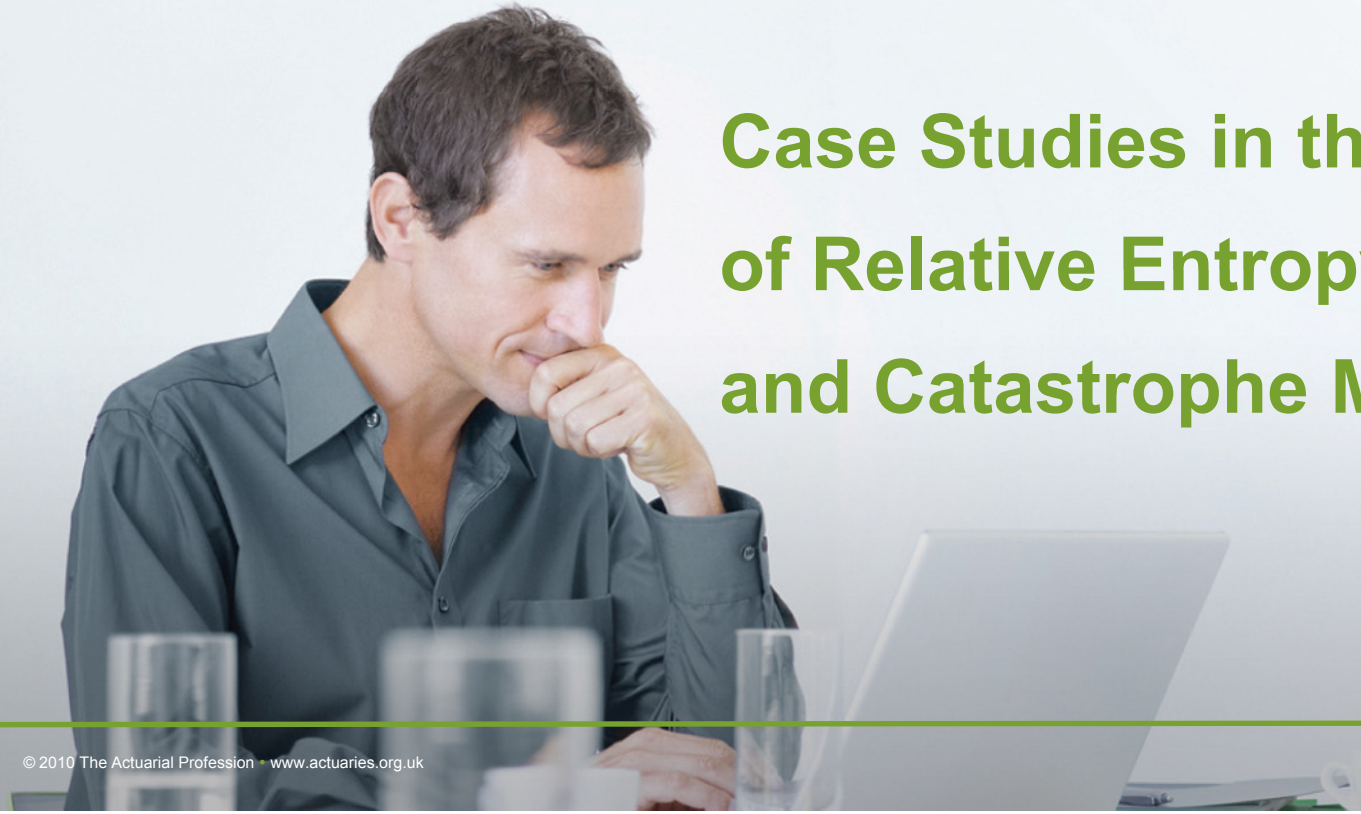
The Actuarial Profession
making financial sense of the future

GIRO conference and exhibition 2010

Alun Marriott, Richard Millns



Case Studies in the application of Relative Entropy to ESGs and Catastrophe Modelling

12-15 October 2010

Introduction
Agenda

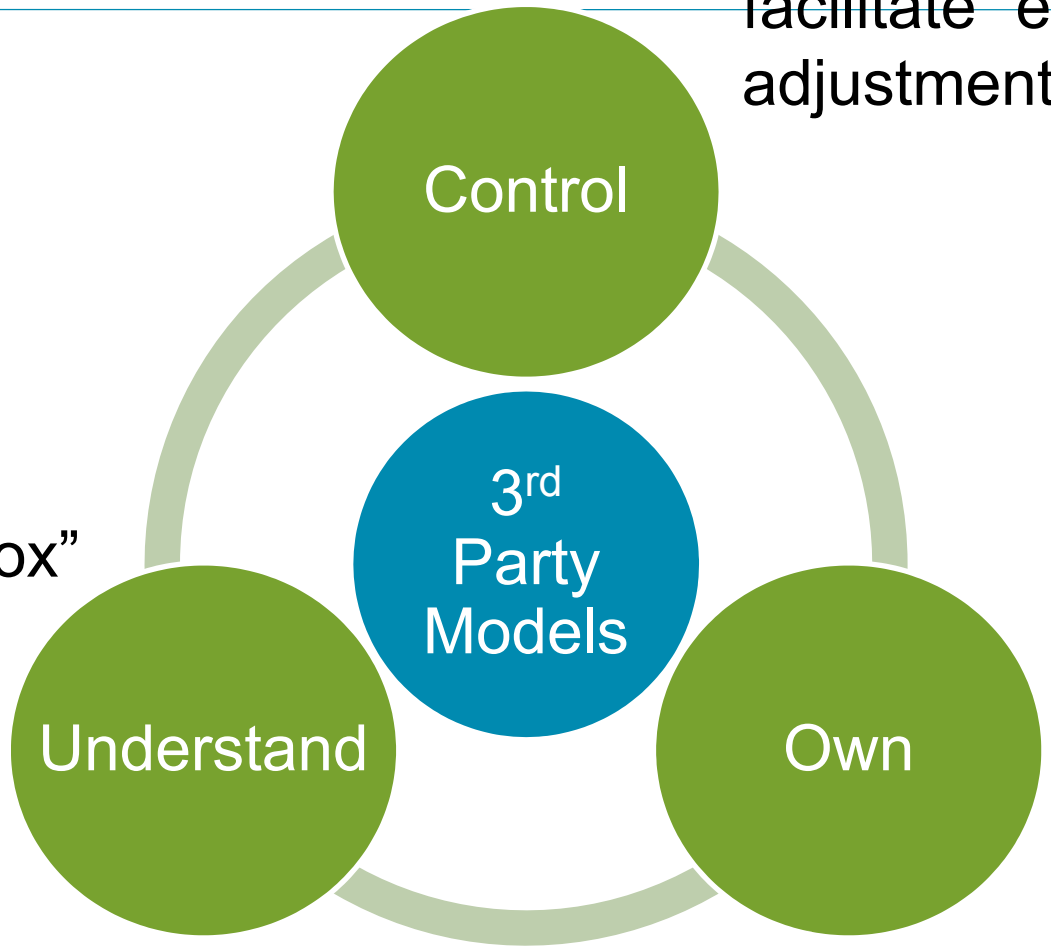
- Review of current modelling environment – particularly with reference to ESGs (Economic Scenario Generators)
- Discussion of Dilemma facing those outsourcing models to 3rd parties
- Review of Relative Entropy techniques
- Application to ESG models
- Application to Catastrophe models.

Introduction
Why this workshop ....

- Many financial models depend on stochastic projections – either in the “Real World” or “Risk Neutral”
- An ESG simulates the various permutations of economics variables – Catastrophe Models simulate the frequency and severity of catastrophe events, and the subsequent insurance losses
- Each are often outsourced and frequently hard to “re-parameterise”
- Relative Entropy provides a way to allow users of such “stochastic” results to modify the distributional properties in a coherent, robust manner – but why do you want to?

Introduction
Background - Solvency 2

Mechanism must facilitate “easy” adjustment



No “Black Box”

Control or Ownership “easy” to implement

Don’t blame the model provider

The Actuarial Profession
making financial sense of the future

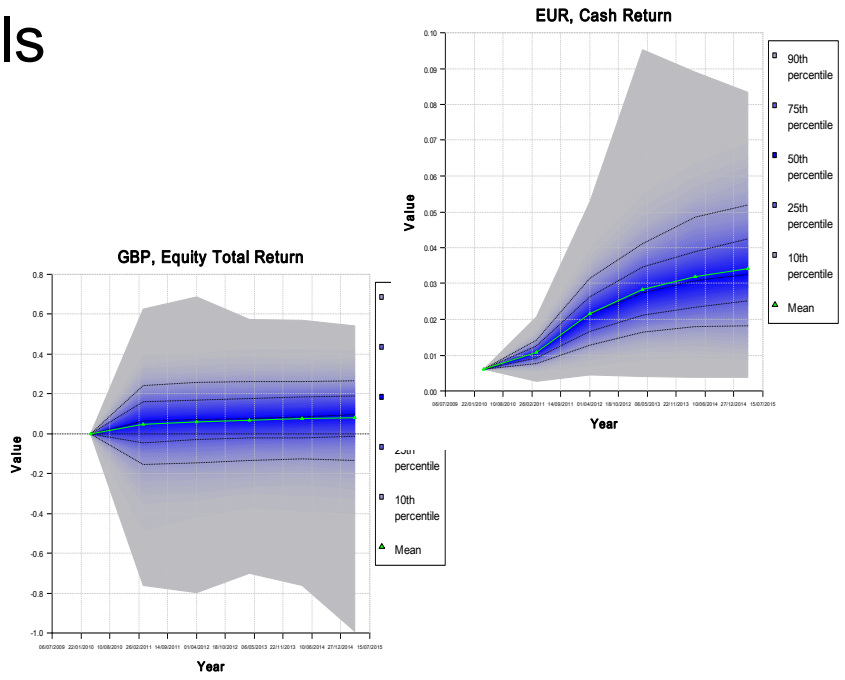
Part 1
REVO & Economic Scenario Generators

Alun Marriott

Head of EMB Investment Consultancy

ESG – A Monte Carlo Approach

- Key Features
  - Many Simulations – 1000 to 1,000,000 simulations
  - Multi Economy – Coherence between economies
  - Coherent “within” an economy:- both “Copula” & “Cascade”
  - Complex “Multi-Year” models
  - Real World vs Risk Neutral

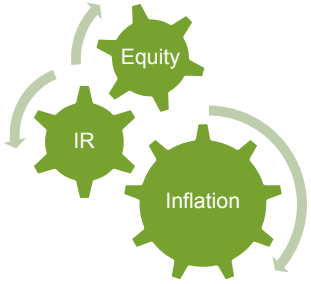


ESG
(for Risk Analysis)

- Risk Neutral vs. Real World – Much debate – but Risk is Real
- The evolution (though time) of Risk must consequently follow a Real World scenario – albeit that embedded options or derivatives will still require Risk Neutral valuation... Another topic!

