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A DISSECTION OF PENSIONS FUNDING

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1. INTRODUCTION

1.1 THIS paper is about actuarial methods of funding pension schemes and follows on from the report of the Working Party of the Pensions Standards Joint Committee on Terminology of Pension Funding Methods[1] (The Terminology Report) published in 1984. It looks at the basic structure of the main methods and at how they behave. We then discuss the question of the suitability of the methods under various conditions. The reader may find it useful to have a copy of the Terminology Report to hand.

The paper is written against a background of uncertainty, as regards the State Earnings-Related Pension Scheme, and of great debate and legislative activity as regards Occupational Pension Schemes. This activity and debate makes it more important than ever before that the actuarial profession explains its methods and approaches to those in the pensions industry who are not actuaries, but who nevertheless rely on actuarial advice.

Consequently, our paper is primarily aimed at those who are not pensions experts, but who would like to know more about the actuarial side of the subject as a result of the widespread public debates that are currently going on about various aspects of pensions.

The paper examines the four main funding methods described by the terminology report:

The Current Unit Method	(CU)
The Projected Unit Method	(PU)
The Attained Age Method	(AA)
The Entry Age Method	(EA)

The fifth funding method described in the Terminology Report, the Aggregate Method, is not specifically examined here, despite its common use, because it can be considered as a special case of either the entry age method or the attained age method, running off surplus/deficit in a defined way (i.e. over the remaining working lifetime of the current membership).

We have tried to make the paper of general application and so, in common with the Terminology Report, we have avoided specific reference as far as possible to the United Kingdom State Pension Schemes, or to U.K. pensions legislation in general. Furthermore we have ignored solvency considerations, a point of particular relevance in Section 5. We believe the omission of these details enables the paper to be of wider application.

We have occasionally rounded figures to ease the presentation, but where necessary have used two decimal places.

2. VARIATIONS IN METHODS

2.1 In this section we look at the results for the standard contribution rate and the standard fund, for each of three model funds, as set out in Section 5 of the Terminology Report.

We illustrate the results by adopting simplified models. We use average values and make assumptions about even distributions. The intention is to show what will usually happen, rather than examine every eventuality.

The model schemes and benefits assumed are set out in Appendix A. The assumptions used in the valuation are set out in Appendix B.

2.2 *Standard contribution rates*

The following table sets out the standard contribution rates expressed as a percentage of pensionable payroll:

		Model		
Method	*Control period*	*A*	*B*	*C*
Current unit	1 year	4·3	9·1	13·3
	20 year	10·9	12·0	18·1
Projected unit	1 year	11·8	12·4	13·2
	20 year	13·0	13·1	13·8
Attained age		13·5	13·5	13·9
Entry age		13·0	13·0	13·0

2.2.1 *Current unit method*

The standard contribution rate, expressed as a percentage of earnings, is found by dividing the sum of:

(*a*) the present value of benefits which will accrue in the year following the valuation date, by reference to service in that year and projected earnings in that year, and

(*b*) the present value of the benefits accrued by the valuation date, multiplied by the expected percentage increase in earnings over the next year,

by the present value of members' total projected earnings over that year.

For the purpose of checking and understanding the results we characterized each model as a single member, with an age equal to the weighted average of the model as a whole, and a past service equal to the weighted past service of the scheme as a whole. The weighting is calculated as the unit pension cost for the scheme thus:

$$\frac{\sum(\text{Earnings}_x \cdot R_x)}{\sum \text{Earnings}_x} = R_{\bar{x}}$$

and

$$\frac{\sum(\text{Earnings}_x \cdot Rx \cdot p_x)}{\sum(\text{Earnings}_x \cdot R_x)} = \bar{p} = \text{average past service}$$

where $\bar{x}$ = weighted average age
P_x is past service at age x
Earnings$_x$ is the earnings at age x
R_x is calculated as a deferred annuity rate based on the assumptions in Appendix B.

The models produced the following results:

	Model		
	A	*B*	*C*
$\bar{x}$	42	50	54
$\bar{p}$	10·2	14	14·7

The first line of the table in § 2.2 shows a progression from 4·3% (model A) to 9·1% (model B) to 13·3% (model C).

We can approximate to the standard contribution rate with the expression:

[Unit pension cost at average age $\bar{x}$] [(Rate of accrual per annum) $[1 + j(\bar{p} + 1)]$

where j = earnings increase rate

We can then estimate the standard contribution rate for model A from the rate for model B with the expression:

$$\frac{[1{\cdot}67\,(1 + 11{\cdot}2 \times {\cdot}08)] \times \text{Rate at age } 42}{[1{\cdot}67\,(1 + 15 \times {\cdot}08)] \times \text{Rate at age } 50} \times 9{\cdot}1$$

We can further approximate the ratio of the deferred annuity rates by:

$$\frac{(RA)_{\bar{x}}}{(RB)_{\bar{x}}} = \frac{1{\cdot}09^{42}}{1{\cdot}09^{50}}$$

where interest is 9%. (Mortality was nullified by the secular trend we employed.)

Thus the rate for model A should be:

$$\frac{1{\cdot}896}{2{\cdot}2} \times \frac{9{\cdot}1}{(1{\cdot}09)^8} = 3{\cdot}9$$

compared to 4·3 and for model C should be:

$$\frac{2{\cdot}256}{2{\cdot}2} \times 9{\cdot}1 \times 1{\cdot}09^4 = 13{\cdot}2$$

compared to 13·3.

The results are reasonable, given a somewhat crude method. The discrepancy for the model A estimate arises from the lack of earnings increase required for the member aged 65 next birthday in model B and model C, i.e. there is no such member in model A.

If we extend the above to the 20 year control period, we need to add a further element to our formulation. The effect of a control period is to cost the accrual, on average, at the mid-point of that period. Thus we need to estimate the average age and past service, for each model, at the end of the control period and then take our grand average from these estimates and those shown above at the start of the projection.

This effect is explored further in Section 5.

2.2.2 *Projected unit method*

The standard contribution rate, expressed as a percentage of earnings, is found by dividing the present value of all benefits which will accrue in the year following the valuation date (by reference to service in that year and projected final earnings) by the present value of members' earnings in that year.

If we examine line 3 of the table in § 2.2 we see a progression from 11·8% (model A) to 12·4% (model B) to 13·2% (model C). The trend follows a similar course to that produced by the unit accrual of the current unit method. There we saw that we could estimate the trend from the change in the average age of the models. Similarly for the projected unit method we have:

$$\frac{\text{Cost of unit accrual in } B}{\text{Cost of unit accrual in } A} = \left(\frac{1+i}{1+j}\right)^f$$

where $i = 9\%$, the assumed rate of interest in deferment
$j = 8\%$, the assumed rate of interest in earnings
$f =$ difference in average age

The introduction of a control period produces a similar effect to that produced in the current unit method. The average age of the models is:

	Model		
Age	*A*	*B*	*C*
At the start	40	47	53
After 20 years	57	57	61
Mean costing age	49	52	57

Thus we could expect model B to produce an increase of $(1{\cdot}01)^5$ and line four of the table in § 2.2 confirms that this is so.

2.2.3 *Attained age method*

The standard contribution rate, expressed as a percentage of earnings, is found by dividing the present value of all benefits which will accrue to present members after the valuation date (by reference to service after the valuation date and projected final earnings) by the present value of total projected earnings for all members throughout their expected future membership.

In line 5 of the table in § 2.2 we see that the contribution rate changes very little from model to model. The method produces a similar pattern to the projected unit method at a slightly higher level. The attained age method is, effectively, a projected unit calculation with a control period of 'infinity'. In practice, as no new entrants are assumed, the control period is equal to the longest term to retirement of any scheme member.

We might conclude that the longer the control period the greater the 'stability' or uniformity of the standard contribution rate. We should also remember that we are beginning to drift away from the ongoing profile of the scheme that is being valued. The method anticipates that the scheme will 'age' and therefore the contribution rate is higher than the cost of 1 year's accrual, leading to surplus initially. If new entrants arise, then the surplus will be utilized to reduce future contributions. If no entrants arise, then the initial surplus will eventually offset the increased standard contribution which occurs with increased average age.

2.2.4 *Entry age method*

The standard contribution rate, expressed as a percentage of earnings, is found by dividing the present value of all future benefits by reference to projected final earnings for a member entering at a 'normal' age, by the present value of his total projected earnings throughout his expected future membership. The 'normal' entry age is either estimated from the actual membership, assumed, or calculated from the decrement table employed.

In line 6 of the table in § 2.2 we see a constant rate of 13·0% for the standard contribution rate for all three models. As the same entry age, 25 years, was employed in all cases, this is not too surprising. In order to avoid the surpluses that could emerge, as mentioned above, from this seemingly most stable of methods, the entry age must be chosen with care.

Model A is a 'young' scheme, and therefore the entry age must be chosen with a view as to how the scheme might be expected to develop. The less the membership is expected to mature, the lower the entry age should be assumed. Thus the models, if expected to remain at their current levels of maturity, would in fact have varying assumptions as to age at entry, commensurate with those levels of maturity.

If we assume the following pattern of entry ages, we have:

	Model		
	A	*B*	*C*
Entry age	25	30	35
Contribution rate	13·0	13·2	13·6

2.3 *Standard funds*

The table below shows the standard fund brought out by each method, using the assumptions set out in Appendix B.

Method	Control period	Model A	Model B	Model C
Current unit	1 year	53,536	187,228	184,112
	20 year	53,536	187,228	184,112
Projected unit	1 year	202,268	371,207	335,209
	20 year	202,268	371,207	335,209
Attained age		202,268	371,207	335,209
Entry age		228,904	402,185	359,228
Assets		152,251	321,190	309,689
Payroll		240,474	311,785	202,291

2.3.1 *Current unit method*

The standard fund is the present value of benefits accrued at the valuation date, by reference to current earnings at the valuation date.

If we examine line 1 of the table in § 2.3 we see that the standard funds progress, as multiples of payroll:

(Model A)	(Model B)	(Model C)
·22	·60	·91

We can characterize this pattern in a manner analogous to that used in 2.2.1. We see that the standard fund operates as:

$$(\bar{p}) \times (R\bar{x}) \times \text{payroll}$$

which is equivalent to

$$\Sigma(p_x \times R_x \times \text{salary})$$

Thus we get a progression from:

	(Model A)	(Model B)	(Model C)
$\bar{p} \cdot R\bar{x}$	10·2	$14 \times 1{\cdot}09^{8}$	$14{\cdot}7 \times 1{\cdot}09^{12}$

which is a similar ratio to the standard fund as a multiple of payroll.

2.3.2 *Projected unit method*

The standard fund is the present value of all benefits accrued at the valuation date, by reference to projected final earnings.

