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A new, cohort-based, mortality model (Work in progress)*

John Kingdom

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*This work represents research undertaken outside the work environment and reflects entirely personal views.

Agenda

- Introduction (5 mins)
- Description of the model (10 mins)
- Estimation of the model (5 mins)
- Model results (15 mins)
- Next steps and discussion (10 mins)

Non-actuaries like mortality too!

"*On a long enough time line, the survival rate for everyone drops to zero*" – **Tyler Durden, Fight Club**

"*It's not that I'm afraid to die, I just don't want to be there when it happens*" – **Woody Allen**

"*Nothing in life is certain except death and taxes*" – **Benjamin Franklin**

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Introduction [1]

- Most existing mortality models analyse dynamics of the ‘age-period’ mortality curve over time.

- For example, in the Lee-Carter model, mortality rates across ages at a given point in time (i.e. the age-period mortality curve) are modelled as a function of a derived mortality index. This mortality index is then projected forward to forecast future period mortality curves:

$$\mathbb{E}\left(\log m\left(x,t\right)\right) = \alpha\left(x\right) + \beta\left(x\right) \cdot \kappa_t$$

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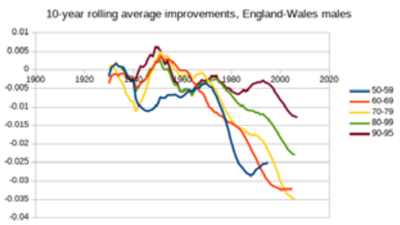
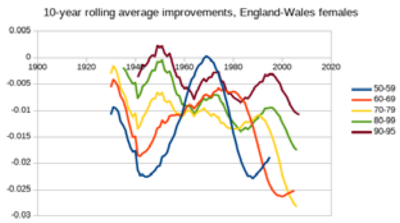
Introduction [2]





- However, patterns in mortality improvements are very difficult to capture as the dynamics in this space seem to vary over time.
- Fitting a simple age-period mortality model such as the Lee-Carter to mortality data will lead to biases across ages and time periods.
- More complex age-period models could produce better results but risk over-fitting and could be difficult to put into context.

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Introduction [3]

- Our proposed model takes a different approach and instead models the 'cohort-period' mortality curve i.e. our focus is on modelling expected mortality rates over the life of a given cohort.
- Central hypothesis: the best predictor for the expected mortality rate for a life aged x at time t is the expected mortality rate of that same life a year earlier i.e. the same life aged $x - 1$ at time $t - 1$.
- In other words, the model projects mortality *advancements* rather than mortality improvements.
- We also add cohort- and period-specific effects by conditioning the following year's expected mortality rate for a particular cohort on current year and year of birth.

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Introduction [4]

- By fitting the model to population mortality data from a range of countries (UK , France, Italy, Japan), we obtain the following observations:
  - our model identifies a new 'golden cohort' in the UK. All else equal, UK males and females born around 1950 - those currently around typical retirement age - exhibit significantly lower rates of mortality advancements than their previous 1900s counterparts; a similar pattern is found in most other countries considered.
  - in every country considered, males have experienced roughly equal or lower mortality advancements than females, particularly in more recent times
  - mortality advancement rates in England and Wales have been relatively high on average but have been the lowest since 2000
  - period effects appear generally to be stronger than cohort effects - particularly so in the UK, where, surprisingly, cohort effects appear particularly weak.

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Introduction [5]

- Areas for future consideration:
  - Backtesting results are good but suggest that future iterations of the model may benefit further from the introduction of an age-related factor
  - The model should also consider the inclusion of a dummy variable to better take into account the effects of WWII in the data
  - Suggestions from the audience.

