Institute
and Facultyof Actuaries

GIRO40

8 – 11 October, Edinburgh

Institute
and Faculty
of Actuaries

110 Years of Ruin Theory: How can it help risk management today?

Corina Constantinescu, IFAM, Liverpool
Jo Lo, Aspen
Meng (Simon) Wang

ertise Onsorship Thought leadership Progress Community Sessional Meetings Education Working parties Volunteering Research Shaping the future Networking Professional support Enterprise and risk Learned society Opportunity International profile Journals Suppo

1. Can ruin theory help?

- Can ruin theory help …,
- … or is it to be forgotten after our studies?

- Are we holding too little capital? Too much?
- Which is better: reinsurance or capital?
- Is there an optimal exposure we should be writing?
- …

1. Aims of this workshop

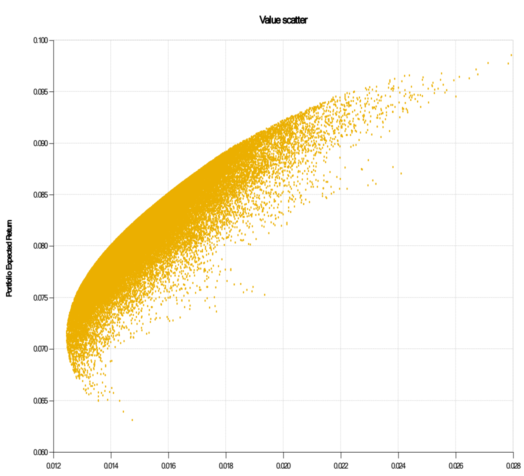
- Can ruin theory help with risk management questions
  - Our conclusion will be: its strength lies in its ability to explore certain problems from different angles
  - What will yours be?

- Is the mathematics too complex? → Go through mathematics
- What can I do with a model for ruin? → Explore risk-return optimisation ideas
- Is it easy to use? → Demonstrate macro-free spreadsheet

1. Optimisation

- Challenging investment and premium rate environments
- Part of modern risk management in G.I. companies
- Are our work being used by others in such exercises correctly?
- If suspicious, can think portfolio *enhancement* rather than optimisation

2a. Risk-return optimisation: classical theory



- Markowitz (1952); Merton (1972); ST5
- "Second stage" of portfolio selection
- Parabolic efficient frontier on the V-E plane

2b. Risk-return optimisation: use

- Part of a toolkit
- Dependent on “first stage” – estimation

- Risk: represented by S.D. of P.V. of returns
- Return: represented by E.P.V.

- Difficult to represent risk and return in one single metric
- Generally: What discount rates?
- For G.I.: Extreme Tails?

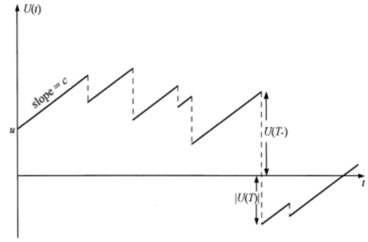
3. Ruin theory as a risk-return tool

- Lundberg (1903); Cramér (1930); CT6

$$U_t = u + ct - \sum_{i=1}^{N_t} X_i$$

- For $N_t \sim \mathrm{Po}(\lambda t)$ and $X_i \sim \mathrm{Exp}(1/\beta)$,

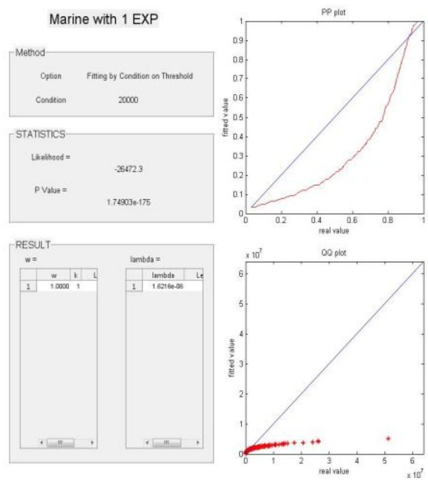
$$\Psi(u) = \frac{\lambda}{\beta c} \exp\left[-u\left(\beta - \frac{\lambda}{c}\right)\right]$$



- Constant dividend $= D.u$ until ruin
- Incorporate into constant *c*
- Risk: represented by $\psi(u)$
- Return: represented by *D*

3a. Hang on… aren't exponential claim severities unrealistic?