In line 3 of the table in § 2.3 we see that the standard funds progress, as multiples of payroll:

(Model A)	(Model B)	(Model C)
·84	1·19	1·66

Like the current unit method we have a pattern that depends on average past service and average age. Thus we get a progression from:

	(Model A)	(Model B)	(Model C)
$\bar{p} \cdot R_{\bar{x}}$	7·4	$9{\cdot}8 \times 1{\cdot}01^{7}$	$12{\cdot}8 \times 1{\cdot}01^{13}$

which mirrors the pattern above.

2.3.3 *Attained age method*

The standard fund definition is the same as that for the projected unit method (see § 2.3.2 above).

2.3.4 *Entry age method*

The standard fund is found by deducting from the present value of total benefits (on projected final earnings) for all members, the value of the standard contribution rate (multiplied by the present value of total projected earnings for all members) throughout their expected future membership.

In line 6 of the table in § 2.3 we see that the standard funds progress, as a multiple of payroll:

(Model A)	(Model B)	(Model C)
·95	1·29	1·78

The standard fund is higher than that for the attained age method, as a lower standard contribution rate is expected in the future for the entry age method (on these assumptions).

If we deduct line 5 from line 6 we find:

(Model A)	(Model B)	(Model C)
26,636	30,978	24,019

This represents the present value of difference in the standard contribution rate, for the attained age method and the entry age method, multiplied by the present value of total projected earnings for all members throughout their expected future membership. Numerically we can show the results as:

(Model A)	(Model B)	(Model C)
(13·47 − 12·96)	(13·51 − 12·96)	(13·90 − 12·96)
× 52,559	× 56,056	× 25,520
= 26,805	= 30,831	= 23,989

The discrepancies (all less than 1%) are due to rounding.

3. VARIATIONS IN ACTUARIAL ASSUMPTIONS

3.1 In this Section we look at the results for the standard contribution rate, as defined, when actuarial assumptions are varied. The range of assumptions investigated are those set out in Appendix 2 of the Terminology Report.

3.2 *Interest rate*

3.2.1 *In possession*

We looked at the interest rate being varied with increases in earnings at different levels.

The table that follows shows the standard contribution rate based on model B, (projected unit 20 year control):

	Interest		
Earnings increase	*7%*	*9%*	*11%*
1% gap	15·6	13·1	11·2
3% gap	11·7	9·9	8·5

As can be seen, the higher the rate of interest assumed in possession, the lower the standard contribution rate. Similar results emerge from all other methods. The progression shown in the table follows the underlying annuity value, and is therefore unaffected by the difference assumed between the interest rate in deferment and the rate of earnings increase.

3.2.2 *In deferment*

As indicated above, the effect of a change in the interest rate is related to the assumed rate of increase in earnings. We examined all methods here, as the gap between interest and earnings increases assumed is the important feature, rather than the absolute levels. If we again use model B we see the following pattern:

Current unit	*Control*	*1% gap*	*3% gap*
7% interest	1 year	11·2	9·5
	20 year	14·7	12·0
9% interest	1 year	9·1	7·9
	20 year	12·0	10·1
11% interest	1 year	7·6	6·7
	20 year	10·1	8·6

The result is similar to that recorded in § 3.2.1 above (i.e. the higher the rate of interest assumed in deferment, the lower the standard contribution rate). In the definition of the standard contribution rate, the 'current unit' of accrual implicitly assumes a zero rate of earnings increase and, hence the 'gap' merely reflects the rate of interest assumed. The past service element of the contribution rate varies, in proportion with the rate of increase in earnings assumed.

The table below shows reductions for all methods, using explicit earnings increase assumptions when the gap is increased. This is explored further in Section 6.

			Interest minus earnings assumption	
Interest	*Method*	*Control*	1% gap	3% gap
	Projected unit	1 year	14·8	10·6
		20 year	15·6	11·7
7%	Attained age		16·1	12·5
	Entry age		15·4	10·1
	Projected unit	1 year	12·4	8·9
		20 year	13·1	9·9

9%	Attained age		13·5	10·5
	Entry age		13·0	8·6
	Projected unit	1 year	10·6	7·7
		20 year	11·2	8·5
11%	Attained age		11·5	9·0
	Entry age		11·1	7·4

As shown in §2.2.2, the standard contribution rate depends on the 'gap' assumed and the average term to retirement of the scheme. The results above are based on model B. We found that the average age was:

Gap (%)	*Average age*
1	47
3	48
7	49 (i.e. under current unit method)
9	50 (i.e. under current unit method)
11	51 (i.e. under current unit method)

Thus we would expect to find that the contribution rate of 12·4% (projected unit 1 year control) becomes:

$$12{\cdot}4 \times \left[\frac{(1{\cdot}09)}{(1{\cdot}08)}\right]^{18} \times \left[\frac{(1{\cdot}06)}{(1{\cdot}09)}\right]^{17} = 9{\cdot}1\%$$

where a 3% gap is assumed, compared to 8·9% from the table above.

3.3 *Earnings increases*

Section 3.2 covers the situation in respect of inflationary or general earnings increases. Some practitioners, however, incorporate a salary or promotional scale in their assumptions. Although listed under financial assumptions in the Terminology Report, salary scales are essentially part of the demographic elements of actuarial assumptions. A salary scale represents the rate at which earnings are expected to increase in real terms (i.e. by way of promotion rather than inflation).

We would expect such a scale to be used in practice only for staff or executive type schemes but, as usual, each scheme will have its own peculiarities in this respect.

The effect on the standard contribution rate of such a scale is similar whatever the funding method adopted. The 'gap' assumed between the rate of interest in deferment and the rate of increase in earnings generally is reduced. The size of the reduction and, hence, the increase caused in the standard contribution rate will depend on the scale assumed. In general, many actuaries adopt a scale around 1% of earnings per annum, being perhaps $1\frac{1}{2}\%$ at younger ages and falling to $\frac{1}{2}\%$

or less by retirement. This can mean that the standard contribution rate does not always increase with average age.

3.4 *Increases in State benefits*

State benefits are often included in benefit formulae for occupational pension where the concept of a target pension is required.

Where included, the actuary usually needs to allow for either or both of the following State benefits:

(i) Basic State Pension;
(ii) State Earnings Related Pension (SERPS).

Related to (ii) are Guaranteed Minimum Pensions (GMPs), which a scheme will accrue when contracted-out under the Social Security Pensions Act 1975.

These benefits increase in line with prices, when in payment, and GMPs revalue in line with earnings generally for in-force members in deferment. As noted in the Terminology Report, most actuaries in practice assume that the rates of increase in State benefits are the same as the rates of increase in general earnings.

We tested the effect of assuming that State benefit increased at a rate 1% lower than earnings increases. The benefits used in our main observations, as shown in Appendix A, were changed so that pensionable earnings were earnings less the Basic State Pension for a single person. We found that the standard contribution rates were:

		Model	
Pensionable earnings	*A*	*B*	*C*
No deduction	11·84	12·42	13·17
Deduction of Basic State Pension (increasing at 8% p.a.)	11·88	12·49	13·19
Deduction of Basic State Pension (increasing at 7% p.a.)	13·07	13·44	13·79

Method: Projected unit, control period 1.
Assumptions: 9% interest, 8% earnings increases, no withdrawals.

The increase in the standard contribution rate as a percentage of pensionable earnings (but a reduction in actual cost) is in two parts:

(*a*) The salary scale employed increases as a result of a fixed deduction and thus increases the average age. The increase in contribution rate is greater under a current unit method, but less using the attained age method.
(*b*) If the Basic State Pension is assumed to increase at less than scheme earnings (i.e. 7% compared to 8%) then pensionable earnings will rise at greater than 8%. This causes pensionable earnings to rise at a higher rate than earnings themselves. Thus we close the 'gap' between pensionable earnings and interest in deferment and cause the standard contribution rate to rise, when expressed in terms of the pensionable payroll.

The results are geared to the assumptions made. The higher the deductive item relative to average earnings, the greater the increase in standard contribution rate.

3.5 *Discretionary pensions increases*

The effect of increasing pensions in payment is to lower the effective rate of interest in possession. As we saw in § 3.2.1 the outcome is a rise in the standard contribution rate under all methods and all models.

3.6 *Mortality*

3.6.1 *In possession*

The effect of allowing for mortality in the assumptions is to increase the effective rate of interest. Thus we can expect the cost to fall. In essence we can show this by simply comparing an annuity certain, $a_{\overline{n}|}$ with its equivalent allowing for mortality, $a_{x:\overline{n}|}$. At any positive rate of interest the former is always higher.

To quantify the effect we need to examine the mortality table assumed. Our basic assumption was PA(90) which ranges from a mortality rate of $2\frac{1}{2}\%$ at 65, to 9% at 80, to 37% at 100. The expectation of life is 14·1 at 65, which suggests that mortality adds perhaps 2% to the rate of interest (i.e. a_{65} at 9% = 6·97 and $a_{\overline{14\cdot 1}|}$ at 11% = 7·00). Hence the force of mortality, μ_x, has increased the effective discount rate by 2%. Strictly speaking, the calculation should be based on the mean discounted term, which depends on the rate of interest employed, but the above gives a general guide.

3.6.2 *In deferment*

The effect is similar to that in § 3.6.1 (i.e. an increase in the effective rate of interest). The aim of funding for pensions is to provide money at retirement and, if mortality is increased, we are expecting fewer scheme members to survive to receive that pension. The effect on the contribution rate will depend on the benefits paid out on death.

3.7 *Early retirement*

Our main runs, shown in the appendices, assume that retirement only takes place at age 65. For many schemes the actuary might make no allowance for early retirement. In that case, when they arise in practice, the actuary would recommend a basis to deal with it. In recognition of the earlier payment date, the accrued pension to date would be actuarially reduced. Weighed against this is the possibly higher than average mortality these members may exhibit. Consideration must also be given to constraints imposed by the Inland Revenue, preservation requirement and other legislation.

Trustees may wish to make explicit allowance for perceived early retirements. We tested model B on the assumptions set out in Appendix B and using the attained age method. Our early retirement pattern assumed was:

Age	*Rate of retirement*
60	25% p.a.
61	12% p.a.
62	12% p.a.
63	12% p.a.
64	12% p.a.

The contribution rate rose as a result, from 13·5% without the allowance, to 14·5% with the allowance. All other methods would show an increase. The benefits emerging were assumed to be accrued penson to date, unreduced for early payment. In general, the earlier the pattern of retirement assumed, the greater the cost emerging.

3.8 *Withdrawal*

The benefit on withdrawal is assumed to be based on earnings at withdrawal, making no allowance for revaluation.