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Description of the model [1]

- Our model begins with the notion that the expected mortality rate for a given cohort *c* at age *x* is a function *f* of the expected mortality rate for that same cohort at age *x* − *1*, the cohort *c*, and the period *t*:

$$\bar{q}_{x,c} = f\left(\bar{q}_{x-1,c}, c, t_{x,c}\right)$$

- Without loss of generality, we can re-express this in multiplicative form:

$$\bar{q}_{x,c} = \Lambda\left(\bar{q}_{x-1,c}, c, t_{x,c}\right) \cdot \bar{q}_{x-1,c}$$

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Description of the model [2]

- The next step, for estimation purposes, is to take the logit of the mortality rate, to restrict this in our model within the interval (0, 1):

$$\xi_{x,c} = \log \frac{q_{x,c}}{1 - q_{x,c}}$$

- Applying the logit to both sides of this equation, we can get the following equation:

$$\bar{\xi}_{x,c} = \lambda \left(\bar{\xi}_{x-1,c}, c, t_{x,c} \right) + \bar{\xi}_{x-1,c}$$

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Description of the model [3]

- We then hypothesize that λ takes the following form:

$$\lambda \left(\bar{\xi}_{x-1,c}, c, t_{x,c} \right) = \beta + \gamma \left(c \right) + \delta \left(t_{x,c} \right)$$

- Given this, we can express expected logit mortality rates as follows:

$$\bar{\xi}_{x,c} = \beta + \gamma \left(c \right) + \delta \left(t \right) + \bar{\xi}_{x-1,c}$$

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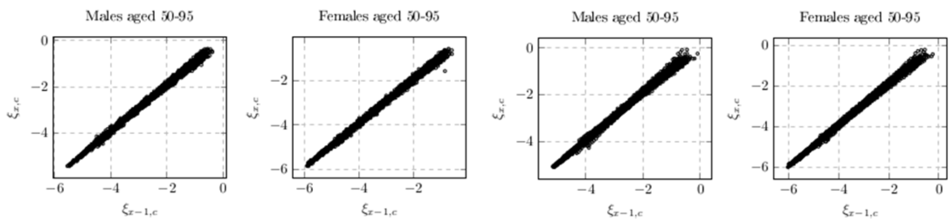
Description of the model [4]

- This linear relationship in the logit of the mortality rate is strongly supported by the data:



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Estimation of the model [1]

- For the purposes of estimating our model, we assume its error terms are *iid* normal:

$$\xi_{x,c} = \beta + \gamma\left(c\right) + \delta\left(t\right) + \xi_{x-1,c} - \tilde{\epsilon}_{x-1,c} + \tilde{\epsilon}_{x,c}$$

$$\tilde{\epsilon}_{x,c} \sim N\left(0, \sigma^2\right) \forall x, c$$

- We estimate the model by least squares.

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Estimation of the model [2]

$$\min_{\beta,\gamma(c),\delta(t),\bar{\xi}_{x_0(c),c}} \sum_x \sum_c \epsilon^2_{x,c}, \forall x, c$$

subject to:

$$\epsilon_{x,c} = q_{x,c} - \alpha - (\beta + 1) \cdot (q_{x-1,c} - \epsilon_{x-1,c}) - \gamma(c) - \delta(t), x \neq x_0(c)$$

$$\epsilon_{x_0(c),c} = \xi_{x_0(c),c} - \bar{\xi}_{x_0(c),c}$$

$$t = x - c$$

$$\sum_c \gamma(c) = 0$$

$$\sum_t \delta(t) = 0$$

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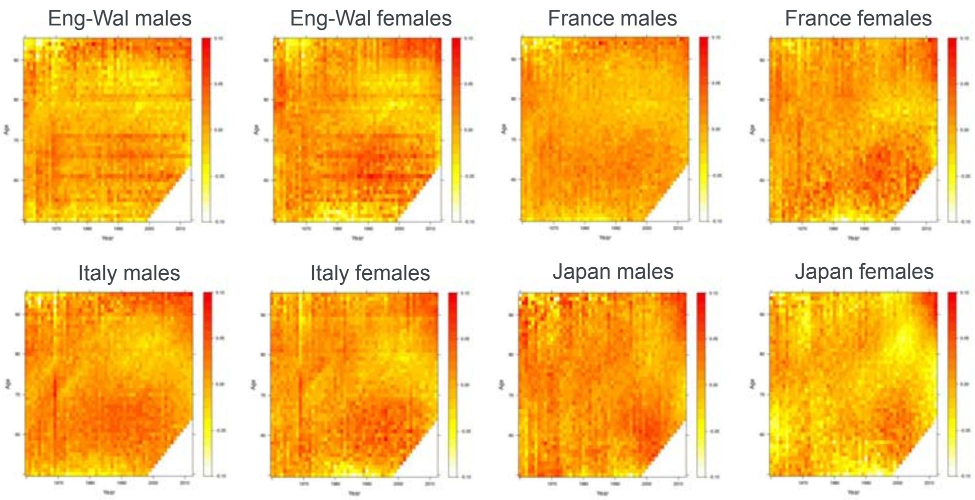
Estimation of the model [3] – residual heatmaps



