The Dynamics of Annuity Pricing, Credit, Mortality Risk & Capital Management for UK Life Offices

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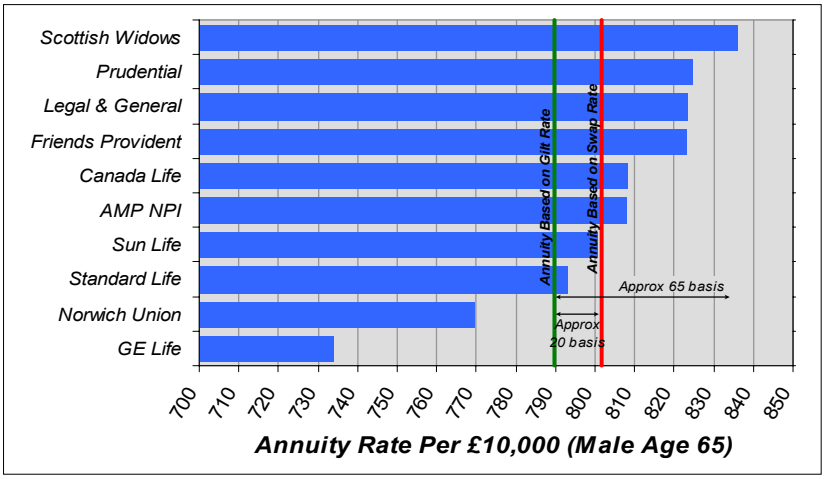
Finance & Investment Conference

Overview

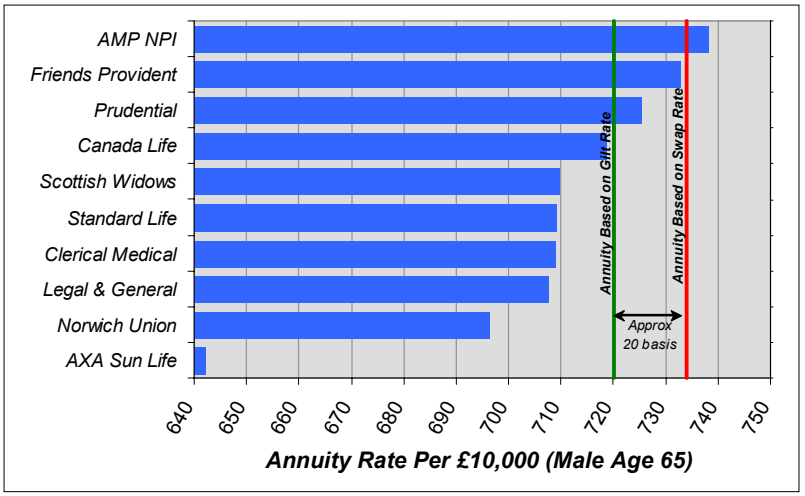
- Some background on the annuity market.
- What policy questions do we want to answer?
- A stochastic model of annuity business:
  - Interest rates
  - Credit behaviour & credit spreads
  - Mortality uncertainty
- A basic case study.

UK Annuity Rates
Analysis at March 2002



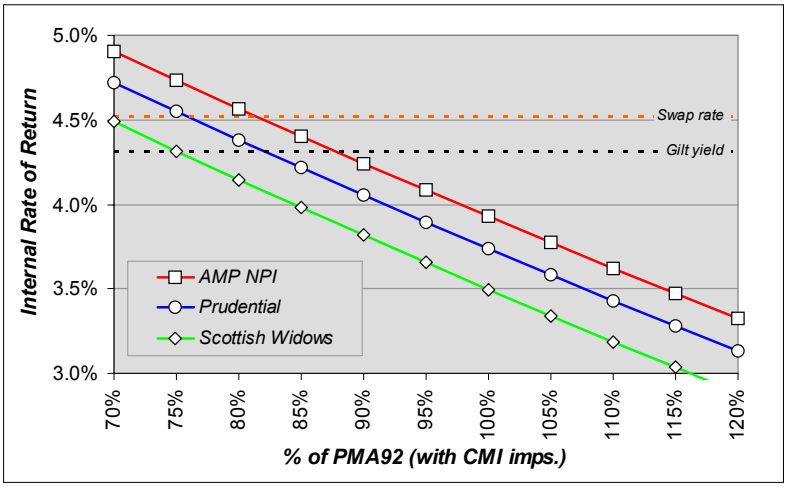
Source: Annuity Direct, Man aged 65, Single Life, No guarantee, level annuity. Assumes 5% expense loading, and 90% PxA92 with CMI 17 improvements.

UK Annuity Rates
Analysis at May 2003



Source: Annuity Direct, Man aged 65, Single Life, No guarantee, level annuity. Assumes 5% expense loading, and 80% PxA92 with CMI 17 improvements.