- Yes, they are!
- But other distributions are allowed…



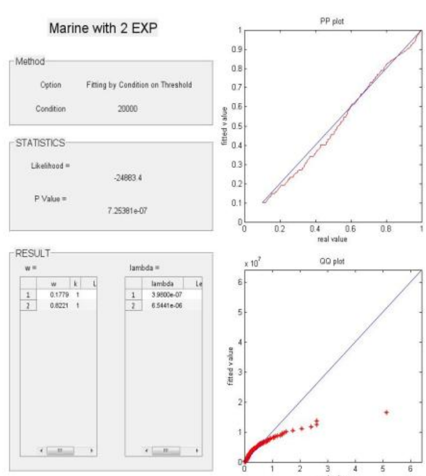
3a. Mixed exponentials give analytic solutions

- Simple PDF:

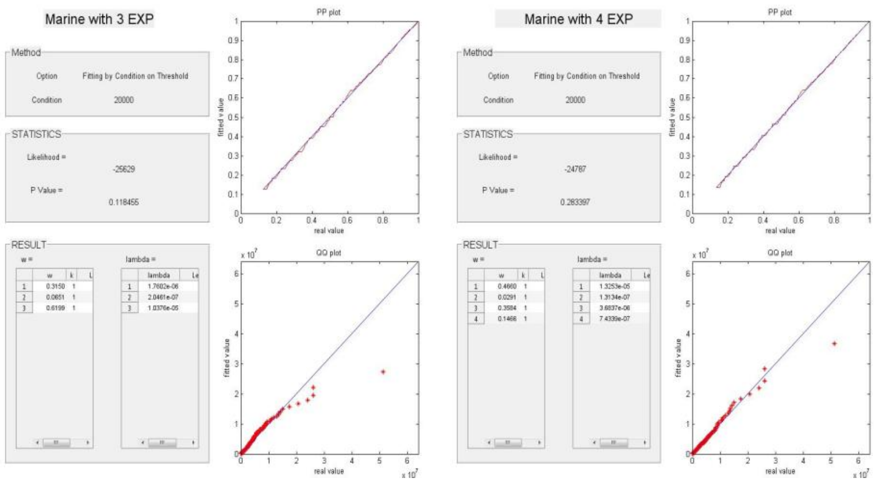
$$f_X(x) = \sum_{i=1}^{n} A_i \beta_i e^{-\beta_i x}$$

- Simple form for ruin:

$$\Psi(u) = \sum_{i=1}^{n} C_i e^{-r_i u}$$



3a. Mixed exponentials are flexible

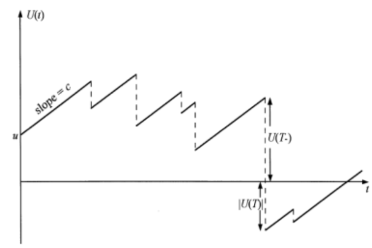


3b. Other advances?

- Many since 1903: shall discuss towards the end
- Simpler models are often better
  - for implementation, and for interpretation
- High-level indications to inform strategic decisions
  - not about detailed and “accurate” predictions

4. Classical risk model

- $U(t) = u + ct - S(t)$
- $S(t) = \sum_{k=1}^{N(t)} X_k$
- $\Psi(u) =$ $\mathrm{P}(U(t) < 0$, for some $t | U(0)$
- $\Psi(\infty) = 0$



