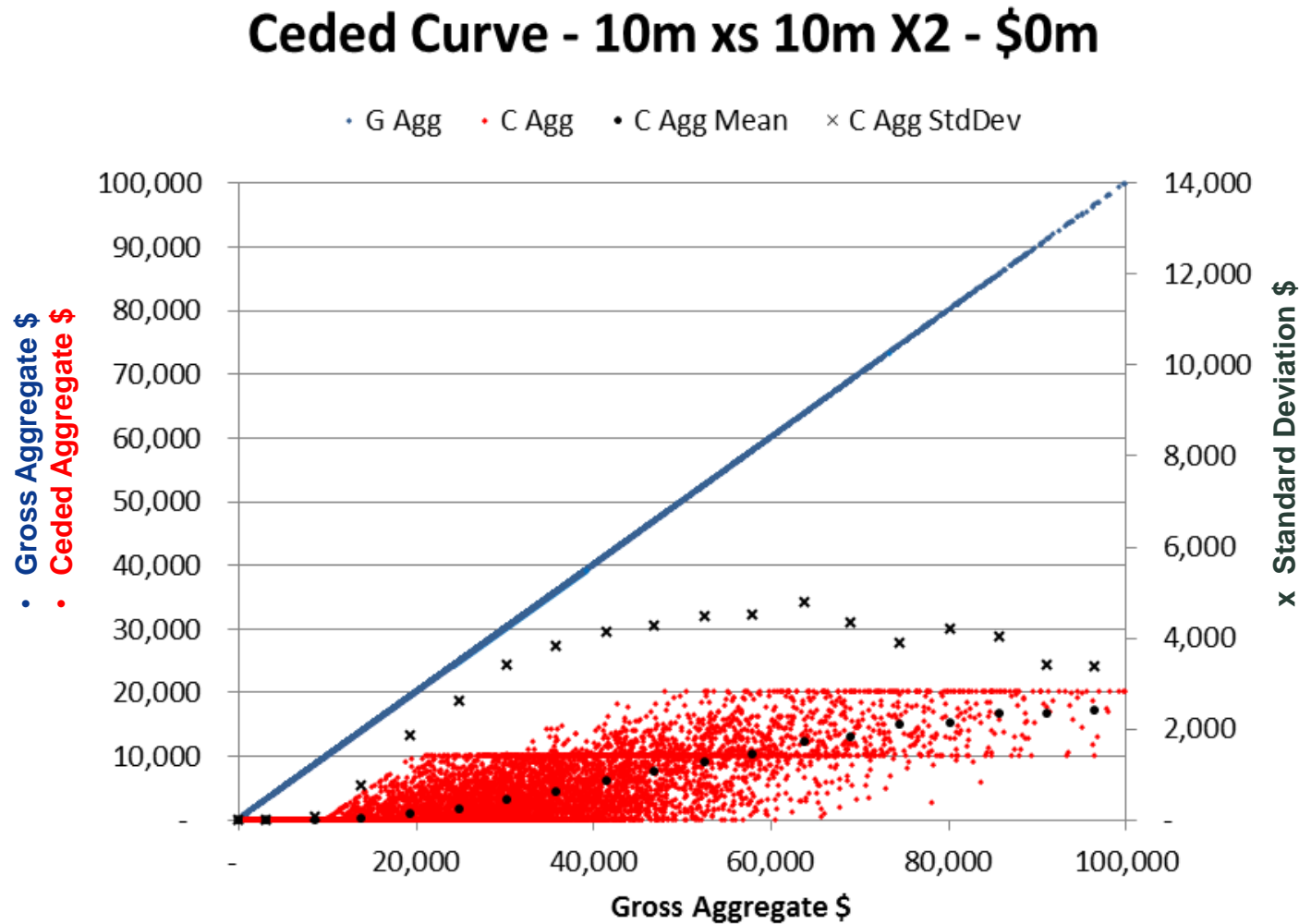
Conditional Ceded Approach - Round 3

No Reinstatements



Conditional Ceded Approach - Round 3

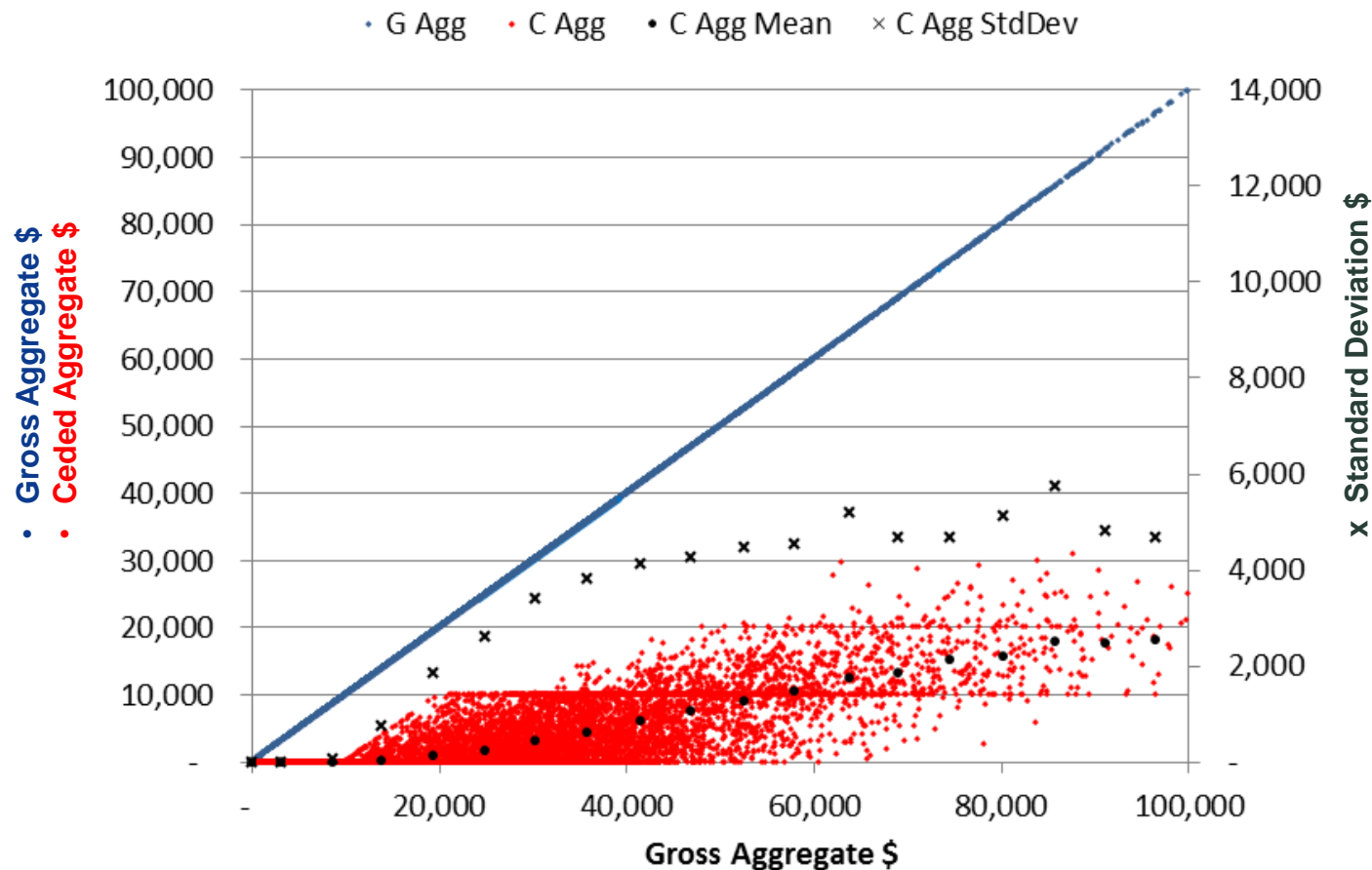
1 Reinstatement



Conditional Ceded Approach - Round 3

Unlimited (99) Reinstatements

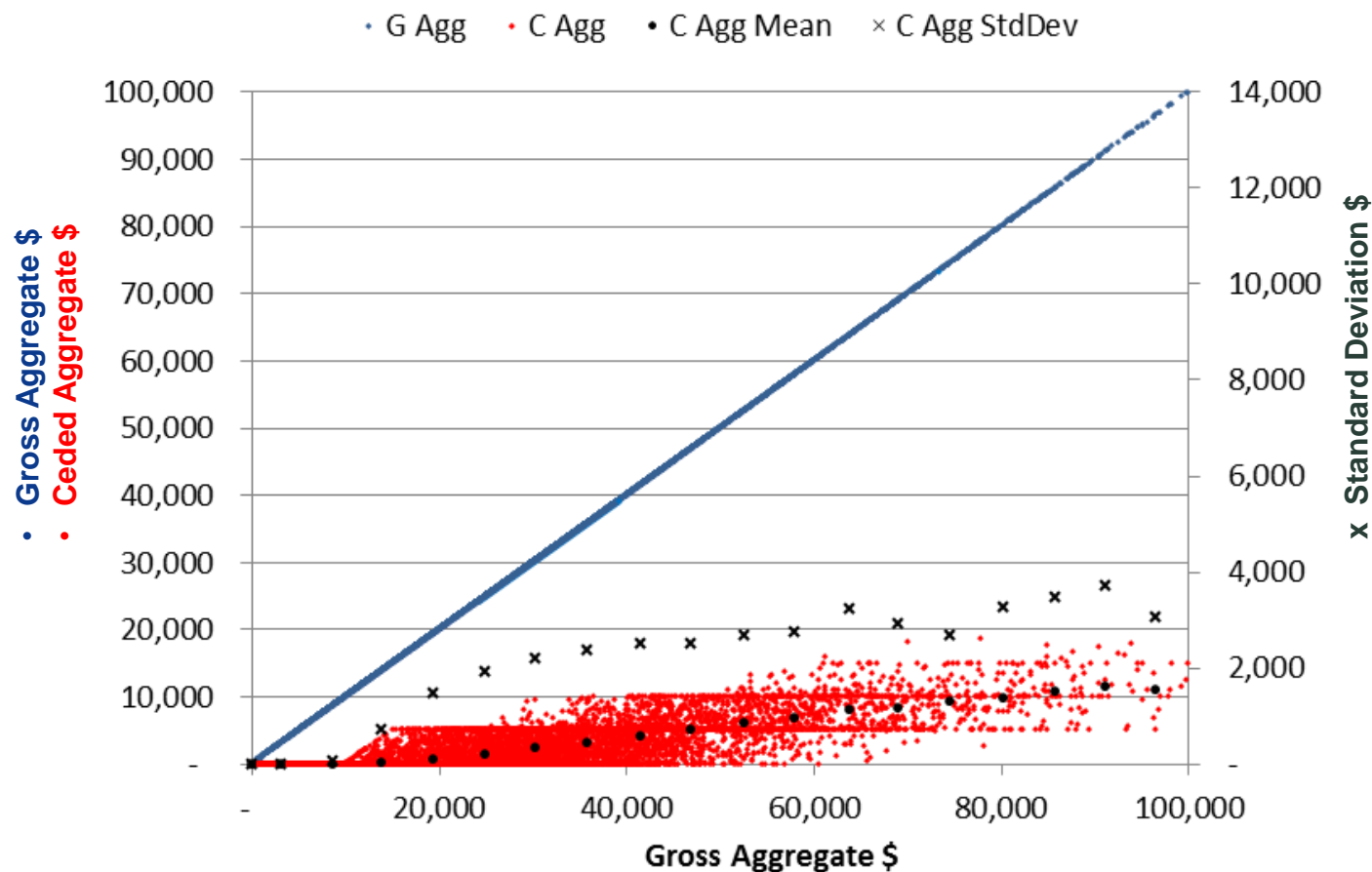
Ceded Curve - 10m xs 10m X100 - \$0m



Conditional Ceded Approach - Round 3

Unlimited Reinstatements; Decrease Limit to \$5m

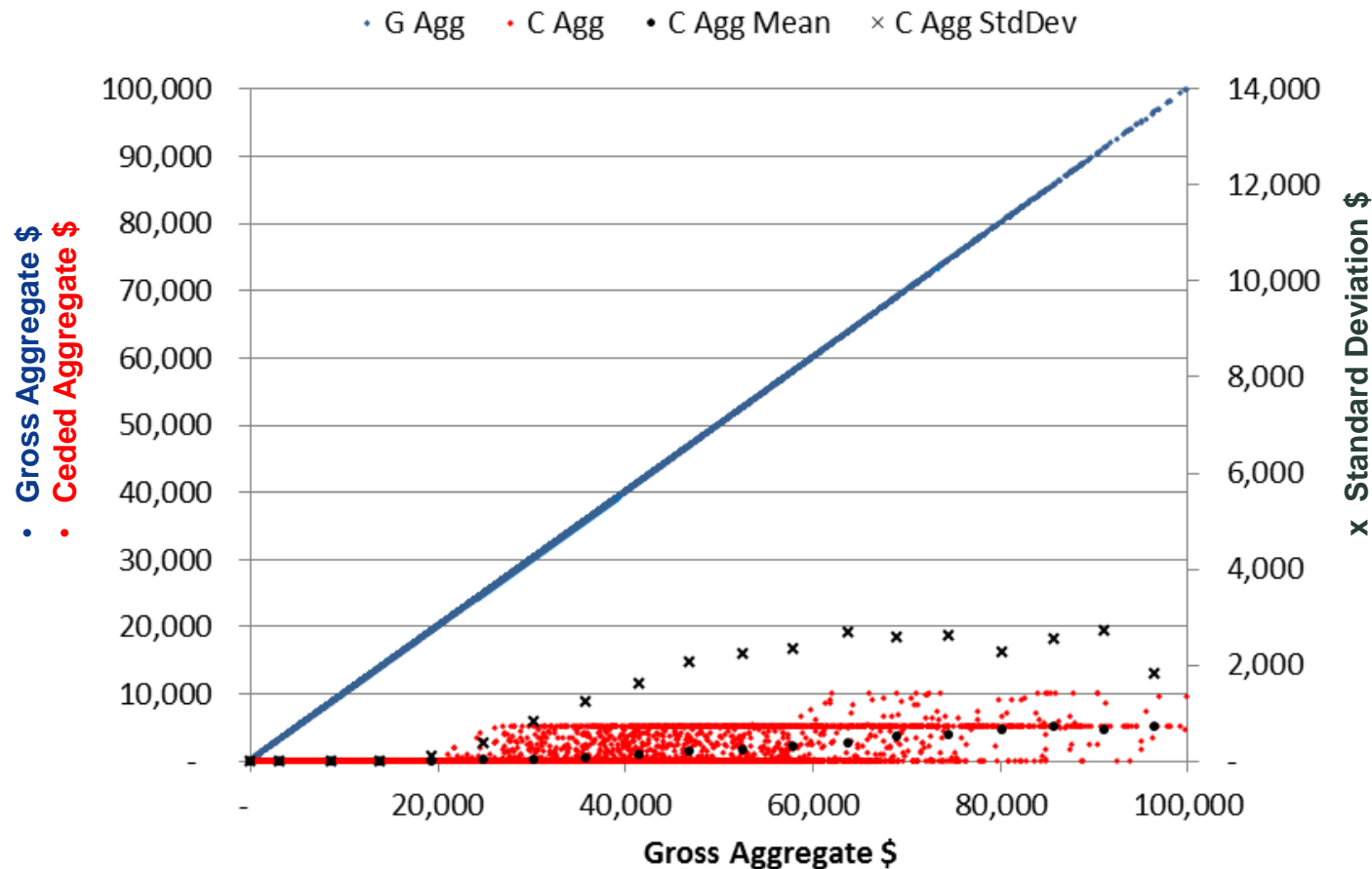
Ceded Curve - 5m xs 10m X100 - \$0m



Conditional Ceded Approach - Round 3

Unlimited Reinstatements; Limit \$5m; Increase Attach to \$20m