Projections – Two Important Components

The Level of Economic Items



Inter-relationships aka Dependencies aka Correlation

ESG
Control & Ownership Issues

- Any “Real World” calibration relies on judgement in the calibration process
  - Which Window, Model, Long Term Economic Target etc.
  - What if we (the users) do not agree with the calibrator?
  - What if we wish to stress the distribution?
  - What if we wish to stress the “path” of the distribution?
- Moreover, model complexity often makes “re-calibration” hard for most “users”

The Options – How do we modify the ESG?

Direct Parameter Adjustment

- Determine which parameter(s) influence the desired output.
- Modify each econometric model parameter directly

Bayesian Approach

- Determine which parameter(s) influence the desired output.
- Introduce parameter uncertainty
- “Bias” the relevant parameter draws in such a way to skew the outcomes

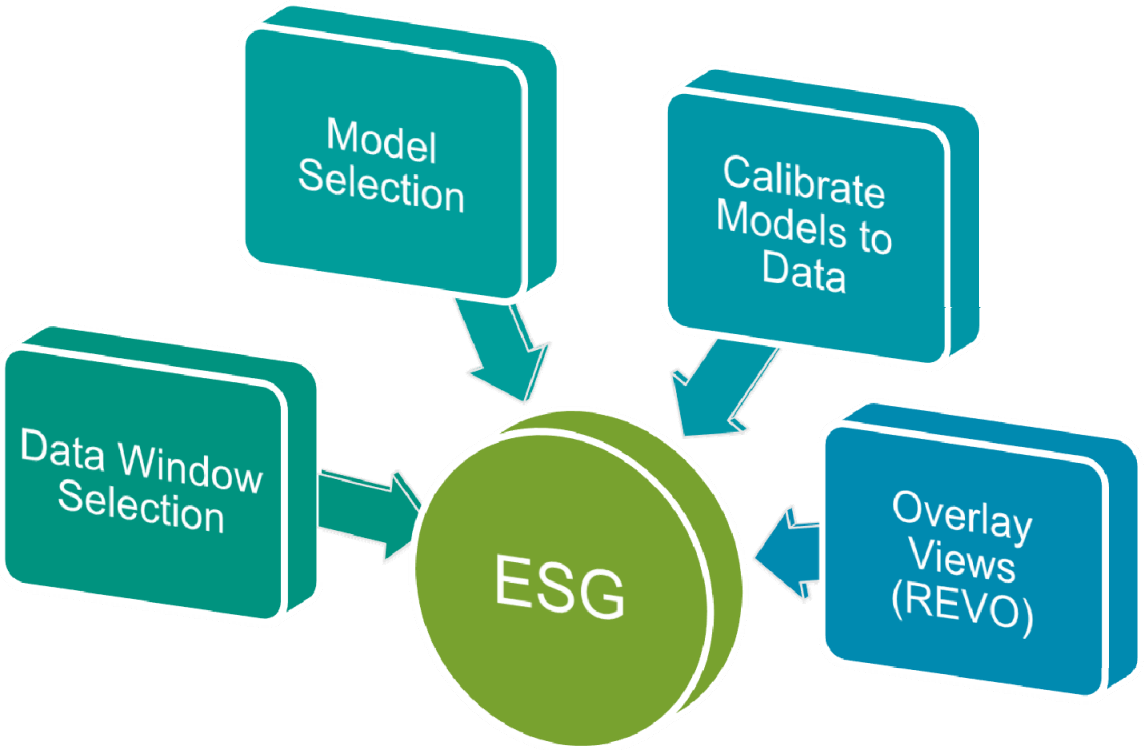
Maximum Entropy (REVO)

- In contrast, this approach *does not* attempt to modify the underlying ESG parameters.
- Instead, the ME approach *rescales* the ESG output simulations to increase the likelihood of some scenarios over others

Relative Entropy
A framework for modifying stochastic data

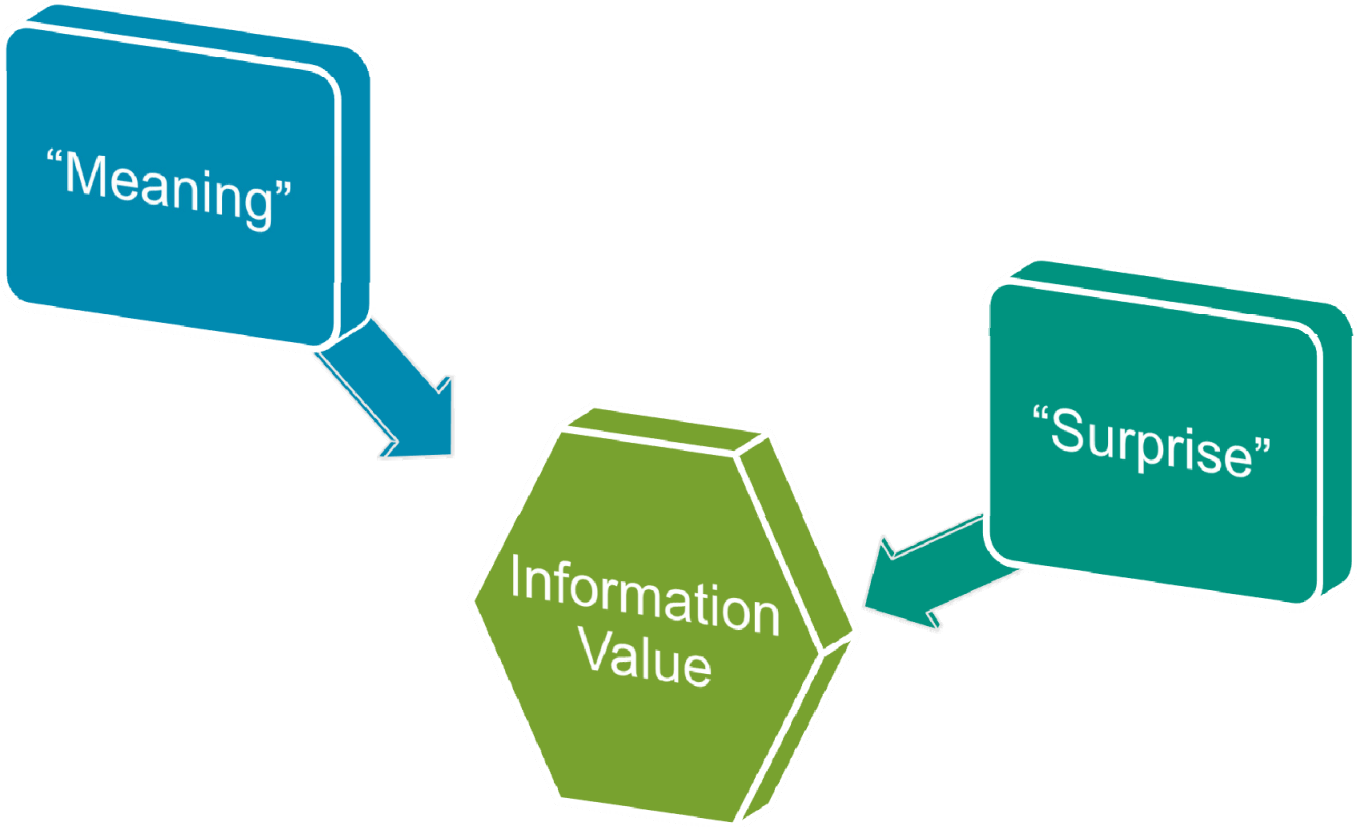
- Each of the questions posed earlier may be translated into "How do we modify a stochastic distribution"?
- We may be interested in modifying a variety of distributional characteristics
  - Mean
  - Volatility / Variance
  - Percentile
  - Correlation?
- But - we want to preserve as much of the original distribution as possible – i.e. REVO is a "Minimum Disturbance" approach