Implied rates of return vs Mortality assumption



Source: Annuity Direct, Man aged 65, Single Life, No guarantee, level annuity. Assumes 5% expense loading, Annuity rates at 25th May 2003.

Background

- Highly competitive annuity market.
- Increasing exposure among issuers to credit-risky bonds.
- Heightened awareness of mortality risk due to past errors in forecasting.
- Scarcity of capital.
- Regulator's focus on risk management.

Policy Issues #1

- How much capital is required to support annuity business?
  - Statutory basis?
  - "Economic" basis ~ Probability of capital exhaustion limited below some threshold level.
- What ROC is achieved?
- How do credit risk & mortality risk fit together?
  - How can credit & mortality hedges be evaluated?
- How much of the bond spread is lost through (expected) default?

Policy Issues #2

Portfolio management:

- How much diversification is required within the bond portfolio?
- Is it appropriate to re-balance to maintain a minimum rating level in the bond portfolio?
- How likely are we to experience default rates above a specified level?

A Model For Annuity Business

Aim to combine models for:

- Liabilities (& Mortality uncertainty)
- Asset model
  - Term structure behaviour
  - Credit behaviour (issuer credit states & credit spreads)
- Valuation rules

Model choice

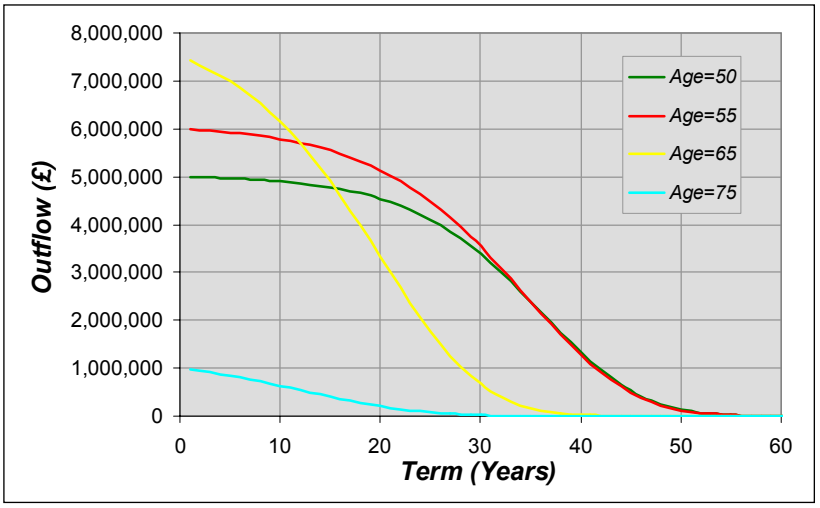
- What sort of model? Closed-form? Historic simulation? *Monte-Carlo* Simulation?
- There is an array of choices for each component of the model
  - Parsimony
  - Transparency
  - Evolution

Model Components: Liabilities

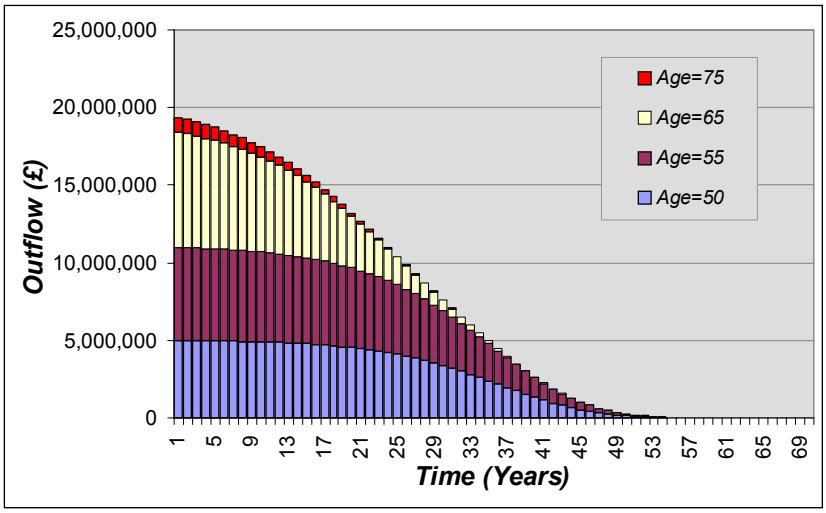
- Analysis of 4 "cohorts" :

Cohort #	Age	Sex (0=Male, 1=Female)	Annuity Payment Per Life	Start Date (Deferral)	Guarantee Period	Type	Smoker Non-Smoker	Number of Lives in Cohort
1	50	0	50	0	0	0	FALSE	100000
2	55	1	60	0	0	0	FALSE	100000
3	65	0	75	0	0	0	FALSE	100000
4	75	1	10	0	0	0	FALSE	100000