Ceded Curve - 5m xs 20m X100 - \$0m



Conditional Ceded Approach - Round 3

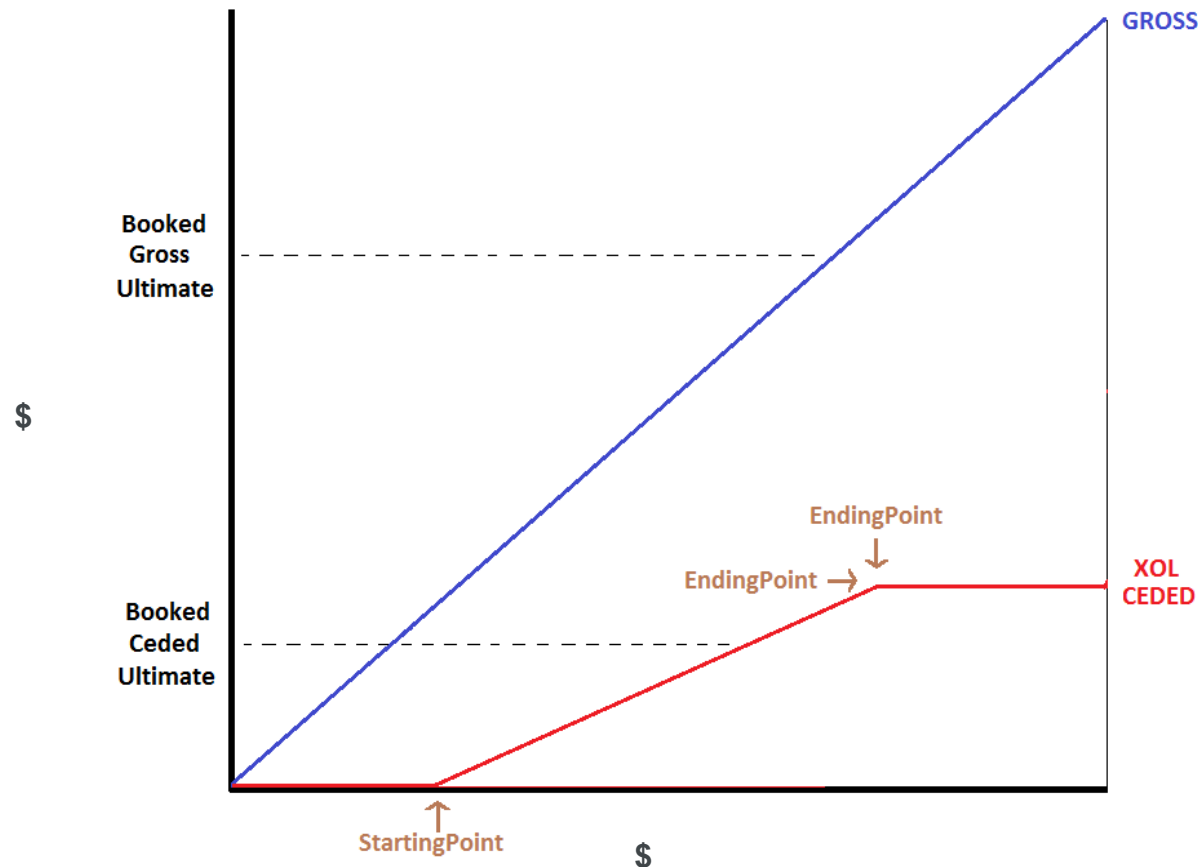
- Every line of business must have a Ceded Curve
- Characteristics of Ceded Curves
 - Starts at zero when Gross is zero : $C(0) = 0$
 - Monotonic increasing: $C(g_1) \leq C(g_2)$ if $g_1 < g_2$
 - Slope at any point can't be greater than slope of Gross curve: $C'(g) \leq 1$ for $g > 0$
- Quota Share
 - Slope is a constant: $C'(g) = k$ for $g > 0$
- Excess of Loss
 - Initially flat
 - Increasing slope then flattened
 - Reflect RI exhaustion



Conditional Ceded Approach - Round 3

- Approximations

- Flat in the beginning stays until a take-off point = *StartingPoint* (constrained by Booked Ceded Ultimate)
- Then flattens off after *EndingPoint* (provided by reserving actuary)



Ceded Ratio Approach



Ceded Ratio Approach

- Generate the Ceded % directly with a distribution
- Require a dependency assumption with the Gross Aggregate \$
- Need a parameter on the ceiling \$

AY 2012 Ultimate		
Sim#	GROSS \$000	Ceded%
1	25,715	?
2	30,833	?
3	31,082	?
4	31,462	?
5	31,671	?
6	32,067	?
7	32,406	?
.	.	.
.	.	.
.	.	.
9999	79,025	?
10000	87,540	?

CEDED \$000
?
?
?
?
?
?
?
.
.
.
?
?

