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Aggregation of credit risk capital and market risk capital assessments

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Background

Aggregation of credit risk capital and market risk capital assessments

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Background

- Growing use of 1-year VaR as a measure of insurance firms' capital requirements
- Capital requirement should allow for joint impact of all risks on the firm's year balance sheet at 1 year:
  - Market risk
  - Credit (default and migration) risk
  - + mortality risk, lapse risk, operational risk etc
- Ideal approach would be to create *joint scenarios* for all risk factors, calculate total impact on balance sheet and resulting overall capital measure

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Background

- However, in practice, scenarios for different risk types are often generated using separate, specialised, models and software systems
  - e.g. specialised credit models typically model credit risk at a far *more granular* level than broad (market + credit) risk scenario generators
  - Integration of such models at a ‘fundamental’ level may be complicated due to differences in model structure

- How to aggregate output from different models?

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Aggregation methods

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Aggregating output from separate models Method 1: Formula

- Measure standalone capital requirements and aggregate using a formula e.g.:

$$EC_{total} = \sqrt{EC_{market}^2 + EC_{credit}^2 + 2\rho_{market,credit} EC_{market} EC_{credit}}$$

- Formula derived under simplifying assumptions (normality, linearity)
  - Under more realistic assumptions, how do we choose the correlation input into the formula?

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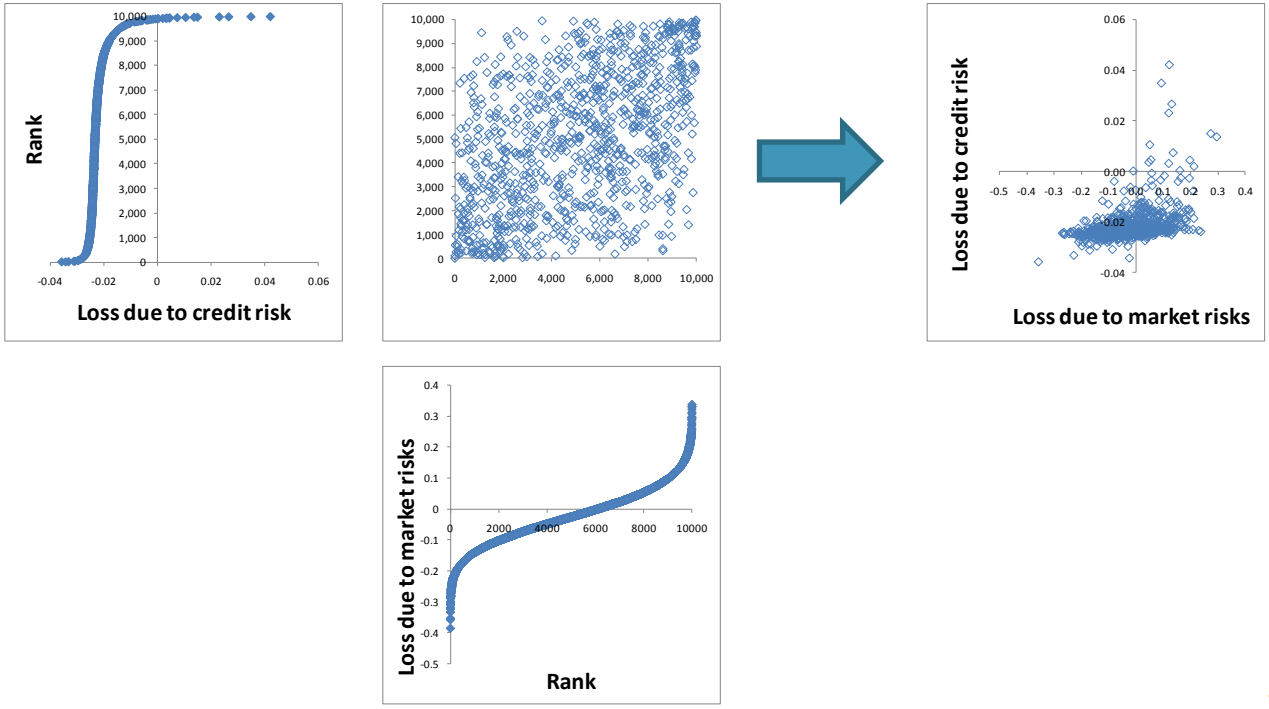
Aggregating output from separate models Method 2: Loss copula