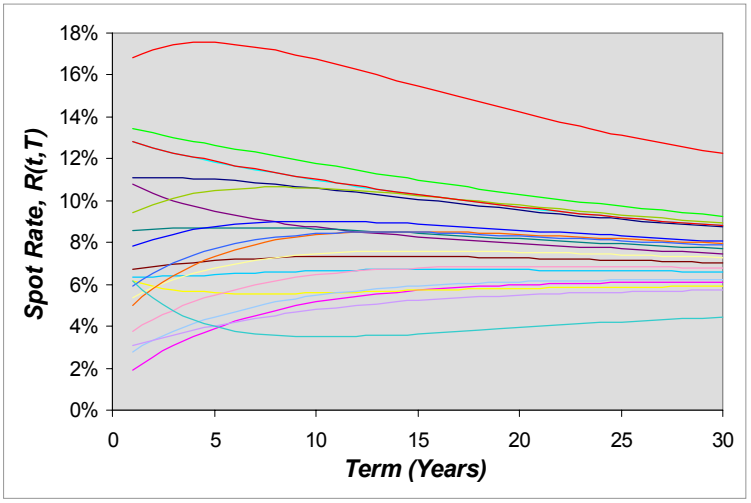
Model Components: Liabilities



Model Components: Liabilities



Model Components:
Default-Free Term Structure Model



Note: α_1=0.250, α_2=0.075, σ_1=0.01, σ_2=0.02, μ=0.096, g=-0.151

Model Components:
Credit Behaviour & Credit Spreads

We have used a modified version of the (CreditMetrics-style) framework of *Jarrow-Lando-Turnbull*.

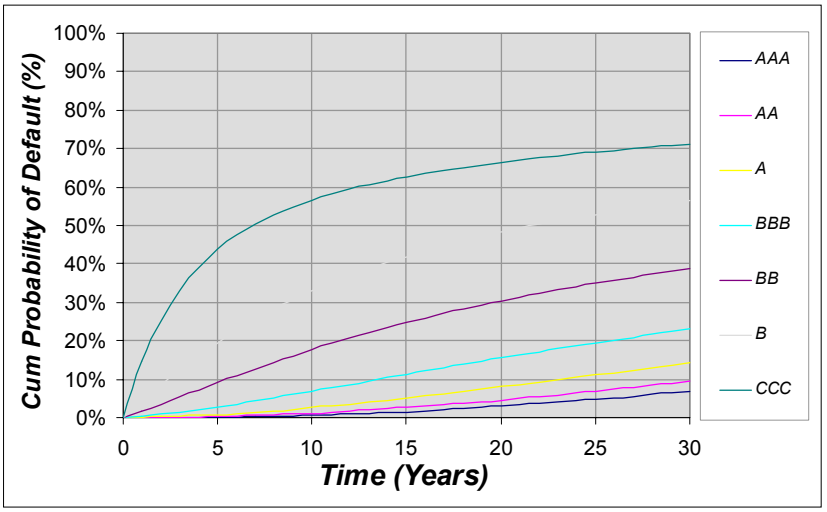
- Provides a method to describe rating changes and defaults.
- Provides a potential method for modelling the behaviour of credit spreads.

A sample 1-year credit transition matrix

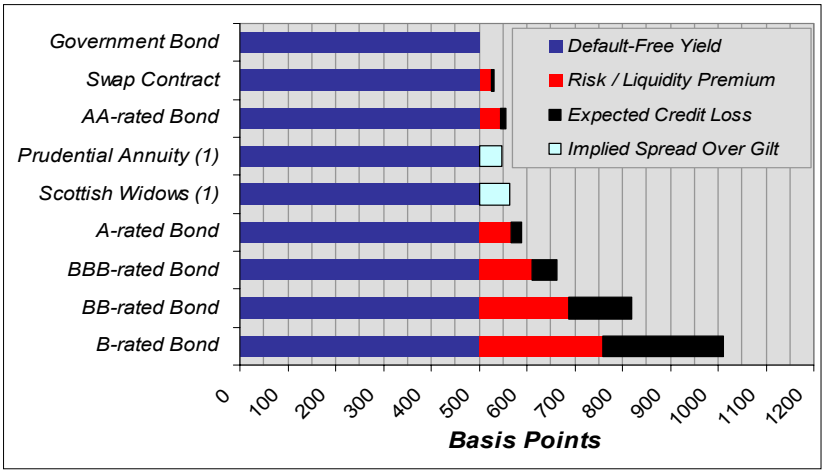
1-Year Credit & Default Transition Matrix