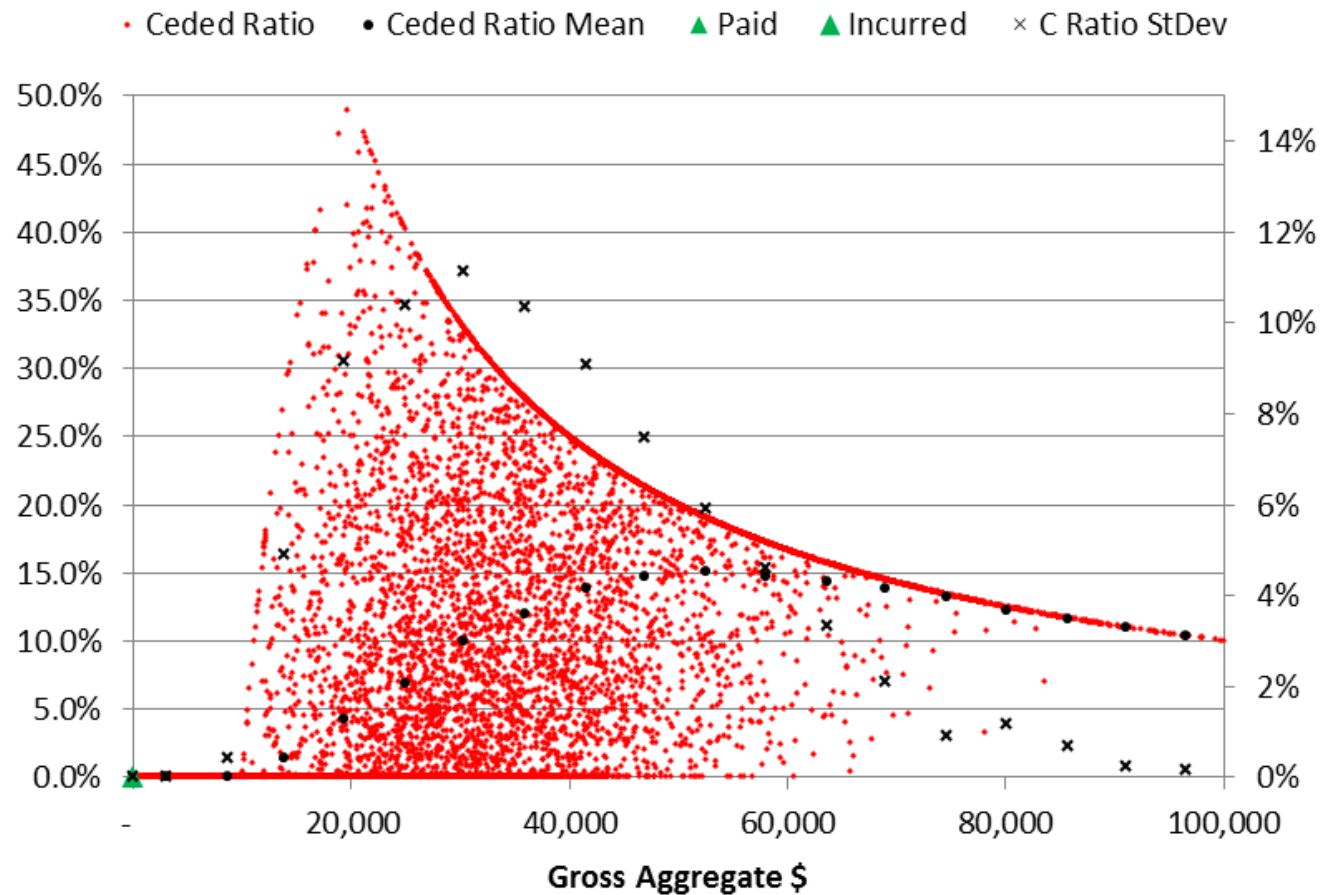
Mean	49,820
	= Gross Booked Ultimate

4,982
= Ceded Booked Ultimate



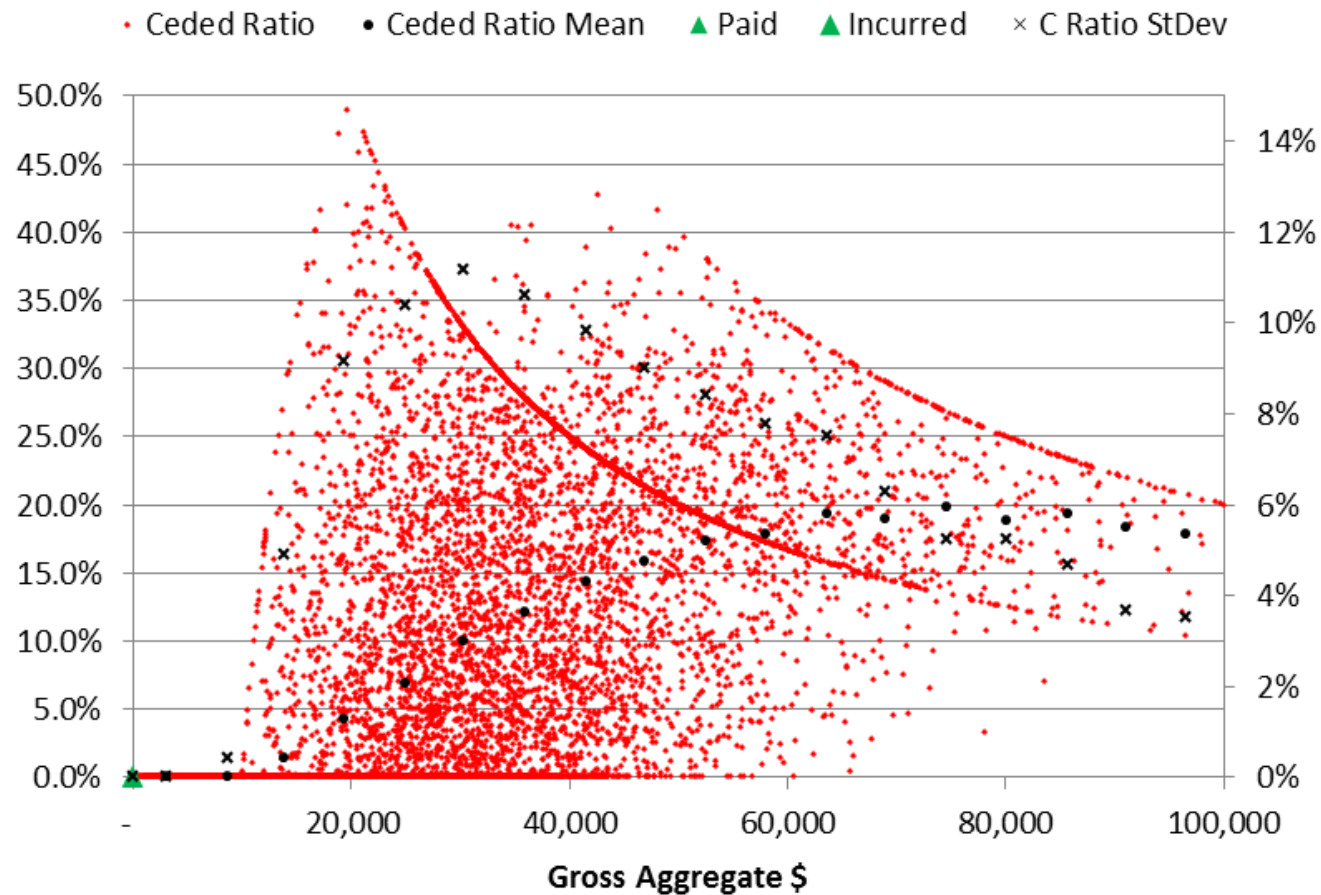
Ceded Ratio Approach

Ceded Ratio - 10m xs 10m X1 - \$0m



