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EQUITY IN BONUS DISTRIBUTION

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In the paper which I submitted to the Institute on 28 April 1944 I discussed the most equitable method of treating appreciation or depreciation in the assets of an office distributing its surplus by means of a uniform reversionary bonus. I believe that the conclusions reached were correct within the limits set, but consideration of the discussion convinced me that these limits were too narrow, since they involved the treatment of the problem of appreciation or depreciation without reference to the other factors affecting equity in bonus distribution and excluded any solution based on the use of a system of distribution of profits other than a uniform reversionary bonus. I felt that the methods used in the paper might produce results of interest if applied to the general problem of securing equity when the conditions experienced differ from those assumed in calculating the premiums, and I was encouraged to proceed on these lines by the suggestion made by Mr H. E. Melville, speaking as President, that the time was suitable for such an inquiry in view of the exceptionally wide fluctuations in the experience rates of interest over the past thirty years.

DEFINITIONS OF EQUITY

Before discussing the problem of equity in bonus distribution it is necessary to arrive at an acceptable definition of equity, but unfortunately there are at least two definitions which can be regarded as reasonable although they are very different in conception and effect.

First Definition: each group of like policies should receive the bonuses it would have received if it had formed a separate and distinct fund.

Second Definition: equity is attained if

(*a*) the profits are determined on the same principles at every valuation and are distributed as uniform bonuses, this fact being known to all new entrants to the fund,

and (*b*) the premiums charged to new entrants correspond to their expectation of bonuses at the date of entry.

The first definition has been implicitly accepted in previous papers on this subject, but in the discussion on my paper submitted to the Institute on 28 April 1944 several speakers adopted the second definition, either explicitly or implicitly. The results of doing so are considered in the later part of this paper, but in the first part it is assumed that the first definition has been accepted.

First Definition

It will be seen that the definition does not give any guidance as to the type of bonus or the rate at which it should emerge. Since a level profits loading is charged for the bonus the obviously equitable method of distribution would

be as a level cash bonus, but it is generally agreed that any type of bonus distribution is equitable provided that

(*a*) the type of bonus to be declared is known to all policyholders before they effect their policies,

(*b*) the premiums are correctly calculated to provide a bonus of the type and rate declared,

(*c*) the conditions experienced are identical with those assumed in calculating the premiums.

Inequity may occur, however, if there is a change in the experience conditions so that the bonus declared differs from that allowed for in the premium calculations. I propose to consider by means of a model office the effect on an office declaring a uniform simple reversionary bonus of a change in conditions under the following headings:

(*a*) Appreciation or depreciation,
(*b*) Change in the net rate of interest,
(*c*) Change in mortality experience,
(*d*) Change in expense ratio,

and also under (*b*) for an office declaring a uniform compound bonus.

Model Office

In choosing the model office I was anxious to reduce the calculations required as far as possible, but it was necessary to have an office containing policies of varying terms and durations. I therefore assumed an office which, for at least 45 years, had issued on 31 December at intervals of five years l_{20} policies at entry age 20, l_{35} policies at entry age 35, and l_{50} policies at entry age 50, each policy for a sum assured of 1, with profits, maturing at 65. I considered a valuation immediately following the quinquennial issue of new policies and payment of maturity claims and assumed that

(1) expenses (unless otherwise stated) and miscellaneous sources of profit can be ignored,

(2) the premiums have always been calculated on the A 1924–29 ultimate table at $3\frac{1}{2}\%$, with a loading for a simple reversionary bonus of £2%,

(3) the mortality experienced has always followed the above table and will continue to do so (unless otherwise stated),

(4) the interest earned on the fund has always been $3\frac{1}{2}\%$ net and will continue at this rate (unless otherwise stated),

(5) annual valuations have always been made by the bonus reserve method, using the A 1924–29 ultimate table at $3\frac{1}{2}\%$, valuing a future rate of bonus of K = £2% and declaring a bonus k = £2%,

(6) death claims are paid at the end of the year of death.

Appreciation and Depreciation

This section is limited to a consideration of the effect of appreciation or depreciation in fixed-interest securities resulting from a fall or rise in the general level of interest rates, and not from any alteration in the intrinsic value of the securities. I shall deal specifically with the treatment of appreciation, since this is an even more controversial problem than the treatment of depreciation.