Rating at Start of Period	Rating at End of Period: AAA	AA	A	BBB	BB	B	CCC	Default
AAA	88.0%	10.6%	0.9%	0.2%	0.3%	-	-	-
AA	3.9%	88.0%	6.7%	0.9%	0.3%	0.3%	-	-
A	0.2%	5.4%	87.8%	5.3%	0.8%	0.4%	-	0.1%
BBB	0.1%	0.6%	9.6%	82.5%	5.3%	1.3%	0.2%	0.4%
BB	0.1%	0.3%	1.1%	9.7%	79.2%	7.2%	0.9%	1.7%
B	-	0.2%	0.4%	0.8%	6.3%	85.4%	2.7%	4.2%
CCC	-	-	1.5%	1.5%	2.6%	9.6%	70.4%	14.5%
Default	-	-	-	-	-	-	-	100.0%

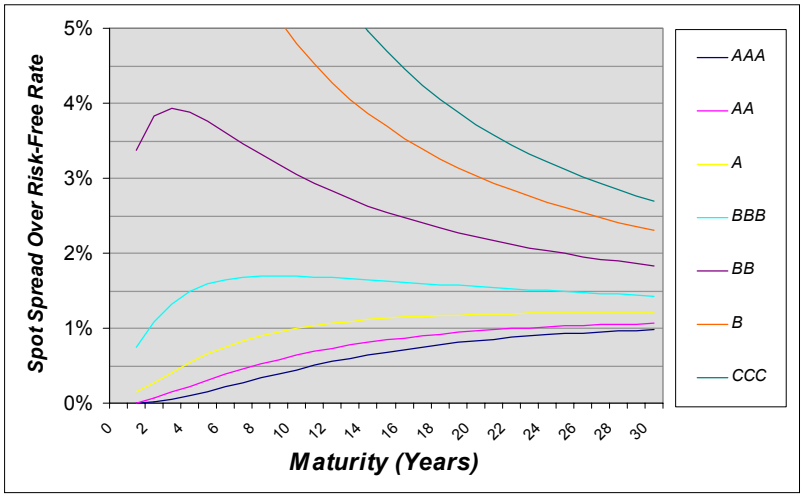
Implied default rates



Bond Yields
Analysis at 1st March 2002

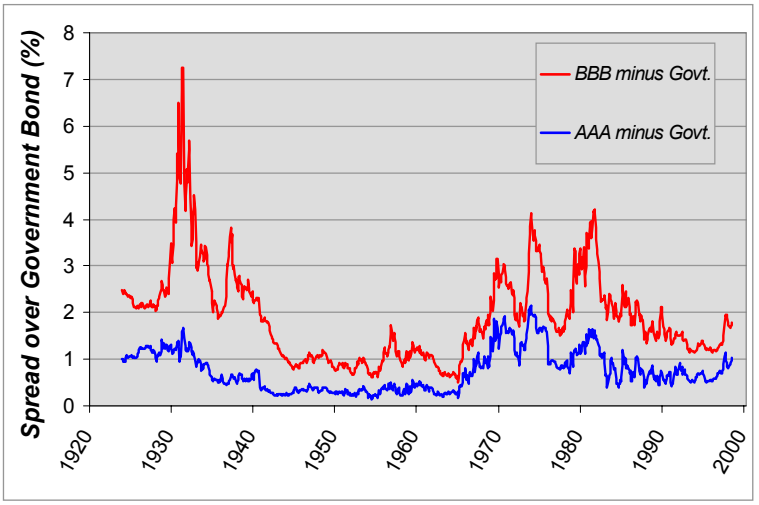


Sample credit spot spreads



Note: B&H best estimate transition matrix, δ=0.333, π=3

Variation in US Credit Spreads



Model Components:
Mortality Model

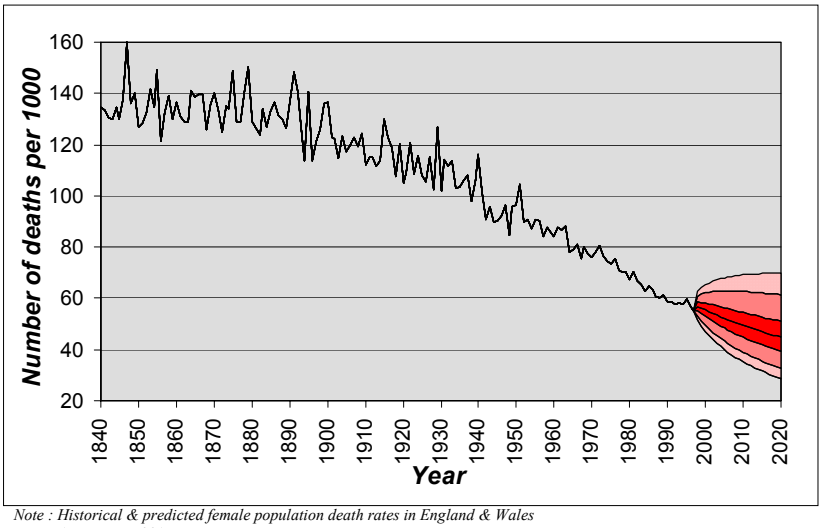
- CMI Tables estimate
- Stochastic Mortality Model
  - Generates random fluctuations around this central estimate.