5. IE → IDE → ODE

- $$\Psi(u) = \mathbf{E}\big(\Psi(u + cT_1 - X_1)\big)$$
- **IE**: $\Psi(u) = \int_0^\infty f_T(t) \int_0^\infty \Psi(u + ct - x) f_X(x) dx\, dt$
  $\boldsymbol{f_T(t) = \lambda e^{-\lambda t}}$
- **IDE**: $\left(-c\frac{d}{du} + \lambda\right)\Psi(u) = \lambda \int_0^\infty \Psi(u - x) f_X(x) dx$
  $\boldsymbol{f_X(x) = \beta e^{-\beta x}}$
- **ODE**: $\left(\frac{d}{du} + \beta\right)\left(-c\frac{d}{du} + \lambda\right)\Psi(u) = \lambda\beta\Psi(u)$

6. Solving the ODE

- $$\left(\frac{d}{du}+\beta\right)\left(-c\frac{d}{du}+\lambda\right)\Psi(u)=\lambda\beta\Psi(u)$$
- Characteristic equation: $(s+\beta)(-cs+\lambda)-\lambda\beta=0$
- Exponents: $s\left(-cs+(\lambda-c\beta)\right)=0$
- Solution (w/out boundary conditions): $\Psi(u)=C_1+C_2e^{-\left(\beta-\frac{\lambda}{c}\right)u}$

$$\boldsymbol{\Psi(u)=\Psi(0)e^{-Ru}}$$

7a. Implementation: obtaining parameters

Planning inputs → Model parameters → Probability of ruin

- *Model parameters can incorporate richer assumptions*
- Inputs for *c*
  - Premium rate (p.a.), expense ratio (as % of premiums), real dividend rate (as % of initial capital, *u*)
  - *c* = premium rates * (1 – expense ratio) – *u* * real dividend rate
- Stochastic inputs – calibrated elsewhere (internal model?)
  - Does not have to be underwriting losses only!
- Inputs for *u*
  - Note maximum *u* check

7b. Implementation: calculations

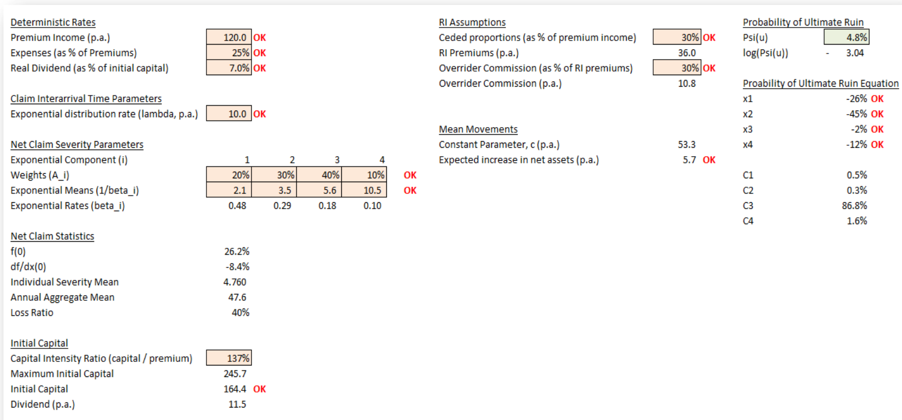
- Planning inputs → Model parameters → Probability of ruin
- *Aim to avoid messy simulations*
- Calculate coefficients of characteristic equation
- Solve equation, assuming distinct roots
  - Using algorithm for quartic equations
- Calculate boundary conditions
- Use inverse matrix to solve them
- Calculate C_i and r_i
- Calculate $\Psi(u)$

7c. Implementation: care when using it

- Planning inputs → Model parameters → Probability of ruin
- Should test it to your satisfaction
  - E.g. use simulations to see if the $\Psi(u)$ matches
  - Have tested it under a handful of assumption sets
- Large numbers could distort answers
  - Inputs in e.g. £m rather than £