I assumed that, immediately before the valuation under consideration was

due to be made, there was a fall of ½% in the general level of interest rates taking into account the redemption terms, with a consequent appreciation of the fund and decrease in the rate of interest from 3½% to 3% making allowance for redemption. The effect upon the fund will depend upon the distribution of the maturity dates of the assets, and for the purpose of illustration I chose two contrasting types of distribution as follows:

Case I. The reserves of each group of like policies invested in assets maturing in the year in which the policies will become payable if they remain in force until maturity.

Case II. Assets held maturing in each future year equal to the difference between the outgo in that year under maturities and death claims and the income in that year from interest and premiums under all policies then in force. It has to be assumed that there will be no withdrawals other than claims and no new entrants after the valuation date.

The percentages of the total assets of the model office maturing in each quinquennial year in the future are shown below for both cases:

Number of years to maturity of asset	Percentage of total assets	
	Case I	Case II
5	32	—
10	24	—
15	16	16
20	12	12
25	8	16
30	4	20
35	3	10
40	1	12
45	—	14

Owing to the assumption that new policies are only issued at intervals of five years and hence that maturity claims only occur at similar intervals, the income of the model office exceeds the outgo in four years in each quinquennium. In calculating the distribution under Case II, I assumed that assets were held maturing in, for example, the 10th year equal to the difference between the total outgo and total income over the 8th to 12th years. As the valuation is being made immediately after the payment of a group of maturities, the income over the first seven years exceeds the outgo over the same period. I set off this excess of income against the excess of outgo in the next period and assumed that no assets were held maturing in either the 5th or 10th years. A similar position would arise in the case of an office transacting an increasing new business. The length of the period during which income would exceed outgo would depend upon the rate of increase and the type of business transacted.

In my previous paper I discussed the alternative methods of dealing with appreciation or depreciation available to an office declaring a uniform reversionary bonus and valuing by the bonus reserve method. I propose to consider further the two methods described in that paper as method A and method D, which are as follows:

Method A. Take K, the future bonus to be valued as a liability, as the rate of bonus which the existing premium scale will support under the new

conditions. This has the effect of treating the appreciation or depreciation as a profit or loss to be dealt with in the valuation period in which it occurs.

In the model office the premiums will support the following bonuses taking the rate of interest as 3%:

Term	Bonus %
	£ s. d.
45 years	1 10 0
30 years	1 11 9
15 years	1 13 3

Since the average term is 30 years, I took K as £1. 11*s.* 9*d.*%, which gives the following rates for k, the bonus to be declared for the valuation period:

Case I. k = £3. 9*s.* 10*d.*%
Case II. k = £7. 8*s.* 6*d.*%

Method D. Take k = K. This has the effect of spreading the profit or loss over the remaining lifetime of the existing policies. In the model office this gives the following results:

Case I. k = K = £1. 14*s.* 3*d.*%
Case II. k = K = £1. 19*s.* 4*d.*%

To consider the equity of the methods it is necessary to show the effect on selected groups of policies of declaring the above bonuses for the whole office, but in order to do so the assets must be allocated among the different groups. As regards case I it is reasonable to assume that the reserves of any selected group are invested in securities maturing in the year in which the survivors of the group will become claims by maturity. For case II, I allocated the assets in rotation, the assets with the shortest term to maturity being allocated to the group nearest to its year of maturity, and so on. It was thus possible to calculate under both cases for selected groups the fund which would be available after the appreciation. I then calculated at 3% the value of the sum assured and bonus, less the value of future premiums, and deducted this from the fund for each selected group. The balance is the amount available to provide the bonuses k and K. The ratio of this available balance to the cost at 3% of the bonuses k and K, at the rates already found for the whole office and quoted above, are shown in Table 1. This table also shows the bonus which could be provided under method D in each group if the groups were treated as separate offices and bonuses k and K declared at the rates appropriate to the individual groups.

A ratio of less than unity indicates that a particular group will be too favourably treated if a uniform bonus is declared, and vice versa. Under a perfectly equitable method the ratio for each group would be unity.

The following conclusions may be drawn from the table:

(1) Method A is much more equitable than method D.

(2) Under a case I distribution, method A attains a fair degree of equity, but it is not satisfactory under a case II distribution. On the basis of the table and my previous paper it seems certain that no method could attain satisfactory results under such a distribution without departing from a uniform bonus.