$$q_x(t) = q_x(0,t)\exp[\, Y_t - \sigma_Y^2/2 - t\sigma_X^2/2 \,]$$

$$Y_t = X_t + \sigma_Y Z_{Yt}$$

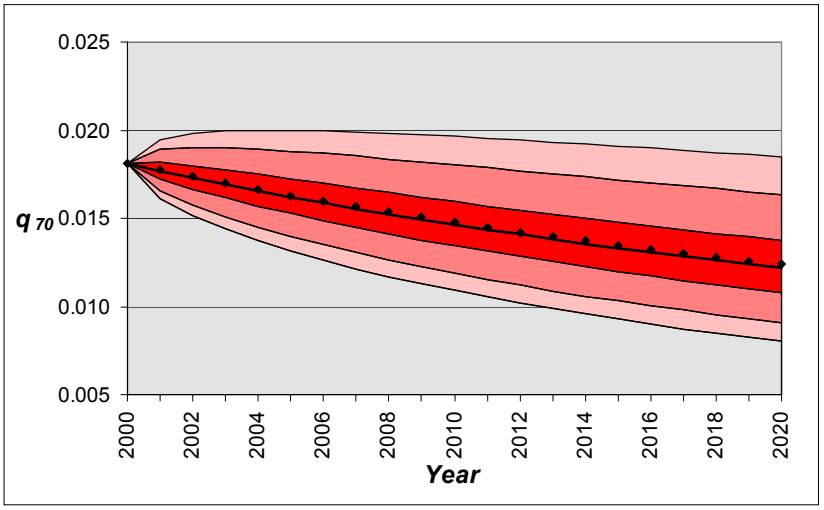
$$X_t = X_{t-1} + \sigma_X Z_{Xt}$$

Stochastic Mortality
The Basic Idea



Note : Historical & predicted female population death rates in England & Wales (75 – 84 yr olds)

Stochastic Mortality
Calibration "A" / 70-year-old Male



Note: $\sigma_X = 4\%$ and $\sigma_Y = 1\%$

Projected Payments Under Stochastic Mortality

Note:Cohort size = 100,000 lives

Model Components: Valuation Rules

"Economic" analysis

- Value assets and liabilities at their market values
- What is the MV of the liabilities? We assume here that they are default-free, i.e. discount at gilt yields.

Model Components: Valuation Rules

Statutory basis

- Different practice among UK life offices
- Typical approach:
  - Calc IRR on bond portfolio cash flows (promised cashflows)
  - Make deduction for 'prudent' default loss (typically 10 -25 bp)
  - Apply 97.5% 'haircut'
- Generally results in statutory yield > gilt yield
  - Does this reflect liquidity or is default allowance inadequate?
- Also results in statutory solvency position less volatile than economic one
  - Credit spread volatility feeds through to liabilities

Model Components: Performance Criteria

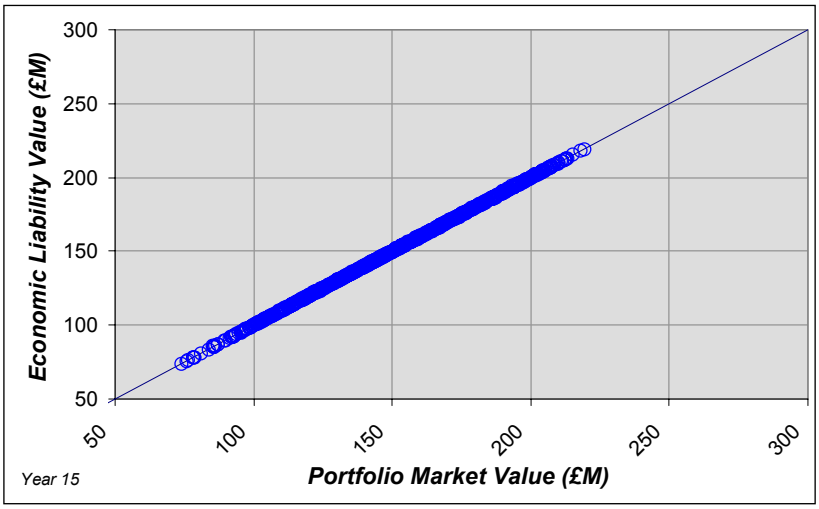
- *Monte Carlo* simulation exercises generate a vast amount of data.
- What criteria do we want to use to compare different management strategies?
  - Probability of insolvency
    - Statutory
    - Economic
  - Fix insolvency probability & estimate required capital support

Case #1

- Expected cash outflows matched with default-free discount (zero-coupon) bonds.
- Single issue per year of annuity outflow.
- Mortality is known and deterministic according to the CMI tables. (Stochastic mortality model is switched OFF)

$\Rightarrow$All cash flows should be perfectly matched.

