The Actuarial Profession
making financial sense of the future

Risk and Investment Conference
Paul Sweeting (University of Kent/J.P. Morgan Asset Management)

# Tail Association and Extreme Loss Calculation and Communication

21 June 2011



## Agenda

**Acknowledgements**

**Calculation**

- Tail Risk
- Extreme Loss

**Communication**

## Acknowledgements


- Work carried out at the University of Kent
- Funded by the UK Actuarial Profession
- Assisted by Fotis Fotiou
- Useful input from
  - The Extreme Events Working Party
  - ERM PEC Research and Thought Leadership Sub-committee






## Calculation of tail risk

- Two broad approaches
  - Tail correlation
  ⇒what is the shape of the relationship in the tail?
  - Tail dependence
  ⇒How much of the distribution is in the tail?
- Expansion to higher dimensions

## Tail correlation

- Broadly speaking, calculate correlation in a tail of the data
- All correlation types can be used
  - Linear only valid for elliptical data
  - Rank versions probably more robust
- Multi-dimensional versions do exist…
  - Collapsing the relationship between more than to variables into a single statistic
- …but often more than one version





## Tail correlation – limitations

- Tail correlation looks – by definition – at the tail
  - What if the extreme values lie off the main diagonal?
  - What if there “arachnitude”?
- Subjectivity also required
  - How is the tail defined?

## Tail dependence

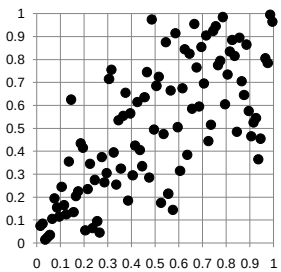
- Broadly speaking, consider
  - The proportion of observations in a tail…
  - …relative to the maximum proportion possible
- Again several versions
- Most popular is the coefficient of tail dependence
- C(u,u)/u, as u tends to zero (for lower measure)
- Main issue: always zero for Gaussian copulas
- Transforms exist…
- …but with similar issues

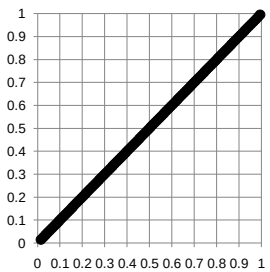




## Tail dependence

- Broadly speaking, consider
  - The proportion of observations in a tail…
  - …relative to the maximum proportion possible…
  - …defined as C(u,u)/u



## Tail dependence

- Again several versions
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## Tail dependence – alternative

- Why not use C(u,u)/u…
- …but use a finite value for u?
- This introduces subjectivity…
- …but can at least be used for all copulas
- Why not also consider for higher dimensions…
- …e.g. C(u,u,u)/u?

## Extreme loss

- Tail association important for model development…
- …but risk of loss often of more concern
- Two key measures
  - Probability of ruin
  - Economic cost of ruin
- Former is proportion of observations below the ruin line…
- …whilst latter is the average value of those observations





## Extreme loss – issue

- Given these measures are in money terms…
- …how do you treat different risks consistently?
- Need a definition of loss
- Approach depends on whether assessing
  - current exposure to loss
  - appropriate exposure to risks

## Definitions of loss – current exposure

- Interested in risks currently faced
- Ruin line parameter L defined as critical level of total loss
- L is maximum loss acceptable from two, three or more, even all, sources
- Can be defined in absolute terms or as change in value.
- Define $X_1, X_2, \ldots, X_N$ where N is number of risks being considered concurrently
- Consider losses from these risks with all others being held constant

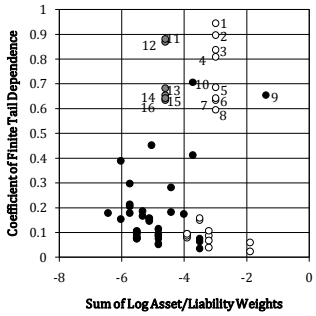




## Definitions of loss – appropriate exposure

- Interested in appropriate allocation between different risks
- Notional threshold loss from risk combinations needed
  - e.g. £10 per £100 invested for two risks
- Distributions of losses for different combinations of risk exposures then be determined
  - e.g. if threshold loss from two risks is £10 per £100 invested and two risks being considered 1 and 2, probability of ruin can be calculated for combinations of £ $W_1$ and £ $W_2$ = £(100-$W_1$)
- Can be extended to higher dimensions…
- …and used in calculation of efficient frontiers