Table 1

Group		Case I			Case II		
		Method A	Method D		Method A	Method D	
		Ratio	Ratio	Bonus % $k=K$	Ratio	Ratio	Bonus % $k=K$
Whole office				£ s. d. 1 14 3			£ s. d. 1 19 4
Age at entry	Valuation age						
20	20	·93	·89	1 10 6	·87	·78	1 10 6
	30	:97	:93	1 12 0	·92	·83	1 12 7
	40	1·02	·99	1 13 11	1·01	·95	1 17 3
	50	1·06	1·07	1 16 7	1·13	1·17	2 5 10
	60	·99	1·15	1 19 3	1·30	1·85	3 12 11
35	40	1·00	·98	1 13 5	·94	·88	1 14 8
	50	1·05	1·06	1 16 2	1·04	1·07	2 2 3
	60	·98	1·14	1 19 0	1·17	1·68	3 6 1
50	50	·99	1·00	1 14 3	·86	·89	1 14 11
	60	·97	1·12	1 18 5	1·01	1·45	2 17 0

Change in Net Rate of Interest

The second problem to be considered is the effect of a change in the net rate of interest without a corresponding change in the value of the assets. Such a change could occur as a result of the conversion of securities bearing a high rate of interest, etc., but probably the most serious problem will be that arising from a change in the rate of income tax.

I assumed that the net rate of interest received in the model office had been reduced from $3\frac{1}{2}\%$ to 3% from the beginning of the valuation year and that the change could be considered permanent. This would be approximately equivalent to an increase of 2*s.* 6*d.* in the £ in the rate of income tax.

Under method A, I took $K=$£1. 11*s.* 9*d.*% as before, and this gave a bonus k at a negative rate of £1. 15*s.* 1*d.*%. In practice it would be necessary to declare no bonus for two years and a reduced rate in the third year.

Under method D, $k=K=$£1. 7*s.* 5*d.*%.

Table 2 gives information similar to that shown in Table 1.

It will be seen that serious inequities occur under both methods and that method A is at least as unsatisfactory as method D. Further, the bonuses resulting from the use of method A would be difficult to justify, since the loss following the change in the rate of interest will fall as heavily on future years as on the year under consideration. Method A is only suitable in circumstances such as appreciation or depreciation where a non-recurring profit or loss has occurred in the valuation year, and this method will not be used in the cases which follow.

Table 2

Group		Method A	Method D	
		Ratio	Ratio	Bonus % $k=K$
Whole office				£ s. d. 1 7 5
Age at entry	Valuation age			
20	20	1·00	1·10	1 10 0
	30	·93	1·00	1 7 6
	40	·88	·92	1 5 3
	50	·85	·83	1 2 10
	60	1·05	·68	18 8
35	40	1·05	1·10	1 10 2
	50	1·01	·98	1 7 0
	60	1·27	·82	1 2 6
50	50	1·24	1·21	1 13 3
	60	1·53	·99	1 7 2

Change in Mortality Experience

I assumed that from the beginning of the valuation year the mortality experienced changed from A 1924–29 ultimate to A 1924–29 Light ultimate and that this change could be considered permanent.

Table 3 gives information similar to that in Tables 1 and 2 but for method D only.

Table 3

Group		Ratio	Bonus % $k=K$
Whole office			£ s. d. 2 2 6
Age at entry	Valuation age		
20	20	1·00	2 2 7
	30	1·00	2 2 6
	40	1·00	2 2 6
	50	1·00	2 2 4
	60	·98	2 1 8
35	40	1·00	2 2 8
	50	1·00	2 2 5
	60	·98	2 1 9
50	50	1·02	2 3 2
	60	1·00	2 2 6

It will be seen that very satisfactory results are obtained in this case, but with a change in mortality it is not safe to generalize from the results of one example, because they will depend upon the incidence of the change.

Change in Expense Ratio

The expense ratio is to a very considerable extent under the control of the office, and for existing policies is unlikely to change sufficiently to have any appreciable effect on the equity of the bonus distribution.

The expense ratio for new policies may change but this ought to be allowed for in the premiums charged and should not affect the bonus rates.

It might, however, be considered desirable to treat the interest income as being liable to income tax at the full rate and to deduct income tax from the expenses before calculating the expense ratio. If this were done an increase in the rate of income tax would reduce the expense ratio and a decrease would increase the ratio.