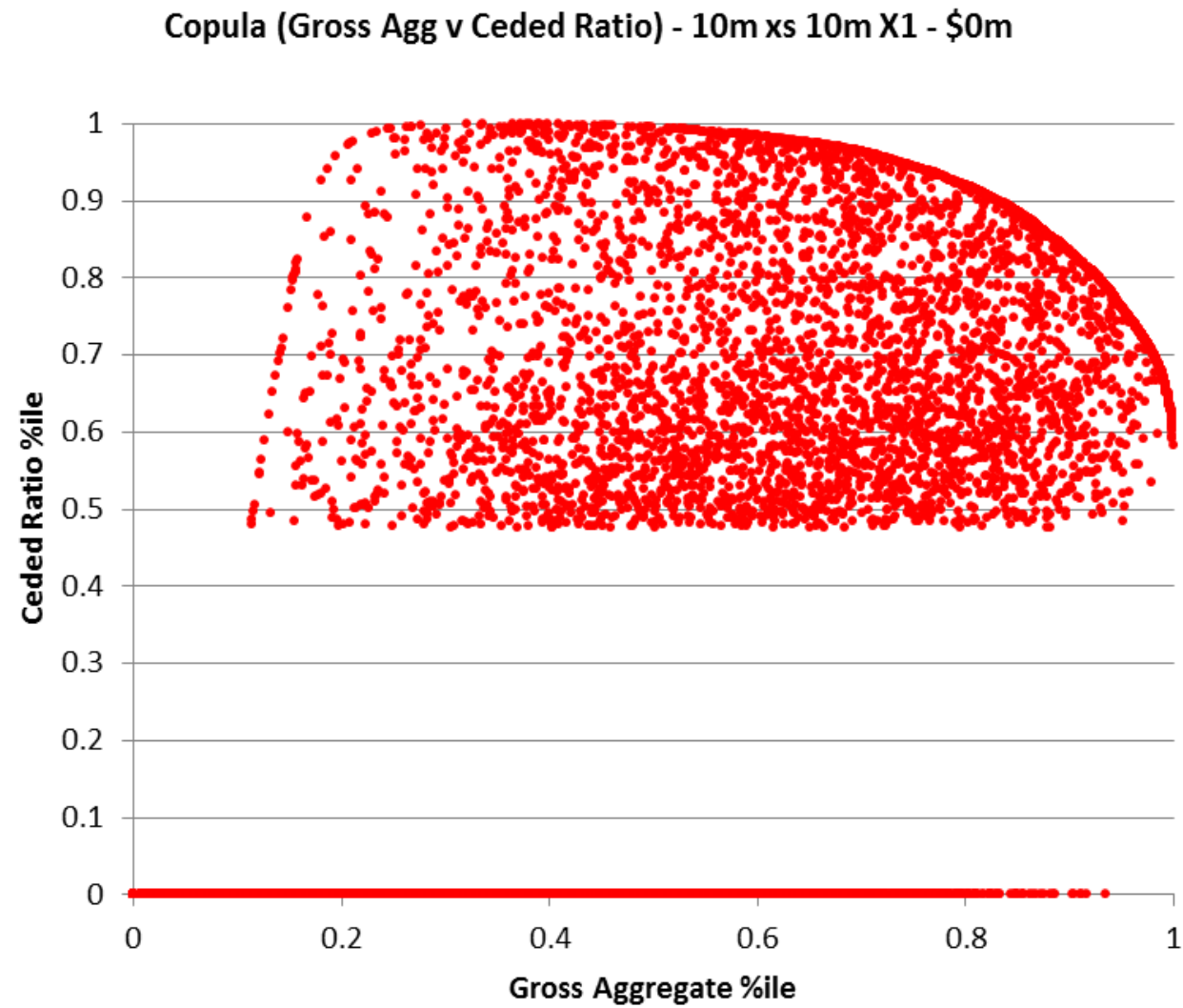
Ceded Ratio Approach

Ceded Ratio - 10m xs 10m X2 - \$0m

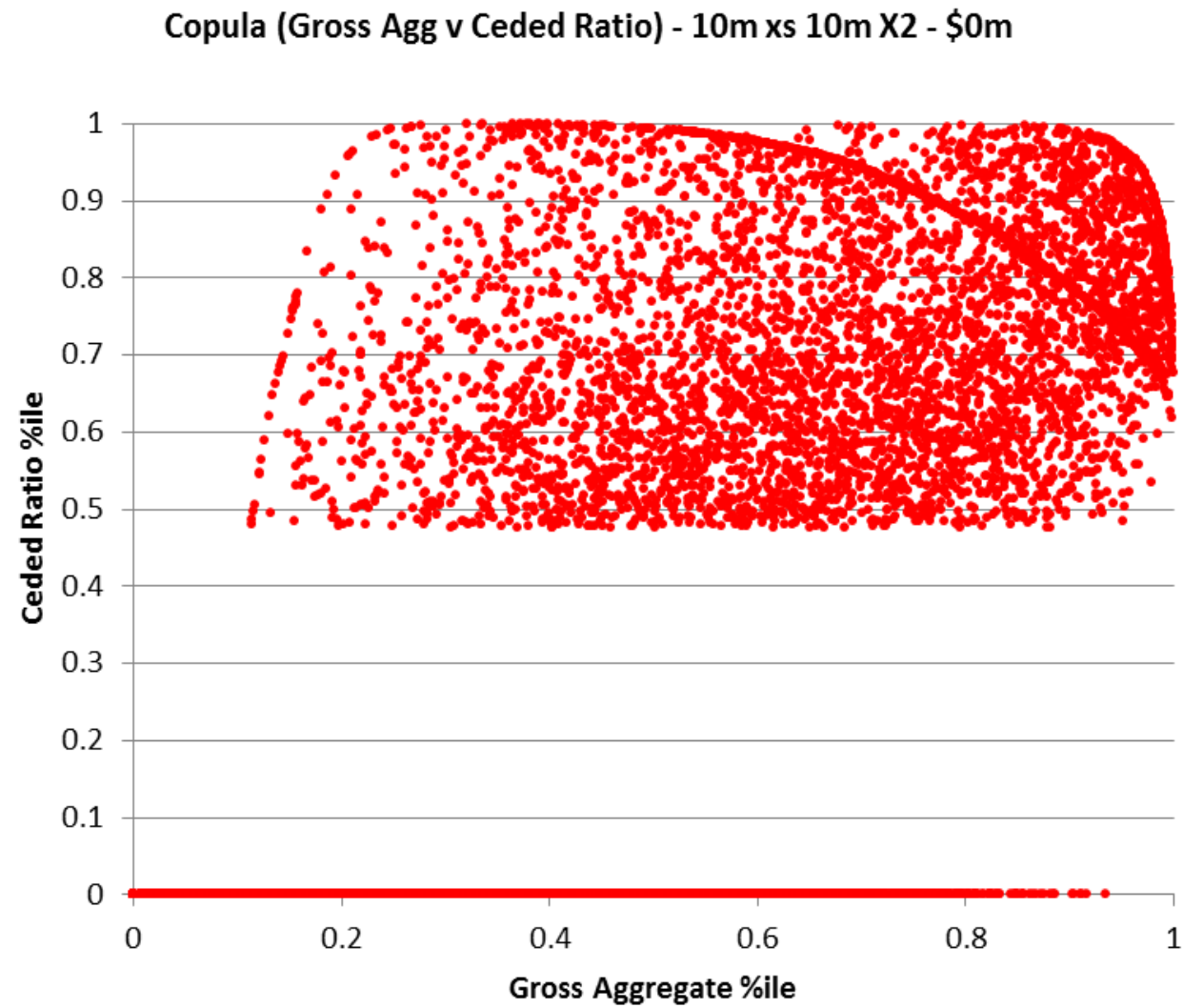


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Ceded Ratio Approach