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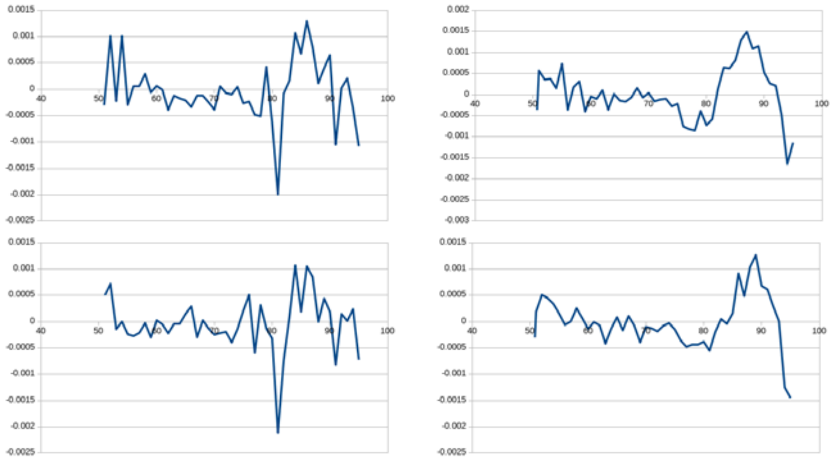
Estimation of the model [4] – average residuals by age

England-Wales France

Males

Females



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Model results [1] – Average mortality advancement rates by country

<u>1922 - 2013</u>

	Males	Females
England-Wales	0.089	0.085
France	0.077	0.078
Italy	0.086	0.089
Japan	0.081	0.081
USA	0.078	0.081

England-Wales has had historically high mortality advancements…

<u>2000 - 2013</u>

	Males	Females
England-Wales	0.075	0.087
France	0.075	0.094
Italy	0.082	0.1
Japan	0.083	0.091
USA	0.077	0.088

… but these have been relatively low in more recent times.

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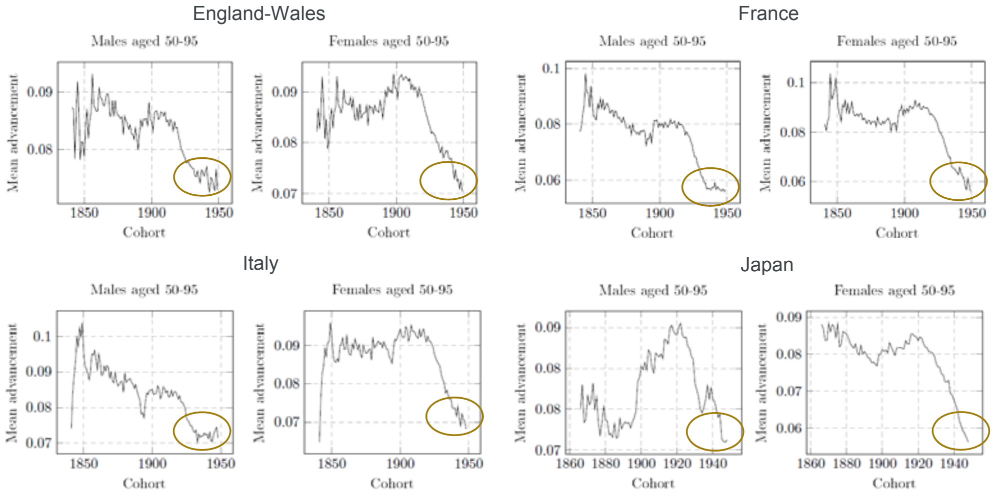
Model results [2] – Average mortality advancement rates by cohort