- Combine scenarios for standalone losses using a copula



- Create total loss as sum of standalone losses

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Aggregating output from separate models Method 2: Loss copula

- More general than formula approach
  - But still assumes standalone losses add to give total loss
- Easy to implement - simply involves *reordering* standalone loss scenarios using ranks sampled from specified copula
- But still have to choose copula and calibrate
  - Copula specifies dependency between portfolio losses and is therefore portfolio specific

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Aggregating output from separate models Method 3: Reordering via bridging factors

- More generally, consider reordering scenarios for underlying risk factors rather than losses
  - Dependency specified at risk factor level; Not portfolio specific
- Reordering specified using ‘bridging factors’

- See also J. C. Garcia Cespedes, J. A. de Juan Herrero, D. Rosen & D. Saunders: *Effective Modeling of Wrong Way Risk, Counterparty Credit Risk Capital and Alpha in Basel II*

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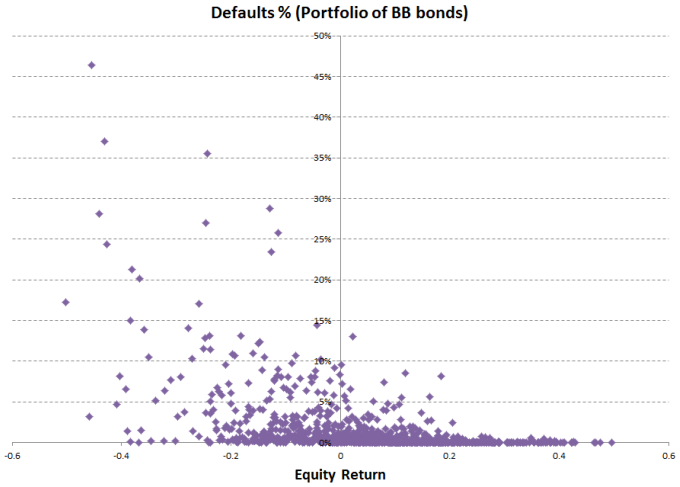
Aggregating output from separate models
Method 3: Reordering via bridging factors

- Retain original scenarios for individual risk factors
  - Marginal distribution assumptions unchanged
- Change order of risk factors relative to each other
  - Changes dependency between such risk factors
- How to choose new order?
  - Proposed approach uses common bridging factors to define the reordering



Bridging factor

MARKET SCENARIOS

Sim	Market Factor 1	MF2 / CF2	…
1	0.20	0.32	…
2	0.23	0.15	…
3	0.18	0.24	…
4	0.15	0.12	…
…	…	…	…

CREDIT SCENARIOS

Sim	Credit Factor 1	CF2 / MF2	…
1	0.31	0.11	…
2	0.29	0.16	…
3	0.26	0.17	…
4	0.20	0.23	…
…	…	…	…

COMBINED SCENARIOS

Sim	Market Factor 1	Market Factor 2	Credit Factor 1	…
1	0.20	0.32	0.20	…
2	0.23	0.15	0.29	…
3	0.18	0.24	0.26	…
4	0.15	0.12	0.31	…
…	…	…	…	…

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Extending the credit risk model to include bridging factors

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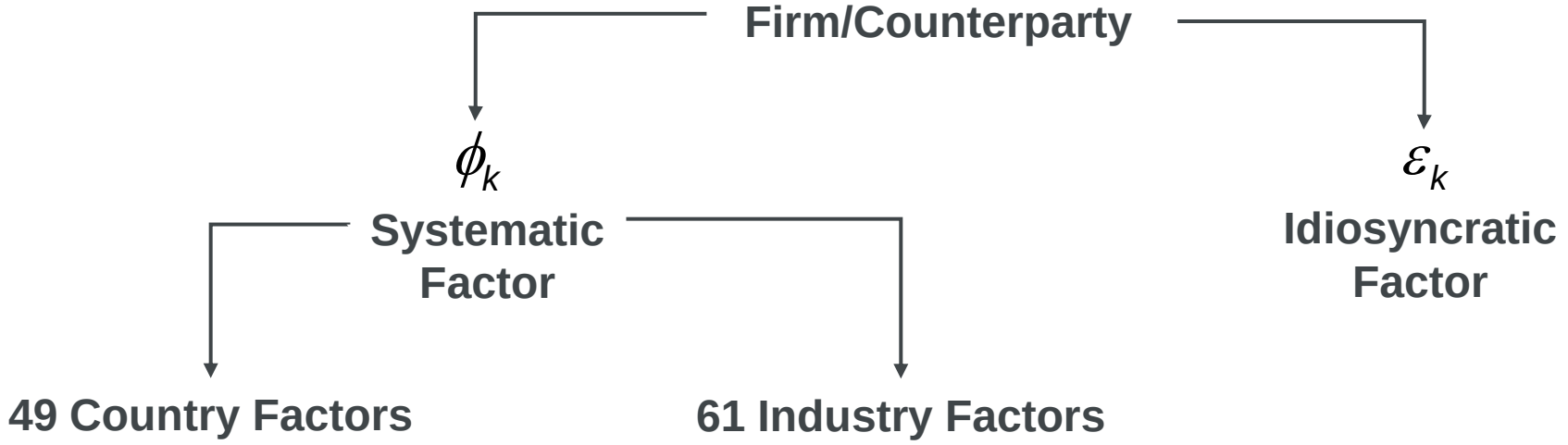
Credit model structure