Tables A and B below show the effect of introducing a withdrawal assumption into the actuarial basis of the table in § 2.2.

Withdrawals were assumed to be age related at:

10% per annum for age less than or equal to 40;
5% per annum for ages 41–50;
0% per annum for ages 51 and over.

Table A

		Model		
Method	*Control*	*A*	*B*	*C*
Current unit	1 year	4·1	8·9	13·2
	20 year	11·9	13·5	18·4
Projected unit	1 year	6·9	8·6	11·5
	20 year	10·3	10·8	13·1
Attained age		10·8	11·2	13·2
Entry age		5·3	5·3	5·3

Table B (= *Table A as a percentage of the table in* § *2.2*)

Method	*Control*	*Model* *A*	*B*	*C*
Current unit	1 year	95	98	99
	20 year	109	113	102
Projected unit	1 year	58	69	87
	20 year	79	82	95
Attained age		80	83	95
Entry age		41	41	41

3.8.1 *Current unit method, 1 year control*

We can see that the effect of introducing the withdrawal assumption is to reduce the standard contribution rate. The greatest reduction occurs to the 'youngest' scheme (i.e. model A, where the effect of withdrawals is greatest). The reduction is relatively small, as the benefit reduction is, on average, only half a year's earnings increase and half a year's accrual for those assumed to leave.

The reader might like to examine the effect of assuming the 35 year old leaves service half-way through the year, as in the example on page 34 of the Terminology Report:

The present value of benefit accruing during the year reduces by 40, and the increase in benefits accrued during the current year by 60 (assuming no further earnings increase). Overall reduction in the standard contribution rate is $100 \div 400 = \cdot 25\%$ of payroll or $4{\cdot}7\%$ of the original standard contribution rate of $5{\cdot}31\%$.

3.8.2 *Current unit method, 20 year control*

At first sight we might have expected a similar result to that under the 1 year control period. The benefits at leaving are still only based on current earnings, which is the basic aim of the method. There is still a loss of accrual and past service increase in the year of leaving.

The increase in the contribution rate occurs because of the loss of the younger contributors due to withdrawal. At the end of the 20 year period the average age of the remaining members is some 3 years higher for model B, where withdrawals are assumed. Over the period the average age is perhaps 1 to 2 years higher, leading to an increase in cost.

3.8.3 *Projected unit method, 1 year control*

The reduction in the standard contribution rate is much greater than for the current unit method. The reserve, in respect of withdrawing members, is based on earnings projected to retirement, whereas the benefit on withdrawal is based on current earnings.

Look again at the examples in the Terminology Report. If we assume the 35-year-old leaves half-way through the year, we see the present value of accruing benefit reduces from 758 to 38. The standard contribution rate reduces from 7·99% to 6·19% of payroll (i.e. by 23%).

3.8.4 *Projected unit method, 20 year control*

There is still a reduction in the standard contribution rate, as we have a release from withdrawals. The effect is less, however, than for the 1 year control period, due partly to the loss of younger contributors and also because the rate of withdrawal is assumed to fall with age. Members aged 25 are assumed to withdraw at 10% per annum at the start of the control period, and at 5% per annum at the end.

3.8.5 *Attained age method*

This method shows a similar result to the projected unit method with a long control period. It represents the 'limiting' case.

3.8.6 *Entry age method*

We assumed an entry age of 25 which, coupled with our withdrawal assumption, suggests that there is only a 12% chance of remaining an active member at age 51, ignoring mortality.

Thus we are funding for a 12% risk of a reserve at 65 based on 40 years earnings increases, and an 88% chance of between 0 and 25 years of earnings increases, ignoring mortality.

In practice we would not expect to see such a heavy rate of withdrawals used with this method.

3.9 *Proportion married*

Our standard assumption was that 90% would be married at retirement and this is a level commonly used in practice. We tested the effect of changing this level to 80%, and again assuming no widow's pension would be paid on death after retirement. The standard contribution rates were:

Assumed proportion married	*Model* *A*	*B*	*C*
90%	11·84	12·42	13·17
80%	11·65	12·22	12·96
NIL	10·15	10·64	11·28

Method: Projected unit, control period 1.
Assumptions: 9% interest, 8% earnings increases.

The results for all three models are consistent (i.e. the 'loading' for widow's pension on death after retirement is 17% for 90% married, and $(17 \times 8)/9 = 15\%$ for 80% married).

3.10 *New entrants*

We introduced new entrants into our assumptions, with three scales, as follows:

0: No new entrants.
1: Each retirement replaced by a new entrant aged 25.
2: Each retirement, death or withdrawal replaced by a new entrant aged 25.

We then re-ran all three models using the current unit and projected unit methods. The control period was 20 years and resulted in the standard contribution rates below:

Table A. *No withdrawals*

		Model		
Method	*Scale*	*A*	*B*	*C*
CU 20	0	10·9	12·0	18·1
	1	10·5	10·3	13·8
	2	10·3	10·1	13·2
PU 20	0	13·0	13·1	13·8
	1	12·9	12·8	13·1
	2	12·8	12·7	13·0

Table B. *Withdrawals*

		Model		
		A	*B*	*C*
CU 20	0	11·9	13·5	18·5
	1	11·3	11·6	14·9
	2	7·5	8·2	12·2
PU 20	0	10·3	10·8	13·1
	1	9·9	9·6	11·0
	2	7·3	7·5	9·4

3.10.1 The general effect was that the introduction of new entrants produced a lower cost. The higher the level of new entrants, the greater the reduction. This falls in line with earlier results, suggesting that a lower average age results in a lower cost.

As the control period is extended, the standard contribution rate emerging will approach an average of the control period 1 figure for the original membership, and a comparable figure for a scheme in a stationary state. The stationary state of the scheme will depend on the new entrant assumptions made. Our scale 1 will reduce a scheme to zero membership.

3.10.2 The curious feature about the results in the tables above is that scale 1 new entrants sometimes produced lower results for model B than model A. This resulted from our initial truncation of model B, in order to produce model A. We removed all members within 10 years of retirement from model B and, hence, for 10 out of the 20 year projection period there were no new entrants in A, but a number entered B. This was not deliberate, but certainly proved interesting when assessing the results!

3.10.3 The effect on the current unit method rates is similar, whether withdrawals are assumed or not. The projected unit method, however, produces much greater reductions when withdrawals are assumed with new entrants than when withdrawals are assumed without new entrants.

For model B, based on a control period of 20 years, the standard contribution rate falls from 13·1% to 10·8% (no new entrants) and from 12·7% to 7·5% (scale 2 new entrants).

The introduction of new entrants produces a greater proportion of potential withdrawals, for whom benefit will be based on only current earnings.

4. TREATMENT OF SURPLUS/DEFICIENCY

4.1 *What is a surplus or deficiency?*

The periodic actuarial valuation of a scheme will place a value on the assets and on the liabilities of the scheme. No matter how the assets are valued and no matter how the liabilities are valued, if the value of assets exceed the value of the liabilities then the scheme, for the purposes of this paper, is said to be in surplus (or in deficiency if liabilities exceed assets).

The size of any surplus or deficiency, and indeed its existence, is notional or temporary for the following reasons:

(*a*) There are several methods of valuing assets (for example, values based on market value, discounted income or written up book value) and of valuing the liabilities (for example, accrued benefit based on current salaries, projected final salaries or total future liability less future contributions). While the methods of valuing assets and liabilities should be chosen to be compatible with each other, many other results are possible.

(*b*) There is a wide variety of possible actuarial bases. Different bases inevitably result in different figures for surplus or deficiency.

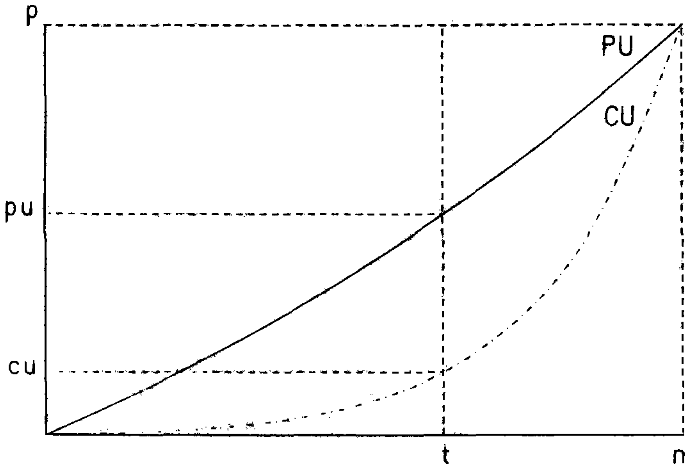
(*c*) The figure is changing materially all the time, owing to underlying financial and demographic fluctuations affecting the scheme.

It is therefore inappropriate at any time to look at a surplus and view it as 'surplus to requirements'. Similarly a deficiency in a scheme is not necessarily a cause for panic.

4.2 *What is the target fund?*

The concept of surplus/deficiency only has relevance when one has a clear idea of the 'target' fund at any point in time. By this is meant: how much had we *expected* to build up in the fund by this time. This concept is discussed in more detail in Section 6, but the chart overleaf gives a simplified illustration.

A member retires at time n with cost of pension P. Assuming P is a final salary pension, then the accrued benefit at any time is represented by line PU if accrued benefit is defined as based on projected earnings, and as CU if defined as based on current earnings. At time t it is clear that there is a very big difference in the two definitions of accrued benefit, and while a scheme funded on the current unit method (CU) may be in surplus, it may be in deficiency if funded on the projected unit (PU) method.



4.3 *Should a surplus/deficiency be 'run off'?*

By 'run off' here we mean: should action be taken to reduce that surplus or deficiency, either by adjusting contributions or by adjusting benefits?

The comments in §4.1 above indicate that it is often not necessary to reduce surplus or deficiency which may arise. This is particularly true where the surplus/deficiency is small, since experience fluctuations could well cause a surplus now to be a deficiency in a few years' time.

Furthermore, there is nothing inherently correct about a funding level of 100% of 'target fund'. This is particularly true if the definition of accrued benefit being used is projected earnings, when the trustees may well decide to 'aim' at building up a fund of, say, 80% of accrued benefit. In this situation, though, the trustees will want to consider the position should there be a winding-up of the scheme, or a partial winding-up on the sale of part of the company.

Nevertheless, there are situations where the trustees may feel that the assets and liabilities are out of line to an extent that requires some redress.

(Government action to control surplus will limit trustees' flexibility here, because there will be conditions under which the trustees do not have a choice but are required to take action on the surplus. Equally the trustees are constrained by the wording of the Trust Deed.)