Case #1
Matched Govt Bonds; SMM=off

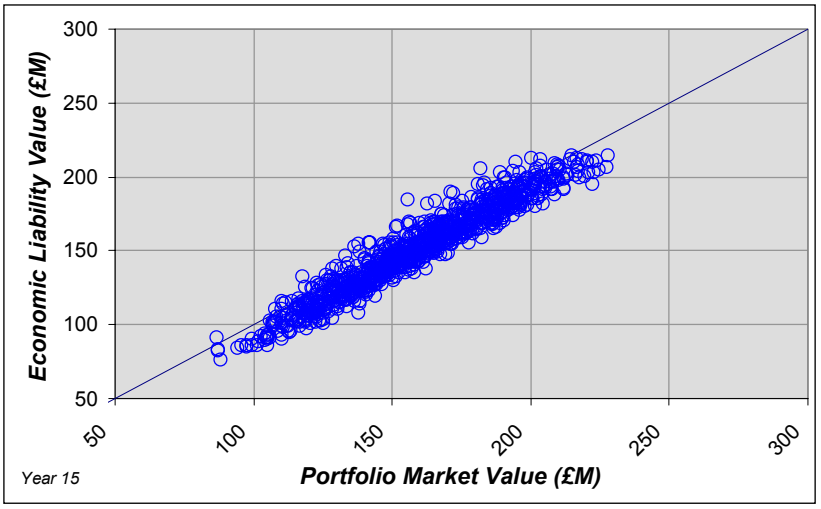


Case #2

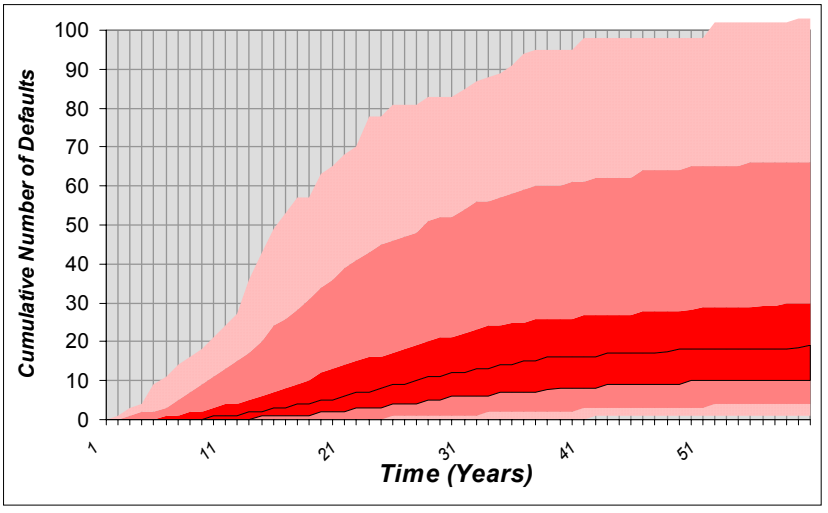
- Portfolio invested in (notional) credit-risky (AAA-rated) bonds. Same portfolio value as for case #1.
- Diversified (notional) portfolio of 500 issuers.
- Mortality is known and deterministic according to the CMI tables. (Stochastic mortality model is switched OFF)

⇒Potential mis-matching due to credit risks.

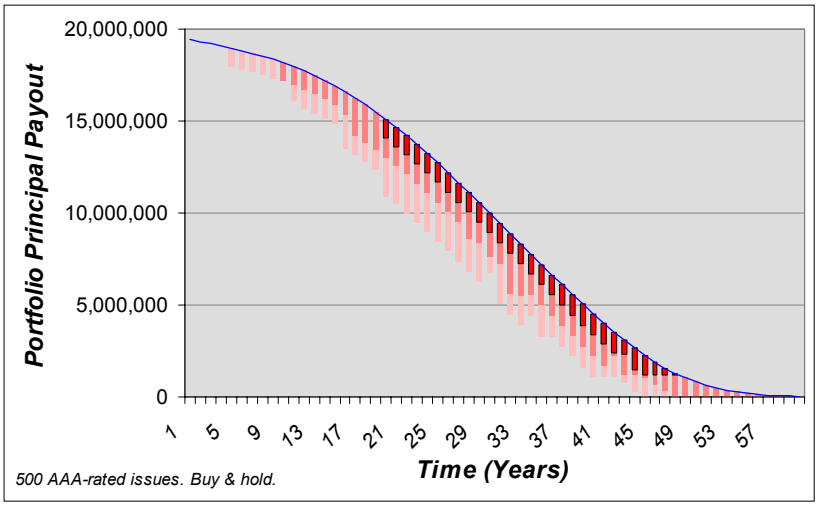
Case #2
AAA-rated Bonds; SMM=off



Simulated default experience
(500 AAA-rated issuers)