I assumed that the rate of income tax had been increased by 2*s.* 6*d.* in the £ from the beginning of the valuation year and that the expenses were those allowed for in the following premium formula:

$$P'_{x:\overline{n}|} = \frac{1}{\cdot 975}\left\{100 P_{x:\overline{n}|} + \frac{3\cdot 5}{1 + a_{x:\overline{n-1}|}} + \cdot 125\right\}.$$

Table 4 gives, for method D only, information similar to that in the previous Tables.

Table 4

Group		Ratio	Bonus % $k=K$
Whole office			£ *s.* *d.* 2 1 8
Age at entry	Valuation age		
20	20	1·01	2 2 2
	30	1·00	2 1 9
	40	1·00	2 1 6
	50	·99	2 1 3
	60	·99	2 1 1
35	40	1·00	2 1 10
	50	1·00	2 1 6
	60	·99	2 1 3
50	50	1·01	2 2 3
	60	1·01	2 1 11

It will be seen that the effect on the bonus of the change in the expense ratio resulting from a given increase in the rate of income tax is very much less than the effect of the change in the rate of interest resulting from the same increase in the rate of income tax.

Compound Bonus

There does not seem to be any reason to expect a uniform compound bonus to be more satisfactory than a uniform simple bonus if the experience conditions change. To test this I adopted the model office with the following modifications:

(1) The premiums have always been calculated on the A 1924–29 ultimate table at 4%, with a loading for a compound bonus of £1. 10*s.* 0*d.*%.

(2) Annual valuations have always been made by the bonus reserve method, using the A 1924–29 ultimate table at 4%, valuing a future rate of bonus K = £1. 10*s*. 0*d*.% and declaring a bonus k = £1. 10*s*. 0*d*.%.

I allowed for future bonuses by valuing the sum assured and bonus at 2½%.

I assumed that from the beginning of the valuation year the rate of interest had been reduced from 4% to 3½%. I calculated by interpolation the rate of bonus k = K which could be declared for the office as a whole and in the selected groups if these were treated as separate offices. The rates are shown in Table 5.

Table 5

Group		Bonus % k = K
Whole office		£ *s*. *d*. 1 1 9
Age at entry	Valuation age	
20	20	1 3 10
	30	1 2 5
	40	1 1 3
	50	1 0 2
	60	18 2
35	40	1 3 1
	50	1 1 2
	60	18 6
50	50	1 4 4
	60	19 7

Though the results are obviously unsatisfactory, it might appear at first sight that they are appreciably more satisfactory than those obtained by a uniform simple bonus in similar circumstances as shown in Table 2. This is, however, almost entirely due to the fact that, for example, a change from a 25*s*. to a 20*s*. compound bonus makes a difference in the total bonus declared on a policy which remains in force for 45 years more than 60% greater than the difference resulting from the same change in a simple bonus.

Contribution Method

The above Tables confirm the results of similar inquiries in the past, viz. that, though a uniform reversionary bonus system can be made to give satisfactory results under stable conditions, it may not be able to deal equitably with changes in the experience conditions and particularly in the rate of interest. The period during which the uniform reversionary bonus system gained almost universal acceptance in this country was one of relative stability, but during the past 30 years changes in the experience conditions of a magnitude as great or greater than those considered in this paper have occurred on several occasions within comparatively short periods. It would seem desirable, therefore, to reconsider the alternatives available and the most obvious is the Sheppard Homans contribution method as used in America and described by J. B. Maclean in *J.I.A.* Vol. LXII.

For an annual valuation Maclean gives the following formula for the theoretical 'contribution' of a policy in its nth year:

$$({}_{n-1}\mathrm{V}+\mathrm{P})(i'-i)+(\mathrm{P}'-\mathrm{P}-e)(1+i')+(q-q')(1-{}_{n}\mathrm{V}),$$

where P′ is the gross office premium, e is the assessed expenses, i' the experience rate of interest, and q' the experience rate of mortality. The other symbols have their usual meanings and are taken on the valuation basis.

It would seem that the contribution method is completely equitable, provided that the expenses, experience rates of mortality and interest used in the formula exactly correspond with the experience of the office. It is obvious from Maclean's paper that in practice the American companies do not attempt to follow their own experience with absolute accuracy as regards mortality and expenses and that the methods adopted in calculating the mortality and loading profit vary considerably between the different offices. It is difficult, therefore, to estimate the errors arising from the approximations used. The calculation of the interest profit is comparatively simple, except for the problem of appreciation or depreciation. On this point Maclean says: 'Another question in regard to the quantity I [i.e. the interest income] is whether or not it should take account of profit or loss from investments. It is evident that any unusually large profit or loss might seriously affect the interest rate, and a sudden material change would upset the even progression of the bonus scale, especially for policies of long duration. Most companies regard the Contingency Fund as being, to a large extent, in the nature of an investment fluctuation fund....'