4.4 *Action against surplus or deficit*

4.4.1 If there is a deficit in the scheme which requires action, this can be done by effectively increasing the contribution rate temporarily (the most extreme being a lump sum payment, though this is rarely appropriate). It may be, in an extreme situation, that this is associated with a reduction in future service benefits.

Where there is a surplus in the scheme which requires action, it can be reduced by a reduction in the contribution rate (similar to the treatment of deficit). Alternatively, it can be reduced by an increase in past service benefits, although this often involves an increase in the scheme's contractual obligation. This increase in obligation can be softened by the trustees agreeing, instead, to be more generous as regards discretionary pension increases in the future, so that the actuary may then strengthen his basis, thus using up the surplus.

4.4.2 Where an adjustment to the contribution rate is the course chosen, the following are some of the more common methods (as described in Section 6 of the Terminology Report). In this paragraph, the word 'deficit' is taken to mean both deficit and surplus.

(*a*) Spread the deficit over a fixed term allowing for interest. Most common terms are 10, 20 or 40 years. This leads to a constant monetary addition. (The factor used to spread the deficit is of the form $a_{\overline{n}|}$ at rate i.)

(*b*) Spread the deficit as in (*a*) but allowing for a decrease in the value of money. This leads to a monetary amount increasing each year by whatever index is used. (The factor used to spread the deficit is of the form $a_{\overline{n}|}$ at rate $(i-e)$.)

(*c*) Spread the deficit over total future expected earnings of the current membership, and express the result as a proportion of the current payroll: it is then applied to total earnings from year to year. (The factor used to spread the deficit is of the form a_x at rate $(i-e)$.)

(*d*) As in (*c*), but restricting projected earnings to those earned in a fixed period of years ahead. (The factor used to spread the deficit is of the form $a_{x:\overline{n}|}$ at rate $(i-e)$.)

(*e*) As in (*b*), using expected earnings growth as the index, and expressing the result as a proportion of current earnings. (The factor used to spread the deficit is similar in form to that used in (*b*) above, though the net rate of interest used may differ.)

(*f*) Payment of a lump sum to eliminate the deficit.

The appropriateness of each method will depend on circumstances. The deficit will need to be erased quickly, for example, if there are a large number of retirements coming up. In this situation method (*c*) may not reduce the deficit quickly enough, and one of the other methods should be used, with a suitable term—the most extreme being (*f*).

Those methods [(*c*), (*d*) and (*e*)] which express the surplus/deficit as a percentage of earnings are vulnerable to changes in real earnings. This is appropriate where the membership is fairly steady, but large numbers of new entrants or withdrawals could destabilize the method. As the contribution is usually paid on total payroll, where a deficit is being run off large numbers of new entrants will cause it to be run off quicker than anticipated. Conversely, large numbers of withdrawals will cause the deficit to be run off more slowly, and may even cause a solvency problem, particularly where transfer values are taken. This is mainly a problem with the current unit method, since with the other methods there is usually a large release on withdrawal anyway.

As a general rule it is convenient, where possible, to match the deficit spread period to the reason for the origin of the deficit.

4.4.3 Throughout the rest of this paper, any adjustments to standard contribution rates to cope with surplus or deficit have been ignored, because the methods used to deal with the surplus/deficit will depend very much on the circumstances, and any method may be used with any of the four basic funding methods.

5. SMALL SCHEME PROBLEMS

5.1 *Introduction*

What is a small scheme? In the context of this Section it is a scheme that is small enough to exhibit unusual features. Those features could be due to:

(i) Domination by a few members in terms of earnings or benefits or both.
(ii) An unusual distribution of members by age and service.
(iii) Some other reason.

This Section looks at the effects on funding methods and standard contribution rates arising from (i) and (ii). In practice, situation (i) is often seen in the small family firm of less than 50 members where one or two directors dominate. Situation (ii) can arise for a variety of reasons and is investigated by looking at various control periods.

Throughout this paper we have ignored any problems arising out of solvency considerations but, clearly, given the structure of models X, Y and Z below, such problems might arise.

5.2 *Models investigated*

We devised three further models, based on our original 'average' model B (shown in Appendix A):

(*a*) Model X was the same as model B, plus one extra member at age 60 earning 5 times the average at that age.
(*b*) Model Y was the same as B, plus two extra members. One was included at age 60 and the other at 55. Both were given earnings of 5 times the average at that age.
(*c*) Model Z increased B to three extra members (i.e. model Y plus one extra member at age 50 with earnings at the higher level).

5.3 The table below shows the standard contribution rates results obtained based on the assumptions in Appendix B.

		Model			
Method	*Control*	*X*	*Y*	*Z*	*B*
Current Unit	1 year	10·8	11·2	11·0	9·1
	20 year	12·6	13·3	14·0	12·0
Projected Unit	1 year	12·6	12·7	12·8	12·4
	20 year	13·1	13·2	13·3	13·1
Attained Age		13·5	13·6	13·6	13·5
Entry Age		13·0	13·0	13·0	13·0

5.3.1 *Current unit method*

The result is clearly affected if we introduce just one dominant member into a 50 life scheme. We have increased the average age and past service dramatically. If the dominant members are static in terms of turnover, as may well be the case for a family concern, then the standard contribution rate will rise in line with the average age and past service. Contribution rates, assessed every three years, will rise until a dominant member leaves service. (See § 5.4.)

5.3.2 *Projected unit method*

The rise in average age caused by the introduction of the dominant members has only a small effect on the standard contribution rate. This makes the method more suitable than the current unit method for small schemes of this type. The increase in contribution rate at successive reviews of the scheme will be slight. (See § 2.2.2 for sensitivity.)

5.3.3 *Attained age method*

A similar pattern emerges to that shown by the projected unit method and similar comments apply.

5.3.4 *Entry age method*

The standard contribution rate is unaffected by the change in membership. If a dominant category exists, however, it may be thought suitable to assess this separately (e.g. assume entry age 40 for these members who might also be on higher benefit levels).

5.4 *Comparison of stability*

In the Terminology Report, control periods were discussed in § 5.16 as follows: "It will be seen that the unit methods result in a stable contribution only if the age structure remains constant. However, particularly for new schemes or schemes with a small number of members, this is unlikely to be a reasonable assumption. For such schemes allowance for variations in the age structure is often achieved by applying a control period to a standard method as described . . . ."

We looked at the dominant member problem, and the unusual distribution problem in the light of variable control periods. The basic model investigated was a scheme with only 4 members, as described in Appendix 1 of the Terminology Report.

Appendix D shows the effect if:

(*a*) *New entrants replace future retirements*
As the control period is increased, the contribution rate for the projected unit method gradually increases from 7·99% at control period 1 to 8·32% at control period 40.
The current unit method, however, rises from 5·31% at control period 1 to 9·11% at control period 40 and has a distinctly cyclical effect. Peaks occur as a retirement approaches and then is replaced by a new entrant.

(*b*) *If new entrants are ignored*
The projected unit method contribution rate rises more steeply than in (*a*), being 8·67 after 40 years. The current unit method contribution rate rises even more steeply, showing how important it is to allow for new entrants as the control period lengthens.
Clearly it is important to choose the control period carefully with small schemes, and to be aware of the likelihood of new entrants.

The assumptions for the two unit methods were chosen with a view to producing a reasonable scale of contribution rates for graphical presentation. The 'crossover' point was at a control period of 8 years and this aspect is considered further in § 6.3.

6. SUITABILITY OF METHODS

6.1 *Introduction*

6.1.1 With a wide variety of funding strategies in use (where funding strategy is taken to mean the combination of funding method and funding assumptions) it is bound to occur to the layman, to non-actuaries in the pensions industry, and even to actuaries who are not pensions experts, that some strategies are more suitable than others. In this Section we examine this issue.

6.1.2 Clearly the topic is a contentious one, and as always the debate has been partially clouded by lack of understanding of the issues involved. Initially, therefore, we ask the question: 'Suitable for what?' It is vital to have the goal posts clearly in sight before kicking the ball.

6.1.3 As a reflex response to § 6.1.1, we can clearly say that some strategies are better than others: we refer to those possible strategies which involve totally inappropriate funding assumptions. Clearly, for example, a projected unit valuation assuming 25% p.a. investment returns and 5% p.a. earnings increases is unlikely to be a sound funding strategy, even if these parameters had been achieved in the last year! Broadly, then, we are not looking at the theoretically possible funding strategies, but more at the range of strategies in regular use. While on the topic of assumptions, however, it is evident that what may be an appropriate assumption for one scheme may be inappropriate for another. Examples of this are the new entrant age assumption in the entry age method, and the assumed level of discretionary pension increases that may be awarded.

6.2 *Suitable for what?*

6.2.1 There are a variety of different reasons for valuing a pension fund, and different strategies may be called for in each case. In this Section we are mainly concerned with the funding strategy required when valuing a scheme for the purposes of determining an appropriate long-term contribution rate, bearing in mind the nature of the liabilities.

We are looking, therefore, for a funding strategy which is suitable for determining a long-term contribution rate as a continuing scheme. So what are the objectives of this contribution rate? What is it trying to achieve? Once we know this, only then can we assess whether the funding strategy underlying the contribution rate is suitable.

6.2.2 In this respect, the actuary must recommend a funding strategy to achieve objectives which are generally accepted by the trustees. These objectives are usually:

(*a*) that the fund should be sufficient at any time to cover accrued liabilities, and
(*b*) that the long-term contribution rate should be broadly stable.

Some trustees may put more emphasis on one objective than on another. Currently, for example, many funds are in surplus, and it is common for contributions to be temporarily reduced or suspended altogether, so as to reduce funding levels towards 100%. This is an example of objective (*a*) having precedence over objective (*b*). It would be a logical extension for trustees and employers to show similar, if not greater, concern where schemes are under-funded. There are other objectives that may be specified, apart from (*a*) and (*b*) above.

Possible examples are:

That the fund should at all times be at least 120% funded on a winding-up basis.

That the cost should be kept to a minimum for the next five years.

That the fund should be less than 105% funded on the Government Actuary's prescribed valuation basis.

Usually, however, trustees will have two objectives, though one may be dominant: to cover accrued liabilities and to have a stable contribution rate.

6.2.3 What is an 'accrued liability'? The phrase does not have a universal actuarial definition, and is the source of a great deal of controversy in the actuarial world. There are two main definitions: one is based on current salary and one on projected salary. The definition included in the PMI/PRAG publication *Pensions Terminology*[(2)] for 'Accrued Benefit' is "The benefits for service up to a given point in time, whether vested rights or not. They may be calculated in relation to current earnings or projected final earnings." (Vested rights essentially means early leavers' benefits.)