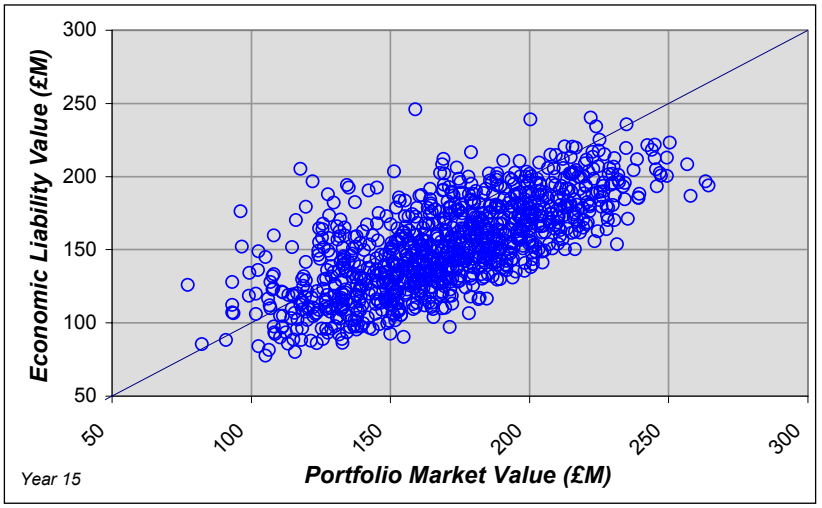


Distribution of year-by-year portfolio delivered cash flows



Case #8

Year 15; A-rated Bonds; SMM=on; No Rebalance



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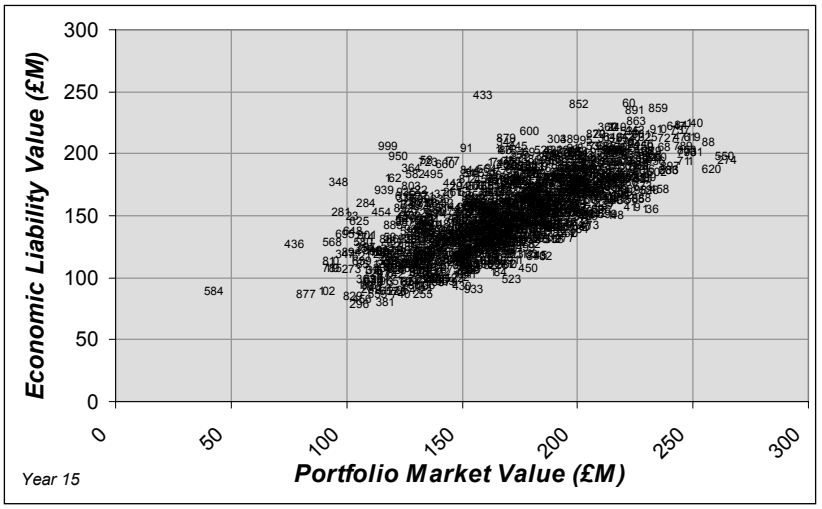

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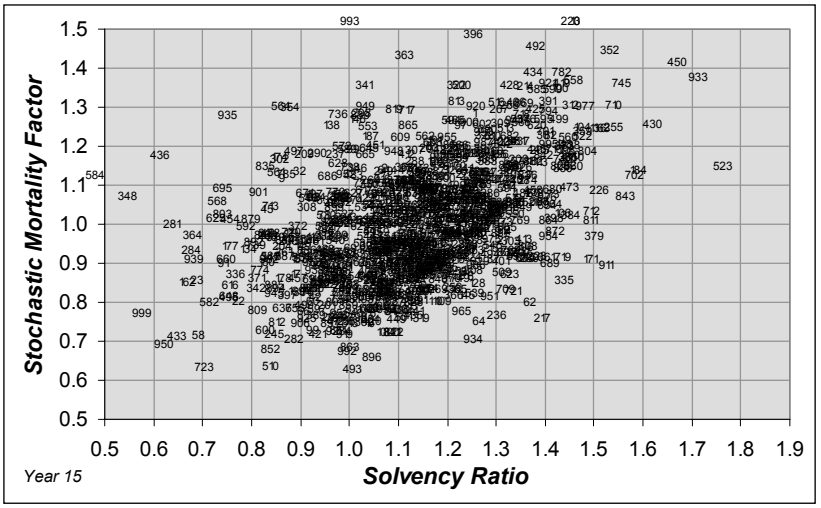