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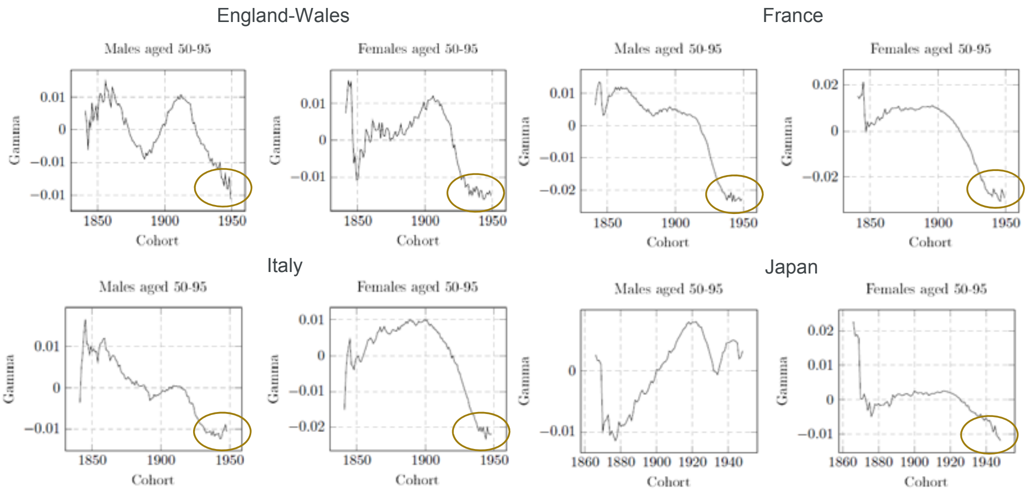
Model results [3] – Breakdown into cohort effects (gamma plots)



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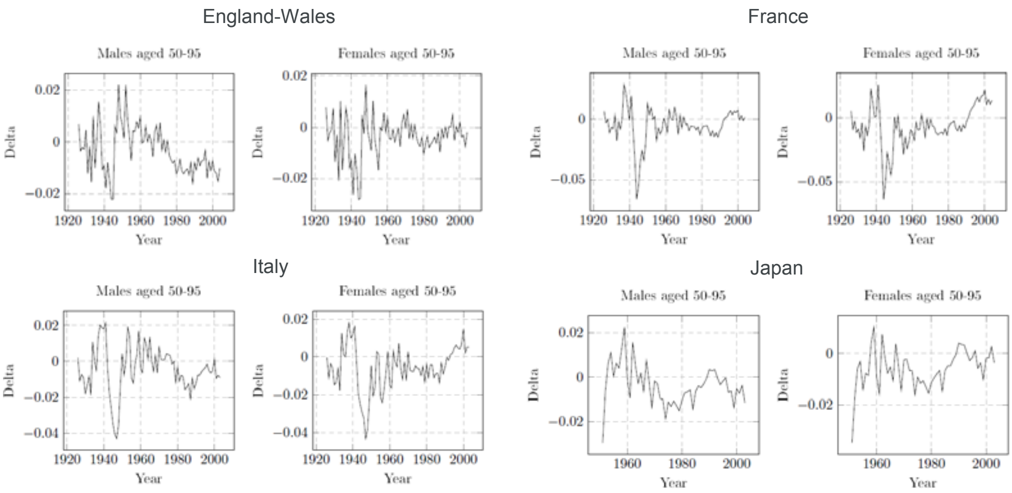
Model results [4] – Breakdown into period effects (smooth delta plots)



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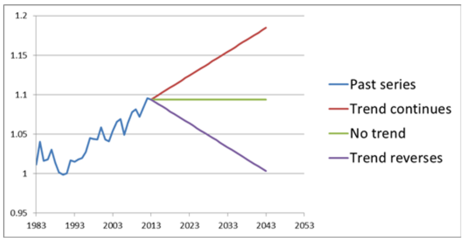
Projection of the model



Illustrative results*

	E-W male	E-W female
65-75	1.9% / 2.0% / 2.1%	2.0% / 2.0% / 2.1%
76-85	2.3% / 2.5% / 2.8%	2.6% / 2.7% / 2.9%
86-95	2.4% / 2.8% / 3.2%	2.9% / 3.1% / 3.4%

* Work in progress – further development required

- The model can be used to forecast future mortality rates with the projection of the single time series variable. For illustration purposes, we have done this simply by assuming that the time series projection assumes that:
  - the trend in the time variable over the previous 30 years is sustained
  - there is no change in the time variable
  - the trend in the time variable over the previous 30 years is reversed.