In symbols the accrued pension in a 60ths scheme may be either:

$$\tfrac{1}{60} \times \text{service} \times \text{current earnings}$$

or

$$\tfrac{1}{60} \times \text{service} \times \text{current earnings} \times (1+j)^{NRA-x}$$

where

$$\begin{aligned} j &= \text{earnings increase assumptions} \\ x &= \text{member's age} \\ NRA &= \text{Normal Retirement Age} \end{aligned}$$

Commonly the difference between these two may be a factor of four (based on a member 20 years from retirement with 7% earnings increases), although it will usually be less for a scheme as a whole, since older members are a higher proportion of the liability and future earnings increases have a lesser effect on their benefits. Nevertheless, the difference could easily be a factor of 2 for a scheme as a whole.

Essentially, the trustees should decide which definition they prefer in funding the scheme, and they will often rely heavily on actuarial advice in making their decision. The distinction should therefore be clear to trustees and the actuary ought not to be in doubt as to the trustees' wishes. For some schemes the trust deed will define a minimum level to be covered at all times in respect of accrued pension. More often a lesser level of solvency is required, being that rquired to cover the priorities on winding up. This latter point has become even more important with the recent disclosure legislation.

The definition used will not only affect the 'target' fund at any time, but will also materially affect the cost of accruing pension as reflected by the standard contribution rate. This is discussed in greater detail in Section 2, as regards the different contribution rates emerging from the projected unit and current unit methods.

6.2.4 What is meant by 'stable contribution rate'? It is common practice in the U.K. to express the funding cost as a proportion of payroll, although this is not essential. It is certainly a convenient method, though, and if the proportion can be stable over time, despite fluctuating experience of the scheme, then the budgeting and financial planning of the employer is greatly assisted.

6.2.5 In conclusion, therefore, we are looking for a funding strategy which copes initially with the objectives of covering 'accrued liability' (however it may be defined) and maintaining a fairly stable contribution rate, despite experience fluctuations.

6.3 *The correct contribution rate*

The layman mentioned at the beginning of this Section may well be heard to say: "Surely there must be a single contribution rate which is correct?"

This statement is true, at least for current members, but of course the only way this rate can be assessed is to wait for 70 years or so until the last of the current members' dependants has finally died. Then one can say: "the absolutely correct contribution to pay 70 years ago would have been . . .".

The cost of a pension scheme is not determined by the amount paid in by way of contribution rate, but by the ultimate experience of the scheme. We cannot predict precisely what that ultimate cost will be, and our contribution rates are therfore only estimates. The complication about the concept of a correct contribution rate is that, if one takes a particular contribution rate, there may be numerous funding strategies which produce a similar rate. Looking at model B in Appendix C, the following strategies all produce a standard contribution rate between 8·5% and 9·0%:

Method	*Interest/Earnings*	*Withdrawals*	*Rate*
CU 1	9/8	Y	8·9
CU 20	11/8	N	8·6
PU 1	9/8	Y	8·6
PU 1	9/6	N	8·9
PU 1	7/4	Y	8·6
PU 20	11/8	N	8·5
PU 20	9/6	Y	8·9
AA	11/8	N	9·0
EA	9/6	N	8·6

It is clear from the above that the long-term contribution rate is not a cast iron figure but an estimate of the long-term cost.

Of course, not all of the literally hundreds of funding strategies that one could put into the above table would give a sensible estimate of the funding level of the scheme.

We conclude, therefore, that while there may be a correct contribution rate, without the benefit of hindsight we have no way of determining what it may be. It is therefore up to the actuary's judgment to select and recommend a funding strategy which will produce suitable results—neither too low, because of the danger of the scheme's solvency, nor too high, because of the diversion of company assets which could possible be put to better use elsewhere in the company.

In the next section we look at the principal characteristics of the four main funding methods and then examine the performance of each, in terms of the funding strategy noted in §6.2.5 above.

6.4 *Funding methods: rationale and characteristics*

In describing the features of the four methods below, we have separately commented on the sensitivity of the methods to new entrants and withdrawals. We have highlighted these areas of the experience, because they mark the principal areas of difference between the methods. Comments on the methods' sensitivity to other factors are found in §6.6.

6.4.1 *Current unit method*

Under the current unit method the contribution rate should pay for benefits accruing in the coming year, based on earnings at the end of that year together with earnings inflation on previously accrued benefits.

The rationale is thus: that at any time the accumulated fund is sufficient to purchase past service benefits based on current earnings.

6.4.1.1 *Sensitivity to new entrants/withdrawals*

The method anticipates a steady flow of new entrants—if the combined effect of the new entrants and exits is to keep the age and past service structure stable, the method will produce a stable contribution rate.

Withdrawals (provided the benefit is based on current salary and service as is usual) do not cause either strain or release on this method: it is therefore common to ignore them in the valuation assumptions. High withdrawals in practice may cause the contribution rate to rise, owing to an increase in the average age. But, in practice, it is likely that such withdrawals would be replaced by new entrants and so keep down the average age, and thus the contribution rate.

6.4.1.2 *Features*

(*a*) The standard fund built up is generally the smallest of the four methods, given the same assumptions, being generally sufficient to purchase withdrawal benefits at any time.

(*b*) For a large scheme with a reasonable prospect of age/past service stability, a stable long-term contribution rate can be achieved, which will be consistent with the maintenance of a fund sufficient to purchase withdrawal benefits on winding-up. If the trustees are happy with an accumulated fund of this size, the method is appropriate.

(*c*) For small schemes, where there is less prospect of age/past service stability, it is advisable to defend against the contribution rate becoming inadequate by building a margin into the contribution rate. This can be done by using conservative assumptions and/or using an explicit margin and/or using a suitable control period.

(*d*) A particular example of lack of stability in the current unit method is where the method is being used to arrive at a long-term contribution rate for a new pension scheme with no past service benefits. In these circumstances, the contribution rate is almost certain to rise as the scheme ages in the first years. Ways of adjusting this feature are mentioned in (*c*) above: for example, including an explicit item in the contribution rate to allow for the immaturity of the scheme.

(*e*) Although, strictly speaking, the method values an accrued benefit based on current earnings, in practice, funding on this basis could mean withdrawal strains owing to compulsory revaluation of early leavers' benefit (through the revaluation of GMPs and/or through Social Security Act 1985 revaluation). By 'strain' is meant an amount to be paid out of the fund which is greater than the reserve held for the member concerned. (A 'release' is a negative strain.) In itself, of course, there is nothing wrong with a strain, but withdrawals experience is unpredictable, and it is an actuarial convention that withdrawals should have a neutral effect on the funding or cause a release. In general, releases are also undesirable, and it may well be that the profession can do more to cope with the withdrawal question.

The current unit method, in practice, treats accrued benefit as withdrawal benefit, rather than current earnings benefit.

(*f*) The method cannot be used to value benefits which are subject to revaluation (such as GMPs or Social Security Act 1985 revaluation). The method is, nevertheless, still useful for schemes not subject to such revaluation, particularly non-U.K. schemes.

6.4.2 *Projected unit method*

The contribution rate on this method is sufficient to purchase benefits based on projected final earnings, which will accrue over the next year.

The rationale of the method is to build up a fund sufficient to purchase past service benefits based on projected final earnings.

6.4.2.1 *Sensitivity to new entrants/withdrawals*

As in the current unit method, this method anticipates a steady flow of new entrants, such that the net effect of all movements is to keep the average age stable.

Withdrawals usually generate a release, since the withdrawal benefit is lower than the accrued benefit. If withdrawals are assumed in the funding strategy, then fewer withdrawals than anticpated may cause a strain. Excess withdrawals cause a release.

6.4.2.2 *Features*

(*a*) The standard fund built up is generally much larger than under the current unit method, and is the same as under the attained age method.

(*b*) The method is far less sensitive to changes in average age than in the current unit method. This is because the effective discount rate in deferment (being the difference between interest and earnings increases accrued) is much lower, generally in the order of 0%–3%.

(*c*) As a result of (*b*) above, the method displays much greater stability, so the conservative measures in § 6.4.1.2(*c*) above are less likely to be necessary. This stability is demonstrated in Section 2.

(*d*) The fund built up is intended to be sufficient to purchase benefits on winding-up, based on earnings at projected date of exit, although its ability to do this will, of course, be subject to experience.

6.4.3 *Attained age method*

The rationale of the attained age method is that there will be no new entrants in the future, and consequently the contribution rate allows for a gradual ageing of the membership. At any point the accumulated fund should be at least equal to the value of the accrued final earnings benefits. The intention is that the contribution rate will remain stable throughout the remaining working lifetime of the current membership.

6.4.3.1 *Sensitivity to new entrants/withdrawals*

(*a*) New entrants at ages younger than the scheme average age cause a contribution to surplus, since the contribution rate will be higher than necessary to fund for their benefits. This contribution to surplus is generally small. The funding level is not immediately affected by new entrants.

(*b*) Withdrawals in the attained age method cause a release, as for the projected unit method. Where there is an explicit withdrawals assumption, too few withdrawals may cause a strain.

6.4.3.2 *Features*

(*a*) The contribution rate is higher than that required to maintain the funding level at 100%. Other things being equal, the funding level therefore rises gradually from the date the contribution rate is established. Once the membership has aged considerably the contribution rate becomes inadequate, the surplus is then drawn on to supplement the contribution rate, and the funding level will drop back towards 100%.

(*b*) The contribution rate will only be stable if either:

   (i) The age structure remains stable, or

   (ii) The valuation assumptions are met; there are no new entrants and the periodic surplus mentioned in (*a*) above is run off over the outstanding working life of the membership.

(*c*) The method is particularly suitable for schemes which are uncertain of new members, or for very small schemes which are prone to changes in average age.

(*d*) A variant of the attained age method is the aggregate method. Under the aggregate method any surplus or deficit is automatically run off over the outstanding working life of the membership (method (*c*) in § 4.4.2). This method produces a stable contribution rate provided there are no new entrants, but new entrants at young ages cause a release and at old ages new entrants cause a strain.

The method has been widely used in the past, but appears to be less popular now, perhaps due to the requirements of Guidance Note 9[3], whereby an ongoing funding level has to be quoted or implied: the aggregate method does not lend itself easily to this.

We have not treated the method as a separate one in this paper, since it is a variant of the attained age method, albeit a widely-used variant. The method can also be considered as the entry age method, running deficits off as per (*c*) of § 4.4.2, or as either of the unit methods with a very long control period—again using method (*c*) of § 4.4.2 in respect of surplus/deficiency in the standard fund.

6.4.4 *Entry age method*

The rationale of the entry age method is that the contribution rate should be set at a level sufficient to purchase benefits for a new entrant. At any point the standard fund required is, therefore, the total expected liability less the value of future contributions.