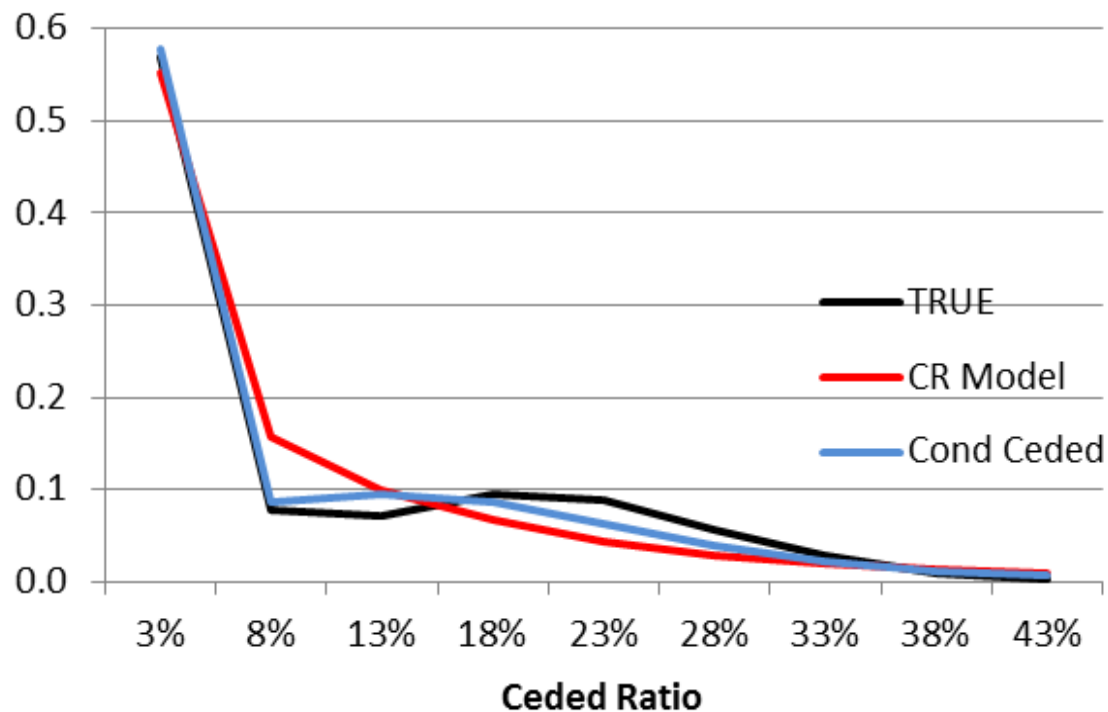
Ceded Ratio Approach



Ceded Ratio Approach

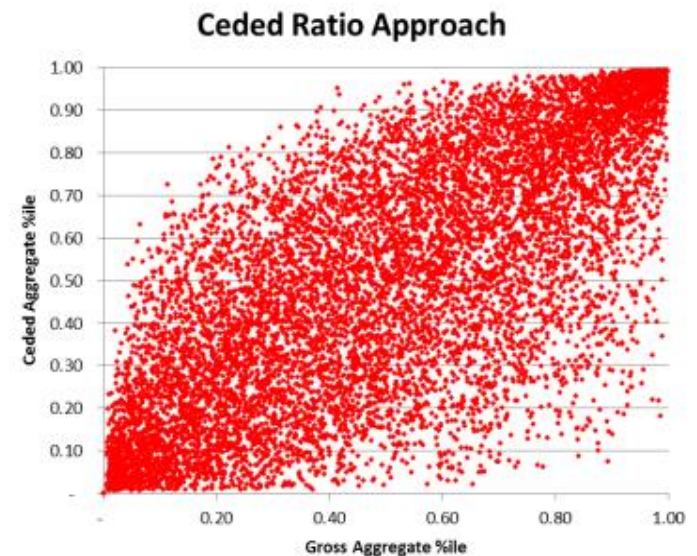
- True Distribution is modelled by a Beta Distribution

Ceded Ratio pdf



➤ 4 Parameters Needed:

