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# E2: Aggregation techniques for Solvency II: a practical example

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# Contents

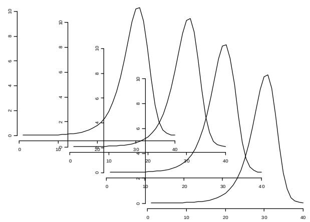


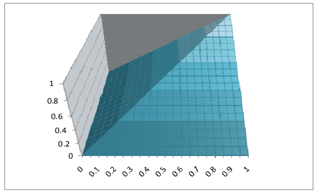
# Building blocks of capital models

## Desired properties

Modelling of individual risks



Modelling of dependence structure




Fat tails
Asymmetry
Non-normality

Tail dependence
Interactions
Diversification

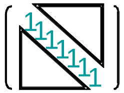
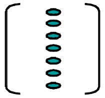
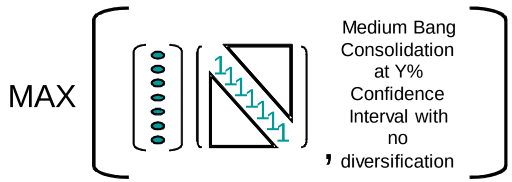
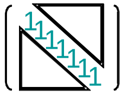
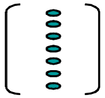
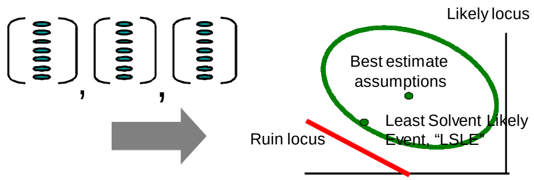
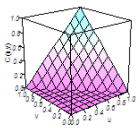
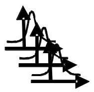
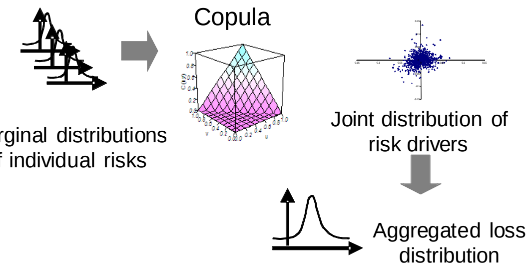
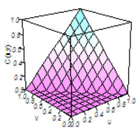
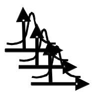
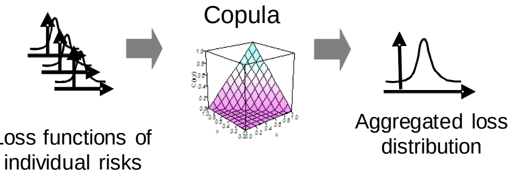
Modelling capital required to survive extreme events

# Key concepts explained

| Concept | Explanation |
|---|---|
| Correlation matrix | Matrix used to describe the dependence between pairs of random variables, eg lapse and interest rates |
| Non linearity/interaction | Non-linearity describes the effect whereby the impact of stresses occurring together differs from the sum of the impacts of the individual stresses |
| Copula | An approach by which the marginal distributions of a set of variables are combined together into a single multivariate distribution |
| Marginal distribution | Distribution of one variable obtained when ignoring all other variables of the joint distribution |
| Tail dependence | Measure of extreme co-movements in the tail of a joint distribution |

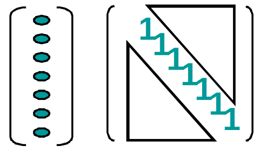
 

# Overview of main approaches

| Method | Individual risk calibration | Dependency structure | Aggregation subject | Transformation | Output |
|---|---|---|---|---|---|
| **Medium Bang** | • Assumed normal distribution.<br>• Mean and standard deviation calibrated. | Correlation Matrix<br> | Vector of net asset stresses by risk factor<br> |  | • Capital requirement at given confidence level.<br>• Non-linearity allowed for through medium bang. |
| **Risk Geographies** | • Elliptically contoured distributions (e.g. Normal, multivariate T) | Correlation Matrix<br> | Vector of net asset stresses by risk factor<br> |  | • As above.<br>• Non-linearity picked up directly. |
| **Input Copula**<br>**(Copula applied to marginal risk distributions)** | • Marginal distribution. | Copula<br> | <br>Marginal distributions of individual risk drivers |  | • Full distribution of capital requirements.<br>• Non-linearity picked up directly |
| **Output Copula**<br>**(Copula applied to loss distributions)** | • As above. | Copula<br> | <br>Loss functions (P&L Impact) of individual risks |  | • Full distribution of capital requirements.<br>• Non-linearity not picked up. |