6.4.4.1 *Sensitivity to new entrants/withdrawals*

(*a*) The method is neutral as regards its attitude to future new entrants. If these new entrants enter at the assumed ages there will be neither strain nor release and the contribution rate will be adequate.

(*b*) Withdrawals in the entry age method cause a release of future service reserve. Where a withdrawal assumption is built into the contribution rate, this release is being anticipated. Fewer withdrawals than expected therefore create a strain—this is similar to other methods, though the size of the strains and releases tends to be larger, because the standard fund for the method is usually greater than any other.

6.4.4.2 *Features*

(*a*) The entry age method is the only one of the four methods where new entrants can create a capital strain or release (i.e. a change in the funding level)—this occurs if new entrants occur at ages other than those assumed.

(*b*) Since, by definition, any realistic entry age assumption will be less than the average age of the members, the standard fund is the largest of the four methods. This is demonstrated in Section 2.

(*c*) On the closure to new entrants of a scheme funded on this method, the contribution rate should remain stable (if, of course, the assumptions are borne out in practice).

(*d*) The determination of the new entrant age assumption is difficult because:

(i) The contribution rate is sensitive to withdrawal assumptions (usually on the light side). A light withdrawals assumption will produce too high a contribution rate, so it may therefore be reasonable to use a slightly younger than experienced new entrant age assumption. This would mean a valuation strain on entry!

(ii) Both new entrant and withdrawal experience is unpredictable and dependent on the financial fortune of the employer, as well as the economy in general.

6.5 *Cover of accrued liability*

How do these four methods compare, as regards suitability, when measured against our two objectives of:

(i) cover of accrued liability, and
(ii) stability?

Each of the four funding methods incorporates a definition of accrued liability, in the sense that this is the value of the liabilities thrown up by the method as defined by the standard fund. There are two broad reasons why this indicator of accrued liability may be unsuitable:

(*a*) The definition may not be compatible with the trustees' feeling of the accrued liability. This is particularly true of the entry age method, where the accumulated fund may be considerably larger than that required to purchase winding-up benefits (whatever these may be).

(*b*) The actuarial assumptions underlying the liability may be suitable for determining a long-term contribution rate, but less suitable for placing a value on the liabilities.

The important question, as always, is "what do the trustees want?". It is likely that one funding level is to be used for two different purposes: namely, the assessment of an on-going funding position and the assessment of the ability of the scheme to meet accrued liabilities on a winding-up.

In the latter case, accrued benefit should be based on current earnings or projected earnings (as the trustees or trust deed direct) and on current service. The actuarial basis should reflect competitive lump sum deferred annuity terms in the market, and asset values should be related to market values.

An on-going funding level, on the other hand, will use the assumptions and definitions of accrued liability inherent in the funding strategy, so as to give a picture of the position of the fund compared to its projected long-term path.

In terms of Guidance Note 9, of course, actuaries give indications of both these figures when preparing a funding report. But, in terms of the objectives of funding discussed in § 6.2.2 it is more likely that what the trustees have in mind is the ability of the scheme to cover accrued liabilities on winding-up, either on current or projected earnings. In this respect, the entry age method involves the building up of a fund of materially larger size than necessary.

The current unit method involves a lower funding target than the projected unit and attained age methods, being based on current rather than on projected earnings. Poor experience can cause the current unit funding level to fall below even current earnings winding-up benefits. But this experience will also cause projected unit and attained age funding levels to fall short of their targets, so that each fund is then underfunded in comparison with the trustees' objectives, although the current unit fund offers less security to its members on winding-up.

Provided, therefore, that the trustees are aware of the issues, it cannot be said that current unit builds up too small a fund, or that projected unit and attained age too large a fund.

6.6 *Stability of contribution rates*

How do the four methods compare as regards stability of contribution rate? By stability is meant a lack of sensitivity to fluctuations in experience and membership: these fluctuations may occur in either the financial or demographic factors.

6.6.1 *Financial fluctuations*

Clearly no method is insensitive to fluctuations in the financial factors. If the fluctuations are short-term ones which do not cause the actuary to change his valuation basis, then the fluctuations will be reflected in a funding level diverging from 100%. Since the entry age method has the largest fund, it could be said to be most sensitive to these fluctuations, and similarly the current unit method the least sensitive. The treatment of surplus and deficit is covered in Section 4, and generally is a matter of judgment: various approaches are possible.

A divergence may be such that a change in the valuation basis is called for. The effect of this on the contribution rate will depend on:

(i) The extent to which the changed liability valuation basis affects the funding level. Either, or both of the liability and asset valuations may be affected. The handling of a changed funding level would follow the methods in Section 4.

(ii) The exact nature of the changes to the valuation basis, and which funding method is being used.

Table A illustrates this second point. In each case no withdrawals are assumed, and the variation is in the interest (i) and earnings increase (e) assumptions. Column 1 shows the contribution rate thrown up by each method on an interest

Table A

Method		*(1)* *9/8*	*(2)* *9/6*	*(a)* %	*(3)* *7/6*	*(b)* %	*(4)* *7/4*	*(c)* %
Current unit	1	9·1	7·9	−13%	11·2	+23%	9·5	+4%
	29	12·0	10·1	−16%	14·7	+23%	12·0	0%
Projected unit	1	12·4	8·9	−28%	14·8	+19%	10·6	−15%
	20	13·1	9·9	−24%	15·6	+19%	11·7	−11%
Attained age		13·5	10·5	−22%	16·1	+19%	12·5	−7%
Entry age		13·0	8·6	−34%	15·4	+18%	10·1	−22%

Column (a) is the percentage differential between (2) and (1).
Column (b) is the percentage differential between (3) and (1).
Column (c) is the percentage differential between (4) and (1).

basis of 9% and on an earnings increase basis of 8% (9/8). Column 2 uses the same interest basis, but reduces earnings increases to 6% (9/6). Column 3 uses a 7/6 basis, and Column 4 a 7/4 basis. The figures are based on model B.
Table A illustrates the sensitivity of the methods to changes in the financial basis, and the following features are of interest:

(i) All column (a) are negative because of the increase in the interest/earnings differential $(i-e)$ from 1% to 3%. 'i' is unchanged.
(ii) All column (b) are positive because of the decrease in 'i' while leaving $(i-e)$ unchanged at 1%.
(iii) Column (c) varies because of the combination of an increased $(i-e)$ and a decreased 'i'.
(iv) The differences between the methods is interesting and repays study: all the differences can be explained by:

* a change in $(i-e)$ leaving 'i' unchanged affects those methods more which have a greater exposure to $(i-e)$;
* a change in 'i', leaving $(i-e)$ unchanged affects these methods more which have a greater exposure to 'i'.

Table B (p. 102) illustrates the exposure (and therefore the sensitivity) of each method to changes in the financial assumptions.

Column (a) shows the exposure to the 'gap' between interest and earnings increases (i.e. during the method's 'projection period'). Column (b) deals with the exposure to the interest assumptions (the table assumes that retirement increases are fixed—as throughout this paper) which affects the period from the end of the 'projection period' onwards. Column (c) records numerically the approximate values of these exposures, given the assumptions below Table B.

The value of Table B is that we can immediately see how sensitive a method is to variation in $(i-e)$ and i, by looking at the figures in column (c). The higher the value, the greater the sensitivity, and vice-versa.

Table B. *Exposure of methods to financial fluctuations*

Method	(*a*) *Exposure to* i − e	(*b*) *exposure to* i	(*c*) *In figures: exposure to* (i − e)	i
CU 1	1 year (i.e. 1 year proejection period but $(i-e)$ will also affect the uprating of past service accrual)	From the end of that year onwards	1	39
CU 20	10 years (the 20 year projection method gives an average projection term of 10 years)	From the end of projection term onwards (on average therefore from 10 years time onwards)	10	30
PU 1	Average term to retirement	From retirement age onwards	20	20
PU 20	Average term to retirement (but projection period ages the scheme, and so reduces the average deferment term)	From retirement age onwards	10	20
AA	Average term to retirement, but this is effectively the same as PU with an 'infinite' projection period, so the term should be at least as low as PU 20	From retirement age onwards	< 10 (say 8)	20
EA	Average term to retirement from entry age. Again, the long projection period ages the scheme, but the average *term should be longer than* AA since entry age assumed will generally be less than the average age of the scheme. It will also generally be more than PU 20 unless the scheme has a very young average age.	From retirement age onwards	say 25	20

Underlying assumptions

Retirement age:	65	Life expectancy at age 45:	40
Average scheme age:	45	Entry age assumptions:	25

We conclude from Table B that:

(i) The entry age method is usually the most sensitive to an increase in $(i-e)$, whereas the current unit (1) is insensitive.
(ii) The current unit method is most sensitive to fluctuations in 'i'.
(iii) The sensitivity to a change in $(i-e)$ varies widely between methods and also depends on the age profile of the scheme.
(iv) Where both $(i-e)$ and i change, the sensitivity will depend on the exact changes made and on the age profile of the scheme.

6.6.2 *Demographic fluctuations*

Deaths, normal retirements, early retirements, withdrawals and new entrants are the principal demographic factors affecting pension funds. Section 3 has examined the effect on results, when varying demographic factors.

As when dealing with financial fluctuations, we need to distinguish between those which affect the funding level and those which affect the periodic calculation of the contribution rate.

The funding level is affected whenever the reserve held in respect of the decrement is different from the benefit paid out.

Deaths usually result in a release if the death benefits are insured or, if they are not, in a strain if there are more deaths than expected. Normal retirements may cause a release or strain if the annuity is insured, insofar as the terms are different from those assumed in the funding strategy. Early retirements may cause a strain or release also, depending not only on the normal reitrement considerations but also on the actuarial reduction (if any) applied to the benefits. Withdrawals cause a release under all methods, except current unit where the effect is neutral. New entrants cause neither strain nor release, except under the entry age method where entry at a younger age than assumed causes a release and vice-versa. All these demographic changes cause funding level changes, depending on the method and circumstances.

So how is the periodic assessment of the contribution rate affected? In two ways:

(i) Insofar as the funding level has moved from its expected level, the treatment of the surplus/deficit may change. This is discussed in Section 4.
(ii) A change in the average age of the scheme (and in the case of current unit method, the average age and past service) will affect the contribution rate at the next review. This is not true of the entry age method, of course, unless the assumed entry age is changed.

Furthermore, in the interim period up to the next review, the continuing contribution rate will be inappropriate to the age/service profile of the scheme, creating an element of surplus/deficit at the next review.