- We consider a granular credit model where credit quality of individual firms depends on systematic country and industry *factors*

$$r_k = \sqrt{RSQ_k}\,\phi_k + \sqrt{1-RSQ_k}\,\varepsilon_k$$



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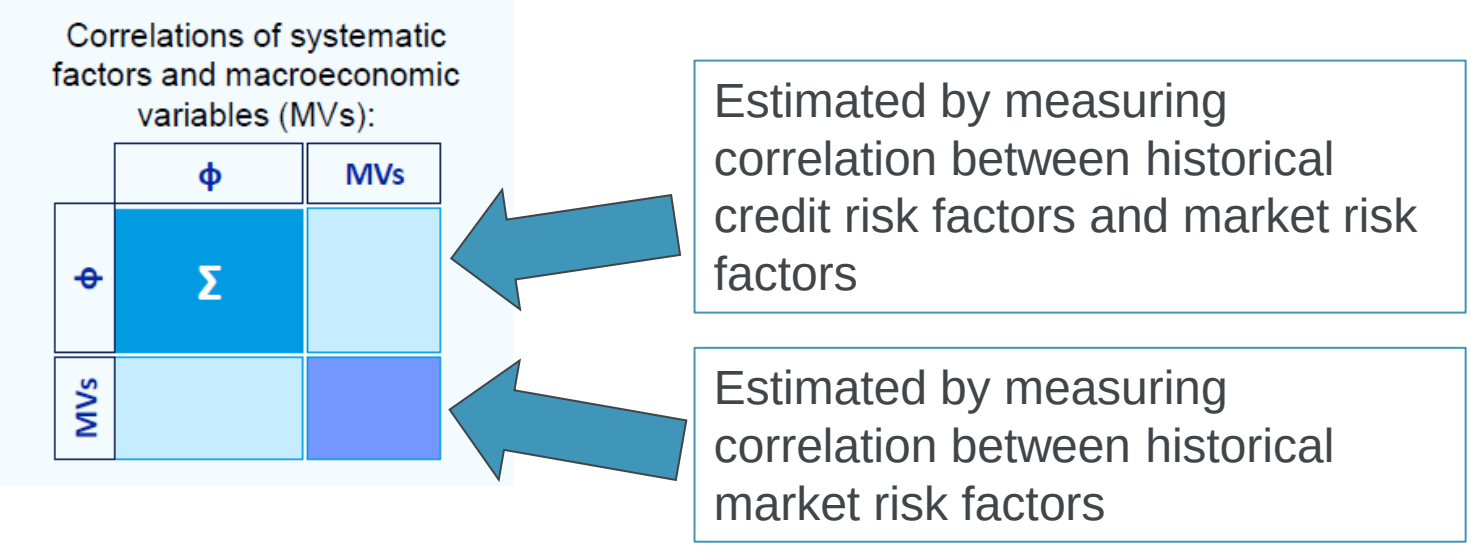
Expansion of the credit model

- Expand the credit model to include additional random variables representing market risk factors:



  - See *Modeling Credit Correlations Using Macroeconomic Variables*, Moody's Analytics Modeling Methodology

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Example:
Single bridging factor