The ESG Parameterisation Process

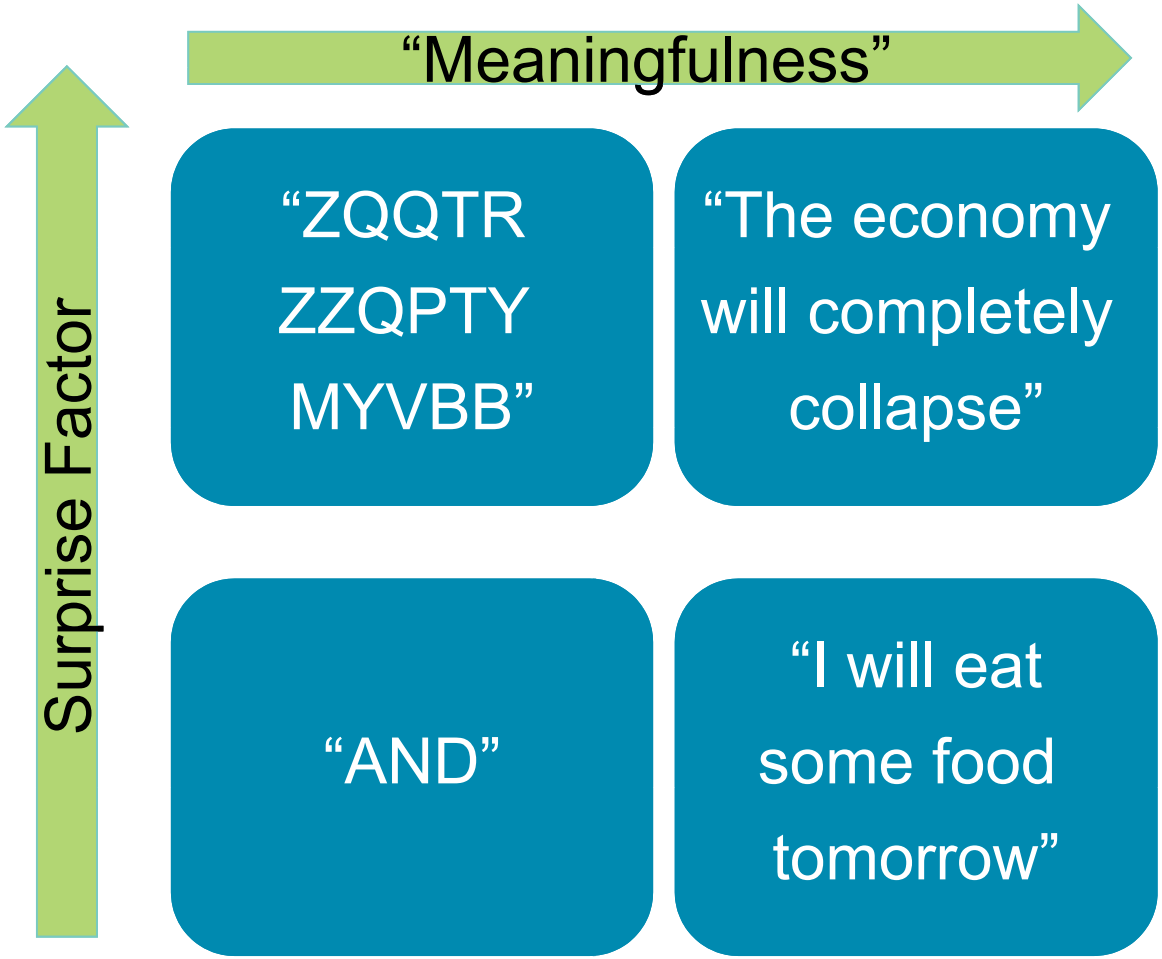


Relative Entropy View Overlay (REVO)
Sources of View (for Ownership / Alignment)

REVO
Quantifying Information (1) – Shannon, 1948



REVO
Surprise Factor vs “Meaningfulness”



Idea courtesy of Applebaum [1]

REVO
Intuitive Properties of Information

Information Value	Description	Log Function Example
$I(x_1 \cap x_2) \geq I(x_2) \geq I(x_1)$	The lower the probability of the event occurring, the greater the Information Value	-Log function is monotonic increasing on (0,1]
$I(x_1 \cap x_2) = I(x_1) + I(x_2)$	The Information Value does not depend on how it is received.	-Log(⅓ x ¼) = -Log(⅓) + -Log(¼)
$I(x_i) \geq 0$ for all i	Information can never be "lost"	-Log function is strictly positive on (0,1]

REVO
Information, $\mathbb{I}(X)$ to Entropy, $\mathbb{H}(X)$

Definition of Information Content

$$I(X) = -K \log_a P(X)$$

Definition of Informational Entropy

Simply the expectation of the Information Value over all events.

$$\mathrm{H}(X) = \mathbb{E}[I(X)] = -K \sum_{j=1}^{n} p_j \log_a p_j$$

We typically assume K = 1, and use the natural logarithm.

REVO
The Maximum Entropy Principle (ME)

Principle of Maximum Entropy

In the absence of any other information, when we are “most uncertain”, entropy is then maximized.

Why is the Principle of Maximum Entropy helpful ?

The Principle of Maximum Entropy allows us to quantify the effect of introducing information – and hence design a scheme for adjusting probabilities as information is added.

REVO
ME in Practice

Monte Carlo

- Each simulation “usually” equally likely
- Instead – we ascribe a “weight” to each simulation
- Each simulation “weight” is calculated so as to achieve the desired view (as closely as possible) whilst reducing entropy the least, i.e. Achieve target view(s) whilst disturbing the distribution the least.
- Effectively we define a new probability measure

Simulation Number	Original Weight	Rescaled Weight
1	100.0%	361.6%
2	100.0%	90.5%
3	100.0%	96.7%
4	100.0%	50.2%
5	100.0%	56.3%
…	…	…

REVO
ME in Practice (2)

Coherent Adjustment of the "Joint Distribution"

- It is worth noting that each simulation represents one projection of the fully calibrated and integrated global economy (in the case of an ESG) – i.e. for all currencies, series, and time steps

- By rescaling one simulation, we modify all its constituent components equally, and hence we implicitly retain any "inter" series dependencies that may exist.
- This does NOT mean that the correlation is unchanged; more that the ex-post correlation reflects the views you have specified.

REVO
A Note on Confidence Measures

Confidence measures give us a way to “scale” the effect of our judgement overlay.

We can assign a confidence to “multiple” providers of overlay, and separately ascribe a confidence to each element of data the provider supplies.

How do Confidence Measures Work?

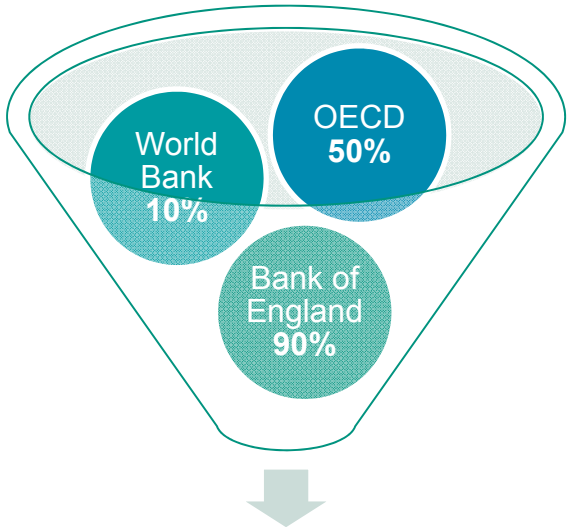
Essentially, we adopt a heuristic approach as given by Meucci (1).

A view with “Zero” confidence is effectively the original distribution, whereas a view with “Full” confidence is effectively fully re-sampled.

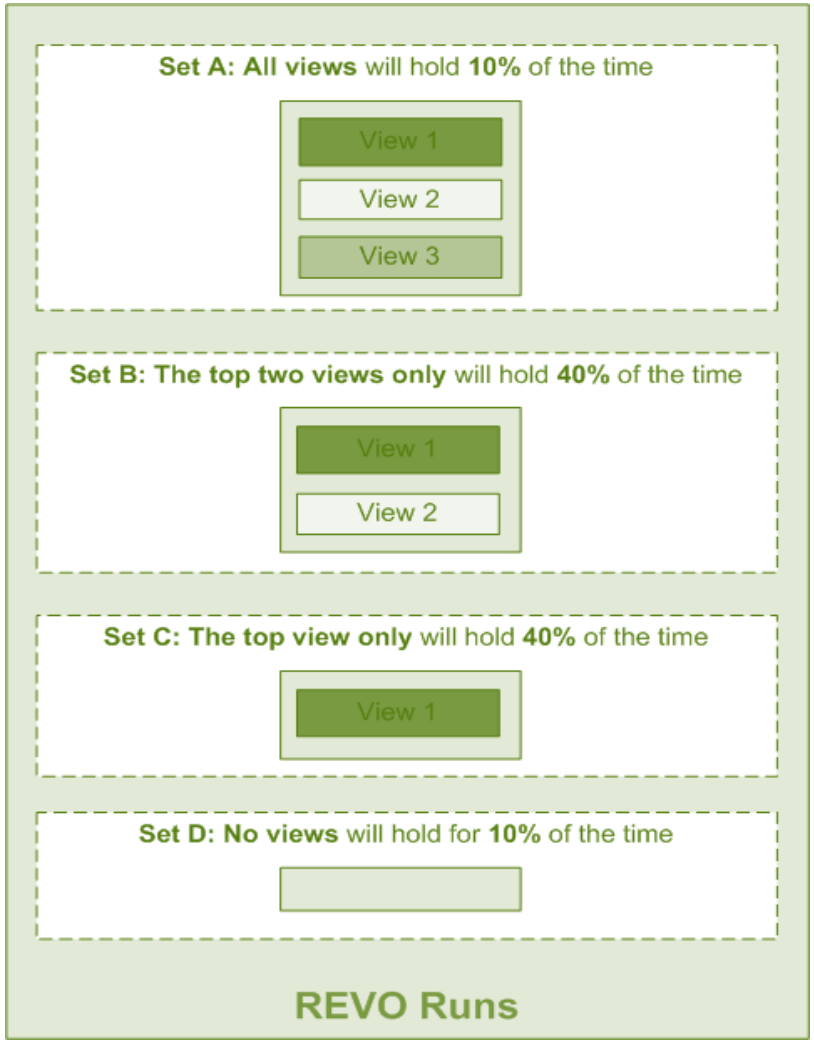
(1) Meucci – “Fully Flexible Views – Theory and Practice”

REVO
A Note on Confidence Measures (2)

They also give us a method for incorporating multiple views, perhaps coming from different sources, some of which we may have more confidence in than others.