The sensitivity of the methods to changes in average age is therefore important. The following table compares contribution rates under model A (a young fund) and model C (an older fund), based on 9% interest and 8% earnings increases with no allowance for withdrawals:

Method		*Model A*	*Model C*	%
CU	1	4·3	13·3	+209
	20	10·9	18·1	+66
PU	1	11·8	13·2	+12
	20	13·0	13·8	+6
AA		13·5	13·9	+3
EA		13·0	13·0	nil
Approximate average age (from § 2.2.1)		42	54	

The answers are sensitive to the age and earnings profiles of the schemes, but it is clear that the current unit method is extremely sensitive. This is explained by the fact that, in the above example the effective discount rate is about 9% p.a., whereas for the other three methods it is only about 1% p.a. (Section 2 goes into this in more detail.)

As a general point, however, it is worth noting that the unit methods assume that the age structure will remain fairly stable and, if it does so, so will the contribution rate. The attained age method assumes there will be no new entrants, and the the average age will gradually increase until finally all the members have retired. This assumption is difficult to defend in a healthy, ongoing scheme though, on closure of the scheme to new entrants, a fairly stable contribution rate is expected, whereas it could rise in the case of the current unit method.

6.7 *Conclusion: suitability of the methods*

The methods exhibit different features and peculiarities, some of which have been highlighted in this chapter. The answer to the question "which method is suitable?" will depend on the circumstances. There are so many different types of fund, funding positions, anticipated future of the company and personnel policies, that all the methods have a role to play.

It is up to the actuary to produce a suitable set of assumptions to apply to a particular method and fund, so as to produce a funding strategy which meets the trustees' funding objectives.

7. ACKNOWLEGEMENTS

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8. REFERENCES

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(3) Institute & Faculty of Actuaries (1984) Guidance Note 9 'Retirement Benefit Schemes—Actuarial Reports'. (Issued to members.)

APPENDIX A

Model schemes

Our starting point was to construct an 'average' all male scheme. We might well have used a basis of uniform membership spread over 40 years but finally settled for a slightly younger design. We judged this to be more realistic in view of the withdrawals that might be expected at younger ages. This is shown in the Paper as model B.

To produce a 'younger' than average model we truncated model B by removing members aged 56 to 65 next birthday. The resulting model therefore has 40 members and a payroll of £240,474. This basis forms model A.

Our older than average model was also created by truncating model B. This time we removed the younger members up to and including age 35 next birthday. The result model therefore has 30 members and a payroll of £202,291. This basis forms model C.

Scheme benefits

The normal retirement date is 65th birthday.

Pensionable earnings are full earnings in the last tax year.

Final pensionable earnings are the average of the last three pensionable earnings.

The pension provided for a member on retirement at normal retirement date is one-sixtieth of final pensionable earnings for each complete year of pensionable service, together with a proportionate amount for each additional month.

On retirement in ill-health the early retirement pension is the member's pension accrued to date of retirement.

There is a widow's pension of one-half of a member's pension, in the event of death after retirement.

All pensions in the course of payment increase at 3% p.a. compound.

Model B

Age next birthday	*Members*	*Current pensionable earnings*	*Average past service*
26	2	10,000	1
27	2	10,200	1
28	2	10,404	2
29	2	10,612	2
30	2	10,824	3
31	2	11,040	3
32	2	11,262	4
33	2	11,486	4
34	2	11,716	5
35	2	11,950	5
36	1	6,035	6
37	1	6,095	6
38	1	6,156	7
39	1	6,218	7
40	1	6,280	8
41	1	6,343	8
42	1	6,406	9
43	1	6,470	9
44	1	6,535	10
45	1	6,600	10
46	1	6,633	11
47	1	6,666	11
48	1	6,699	12
49	1	6,733	12
50	1	6,767	13
51	1	6,800	13
52	1	6,834	13
53	1	6,869	14
54	1	6,903	14
55	1	6,938	14
56	1	6,972	15
57	1	7,007	15
58	1	7,042	15
59	1	7,077	16
60	1	7,113	16
61	1	7,148	16
62	1	7,184	17
63	1	7,220	17
64	1	7,256	17
65	1	7,292	17
Total	50	311,785	

APPENDIX B

Core actuarial assumptions

Rate of interest:	In possession and in deferment—9% p.a.
Earnings increases generally:	8% p.a.
Pension increases in payment:	3% p.a.
Mortality:	After retirement PA(90) unrated. Before retirement A67/70 age rating − 1. Spouse mortality Female PA(90) age rating − $\frac{1}{2}$.
Early retirement rates:	None assumed.
Withdrawals:	None assumed.
Proportions married:	90% at age 65 for males. Husbands are assumed to be 3 years older than their wives.
New entrants:	None assumed. Entry age method assumed entry age 26 next birthday.

Variations from the above assumptions used in this Paper are shown in the appropriate text.

APPENDIX C1

Method = Current unit

			WITHDRAWALS											
			NIL						SCALE1					
			INTEREST						INTEREST					
			7		9		11		7		9		11	
			EARNINGS		EARNINGS		EARNINGS		EARNINGS		EARNINGS		EARNINGS	
			4	6	6	8	8	10	4	6	6	8	8	10
PROJ	MODEL													
1	A	CONT RATE	5.53	6.35	3.80	4.31	2.71	3.04	5.16	5.95	3.60	4.10	2.59	2.91
	B	CONT RATE	9.51	11.16	7.87	9.06	6.71	7.61	9.22	10.85	7.72	8.90	6.62	7.51
	C	CONT RATE	13.44	15.89	11.53	13.35	10.04	11.40	13.31	15.74	11.44	13.22	9.98	11.33
20	MODEL													
	A	CONT RATE	10.96	13.60	8.99	10.92	7.51	8.96	11.49	14.37	9.72	11.91	8.34	10.06
	B	CONT RATE	12.03	14.67	10.11	12.03	8.63	10.06	12.98	15.89	11.29	13.46	9.88	11.59
	C	CONT RATE	17.04	20.89	15.11	18.05	13.43	15.73	17.31	21.16	15.47	18.45	13.86	16.22

APPENDIX C2

Method = Projected unit

			WITHDRAWALS											
			NIL						SCALE1					
			INTEREST						INTEREST					
			7		9		11		7		9		11	
			EARNINGS		EARNINGS		EARNINGS		EARNINGS		EARNINGS		EARNINGS	
			4	6	6	8	8	10	4	6	6	8	8	10
PROJ	MODEL													
1	A	CONT RATE	8.97	14.09	7.59	11.84	6.55	10.13	6.37	8.65	5.01	6.86	4.11	5.65
	B	CONT RATE	10.58	14.79	8.93	12.42	7.68	10.63	8.57	10.60	6.94	8.58	5.80	7.17
	C	CONT RATE	12.54	15.69	10.56	13.17	9.06	11.25	11.61	13.95	9.60	11.53	8.13	9.75
20	MODEL													
	A	CONT RATE	11.26	15.44	9.50	12.96	8.17	11.07	9.99	12.58	8.16	10.30	6.86	8.65
	B	CONT RATE	11.72	15.57	9.88	13.06	8.48	11.16	10.83	13.13	8.89	10.79	7.50	9.09
	C	CONT RATE	13.81	16.46	11.61	13.80	9.94	11.78	13.50	15.69	11.26	13.07	9.58	11.10

APPENDIX C3

Method = Attained age

		WITHDRAWALS											
		NIL						SCALE1					
		INTEREST						INTEREST					
		7		9		11		7		9		11	
		EARNINGS		EARNINGS		EARNINGS		EARNINGS		EARNINGS		EARNINGS	
		4	6	6	8	8	10	4	6	6	8	8	10
MODEL													
A	CONT RATE	12.19	16.06	10.28	13.47	8.82	11.51	10 52	13.19	8.63	10.83	7.27	9.13
B	CONT RATE	12.49	16.11	10.52	13.51	9.03	11.54	11 23	13.61	9.25	11.21	7.81	9.46
C	CONT RATE	14.04	16.59	11.80	13.90	10.10	11.87	13 65	15.80	11.38	13.16	9.69	11.19

APPENDIX C4

Method = New entrant (at age 25)

		WITHDRAWALS											
		NIL						SCALE1					
		INTEREST						INTEREST					
		7		9		11		7		9		11	
		EARNINGS		EARNINGS		EARNINGS		EARNINGS		EARNINGS		EARNINGS	
		4	6	6	8	8	10	4	6	6	8	8	10
MODEL													
A	CONT RATE	10.13	15.43	8.58	12.96	7.40	11.08	4.43	6.91	3.31	5.34	2.63	4.32
B	CONT RATE	10.13	15.43	8.58	12.96	7.40	11.08	4.43	6.91	3.31	5.34	2.63	4.32
C	CONT RATE	10.13	15.43	8.58	12.96	7.40	11.08	4.43	6.91	3.31	5.34	2.63	4.32

APPENDIX C5

Method = C.U. with new entrants (Type 1)

			WITHDRAWALS											
			NIL						SCALE1					
			INTEREST						INTEREST					
			7		9		11		7		9		11	
			EARNINGS		EARNINGS		EARNINGS		EARNINGS		EARNINGS		EARNINGS	
			4	6	6	8	8	10	4	6	6	8	8	10
PROJ	MODEL													
20	A	CONT RATE	10.59	13.07	8.67	10.46	7.22	8.57	11.01	13.65	9.29	11.28	7.96	9.51
	B	CONT RATE	10.62	12.79	8.81	10.34	7.45	8.58	11.48	13.85	9.86	11.63	8.60	9.06
	C	CONT RATE	13.59	16.26	11.83	13.79	10.39	11.89	14.39	17.25	12.73	14.89	11.33	13.01

APPENDIX C6

Method = C.U. with new entrants (Type 2)

			WITHDRAWALS											
			NIL						SCALE1					
			INTEREST						INTEREST					
			7		9		11		7		9		11	
			EARNINGS		EARNINGS		EARNINGS		EARNINGS		EARNINGS		EARNINGS	
			4	6	6	8	8	10	4	6	6	8	8	10
PROJ	MODEL													
20	A	CONT RATE	10.45	12.87	8.54	10.28	7.11	8.42	7.81	9.38	6.37	7.51	5.35	6.22
	B	CONT RATE	10.39	12.48	8.59	10.07	7.26	8.34	8.56	10.05	7.15	8.22	6.14	6.94
	C	CONT RATE	13.09	15.60	11.35	13.19	9.95	11.34	12.07	14.25	10.55	12.15	9.33	10.55

APPENDIX C7

Method = P.U. with new entrants (Type 1)