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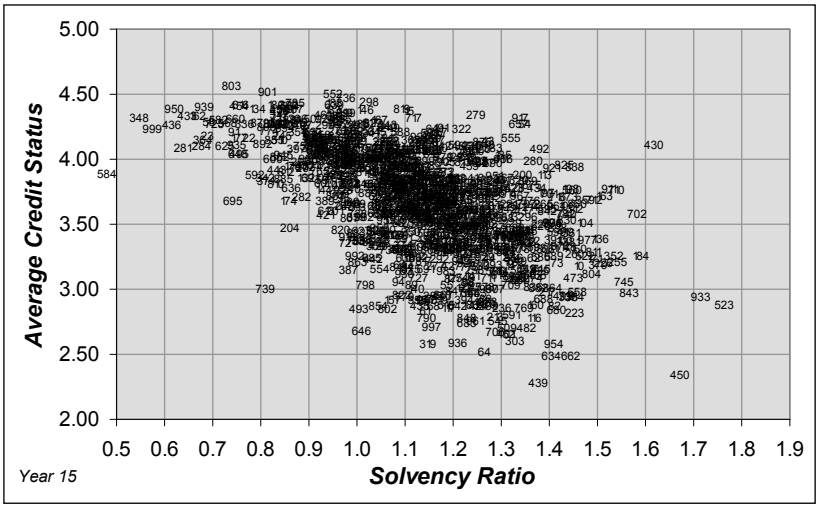

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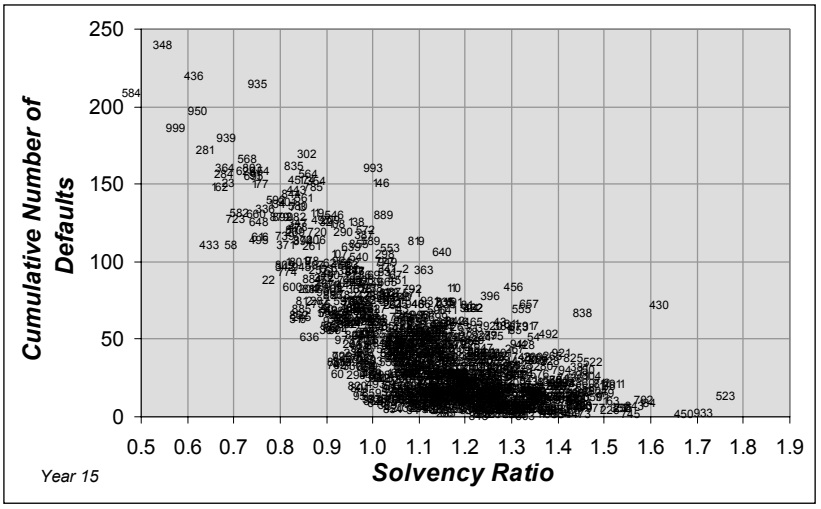
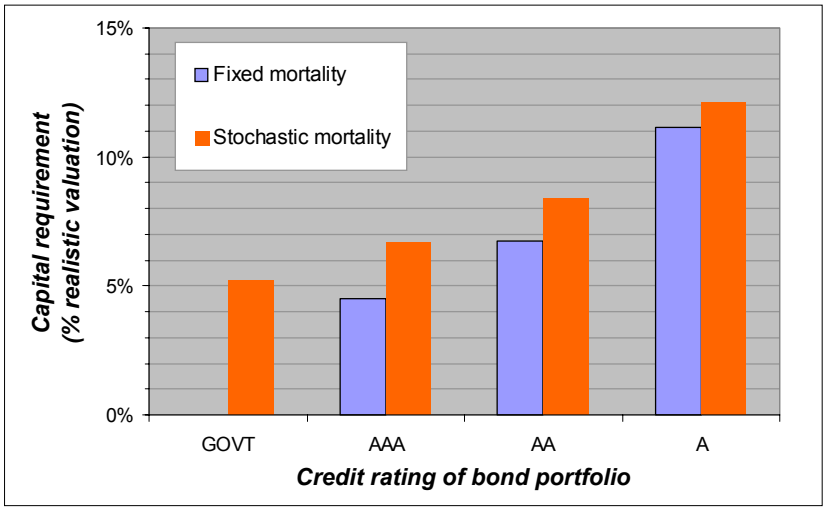


Assessing risk-based capital

- What do we mean by *risk-based capital*?
- The capital required to fund the liabilities with a given level of confidence.
- Two components:
  - Fair/realistic value
  - Mis-match reserve
- Two approaches:
  - 'run-off' approach
  - VAR-style percentile calculation

Capital requirements to support various credit strategies



Summary

- Annuity providers are operating in an environment of intense competition.
- Liabilities are increasingly supported by credit-risky asset portfolios.
- Profitability hinges on a number of uncertain factors:
  - The future realised credit experience
  - The future realised mortality experience
  - The level of anticipated cash flow matching
  - Interest rate variability
- Stochastic models enable providers to investigate & quantify these factors providing a path towards:
  - A better understanding of total portfolio exposure
  - Better risk management & capital allocation

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