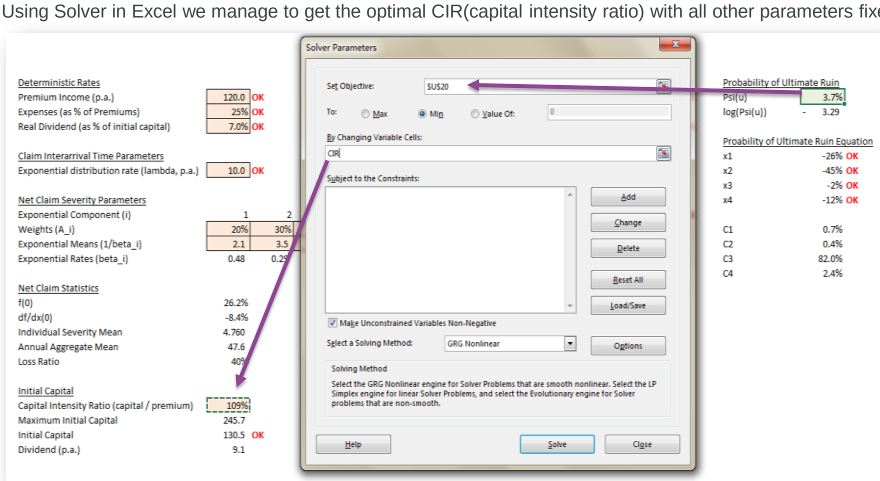
8. Capital Setting Example

Screenshot of spreadsheet

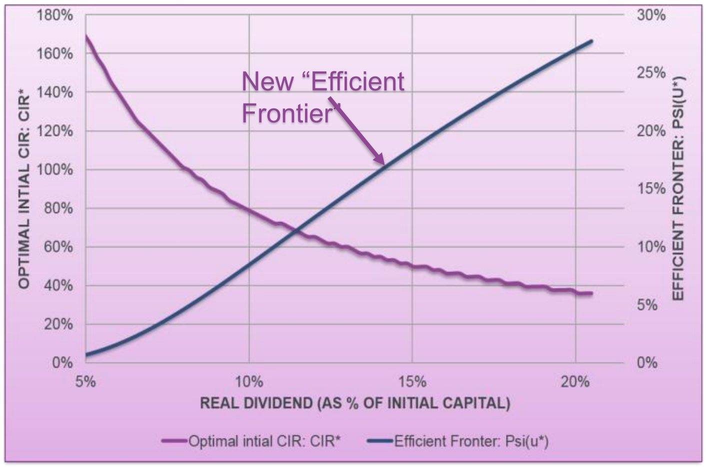


8. Capital Setting Example

Using Solver in Excel we manage to get the optimal CIR(capital intensity ratio) with all other parameters fixed.

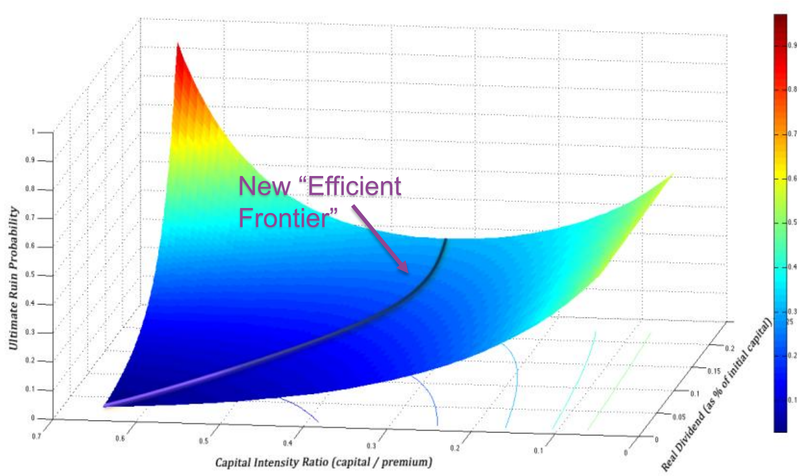


9. A New “Efficient Frontier”



This is an “Efficient Frontier” drawing with: Premium Income (p.a.):120.0; Expenses (as % of Premiums): 25%; Real Dividend (as % of initial capital): 0% to 20%; Exponential distribution rate (lambda, p.a.):10; Capital Intensity Ratio (capital / premium): 0% to 170%; Ceded proportions (as % of premium income): 30%; Overrider Commission (as % of RI premiums): 30%.