It seems clear from this quotation that appreciation or depreciation of the type discussed earlier in this paper would not be treated as profit or loss in the valuation period in which it occurred. On the other hand, Maclean does not suggest the use of different values of i' for different groups of policies, and inequity must, therefore, occur if there is a change in the general level of interest rates. For example, consider a model office similar to that used earlier in this paper but with the following modifications:

(1) The premiums charged are the A 1924–29 ultimate $2\frac{1}{2}\%$ net premiums (expenses being ignored).

(2) Valuations have always been made by the same table, the bonuses being calculated by the contribution method and distributed in cash.

If a fall of $\frac{1}{2}\%$ in the general level of interest rates were to occur and if the assets were not written up to their new market values or the appreciation were to be transferred to a contingency fund, the interest on which was carried to the main fund, the interest income in subsequent years would consist of interest at $3\frac{1}{2}\%$ on the assets held at the date when the appreciation occurred and interest at 3% on the investments made after that date. If i' were taken as the average rate of interest for the whole office, the policies for which large reserves were held at the date when the appreciation occurred would be credited with less than the interest earned by the assets corresponding to their reserves, while the reverse would apply to policies effected after the date of the appreciation or to policies with small reserves at that date. Assuming a case I distribution of the assets, Table 6 shows, for the sixth year after the appreciation, the ratio of the true surplus in selected groups to the surplus which would be brought out if an average rate of interest (in this case $3{\cdot}34\%$) were allotted to all policies.

It will be seen that the inequities are more serious than those occurring in a case I distribution under the uniform simple reversionary bonus system if method A is used.

Table 6

Age at entry	Valuation age	Ratio
20	21	·58
	31	·90
	41	1·03
	51	1·08
	61	1·12
35	41	·70
	51	1·00
	61	1·08
50	51	·58
	61	·93

Difficulty of applying Contribution Method

The preceding remarks are criticisms of the practical application of the contribution method in America and not of the method itself, which can undoubtedly be made to produce completely equitable results at the cost of a very serious amount of work. It has, however, usually been held that the volume of work required is such that it is not practicable to use the method in this country, the principal reasons given for this opinion being as follows:

(*a*) The contribution method requires classification by
(1) original term,
(2) duration,
(3) age at entry.

For most companies transacting ordinary life assurance business in this country this would result in many small groups and, indeed, in the individual valuation of many policies.

(*b*) The returns under the Assurance Companies Act, 1909, require the values of the sums assured and bonuses, office premiums and net premiums to be shown separately. These would have to be calculated in addition to the contribution formulae.

(*c*) The contribution method is designed for use with a cash bonus and, if reversionary bonuses were declared, the work would be further increased by the necessity of allowing for vested bonuses.

Modified Contribution Method

It does not appear to have been pointed out that the contribution method can be modified for use in conjunction with the ordinary group valuation, thus reducing the work required in its application and entirely removing the objections listed above as (*b*) and (*c*).

Consider first the whole-life assurance classes of an office making an annual valuation. The policies will be grouped by office years of birth, giving the attained ages at the valuation date. The ordinary valuation working will give for each age group the value of the sums assured and bonuses and the value of the net premiums. A further column can then be added giving the difference between the two previous columns. This will, of course, be the reserve for the group and must be exactly equal to the sum of the reserves of the individual

policies forming the group. These reserves may be the reserves at the beginning of the policy year after payment of the premium or the reserves at the end of the policy year or at some intermediate point, while under the contribution method the reserve at the beginning of the policy year after payment of the premium is used in calculating the interest profit and the reserve at the end of the policy year for the mortality profit. The error resulting from the use of the reserve at the valuation date will be very slight.

The interest profit for the group will, therefore, be

$$\text{Valuation reserve} \times (i' - i).$$

The mortality profit for the group will be

$$(q - q') \text{ (sums assured and bonuses} - \text{valuation reserve).}$$

In order to obtain the loading profit it will be necessary to introduce an additional valuation factor, and for each policy at entry there will be recorded e, the expense loading in the office premium.

The loading profit for the group will then be

(office premiums – net premiums – the total of e recorded for the group).