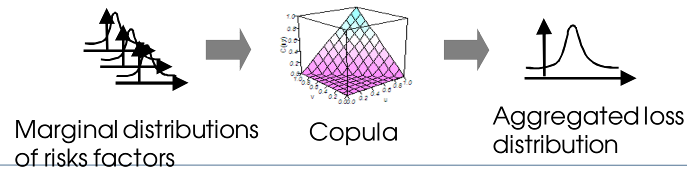
# Aggregation with Variance/Covariance



| Method | Variance / Covariance |
|---|---|
| **Brief description** | • Similar to Standard Formula under Solvency II, based on **a correlation matrix**<br>• Most common method currently used by insurance companies to perform capital calculations (see CRO Forum, Jan. 2009), but leading companies are revisiting their choice of this method |
| **Risk factor modeling & calibration** | • Risks are **elliptically distributed** and are combined by using a variance / co-variance approach<br>• Assumptions required regarding the mean and standard deviations of the individual risks |
| **Process** | • **Closed-form formula** combining the capital requirement calculated for each risk factor at a given level of confidence using a correlation matrix<br>• Approximate adjustment can be made to allow for the non-linearity of different risks (so-called **Medium Bang** approach used in the UK, involving a combined scenario), but it remains ultimately arbitrary |
| **Outputs** | • The method produces a **capital result at the required level of confidence**. Results at different levels of confidence can be produced by repeating the calculation process<br>• Results fairly stable over time due to the simplicity of the method |
| **Key technical features** | • Non-linearity, fungibility restrictions and asymmetry are not picked up by the method<br>• Assumes that stress tests for risk factors correspond to the tail of the capital distribution |

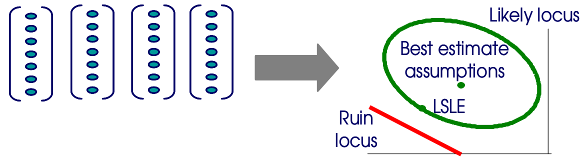
 

# Aggregation with Input Copulas




| Method | Copulas |
|---|---|
| **Brief description** | • **Monte Carlo approach**<br>• This method combines the marginal distributions of  risk factors using copula functions. Fed through an ALM model, these simulations are then used to produce a full distribution of capital requirements |
| **Risk factor modelling & calibration** | • Flexibility in assumed distributions of risk factors (including non-normal) and dependency structure<br>• A key challenge will be obtaining **reliable data to calibrate each copula** |
| **Process** | • A large number (e.g. 100,000) of scenarios are produced from the risk factor distributions<br>• Marginal distributions of risk factors are combined using copula functions<br>• The ALM model used for generating capital requirements is usually a **simplified model** (for example based on Replicating Portfolios or Formula Fitting), as many scenarios need to be run to achieve statistical convergence |
| **Outputs** | • **Full distribution** of capital requirements (does not assume that stress tests for risk factors correspond to the tail of the capital distribution)<br>• Results tend to be **less stable** over time as the approach is simulation based. They will need to be smoothed, which gives rise to a potential issue of managing the impact of smoothing |
| **Key technical features** | • Allowance can be made for **non-linearity** between risk factors through the use of an appropriate simplified ALM model<br>• **Tail dependence** and **fat-tailed distributions** can be allowed for<br>• **Fungibility restrictions** can be allowed for as the method is simulation based |

# Aggregation with Risk Geographies




| Method | Risk Geographies |
| --- | --- |
| **Brief description** | • Basically an **enhancement of the Variance / Covariance + Medium Bang approach** which enables the insurer to determine the **exact scenario** which would have the most severe financial impact on its balance sheet at a given level of confidence |
| **Risk factor modeling & calibration** | • Not restricted by particular choices of distributions or copulas<br>• However, most of the implementations to date have been in the variance-covariance framework |
| **Process** | • The most onerous scenario is determined by means of an **iterative process of stress tests**<br>• The result does not require Monte Carlo simulations of capital requirements and runs with **full ALM model** |
| **Outputs** | • The method produces a **capital result at the required level of confidence**. Results at different levels of confidence can be produced by repeating the calculation process<br>• The other main outputs are the **risk scenarios** (LSLE, MLRE)<br>• Results tend to be **fairly stable** over time as the approach is not simulation based |
| **Key technical features** | • Risks are combined in such a way that **non-linearity** is allowed for<br>• Allowance for fat tails, tail dependencies and fungibility restrictions is possible but sometimes requires adaptations |