REVO
A Note on Confidence Measures (3)



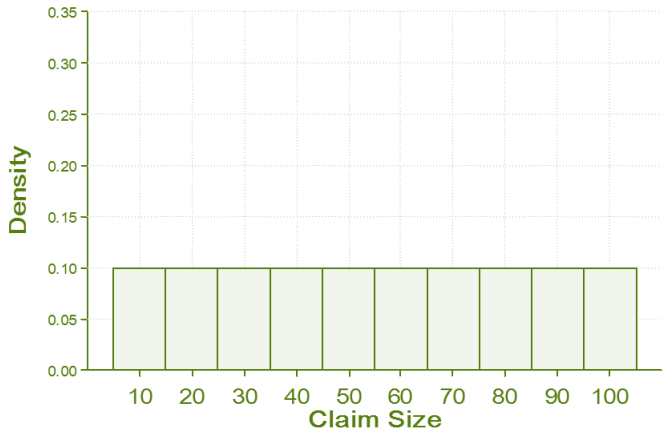
Perform a REVO on each set of views A to D, obtaining the probability weights for each run.

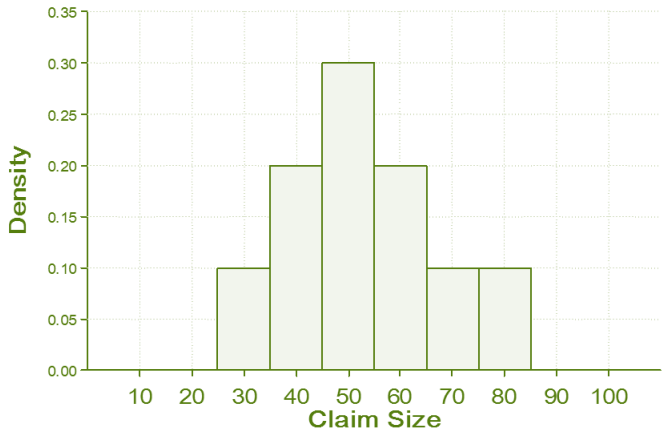
Note that the run for set D here is no run at all: we simply retain the original probability weights.

10% * set A weights
+
40% * set B weights
+
40% * set C weights
+
10% * set D weights
=
Final weights

REVO
Effective Number of Simulations

- **Entropy Decreases** as a consequence of new information.
- This decrease in entropy corresponds to a **decrease in the effective number of simulations** being used.

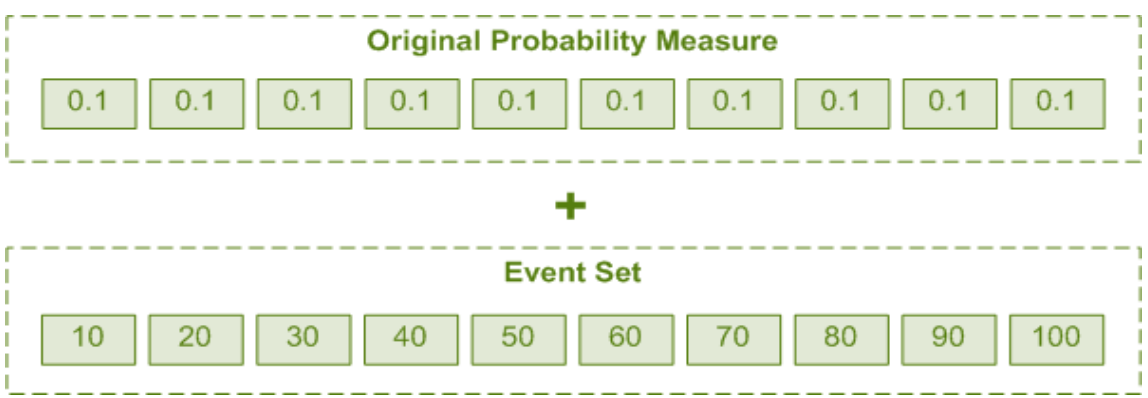


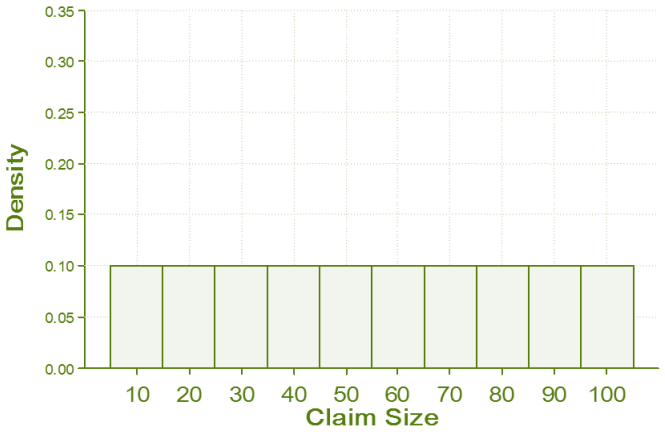


- In this case, by re-weighting we have gone from 10 simulations to effectively using only **six**.

REVO
Over-Extreme Views

The more extreme the views applied, the lower the effective number of output simulations.





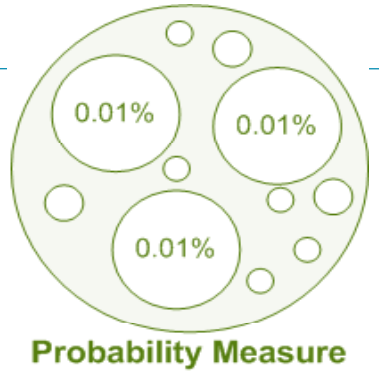
- A mean claim size of 110 would be over-extreme.
- Cannot reweight probabilities to achieve this view.
- REVO is simply re-using the set of possibilities given to it by the original ESG: if the possibility of the extreme view is not present in the original set of possibilities, then it cannot be satisfied.

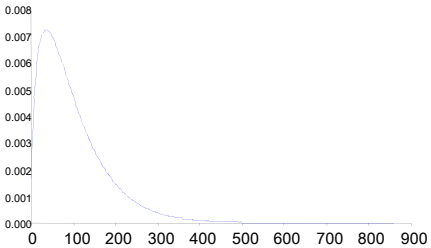
REVO
Probability Measures & Event Sets

- Monte-Carlo simulations are defined by two quantities:
- The **Probability Measure** (usually uniform).
- The **Event Set** (the variables modelled)

- The low-level REVO process determines a new probability measure (probability weights) but does **not** change the event set.

- However, most downstream systems assume uniform => Resample



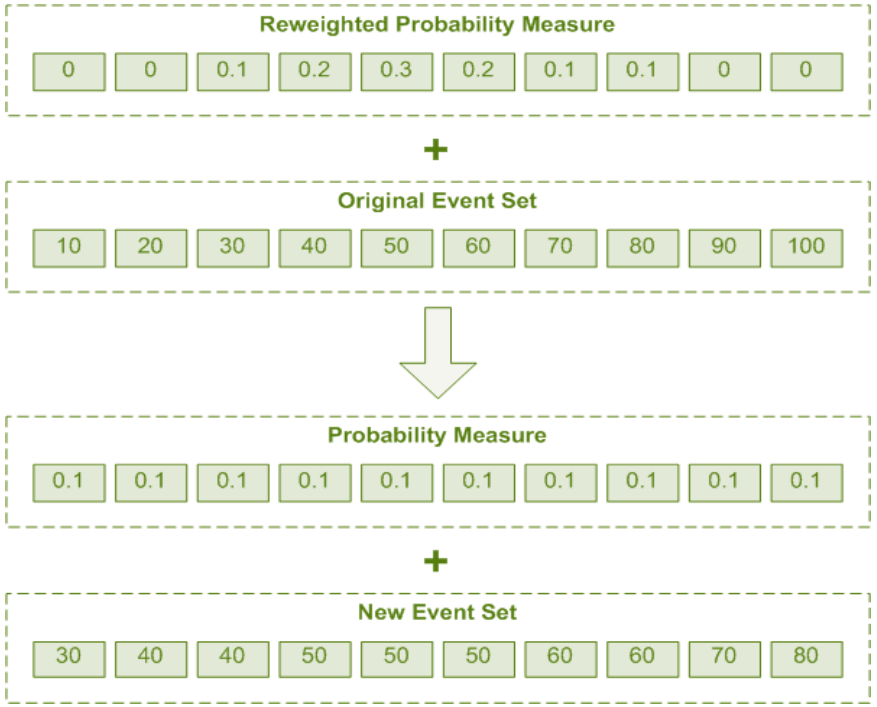


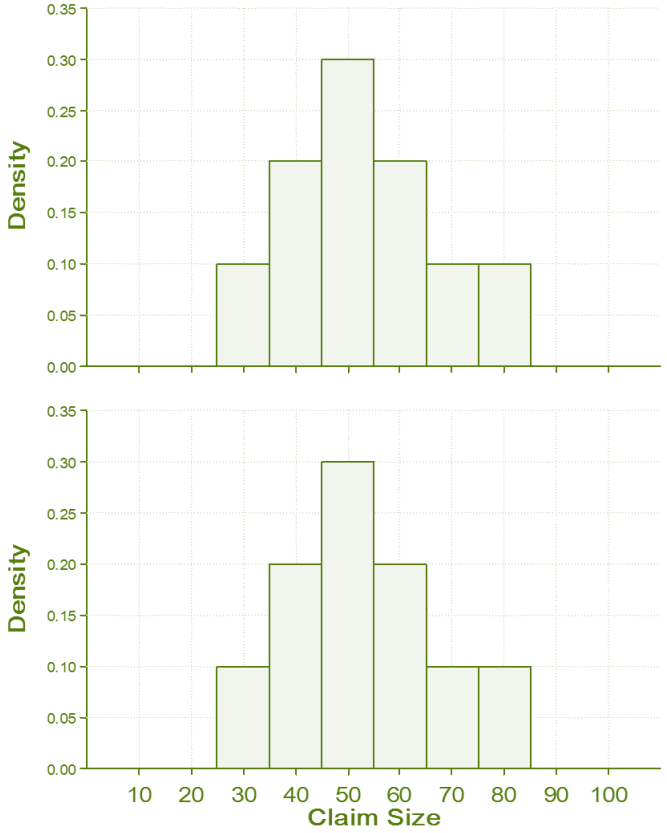
REVO
Resampling (2)

From: A reweighted probability measure and original event set.