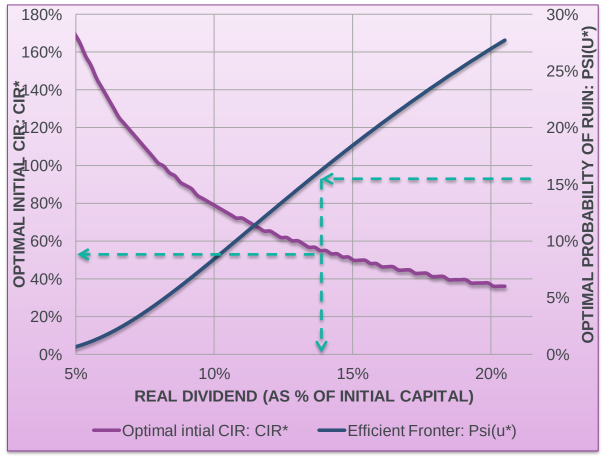
9. A New “Efficient Frontier”



This is a 3D “Efficient Frontier” drawing with: Premium Income (p.a.):120.0; Expenses (as % of Premiums): 25%; Real Dividend (as % of initial capital): 0% to 25%; Exponential distribution rate (lambda, p.a.):10; Capital Intensity Ratio (capital / premium): 0% to 70%; Ceded proportions (as % of premium income): 30%; Overrider Commission (as % of RI premiums): 30%.

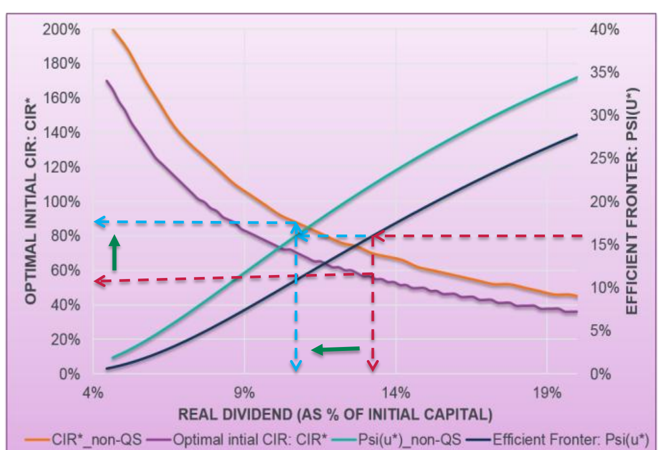
9a. Optimal dividends and capital

- If we want at most 15% chance of ruin, what is the optimal combination initial capital and dividend ratio?
- How about if we want a dividend at 20% of initial capital?



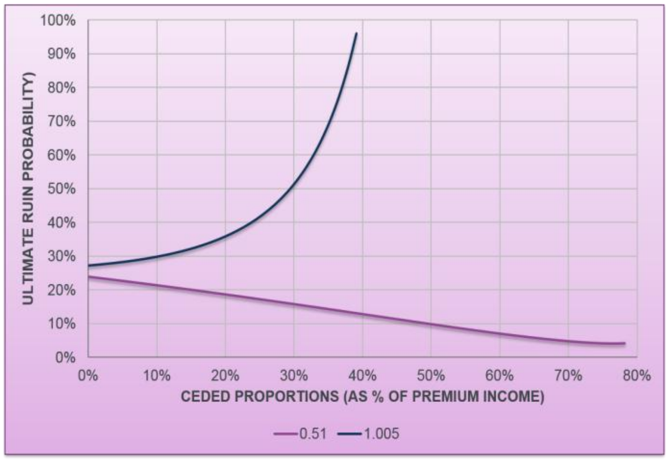
9b. What if we no longer have QS?

- What if Quota-share reinsurance no longer available?