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Model validation [1] – Correlations between time indices (delta plots)

- The table below provides the correlation of period effects, taken over the time period considered (1922-2013). The correlation is measured for changes in the (t) variable of each fitted model. Where there is less data available, correlations are measured over the longest data available.

	EW_m	EW_f	Fr_m	Fr_f	It_m	It_f	Jp_m	Jp_f	US_m	US_f
EW_m	1									
EW_f	0.97	1								
Fr_m	0.42	0.46	1							
Fr_f	0.51	0.56	0.97	1						
It_m	0.28	0.25	0.77	0.72	1					
It_f	0.22	0.21	0.76	0.72	0.98	1				
Jp_m	0.32	0.25	0.31	0.33	0.4	0.33	1			
Jp_f	0.31	0.23	0.28	0.3	0.38	0.33	0.99	1		
US_m	0.38	0.38	0.42	0.4	0.42	0.41	0.43	0.45	1	
US_f	0.38	0.38	0.38	0.38	0.38	0.37	0.42	0.43	0.97	1

Sources of comfort

Intra-country correlations are high

Correlations are higher between countries which are geographically close

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Model validation [2] – Backtesting

- The model was fitted on England-Wales data for males and females for the data period 1922-1983. Mortality rates were then projected forward by selecting a constant trend parameter (delta) which broadly matches observed *average* advancement rates.
- The projected output was then compared to actual mortality rates observed to assess whether any strong patterns can be observed in the residual across ages and over time.
- The backtesting performs well, but suggests that the model may benefit further with the introduction of an age-related factor.

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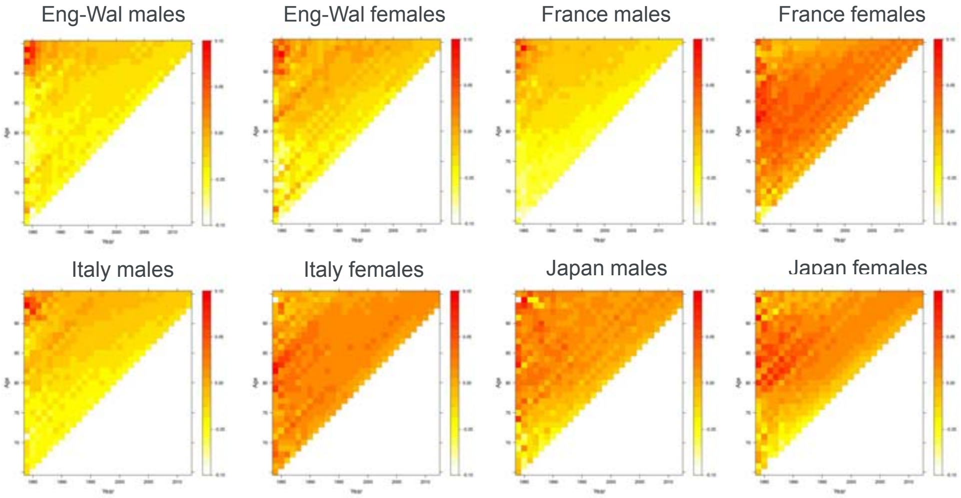
Model validation [3] – Backtesting results: annualised residuals



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Conclusions and next steps

- New way to consider how mortality rates can be analysed and projected forward – by considering mortality rate advancements.
- Ability to capture complex mortality trends and project rates forward by projecting only a single parameter.
- Cohorts born in around 1950 generally experience lower mortality rate advancements than their counterparts.
- In every country considered, males have experienced roughly equal or lower mortality advancements than females, particularly in more recent times.
- Backtesting results are good but suggest that the performance of the model may benefit further by introducing an age-related factor.

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