To: A uniform probability measure and new event set.

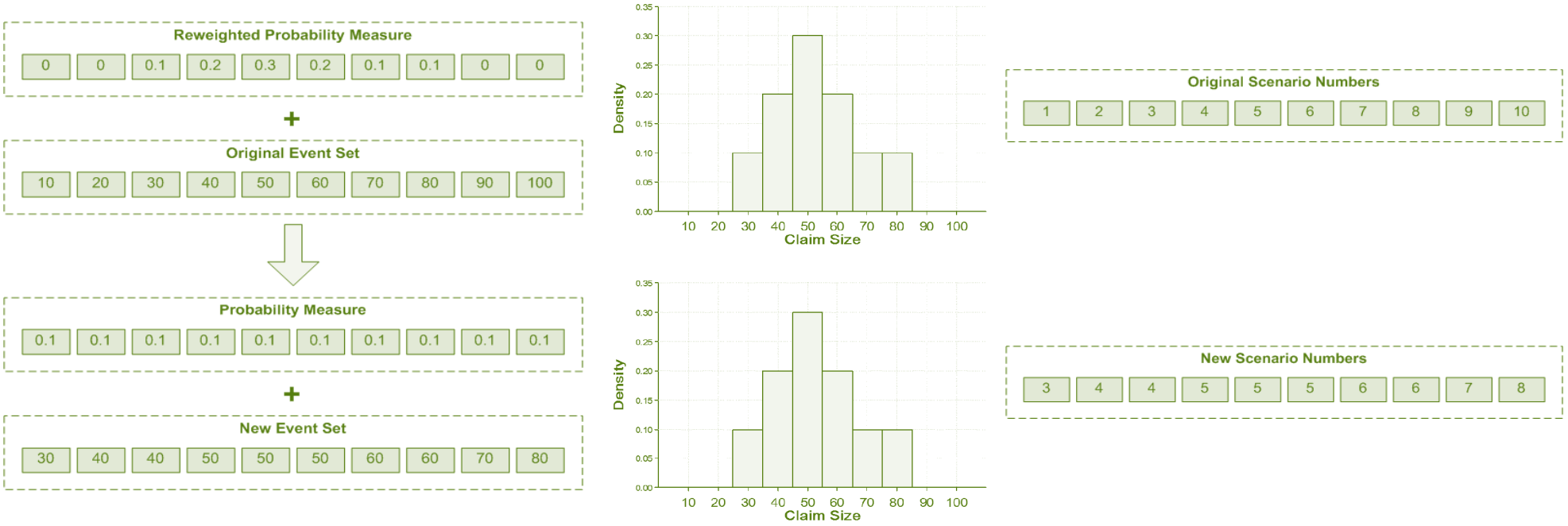
…so that the variables involved do not change their distributions.





REVO
Effect of Resampling on Scenario Numbers

- We can see duplicate simulations
- Take care regarding scenario numbers



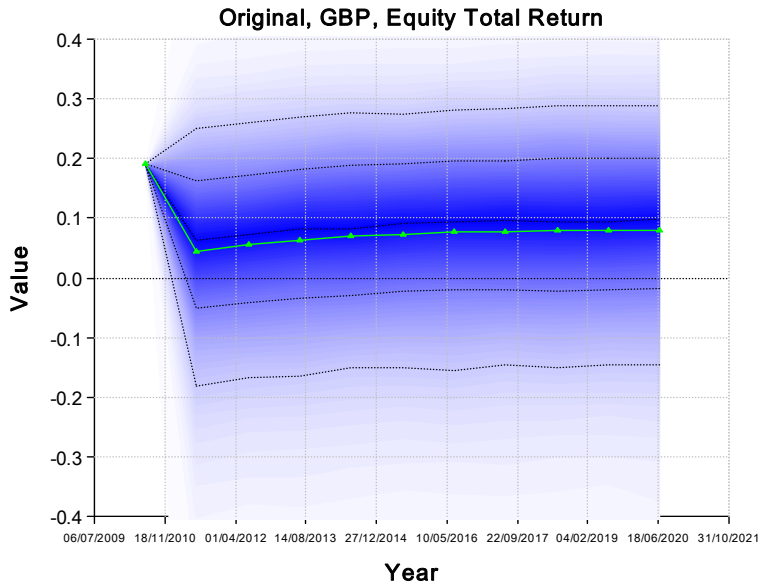
- This is undesirable and so we do **not** do this.
- Instead, the original scenario numbers are retained.

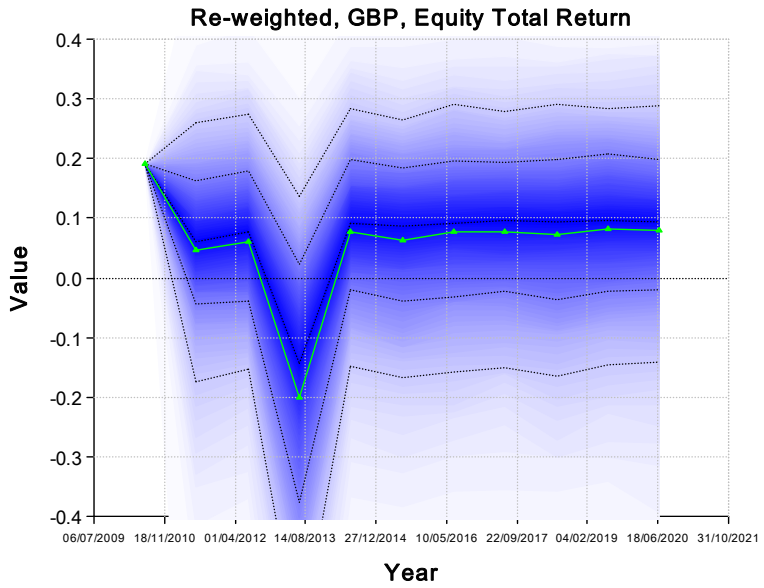
The Actuarial Profession
making financial sense of the future

REVO: Practical Examples

REVO
Results - Economic Crash View

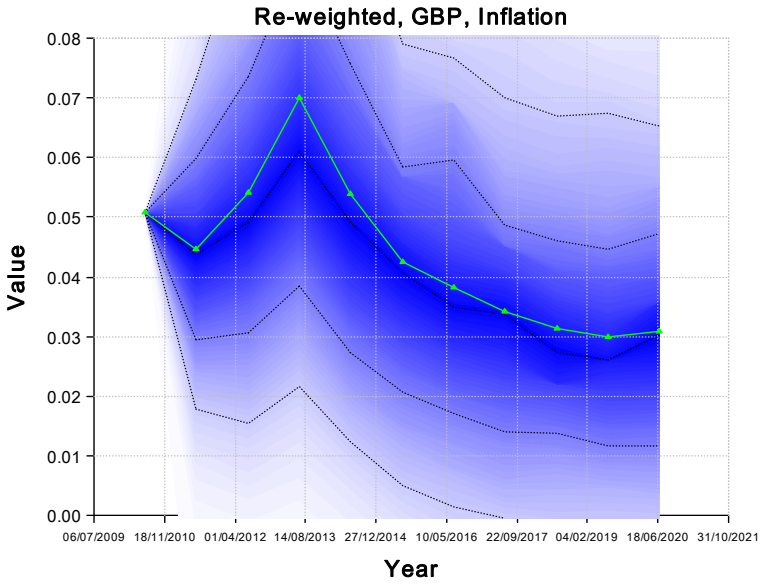
Here, we have specified a view of an equity total return of -20% in March 2013.





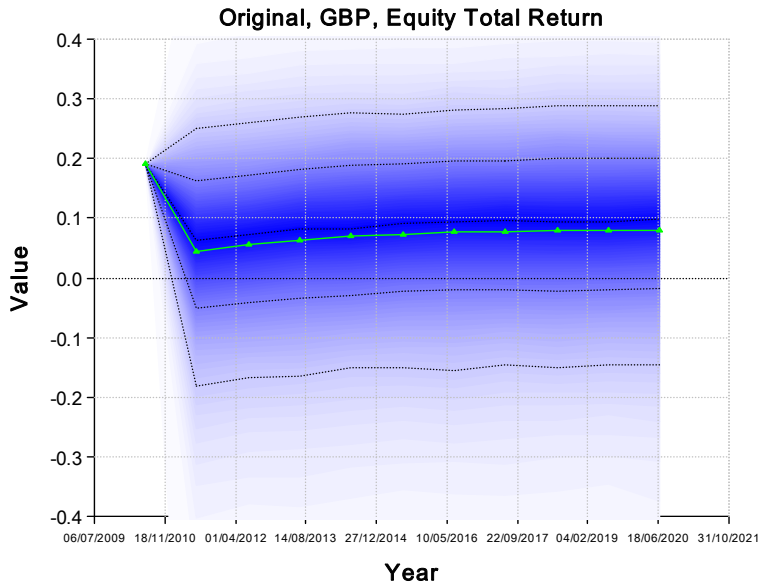
REVO
Results - Hyperinflation View

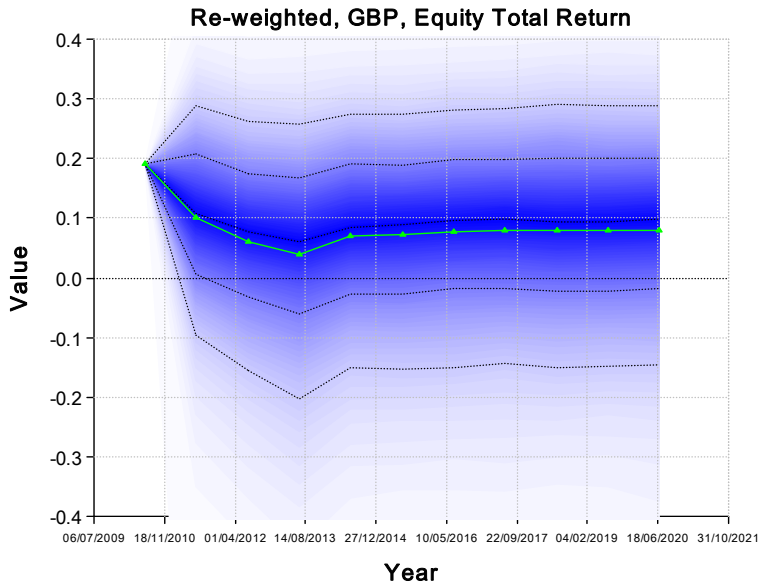
Here, we have specified a view on inflation of 9% in March 2013.