# Illustration of Risk Geographies & Input Copulas

Input Copula produce relatively smooth capital requirements .....

| Business Unit | Risk Geographies | Input Copulas | | |
|---|---|---|---|---|
| | | Highest of 5 runs | Lowest of 5 runs | Lowest / Highest |
| BU 1 – Term Assurances | 5,746 | 5,744 | 5,729 | 99.7% |
| BU 2 – Annuities | 15,909 | 16,102 | 15,921 | 98.9% |
| BU 3 – Pensions | 8,659 | 8,896 | 8,708 | 97.9% |
| Diversification benefit | -11,761 | -12,053 | -11,754 | n/a |
| **Total – post diversification (Group)** | **18,553** | **18,689** | **18,604** | 99.5% |

However, identification of biting scenario is difficult .....

Scenarios driving capital requirements (100,000 simulations)

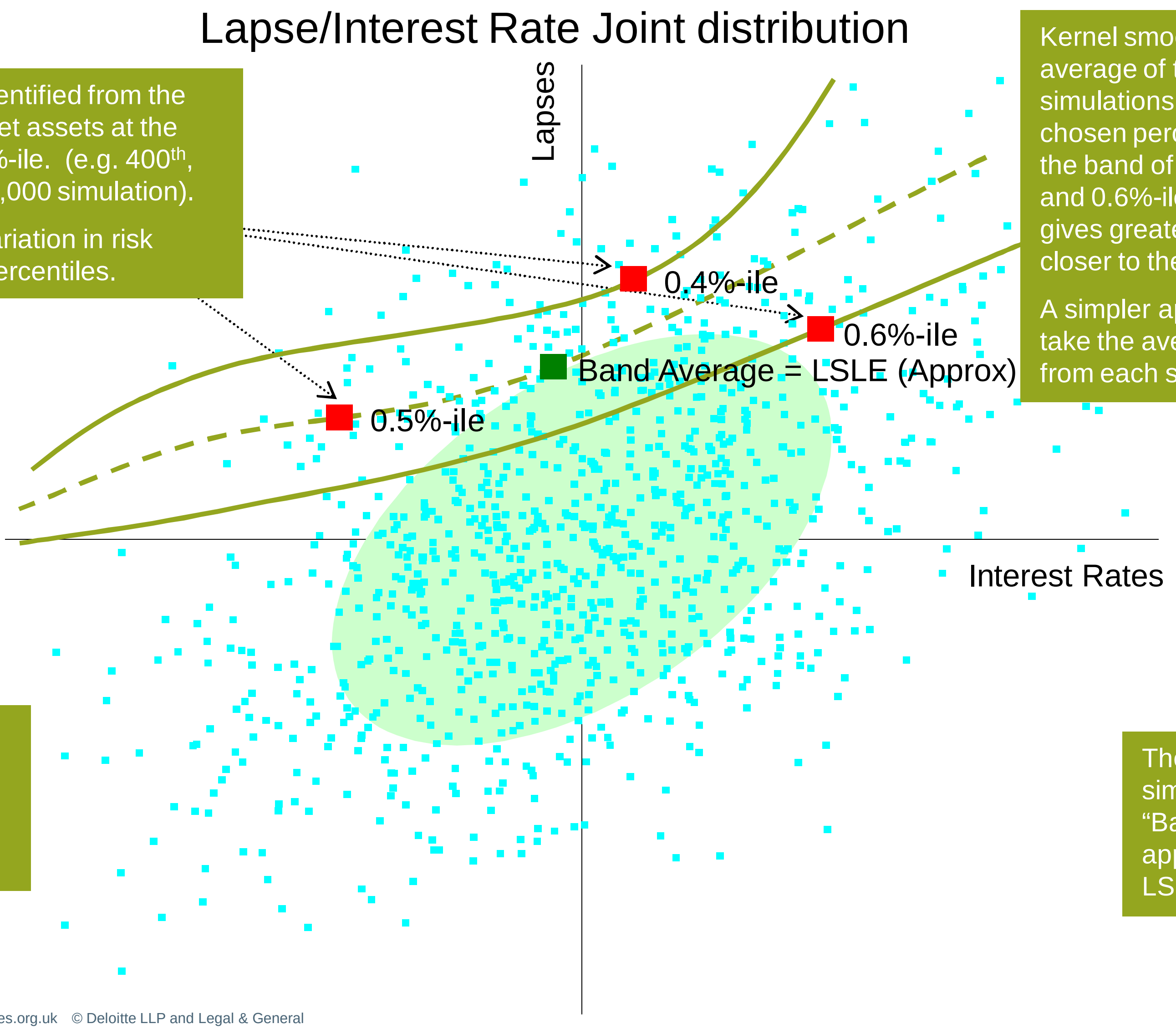
| Risk | Risk Geographies | Input Copulas | | | | | |
|---|---|---|---|---|---|---|---|
| | LSLE | Run 1 | Run 2 | Run 3 | Run 4 | Run 5 | Range |
| Interest | 0.15% | 0.77% | 0.68% | -0.60% | 1.80% | 1.93% | 2.53% |
| Credit Spreads | 2.24% | 2.30% | 2.06% | 1.70% | 2.12% | 3.25% | 1.55% |
| Equity Levels | -3.14% | 0.86% | -7.11% | -19.35% | -4.45% | 8.95% | 28.31% |
| Asset Share Volatility | 5.43% | 6.27% | 7.59% | 5.33% | 13.70% | 2.20% | 11.49% |
| Mortality Level | 2.77% | 7.48% | 5.83% | 0.60% | 5.06% | 3.04% | 6.88% |

