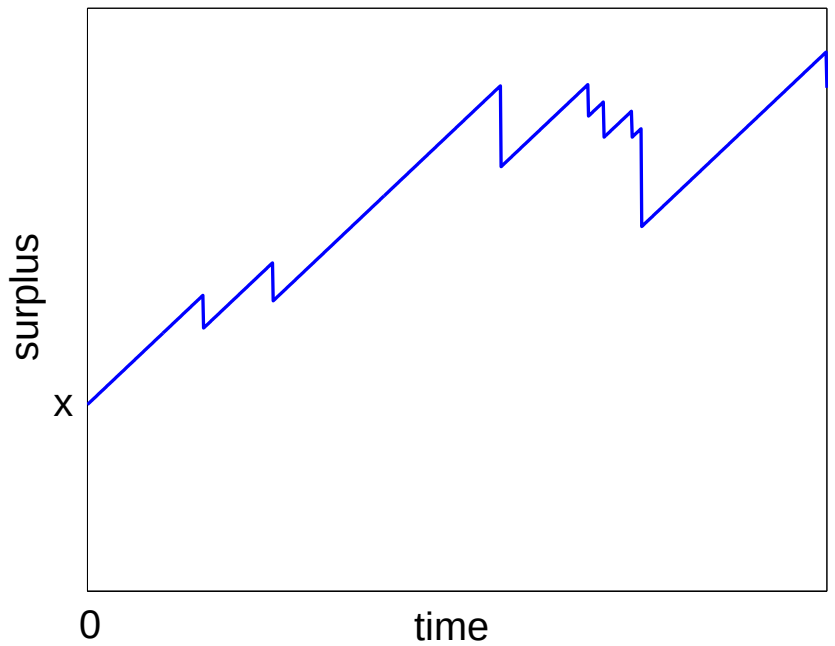
Model assumptions

Working party: ‘Practising ruin’

September 26, 2014 - Giro conference

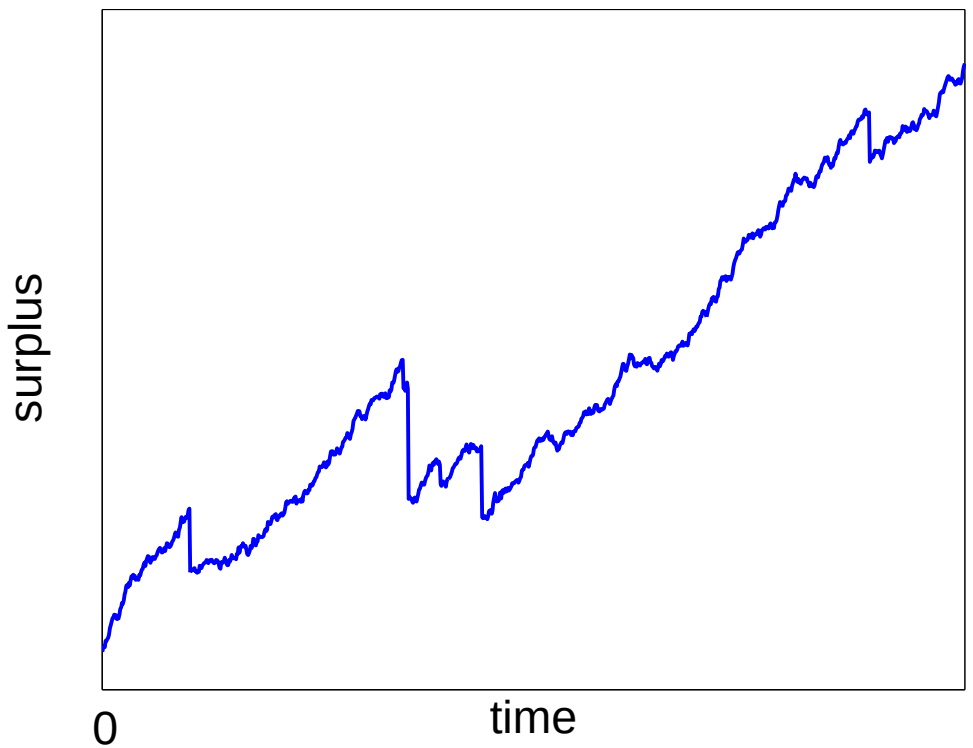
Classical risk process

- Initial capital $x > 0$.
- Constant premium rate/drift $c > 0$.
- Time between claims is exponentially distributed with parameter $\lambda > 0$.
- f denotes the (common) probability distribution function of each claim.



Including extra uncertainty

- Perturb the risk process by adding a Brownian motion which has a parameter $\sigma > 0$.
- Roughly speaking, on each time interval of infinitesimal length Δ, a normally distributed random variable with mean 0 and variance $\sigma^2\Delta$ is added.