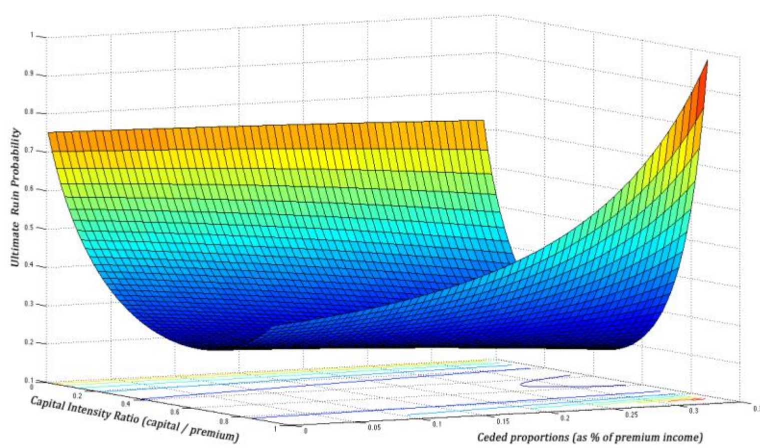
This is an "Efficient Frontier" drawing with: Premium Income (p.a.):120.0; Expenses (as % of Premiums): 25%; Real Dividend (as % of initial capital): 4% to 20%; Exponential distribution rate (lambda, p.a.):10; Capital Intensity Ratio (capital / premium): 0% to 200%; Ceded proportions (as % of premium income): 0% & 30%; Overrider Commission (as % of RI premiums): 0% & 30%.

9c. Reinsurance or not?



This is a Ruin probability drawing with: Premium Income (p.a.):120.0; Expenses (as % of Premiums): 25%; Real Dividend (as % of initial capital): 13%; Exponential distribution rate (lambda, p.a.):10; Capital Intensity Ratio (capital / premium): 51% and 100.5%; Ceded proportions (as % of premium income): 0% to 78%; Overrider Commission (as % of RI premiums): 30%.

9c. Reinsurance or not?



This is a 3D Ruin probability drawing with: Premium Income (p.a.):120.0; Expenses (as % of Premiums): 25%; Real Dividend (as % of initial capital): 15%; Exponential distribution rate (lambda, p.a.):10; Capital Intensity Ratio (capital / premium): 0% and 100%; Ceded proportions (as % of premium income): 0% to 35%; Overrider Commission (as % of RI premiums): 30%.

10. Can Ruin Theory be helpful?

- Simplified version of reality…
  - … but what model isn't?
- The key is:
  - When used properly, …
  - … can it help answer key questions in decision making?
- Considers problems through very different point of view, …
  - … which can be helpful
- Simple assumptions also helps implementation…
  - … the work is in calibration – leverage off S2 work?

10a. Can it contribute to capital setting?

- Current approaches considers
  - internal risk appetites
  - external requirements
  - general market environments
- Presented approach contributes by
  - Considering from risk-return optimality perspective…
  - ... with tail-sensitive risk metrics; avoids use of remote percentiles …
  - … and with model assumptions, of course…
  - … but at least can provide a starting point to answering the problem

10b. How about reinsurance decisions?

- Current approaches involve
  - Quantitative evaluations of quoted prices; impacts on P&L and BS
  - Consideration of commercial environments, market practice and external requirements
- Risk-return considerations / optimisation increasingly popular
- Presented approach contributes by
  - Considering long-term stable relationship with reinsurers…
  - … gives additional information via optimality

11. What we have not discussed

- Renewal risk models – *N(t)* renewal process -

$$f_T(x) = \sum_{i=1}^{n} A_i \lambda_i e^{-\lambda_i x}$$

- Risk models perturbed by a diffusion

$$U_t = u + ct - \sum_{i=1}^{N_t} X_i + \sigma B_t$$

- Risk models with dividends paid according to a barrier strategy
- Summer collaboration experience (MSc projects written in cooperation with insurance industry)

12. Summary

- *Can ruin theory help answer risk management questions?*
- Went through mathematics (which was quite straightforward?)
- Evaluated model in two situations
  - helps giving another viewpoint …
  - …through optimality and long-term considerations
  - beware of spurious accuracy
  - simplifying assumptions can help …
  - … or can sometimes be improved on
- Demonstrated macro-free spreadsheet
  - simple enough to use solver or to give multiple scenarios

Questions Comments

Expressions of individual views by members of the Institute and Faculty of Actuaries and its staff are encouraged.

The views expressed in this presentation are those of the presenter.