Requires a smoothing mechanism .... Kernel smoothing

# Kernel smoothing explained

These are the scenarios identified from the simulations that drive the net assets at the 0.4%-ile, 0.5%-ile and 0.6%-ile. (e.g. 400th, 500th and 600th runs of 100,000 simulation).

It highlights the potential variation in risk factors between different percentiles.




Kernel smoothing takes a weighted average of the net assets from the simulations that lie between 2 chosen percentiles (e.g. shown by the band of points between 0.4%-ile and 0.6%-ile). The kernel function gives greater weighting to points closer to the mid percentile.

A simpler approach could be just to take the average of the net assets from each simulation.

The curves on the distribution represent contours of equal net assets for different percentiles.

The average of the simulations results is the “Band Average” which is approximately equal to the LSLE.

# An example of Kernel smoothing

Attribution using Euler method and driving scenarios produces widely varying capital amounts

| Business Unit | Risk Geographies | Input Copula (100,000 Simulations, Unsmoothed) | | | | | |
|---|---|---|---|---|---|---|---|
| | | Run 1 | Run 2 | Run 3 | Run 4 | Run 5 | Size of range |
| BU 1 – Term Assurances | 2,837 | 4,636 | 3,757 | 2,086 | 2,303 | 1,854 | 2,781 |
| BU 2 – Annuities | 13,601 | 13,219 | 12,289 | 10,112 | 14,143 | 19,914 | 9,802 |
| BU 3 – Pensions | 2,115 | 762 | 2,558 | 6,452 | 2,242 | -3,081 | 9,533 |
| **Group** | **18,553** | **18,617** | **18,604** | **18,650** | **18,689** | **18,687** | **85** |