REVO
Results – Path Stress (1)

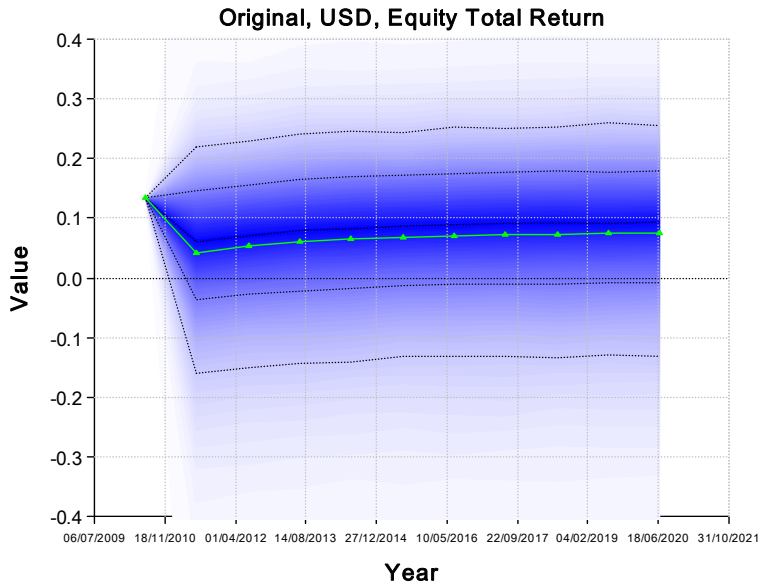
Here, we have specified a path view on GBP Equity, 2011-2013 (10%, 6%, 4%)

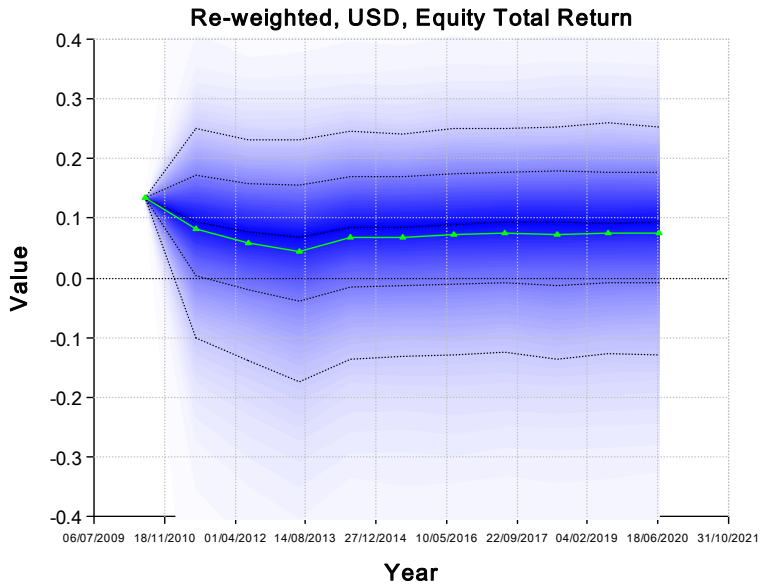




REVO
Results – Path Stress (2) - Coherence

As previous slide, but now looking at impact on USD from a GBP Equity path view





The Actuarial Profession
making financial sense of the future

REVO: Further Applications & Case Studies

REVO
Application 1: Alignment to Corporate Strategy

- Align business plan to risk framework
- For example – Ensure that inflation targets in the business plan are consistent with those in the risk management system
- Business hence gains “trust” in the modelling framework as it “makes sense” hence, they “use” the model (supporting the S2 “Use Test”)

REVO
Application 2: Stress Testing

- There are many cases where we may wish to stress our assumptions – including Means, Percentiles, Volatilities
- We may also wish to not only stress the distribution at a single snapshot through a projection, but also a path of such events through time

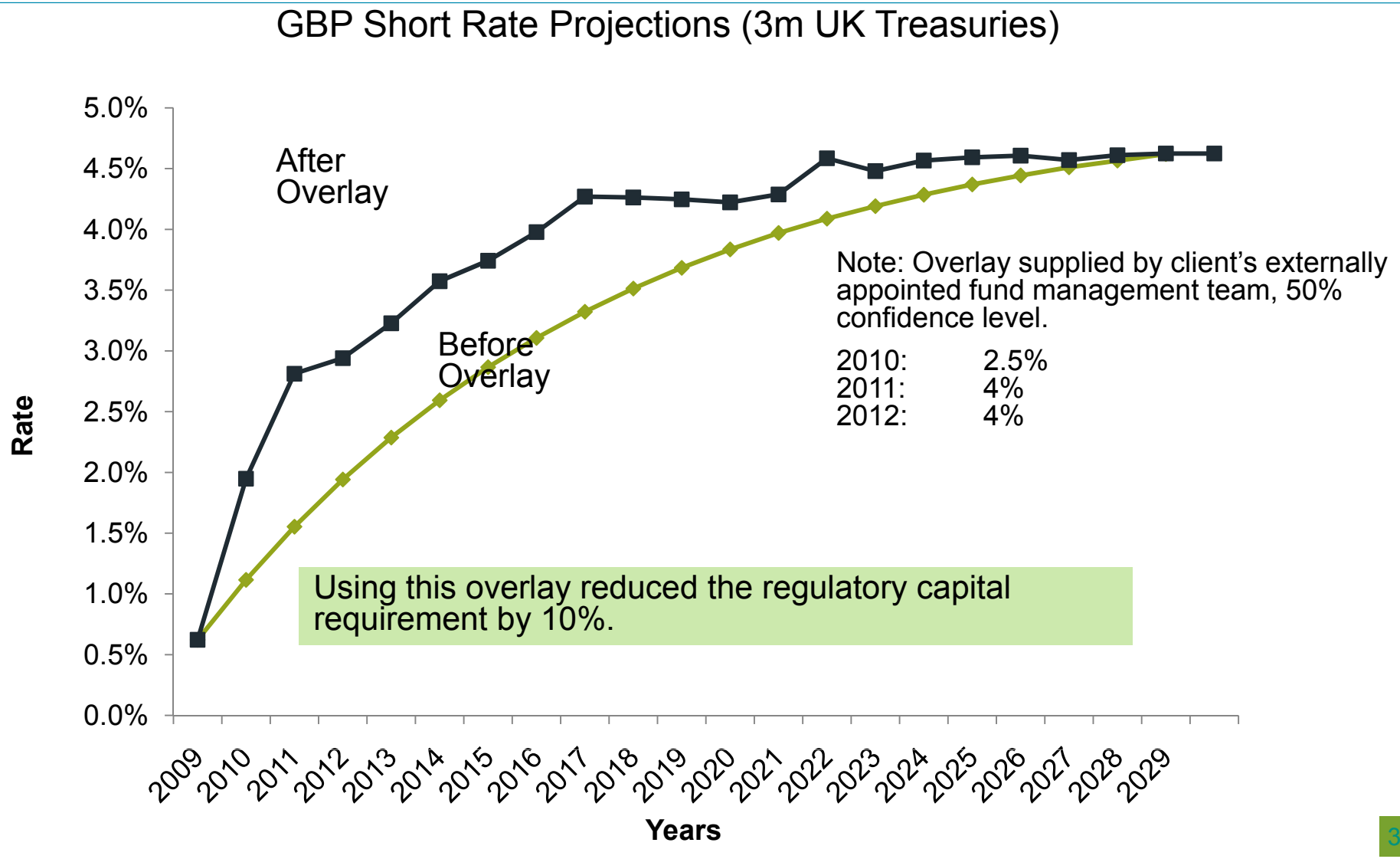
REVO
Application 3: Investment Strategy

- Asset Liability Modelling, Portfolio Construction (and Business Strategy) aligned to corporate views (or views provided by asset management providers)
- Bond ALM with Overlay – Strategic allocation able to reflect strategic market views
- Risk Budgeting
- Setting Investment Guidelines & Benchmarks

The Actuarial Profession
making financial sense of the future

REVO: Case Study – Non Life

REVO
Case Study 1: Short Rate Overlay (2009 ICA)



The Actuarial Profession
making financial sense of the future

Part 2
REVO & Catastrophe Modelling

Richard Millns, FIA

REVO
Catastrophe Modelling

1. Adjusting output from third-party cat models
2. Sensitivity testing
3. Stress testing

REVO - Catastrophe Modelling
Adjusting output from third-party models

- Why?
  - Cat model output does not reflect internal/broker views, or appears misaligned with historical experience
  - Wish to blend results from multiple models, or reflect other multiple views / opinions in the output
  - Load output for model uncertainty / data quality issues etc.