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Example

- Aim to calculate economic capital requirements (1-year VaR) for an asset portfolio
  - Corporate Bonds (50%); Equities (30%); Real Estate (20%)
- 10,000 scenarios generated for market risk factors at one year horizon:
  - Equity returns, real estate returns, risk-free yield curves, corporate credit spread curves
- *Independently* 10,000 scenarios generated for credit risk factors at one year horizon:
  - Credit ratings of each issuer in the corporate bond portfolio

Example reordering scheme: Align credit and market scenarios using single bridging factor

- Expand credit risk model to include a single bridging factor: an equity index representing the firms underlying the corporate bond portfolio

- Reorder market risk scenarios so that ranks of equity index are aligned between market and credit risk scenario sets

Example reordering scheme: Align credit and market scenarios using single bridging factor

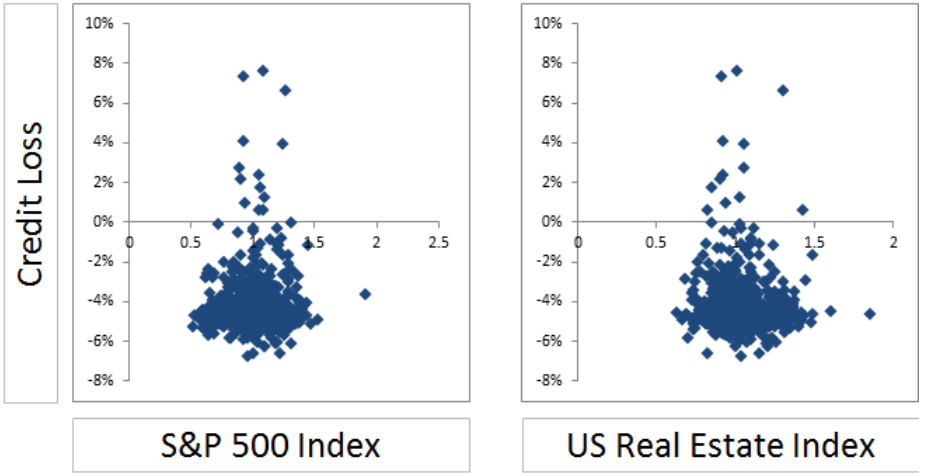
- Key properties of reordered scenario set:
  1. No change to marginal (market and credit) scenarios
  2. No change to joint market risk scenarios
  3. Changes credit-equity dependency to agree with that produced by expanded credit model
  4. Other credit-market risk dependencies *induced*

Dependency: reordered scenario sets

- Original:

	S&P500	Real Estate	US Govt 10yr rate	US A 10yr spread
Credit loss	-0.01	0.00	0.00	0.02
S&P500		0.31	-0.13	-0.31
US Real Estate			-0.09	-0.15
US Govt 10yr rate				-0.19

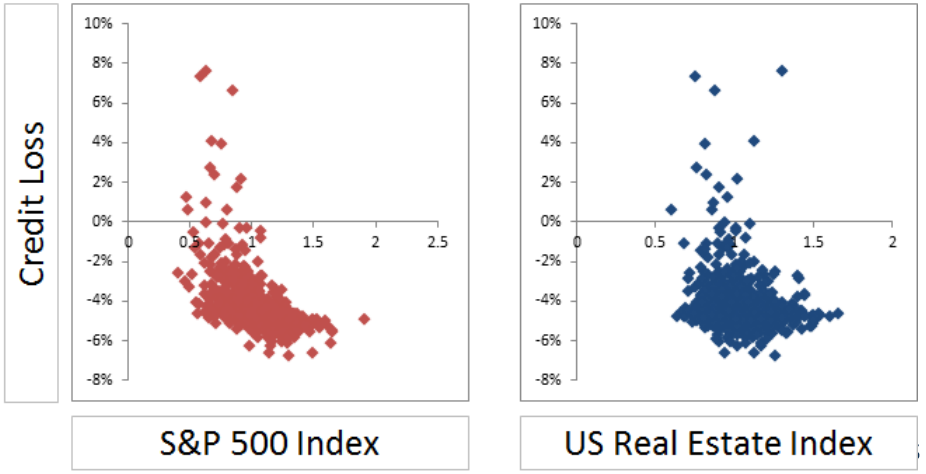
Dependency controlled within ESG
Dependency induced



- Reordered:

	S&P500	Real Estate	US Govt 10yr rate	US A 10yr spread
Credit loss	-0.56	-0.17	0.07	0.18
S&P500		0.31	-0.13	-0.31
US Real Estate			-0.09	-0.15
US Govt 10yr rate				-0.19