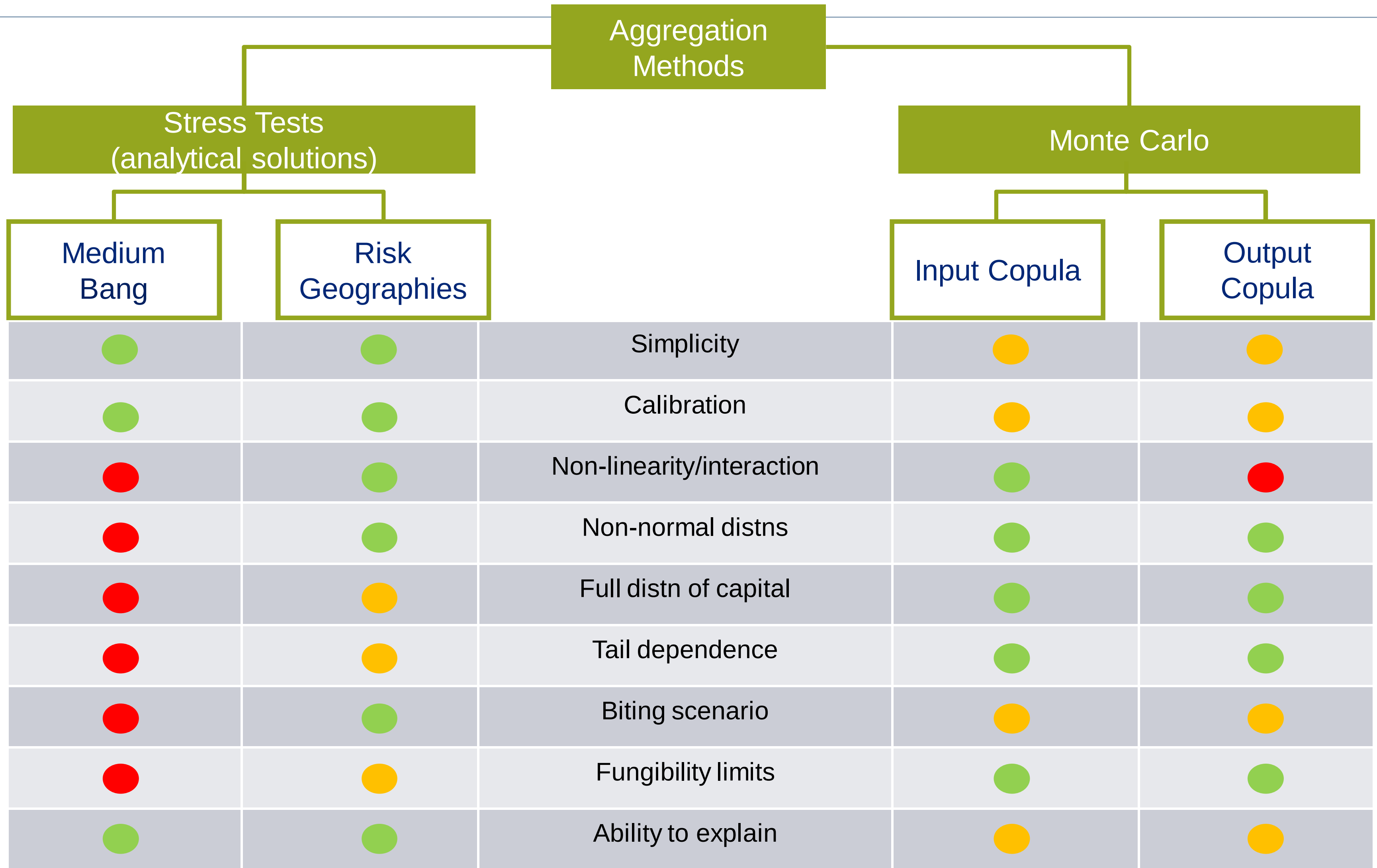
Kernel smoothing reduces variability ....

| Business Unit | Risk Geographies | Input Copula (100,000 Simulations, Smoothed) | | | | | |
|---|---|---|---|---|---|---|---|
| | | Run 1 | Run 2 | Run 3 | Run 4 | Run 5 | Size of range |
| BU 1 – Term Assurances | 2,837 | 2,832 | 2,864 | 2,759 | 2,842 | 2,854 | 105 |
| BU 2 – Annuities | 13,601 | 13,531 | 13,571 | 13,809 | 13,567 | 13,689 | 278 |
| BU 3 – Pensions | 2,115 | 2,148 | 2,064 | 2,013 | 2,130 | 2,065 | 135 |
| **Group** | **18,553** | **18,512** | **18,500** | **18,581** | **18,539** | **18,608** | **108** |

... meaning we are less exposed to random variations

# Comparison of methods considered




**Risk Geographies & Input Copula selected**

# L&G experience to date

**Selection:**

- Monte Carlo with Input Copula and Risk Geographies chosen methods
- Monte Carlo approach was prioritised due to automatic provision of full distribution of capital amounts, and prevalence of the approach
- Also considered to be the most flexible method to implement and to address challenging Solvency II requirements such as extreme events and fungibility constraints

# L&G experience to date (continued)

**Monte Carlo with Input Copula:**

- Randomness of scenarios makes identification of “biting scenario” complex
- Creates challenge for both attribution and communication to Boards
- Algo out-of-the-box smoothing methodology initially used in place of flexible Kernel smoothing

**Risk geographies:**

- Build on hold pending completion of Monte Carlo
- May be used as a validation tool