REVO - Catastrophe Modelling
Adjusting output from third-party models

- What options are available:
  - Re-run cat models with different exposure or event set assumptions etc.
  - Use alternative model
  - Somehow ‘scale’ the output appropriately
  - Re-weight simulations to produce desired results (REVO)

REVO - Catastrophe Modelling
Adjusting output from third-party models

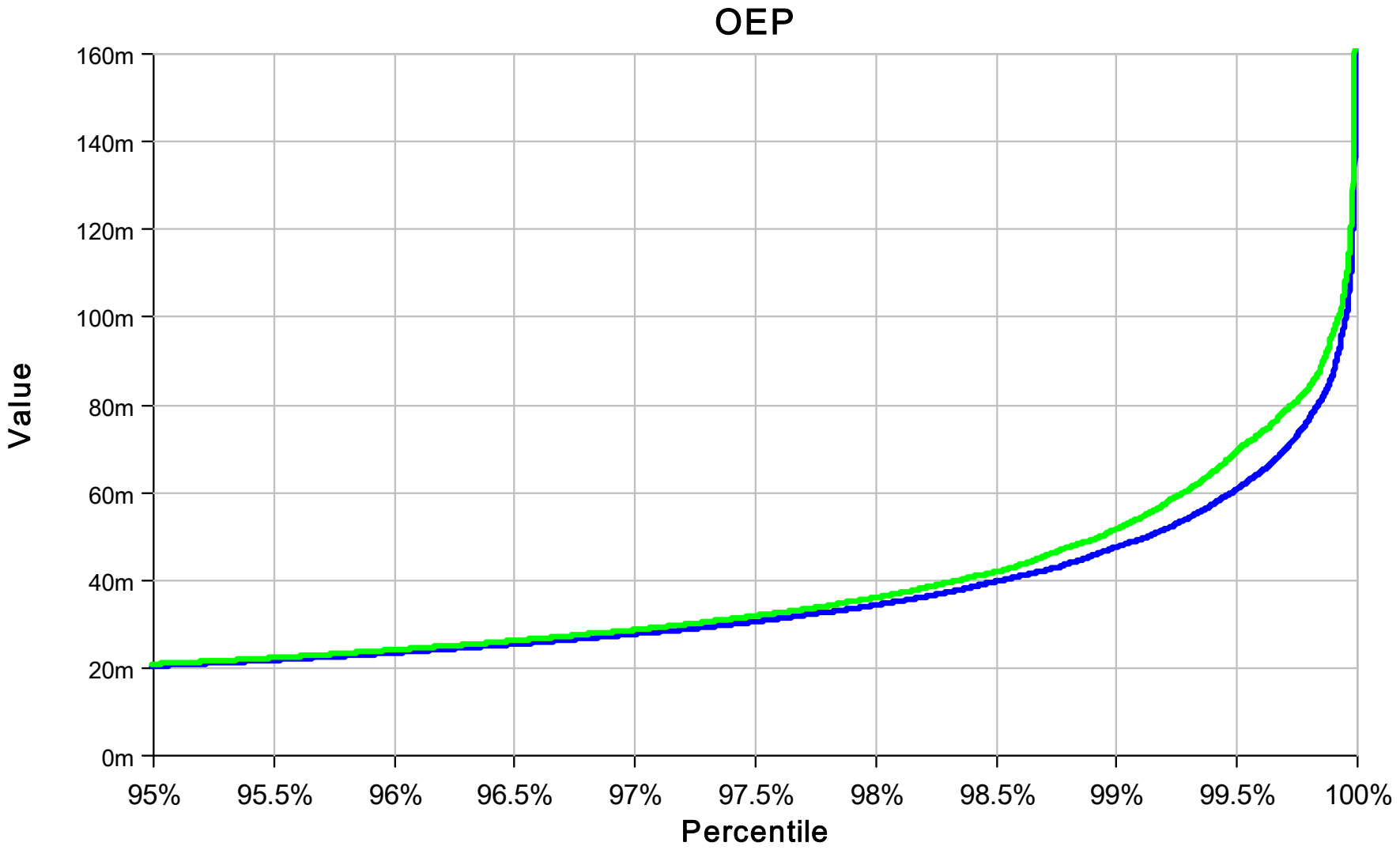
- Use REVO approach to specify mean and/or percentiles for AEP/OEP at class or portfolio level
- Apply separately to each independent peril
- For multi-year output, apply to one-year then re-shuffle
- Start with larger number of sims
- Sample down to desired number of sims (stratified)

REVO - Catastrophe Modelling
Adjusting output from third-party models

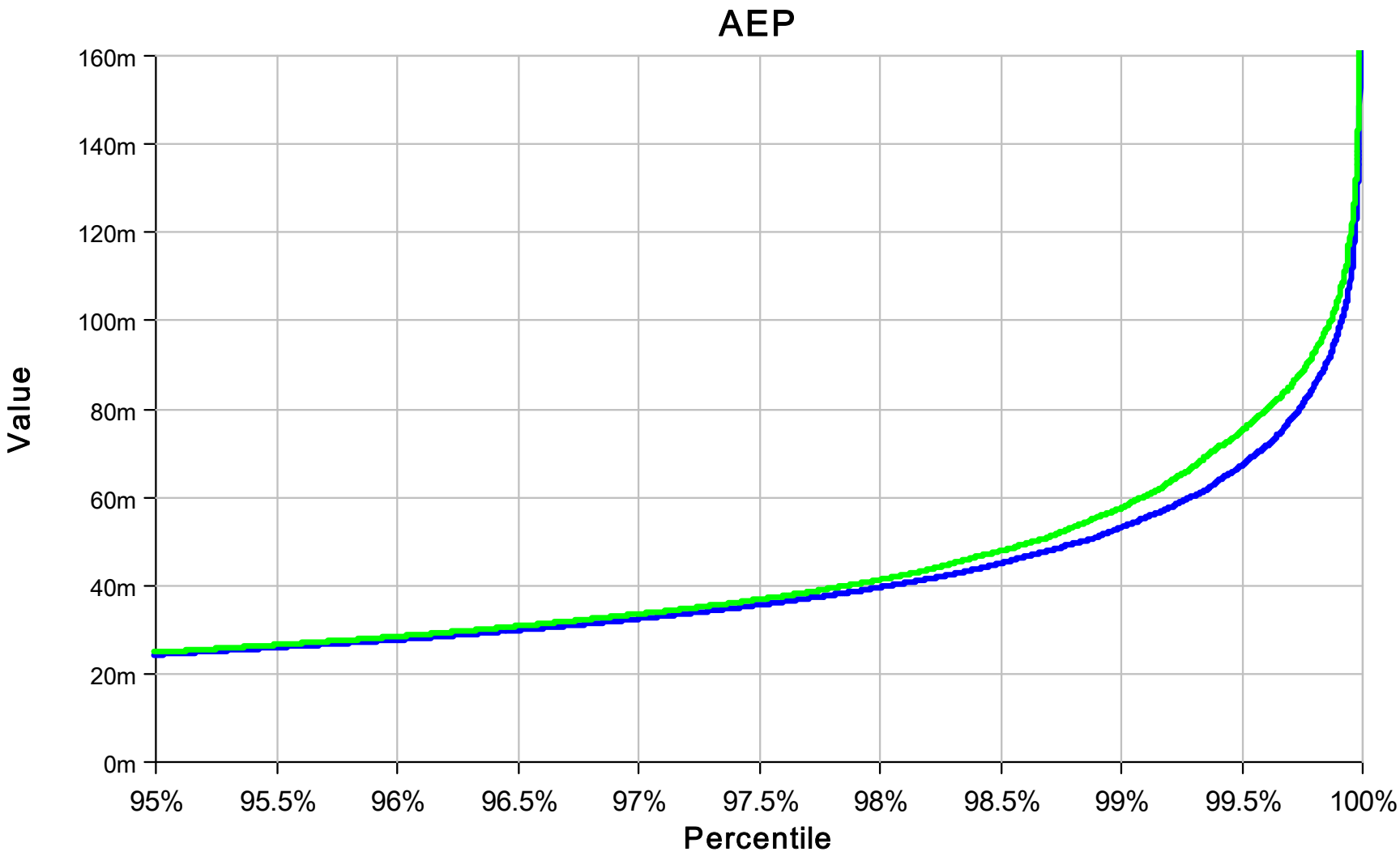
- Example
  - Load model for model uncertainty / data quality at high return periods
  - Increase OEP at 99.5th percentile by 15% at portfolio level

Variable	Measure	Percentile	Original Value	View	Confidence
OEP	Percentile	99.5th	£60.8m	£69.9m	100%

REVO - Catastrophe Modelling
Adjustment of OEP



REVO - Catastrophe Modelling
Effect on AEP



REVO - Catastrophe Modelling
Before and After OEP Adjustment

Statistics for Variables				
	AEP		OEP	
	Original	Re-weighted	Original	Re-weighted
Mean	5.4	5.6	4.6	4.7
SD	10.8	11.6	9.5	10.3
10th percentile	0.0	0.0	0.0	0.0
25th percentile	0.0	0.0	0.0	0.0
50th percentile	1.3	1.3	1.1	1.1
60th percentile	2.5	2.6	2.1	2.1
70th percentile	4.5	4.5	3.7	3.7
80th percentile	7.9	8.0	6.4	6.5
90th percentile	15.1	15.4	12.5	12.6
95th percentile	24.3	24.9	20.4	21.0
99th percentile	53.2	58.2	47.6	52.0
99.5th percentile	67.2	75.7	60.8	70.0
99.75th percentile	80.9	88.8	73.1	81.5
99.9th percentile	97.9	104.8	87.5	96.4

REVO - Catastrophe Modelling
Add sensitivity test constraint

- Load model for model uncertainty / data quality at high return periods
- Increase OEP at 99.5th percentile by 15% at portfolio level
- Do not adjust AEP mean amount