Dependency controlled within credit risk model
Dependency controlled within ESG
Dependency induced

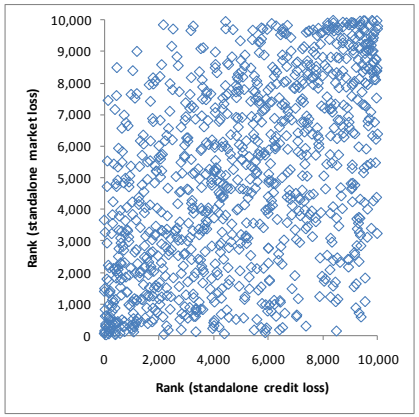


Economic Capital

- Use joint scenarios to calculate Economic Capital (defined as 99.5% VaR of losses on the asset portfolio, less expected loss)

	Economic Capital
Credit risk	0.0283
Market risk	0.2523
Total	**0.2592**

- For comparison:
  - Rank correlation between standalone losses = 0.39



  - 'Implied' correlation = 0.16

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Example:
Multiple bridging factors

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Reordering using multiple common bridging factors

- So far have considered aligning credit and market scenarios using single bridging factor: an equity index
- Suppose our asset portfolio contains two corporate bond sub-portfolios (US and UK) and two equity sub-portfolios (US and UK)
  - 30% US bonds, 30% UK bonds, 20% US equities, 20% UK equities
- Which equity index to choose as bridging factor?

Reordering using multiple common bridging factors

- For example, suppose we choose the S&P 500:

	Credit loss (UK)	S&P500	FTSE100	US 10yr Rate	UK 10yr Rate
Credit loss (US)	0.66	-0.73	-0.50	0.09	0.09
Credit loss (UK)		-0.47	-0.33	0.06	0.07
S&P500			0.69	-0.13	-0.13
FTSE100				-0.14	-0.14
US 10yr Rate					0.64

Dependency controlled within expanded credit risk model

Dependency controlled within ESG

Dependency induced

  - In this case, the induced correlation between the FTSE 100 and losses on the US bond portfolio is larger than the induced correlation between the FTSE 100 and losses on the UK bond portfolio

Reordering using multiple common risk factors

- Reordering technique extends to multiple bridging factors:

1. Expand and calibrate the credit model to include multiple bridging factors

   e.g. S&P 500 and FTSE 100

2. Group market risk factors according to which bridging factor they are attached to
   - In this case, align all risk factors with the S&P 500, except the FTSE 100
3. Separately reorder each market risk group

Reordering using multiple common risk factors

- Key properties of reordered scenario set:
  1. No change to marginal (market and credit) scenarios
  2. No change to joint market risk scenarios *within* groups
  3. Changes dependency between bridging factors to agree with that produced by expanded credit model
  4. Changes dependency between credit and all chosen common risk factors to agree with that produced by expanded credit model
  5. Other dependencies induced

Dependency: reordered scenario sets

- Single bridging factor (S&P 500):

	Credit loss (UK)	S&P500	FTSE100	US 10yr Rate	UK 10yr Rate	US A 10yr Spread	UK A 10yr Spread
Credit loss (US)	0.66	-0.73	-0.50	0.09	0.09	0.23	0.23
Credit loss (UK)		-0.47	-0.33	0.06	0.07	0.16	0.16
S&P500			0.69	-0.13	-0.13	-0.31	-0.31
FTSE100				-0.14	-0.14	-0.31	-0.31
US 10yr Rate					0.64	-0.19	0.00
UK 10yr Rate						0.01	-0.17
US A 10yr Spread							0.77

Dependency controlled within expanded credit risk model
Dependency controlled within ESG
Dependency induced

- Two bridging factors (S&P 500 and FTSE 100):

	Credit loss (UK)	S&P500	FTSE100	US 10yr Rate	UK 10yr Rate	US A 10yr Spread	UK A 10yr Spread
Credit loss (US)	0.66	-0.73	-0.67	0.09	0.09	0.23	0.23
Credit loss (UK)		-0.47	-0.67	0.06	0.07	0.16	0.16
S&P500			0.69	-0.13	-0.13	-0.31	-0.31
FTSE100				-0.09	-0.09	-0.23	-0.22
US 10yr Rate					0.64	-0.19	0.00
UK 10yr Rate						0.01	-0.17
US A 10yr Spread							0.77