- Mean
- SD/CV
- Max Recovery Cap
- Correlation Factor

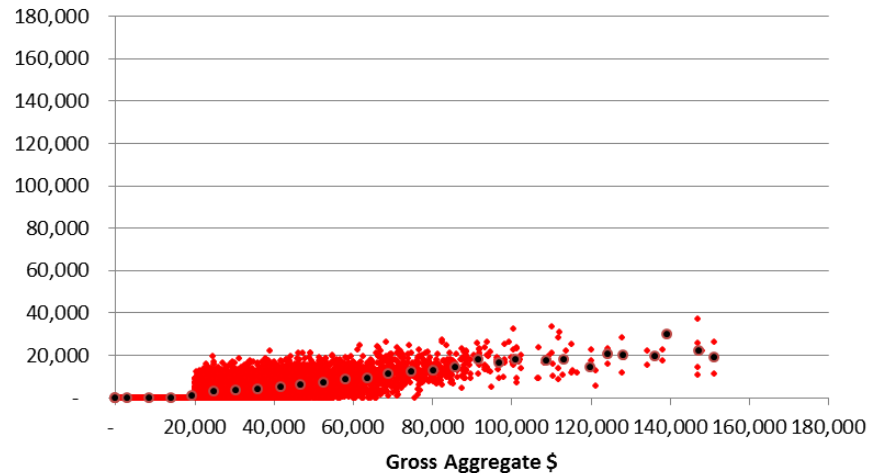


Comparison of Methods

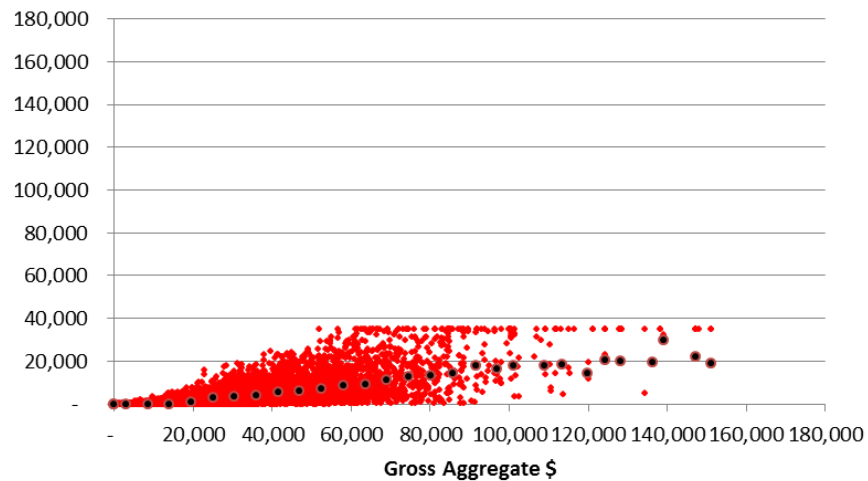


Comparison of Methods – Ceded vs Gross Aggregate \$

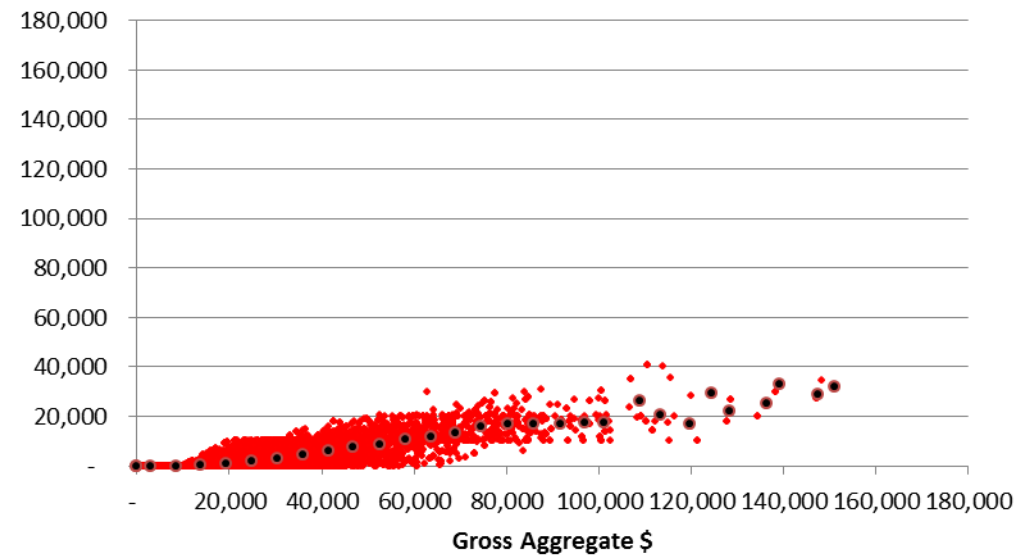
Conditional Ceded Approach



Ceded Ratio Approach

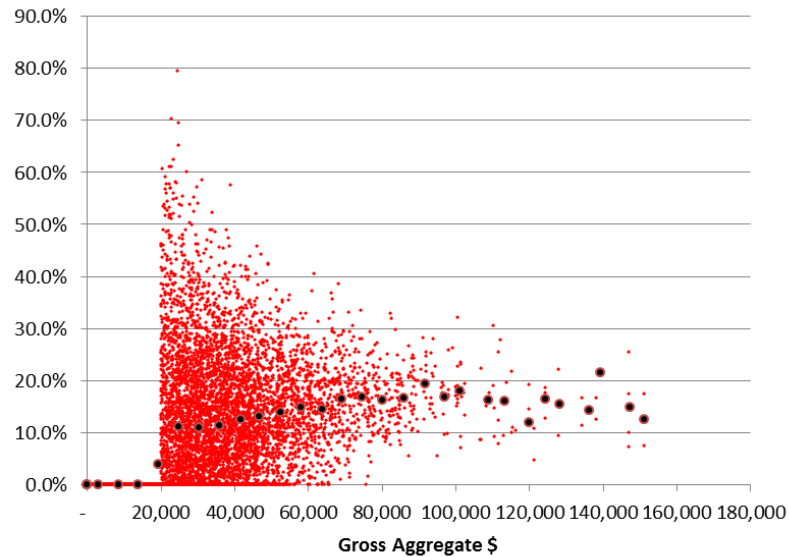


True Result

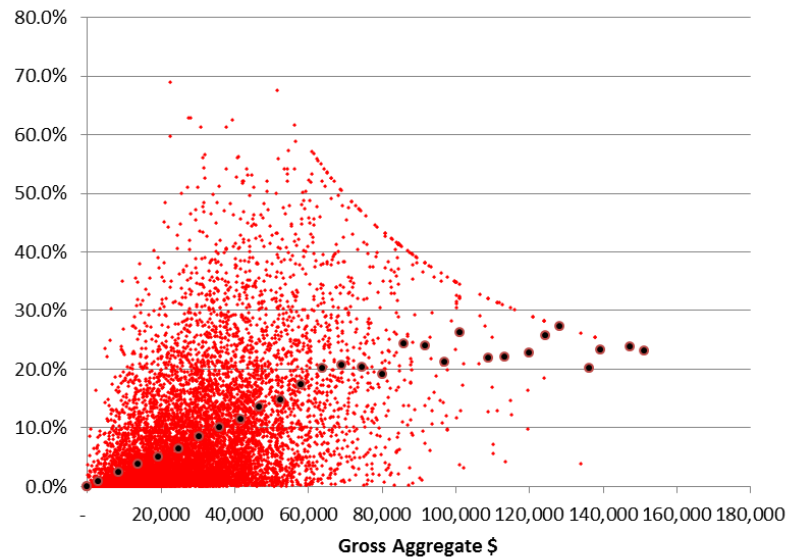


Comparison of Methods – Ceded Ratio vs Gross Aggregate \$

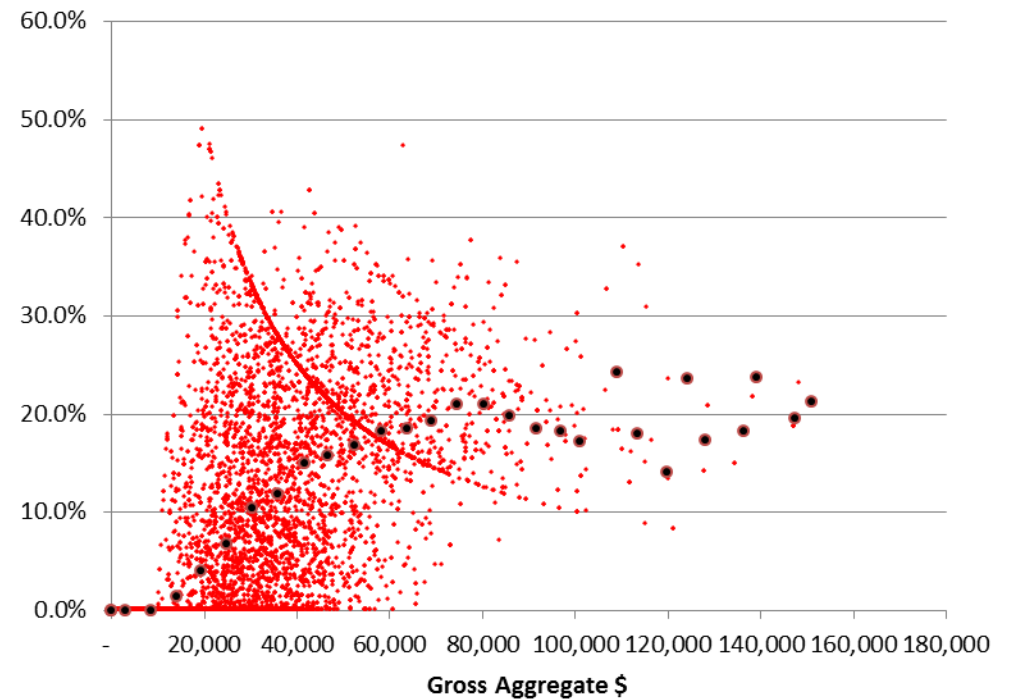
Conditional Ceded Approach



Ceded Ratio Approach

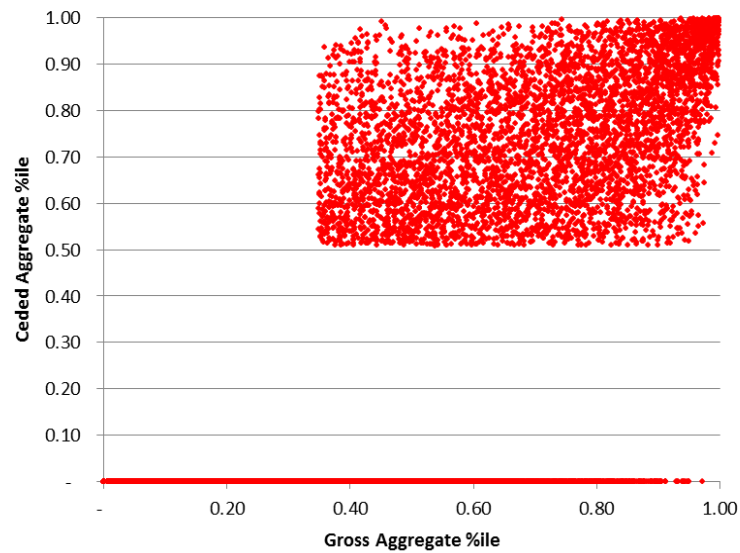


True Result

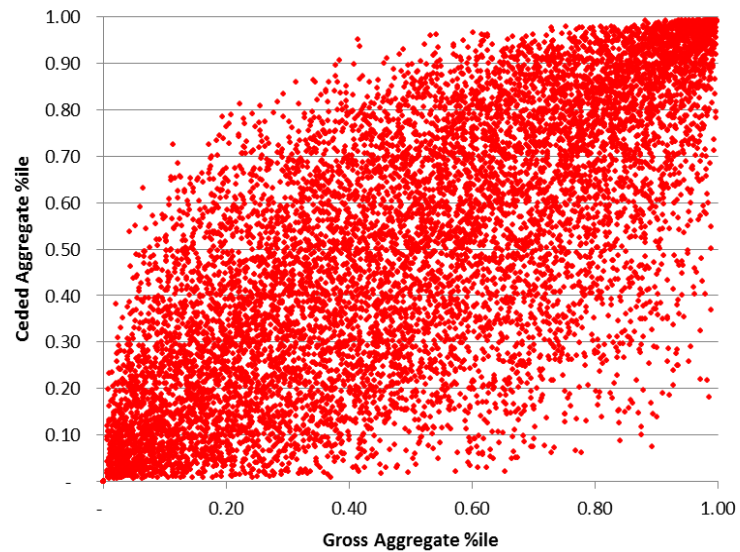


Comparison of Methods – Copulas

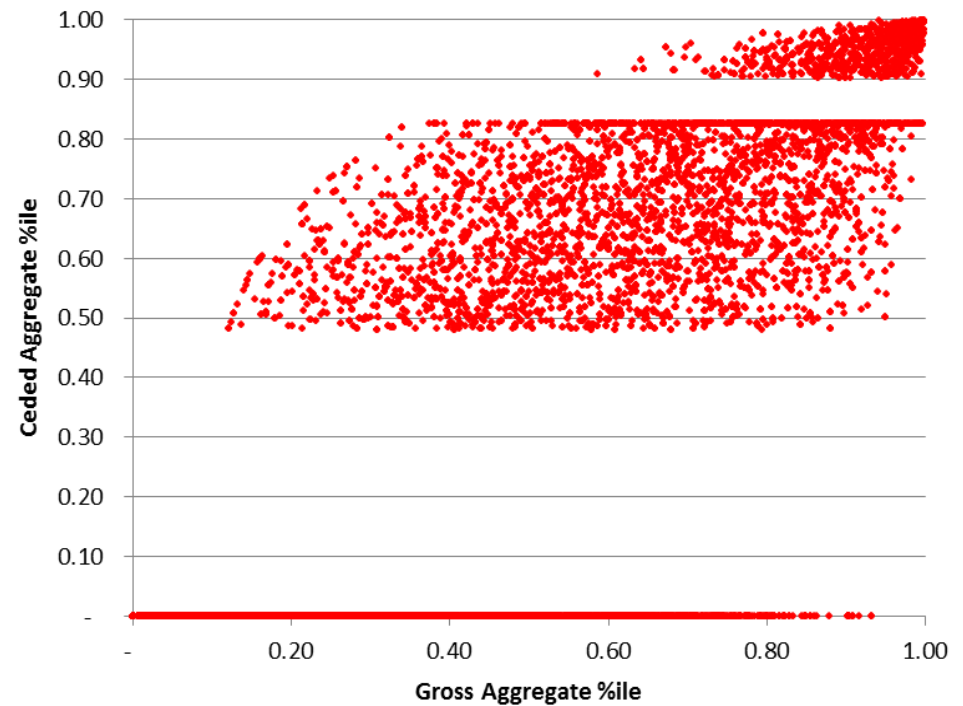
Conditional Ceded Approach



Ceded Ratio Approach



True Result



Comparison of Methods

	True Aggregate Net	Conditional Ceded Approach	Ceded Ratio Approach	Fixed Ceded Ratio Approach
Mean	26,060	26,385	26,244	26,087
StDev	15,267	16,007	15,477	16,775
CoV	58.6%	60.7%	59.0%	64.3%
10.0%	8,937	8,889	8,680	7,861
25.0%	15,154	15,155	15,064	14,009
50.0%	23,937	23,562	24,232	23,289
75.0%	34,021	34,794	34,808	34,667
90.0%	45,086	46,896	46,107	47,508
95.0%	53,704	56,052	54,236	57,637
99.5%	86,564	84,192	83,516	89,197

Comparison of Methods

	Advantages	Disadvantages
Conditional Ceded Approach	<ul style="list-style-type: none"> • Capture the true dynamics of reinsurance • Produce Gross and Net trial consistent results • Useful on Deterministic basis in rationalising the ceded reserves • Force actuaries