Characteristics of the risk process

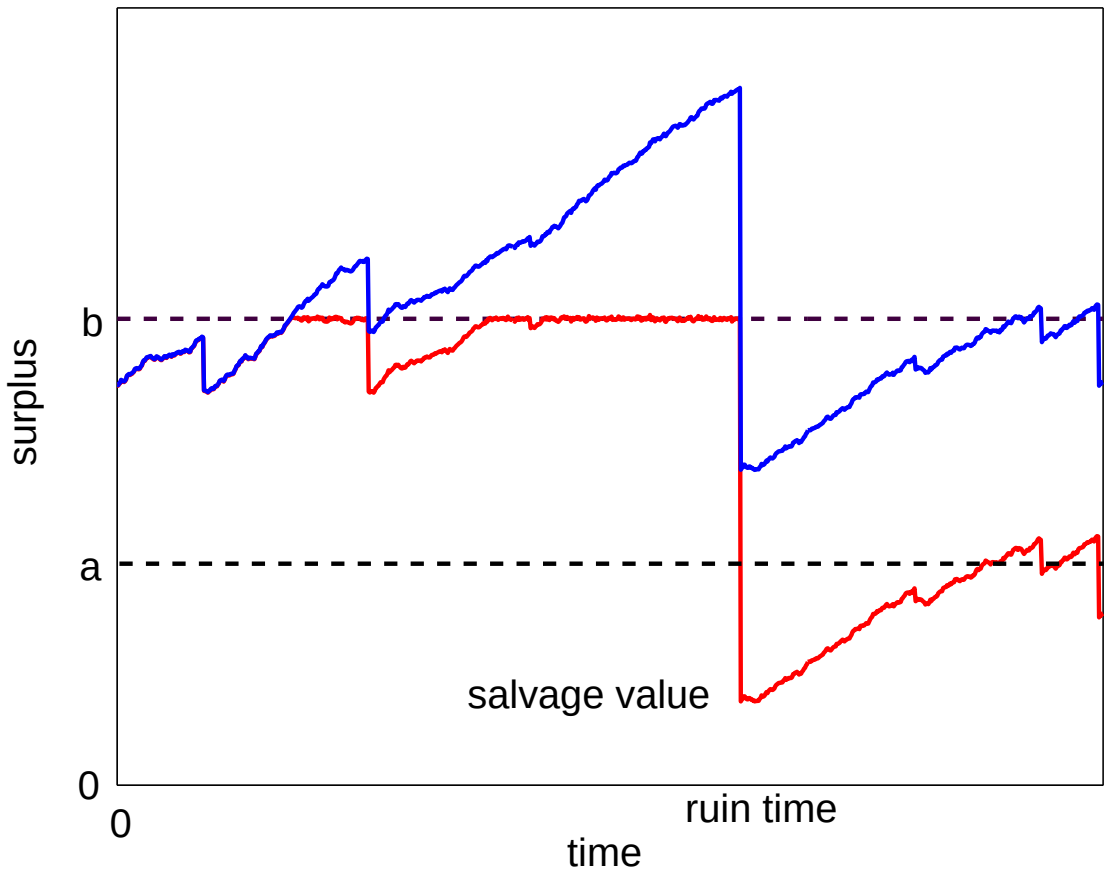
- Local behaviour does not depend on
  - level of the surplus,
  - time,
  - length of time since last claim.
- There are downwards jumps but no upward jumps.

Including dividends

- Dividends can be paid out to shareholders dynamically.
- Ruin level $a \geq 0$: ruin occurs when surplus $< a$.
- Salvage value: at ruin, residual value goes to shareholders.
- Firm value: expected value of cashflow to shareholders, discounted at rate $q > 0$.
- Optimal dividend strategy?

Barrier strategy

- We focus primarily on barrier strategies, which are parametrised by a barrier level $b > a$.



Claim distribution

- Mixture of exponentials:

$$f(z) = \sum_{i=1}^{n} \frac{p_i}{m_i} \mathrm{e}^{-z/m_i}, \quad z > 0,$$

  where n is a positive integer, $m_i, p_i > 0$ and $\sum_{i=1}^{n} p_i = 1$.

- Expected value of a claim is given by

$$\sum_{i=1}^{n} p_i m_i.$$

Why this choice?

- Relatively flexible.
- Very easy and fast to compute firm value under a barrier dividend strategy:
  - Explicit formula up to computing the roots of a certain polynomial.
  - Developed bounds for these roots.
- Barrier strategy is optimal (at least when $a = 0$).

Possible extensions

- Different penalty at ruin.
- Include capital injections (can increase firm value).
- Different definition of ruin.
- Allow parameters to change when surplus drops below a certain level.
- Allow parameters to change after an exponentially distributed amount of time.
- Include (optimal) reinsurance.

References

Kyprianou, Andreas E.
Gerber-Shiu Risk Theory.
Springer, Cham 2013.

Major, John A.
The firm-value risk model.
Working paper, 2009.
`http://www.cb.wsu.edu/aria2009/aria2009.htm`

Inputs and output of spreadsheet

- Input parameters
  - initial capital $x > 0$,
  - premium rate $c > 0$,
  - Gaussian parameter $\sigma \geq 0$,
  - claim intensity $\lambda > 0$,
  - discount rate $q > 0$,
  - 'means' of claim distribution $m_1, m_2, \ldots, m_n > 0$,
  - weights of claim distribution $p_1, p_2, \ldots, p_n > 0$, $\sum_{i=1}^{n} p_i = 1$,
  - ruin level $a \in [0, x]$,
  - barrier level $b \geq x$.
- Output
  - firm value with dividend barrier b and ruin level 0,
  - firm value with dividend barrier b and ruin level a.