Dependency controlled within expanded credit risk model
Dependency controlled within ESG
Dependency induced

Comparison of correlations using different reordering schemes

- Independent

	Credit loss (UK)	S&P500	FTSE100	US 10 Year Rate	UK 10 Year Rate	US A 10-year credit spread	UK A 10-year credit spread
Credit loss (US)	0.66	0.00	0.00	0.01	0.00	0.00	-0.01
Credit loss (UK)		0.00	0.00	-0.01	0.00	0.00	0.00
S&P500			0.70	-0.13	-0.13	-0.31	-0.31
FTSE100				-0.15	-0.15	-0.31	-0.31
US 10 Year Rate					0.64	-0.19	0.00
UK 10 Year Rate						0.01	-0.17
US A 10-year credit spread							0.77

- US equities

	Credit loss (UK)	S&P500	FTSE100	US 10 Year Rate	UK 10 Year Rate	US A 10-year credit spread	UK A 10-year credit spread
Credit loss (US)	0.66	-0.73	-0.51	0.11	0.10	0.23	0.23
Credit loss (UK)		-0.47	-0.33	0.08	0.07	0.16	0.16
S&P500			0.70	-0.13	-0.13	-0.31	-0.31
FTSE100				-0.15	-0.15	-0.31	-0.31
US 10 Year Rate					0.64	-0.19	0.00
UK 10 Year Rate						0.01	-0.17
US A 10-year credit spread							0.77

- US and UK equities

	Credit loss (UK)	S&P500	FTSE100	US 10 Year Rate	UK 10 Year Rate	US A 10-year credit spread	UK A 10-year credit spread
Credit loss (US)	0.66	-0.73	-0.67	0.11	0.10	0.23	0.23
Credit loss (UK)		-0.47	-0.67	0.08	0.07	0.16	0.16
S&P500			0.69	-0.13	-0.13	-0.31	-0.31
FTSE100				-0.09	-0.09	-0.23	-0.22
US 10 Year Rate					0.64	-0.19	0.00
UK 10 Year Rate						0.01	-0.17
US A 10-year credit spread							0.77

- US and UK equities and rates

	Credit loss (UK)	S&P500	FTSE100	US 10 Year Rate	UK 10 Year Rate	US A 10-year credit spread	UK A 10-year credit spread
Credit loss (US)	0.66	-0.73	-0.67	0.11	0.31	0.23	0.23
Credit loss (UK)		-0.47	-0.67	0.12	0.25	0.16	0.16
S&P500			0.69	-0.15	-0.27	-0.31	-0.31
FTSE100				-0.01	-0.16	-0.23	-0.22
US 10 Year Rate					0.74	0.03	0.03
UK 10 Year Rate						0.08	0.08
US A 10-year credit spread							0.77

Dependency controlled within expanded credit risk model

Dependency controlled within ESG

Dependency induced

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Comparison of capital requirements using different reordering schemes

Bridging factors	Economic Capital			Rank correlation between standalone losses
	Credit	Market	Total	
None (independent scenarios)	0.028	0.276	0.277	0
US equities	0.028	0.276	0.287	0.526
US and UK equities	0.028	0.272	0.282	0.632
US and UK equities and risk-free rates	0.028	0.277	0.287	0.655

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Summary

Aggregation of credit risk capital and market risk capital assessments

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Summary

- Aggregation of market and credit risk capital assessments is complicated by use of different models and software systems
- Simple approaches to capital aggregation (e.g. formula, loss copula) rely on correlation inputs that are hard to measure
- Bridging factors can be used to capture dependency between underlying market and credit risk factors and allow post-processing of separate credit and market scenarios via reordering

Questions

Comments

Expressions of individual views by members of the Institute and Faculty of Actuaries and its staff are encouraged.

The views expressed in this presentation are those of the presenter.

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Further reading

- S. Morrison: *Aggregation of market and credit risk capital requirements via integrated scenarios*, Barrie & Hibbert Research report (2013)

- J. C. Garcia Cespedes, J. A. de Juan Herrero, D. Rosen & D. Saunders: *Effective Modeling of Wrong Way Risk, Counterparty Credit Risk Capital and Alpha in Basel II*, The Journal of Risk Model Validation 4(1), 71-98 (2010)

- L. Pospisil, N. Patel, A. Levy: *Modeling Credit Correlations Using Macroeconomic Variables*, Moody's Analytics Modeling Methodology (2012)