			WITHDRAWALS											
			NIL						SCALE1					
			INTEREST						INTEREST					
			7		9		11		7		9		11	
			EARNINGS		EARNINGS		EARNINGS		EARNINGS		EARNINGS		EARNINGS	
			4	6	6	8	8	10		6	6	8	8	10
PROJ	MODEL													
20	A	CONT RATE	11.07	15.32	9.34	12.86	8.03	10.99	9.62	12.06	7.84	9.85	6.58	8.27
	B	CONT RATE	11.04	15.21	9.31	12.77	8.00	10.92	9.73	11.80	7.94	9.62	6.66	8.07
	C	CONT RATE	12.21	15.66	10.28	13.14	8.82	11.23	11.47	13.32	9.48	10.99	8.03	9.28

APPENDIX C8

Method = P.U. with new entrants (Type 2)

			WITHDRAWALS											
			NIL						SCALE1					
			INTEREST						INTEREST					
			7		9		11		7		9		11	
			EARNINGS		EARNINGS		EARNINGS		EARNINGS		EARNINGS		EARNINGS	
			4	6	6	8	8	10	4	6	6	8	8	10
PROJ	MODEL													
20	A	CONT RATE	11.00	15.29	9.28	12.83	7.98	10.97	7.21	9.13	5.74	7.29	4.75	6.04
	B	CONT RATE	10.92	15.15	9.22	12.72	7.92	10.88	7.61	9.32	6.08	7.46	5.05	6.18
	C	CONT RATE	11.98	15.55	10.09	13.04	8.65	11.15	9.86	11.50	8.06	9.39	6.79	7.89

APPENDIX C9

Unstable schemes
Method = Current unit

			WITHDRAWALS											
			NIL						SCALE1					
			INTEREST						INTEREST					
			7		9		11		7		9		11	
			EARNINGS		EARNINGS		EARNINGS		EARNINGS		EARNINGS		EARNINGS	
			4	6	6	8	8	10	4	6	6	8	8	10
PROJ	MODEL													
1	X	CONT RATE	10.95	12.92	9.32	10.77	8.11	9.21	10.69	12.64	9.18	10.62	8.03	9.13
	Y	CONT RATE	11.36	13.42	9.65	11.16	8.36	9.50	11.13	13.17	9.53	11.02	8.29	9.42
	Z	CONT RATE	11.30	13.35	9.53	11.01	8.18	9.29	11.06	13.08	9.39	10.86	8.10	9.20
20	MODEL													
	X	CONT RATE	12.53	15.17	10.64	12.55	9.16	10.59	13.65	16.57	11.95	14.16	10.57	12.29
	Y	CONT RATE	13.18	15.90	11.30	13.28	9.82	11.30	14.45	17.49	12.76	15.07	11.35	13.16
	Z	CONT RATE	13.75	16.64	11.88	14.01	10.37	11.99	15.02	18.27	13.32	15.80	11.89	13.84

APPENDIX C10

Unstable schemes
Method = Projected unit

			WITHDRAWALS											
			NIL						SCALE1					
			INTEREST						INTEREST					
			7		9		11		7		9		11	
			EARNINGS		EARNINGS		EARNINGS		EARNINGS		EARNINGS		EARNINGS	
			4	6	6	8	8	10	4	6	6	8	8	10
PROJ	MODEL													
1	X	CONT RATE	11.11	15.06	9.37	12.64	8.05	10.81	9.29	11.25	7.56	9.15	6.34	7.67
	Y	CONT RATE	11.34	15.17	9.56	12.73	8.21	10.89	9.67	11.68	7.90	9.54	6.64	8.01
	Z	CONT RATE	11.37	15.19	9.59	12.75	8.24	10.90	9.79	11.90	8.02	9.74	6.75	8.19
20	MODEL													
	X	CONT RATE	11.93	15.63	10.05	13.11	8.63	11.21	11.19	13.37	9.20	11.00	7.77	9.27
	Y	CONT RATE	12.20	15.73	10.27	13.20	8.81	11.28	11.63	13.72	9.59	11.31	8.11	9.55
	Z	CONT RATE	12.41	15.83	10.45	13.28	8.96	11.34	11.95	14.06	9.88	11.61	8.36	9.82

APPENDIX C11

Unstable schemes
Method = Attained age

		WITHDRAWALS											
		NIL						SCALE1					
		INTEREST						INTEREST					
		7		9		11		7		9		11	
		EARNINGS		EARNINGS		EARNINGS		EARNINGS		EARNINGS		EARNINGS	
		4	6	6	8	8	10	4	6	6	8	8	10
MODEL													
X	CONT RATE	12.64	16.15	10.64	13.54	9.13	11.57	11.54	13.80	9.52	11.38	8.04	9.61
Y	CONT RATE	12.82	16.20	10.79	13.58	9.25	11.60	11.92	14.09	9.85	11.63	8.33	9.83
Z	CONT RATE	12.96	16.25	10.90	13.62	9.35	11.64	12.20	14.36	10.09	11.87	8.55	10.05

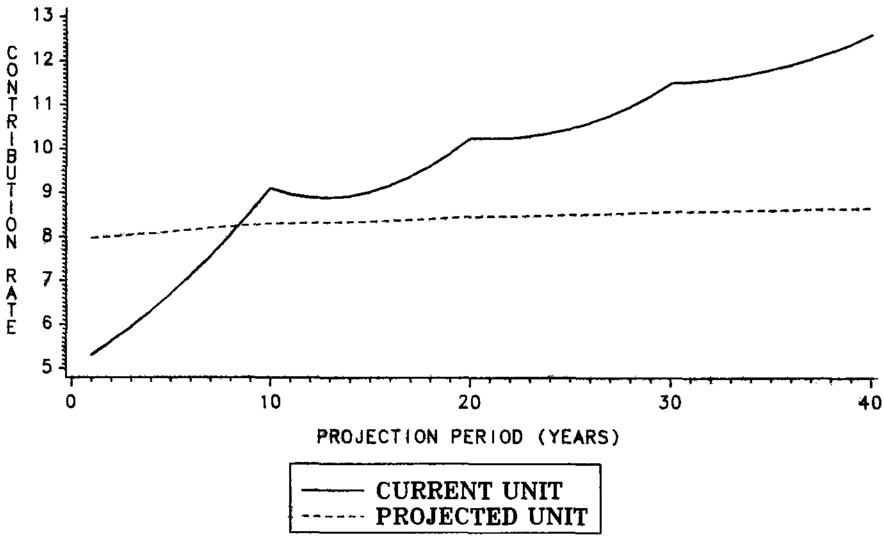
APPENDIX C12

Unstable schemes
Method = New entrant (at age 25)

		WITHDRAWALS											
		NIL						SCALE1					
		INTEREST						INTEREST					
		7		9		11		7		9		11	
		EARNINGS		EARNINGS		EARNINGS		EARNINGS		EARNINGS		EARNINGS	
		4	6	6	8	8	10	4	6	6	8	8	10
MODEL													
X	CONT RATE	10.13	15.43	8.58	12.96	7.40	11.08	4.43	6.91	3.31	5.34	2.63	4.32
Y	CONT RATE	10.13	15.43	8.58	12.96	7.40	11.08	4.43	6.91	3.31	5.34	2.63	4.32
Z	CONT RATE	10.13	15.43	8.58	12.96	7.40	11.08	4.43	6.91	3.31	5.34	2.63	4.32

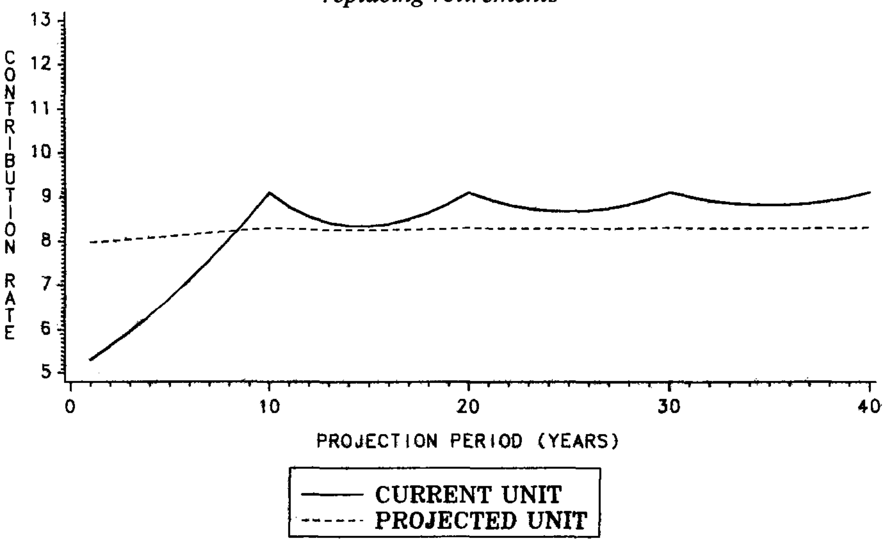
APPENDIX D

Plot of contribution rates over varying projection periods



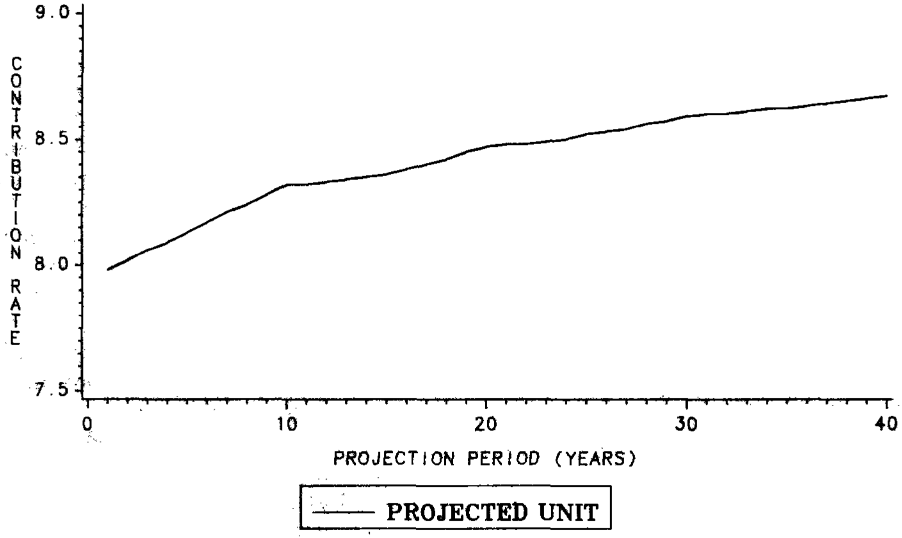
APPENDIX D

Plot of contribution rates over varying projection periods with new entrants replacing retirements



APPENDIX D

Plot of contribution rates over varying projection periods



APPENDIX D

Plot of contribution rates over varying projection periods with new entrants replacing retirements