Variable	Measure	Percentile	Original Value	View	Confidence
OEP	Percentile	99.5th	£60.8m	£69.9m	100%
AEP	Mean		£5.4m	£5.4m	100%

REVO - Catastrophe Modelling
Before and After Adjustment

Statistics for Variables				
	AEP		OEP	
	Original	Re-weighted	Original	Re-weighted
Mean	5.4	5.4	4.6	4.6
SD	10.8	11.2	9.5	10.0
10th percentile	0.0	0.0	0.0	0.0
25th percentile	0.0	0.0	0.0	0.0
50th percentile	1.3	1.2	1.1	1.1
60th percentile	2.5	2.5	2.1	2.1
70th percentile	4.5	4.4	3.7	3.6
80th percentile	7.9	7.7	6.4	6.3
90th percentile	15.1	14.9	12.5	12.2
95th percentile	24.3	24.1	20.4	20.2
99th percentile	53.2	56.8	47.6	50.9
99.5th percentile	67.2	75.3	60.8	70.1
99.75th percentile	80.9	88.4	73.1	80.9
99.9th percentile	97.9	104.0	87.5	95.9

REVO - Catastrophe Modelling
Sensitivity Testing

- Suppose we want to sensitivity test cat assumptions e.g. by adjusting the overall 99.5th percentile OEP curve amount
- Standard approach:
  - Create multiple sets of cat results (using REVO or other technique)
  - Re-run the entire internal model for each one
- However, since each sensitivity test is simply a re-weighting of the original cat simulations there is no need to do it this way!
- Instead, apply re-weighting of simulations at the END of the base model run
- We can now perform all the sensitivity tests in a few minutes rather than several hours

REVO - Catastrophe Modelling
Sensitivity Testing of 99.5 percentile OEP

- Model run once on the Base case
- Entropy re-weighting used to perform sensitivity tests

	Cat Sensitivity Testing Results (£m)			
	Mean Profit	SCR	Economic Capital	RI Value Creation
-10%	7.0	24.5	26.8	-0.2
Base case	6.9	25.0	27.8	0.4
+10%	6.9	25.2	28.4	1.0
+20%	6.8	25.8	29.8	1.4

REVO - Catastrophe Modelling
Stress Testing

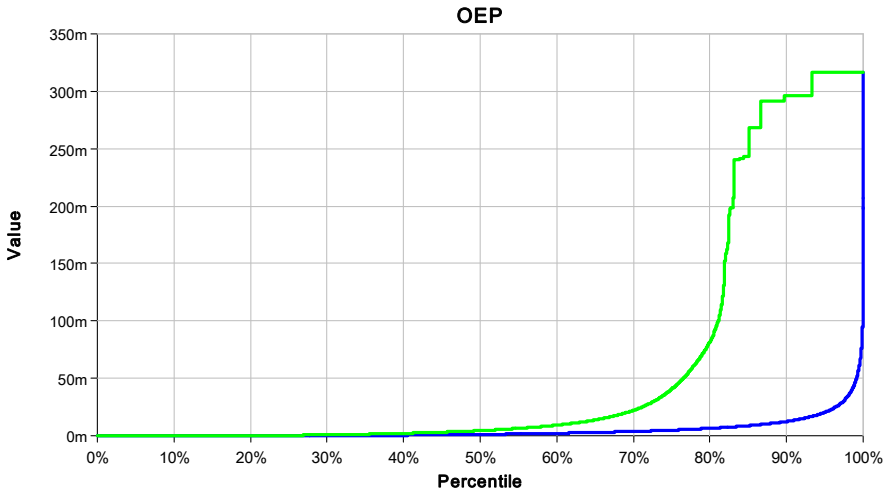
- E.g. What is the impact of a 1 in 200 cat event?
- One approach: generate single 99.5% OEP percentile amount from cat model and run reinsurance model to generate net impact
- Problems
  - Can perform this for each peril, but what about overall 1 in 200 event across perils (and entities)?
  - Reinsurance may respond differently depending on how loss is spread across classes, perils, entities
  - Awkward to pick up dependencies with other risks e.g. reinsurer credit risk, operational risk
  - May also wish to include expected effect of other risks in the stress test result
  - Requires model re-run

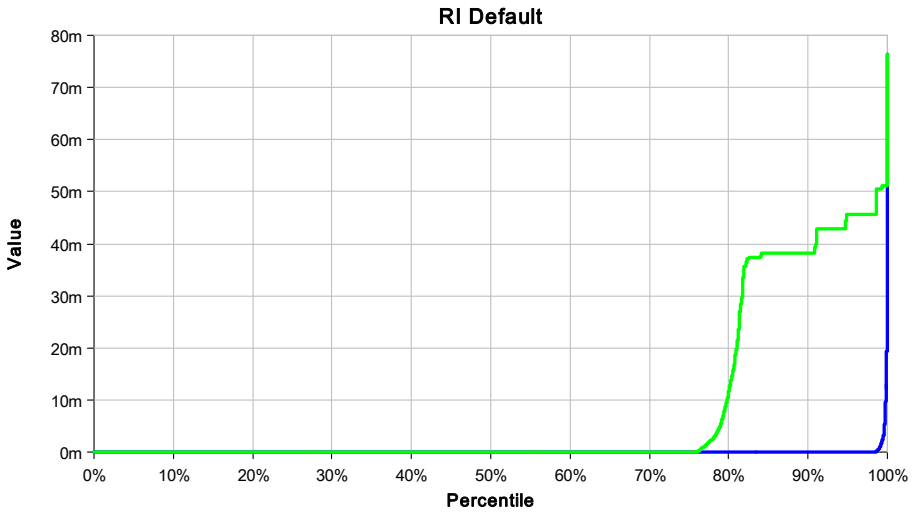
REVO - Catastrophe Modelling
Stress Testing

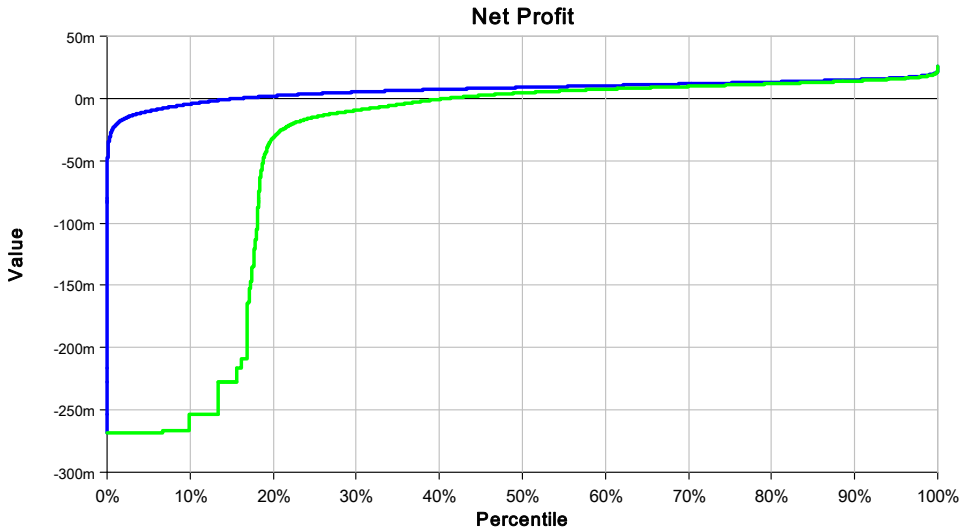
- Alternative approach
  - Set adjusted Cat OEP mean equal to the stress test amount
  - Apply simulation re-weighting
  - Take expected value of all results of interest

Variable	Measure	Original Value	Stress	Confidence
OEP	Mean	£4.6m	£60.8m	100%

REVO - Catastrophe Modelling
Stress Testing: Results Before and After







REVO - Catastrophe Modelling
Stress Testing: Results Before and After

Statistics for Variables								
	AEP		OEP		Net Profit		RI Default	
	Original	Re-weighted	Original	Re-weighted	Original	Re-weighted	Original	Re-weighted
Mean	5.4	62.9	4.6	60.8	6.8	-42.2	0.1	8.1
SD	10.8	110.6	9.5	109.0	8.6	98.3	1.0	16.1
10th percentile	0.0	0.0	0.0	0.0	-4.4	-253.1	0.0	0.0
25th percentile	0.0	0.4	0.0	0.3	3.8	-15.0	0.0	0.0
50th percentile	1.3	5.4	1.1	4.5	8.8	4.5	0.0	0.0
60th percentile	2.5	11.2	2.1	9.2	10.3	7.4	0.0	0.0
70th percentile	4.5	25.9	3.7	22.1	11.6	9.6	0.0	0.0
80th percentile	7.9	88.6	6.4	81.7	13.1	11.7	0.0	11.5
90th percentile	15.1	295.9	12.5	295.9	15.0	14.0	0.0	38.1
95th percentile	24.3	316.3	20.4	316.3	16.4	15.6	0.0	45.7
99th percentile	53.2	316.3	47.6	316.3	18.9	18.3	0.8	50.6
99.5th percentile	67.2	316.3	60.8	316.3	19.7	19.1	3.2	51.0
99.75th percentile	80.9	316.3	73.1	316.3	20.4	19.9	7.9	51.0
99.9th percentile	97.9	316.3	87.5	316.3	21.2	20.9	15.9	51.0

REVO - Catastrophe Modelling Summary

- Entropy approach can help in 3 key areas:
  - Adjustment of model output
  - Sensitivity testing
  - Stress testing
- In many cases, model re-runs can be avoided
- Preserves dependencies in the model
- Can easily perform sensitivity and stress tests which do not directly relate to model parameters e.g.
  - Catastrophe claims
  - Lloyd's ICA ULR and claims reserve stress tests

Questions or comments?

Expressions of individual views by members of The Actuarial Profession and its staff are encouraged.

The views expressed in this presentation are those of the presenter.