# Scatter Plot



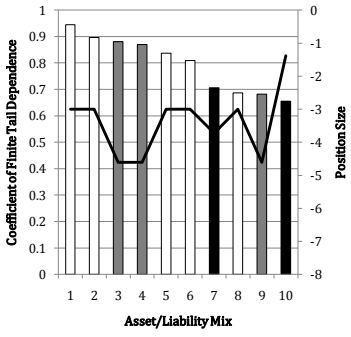

1. Real Liabilities/All IL Gilts
2. Nominal Liabilities/All Gilts
3. Real Liabilities/+15 Year IL Gilts
4. Nominal Liabilities/+15 Year Gilts
5. Real Liabilities/All Gilts
6. Real Liabilities/+15 Year Gilts
7. Nominal Liabilities/All IL Gilts
8. Nominal Liabilities/+15 Year IL Gilts
9. Real Liabilities/Nominal Liabilities
10. UK Equities/Europe ex UK Equities
11. All IL Gilts/ +15 Year IL Gilts
12. All Gilts/+15 Year Gilts
13. All IL Gilts/ All Gilts
14. +15 Year IL Gilts/All Gilts
15. All IL Gilts/+15 Year Gilts
16. +15 Year IL Gilts/+15 Year Gilts

- Black = two risk assets
- White = one asset, one liability
- Grey = two matching assets





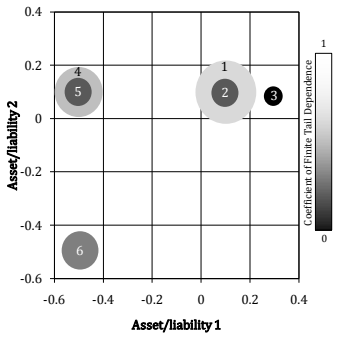
# Bar Chart




1. Real Liabilities/All IL Gilts
2. Nominal Liabilities/All Gilts
3. All IL Gilts/+15 Year IL Gilts
4. All Gilts/+15 Year Gilts
5. Real Liabilities/+15 Year IL Gilts
6. Nominal Liabilities/+15 Year Gilts
7. Europe ex UK Equities/UK Equities
8. Real Liabilities/All Gilts
9. All IL Gilts/All Gilts
10. Real Liabilities/Nominal Liabilities

- Black = two risk assets
- White = one asset, one liability
- Grey = two matching assets

# Balloon Plot



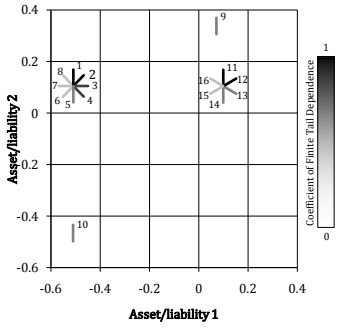

1. All IL Gilts/+15 Year IL Gilts
2. All IL Gilts/All
3. Europe ex UK Equities/UK Equities
4. Nominal Liabilities/+15 Year Gilts
5. Nominal Liabilities/+15 Year Gilts
6. Real Liabilities/Nominal Liabilities

- Black = low risk
- White = high risk





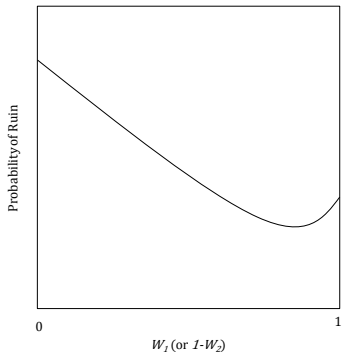
# Sunflower Plot




1. Real Liabilities/All IL Gilts
2. Nominal Liabilities/All Gilts
3. Real Liabilities/+15 Year IL Gilts
4. Nominal Liabilities/+15 Year Gilts
5. Real Liabilities/All Gilts
6. Real Liabilities/+15 Year Gilts
7. Nominal Liabilities/All IL Gilts
8. Nominal Liabilities/+15 Year IL Gilts
9. Real Liabilities/Nominal Liabilities
10. UK Equities/Europe ex UK Equities
11. All IL Gilts/ +15 Year IL Gilts
12. All Gilts/+15 Year Gilts
13. All IL Gilts/ All Gilts
14. +15 Year IL Gilts/All Gilts
15. All IL Gilts/+15 Year Gilts
16. +15 Year IL Gilts/+15 Year Gilts

- White = low risk
- Black = high risk

## Ruin Plot





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