to have a view of potential recoveries • Features such as Aggregate Deductibles can be explicitly captured • Can be used to help set the mean for the Ceded Ratio Approach 	<ul style="list-style-type: none"> • Not trivial to parameterise the Ceded Curve • Tricky to modelling the noise around the Ceded Curve
Ceded Ratio Approach	<ul style="list-style-type: none"> • Simple to Implement • Produce Gross and Net Trial Consistent results • Assumptions can be validated 	<ul style="list-style-type: none"> • Not trivial to parameterise the Ceded Ratio distribution • Tricky to set dependency structure between Ceded Ratio and Gross Aggregate \$ • Can only cap maximum recovery
Fixed Ceded Ratio Approach	<ul style="list-style-type: none"> • Simple to implement • Simple to explain 	<ul style="list-style-type: none"> • Incorrect distribution for Excess of Loss reinsurance • Can be very wrong at the tail (but prudent)
Model Net Directly	<ul style="list-style-type: none"> • Simple to implement • Simple to explain 	<ul style="list-style-type: none"> • Cannot produce Gross and Net trial consistent results without additional assumptions and modelling Gross as well. • <i>All issues in Slide 3</i>
Full Individual Claim modelling	<ul style="list-style-type: none"> • Potentially the most accurate • Netting down strategy simple 	<ul style="list-style-type: none"> • Large Computational power needed • Many more assumptions needed



Questions

Comments

Thank you for your attention!



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