The reversionary bonus to be added to the group in respect of these three sources of profit will be the sum of the above divided by A_y, where y is the attained age of the group.

In order to obtain the bonuses to be allotted to the individual policies it will be necessary to calculate bonus scales for each age at entry. In doing this it will be essential that the reserves used should exactly coincide with the valuation reserves, e.g. in respect of office years of birth in place of true years of birth and the allowance (if any) made for the payment of the premium in the first or second half of the year.

Consider age at entry x and duration n, and assume that the valuation reserve will be reproduced by the formula

$$A_{x+n}(S+B) - \bar{a}_{x+n} P_x,$$

where (S + B) represents the sum assured and vested bonus.

The interest profit will be

$$[A_{x+n}(S+B) - \bar{a}_{x+n} P_x](i' - i),$$

and the corresponding reversionary bonus

$$[A_{x+n}(S+B) - \bar{a}_{x+n} P_x](i' - i)\frac{1}{A_{x+n}} = \left[(S+B) - P_x \frac{\bar{a}_{x+n}}{A_{x+n}}\right](i' - i).$$

The reversionary bonus corresponding to the mortality profit will be

$$(q - q')[(S+B) - A_{x+n}(S+B) + \bar{a}_{x+n} P_x]\frac{1}{A_{x+n}}$$

$$= (q - q')\left[\frac{(S+B)}{A_{x+n}} - (S+B) + P_x \frac{\bar{a}_{x+n}}{A_{x+n}}\right].$$

The reversionary bonus corresponding to the loading profit will be

$$(P' - P - e)\frac{1}{A_{x+n}},$$

omitting the factor $(1 + i')$ as an excessive refinement which is not strictly appropriate to a valuation being made at some intermediate point in the policy year.

Extensive bonus scales will be required giving the bonus at each duration for each age at entry, but the calculations will be extremely simple. (S + B) will be immediately available from the office records, while $\frac{1}{A_{x+n}}$, $\frac{\bar{a}_{x+n}}{A_{x+n}}$ and $P_x \frac{\bar{a}_{x+n}}{A_{x+n}}$ depend upon the valuation basis only and will be unaltered so long as the valuation basis is unchanged. The bonuses arising from the loading profit will be unaltered so long as the rates of office premium and valuation basis remain unchanged, while a change in the rates of office premium charged would involve only a gradual change in the scale of bonuses arising from the loading profit. The preparation of the bonus scales could proceed independently of the valuation work as soon as the rates to be used for i' and q' had been decided.

For endowment assurances the policies will be grouped by office years of maturity and the valuation ages obtained by adopting Sir William Elderton's suggestion and using a fixed maturity age. This greatly reduces the number of bonus scales required, since the bonus will depend upon the original term and the duration only and not upon age at entry. The use of an average age for each group may distort the mortality profit as between the individual policies forming a group, but this distortion is unlikely to be serious unless a very unsuitable mortality table is used in the valuation, and the difficulty will be entirely removed if the valuation table so closely represents the mortality experienced that the mortality profit is negligible and can be ignored. This point will be discussed again later.

The bonuses allotted to the individual policies by means of the bonus scales calculated as above will exactly absorb the total of the profits calculated for each group in respect of interest, mortality and loading, but it is improbable that the total of these profits will exactly equal the surplus for the whole office disclosed by the valuation, the difference arising from miscellaneous sources, e.g. profit or loss from lapses, surrenders and without-profit business. Various methods could be used for allotting this miscellaneous profit or loss among the individual policies, but it would seem most reasonable to distribute it in the form of a level reversionary bonus, which would be added to or deducted from the bonus scales calculated as above.

Changes in the net rate of interest unaccompanied by appreciation or depreciation and changes in the mortality experience will be dealt with automatically, while, if it were desired to allow for the effect on the net expenses of a change in the rate of income tax, a percentage adjustment of e could be made. The only difficulty is the treatment of appreciation or depreciation. It has been shown in this discussion of the application of the contribution method in America that inequity arises unless appreciation or depreciation is treated as profit or loss in the valuation period in which it occurs, or unless varying rates of i' are used at subsequent valuations in accordance with the reserves of the individual policies at the date when the change in the value of the assets occurs. The latter method could not be used with the suggested modified contribution method, so the profit or loss must be distributed immediately. The amount of this profit or loss will depend upon the distribution of the maturity dates of the assets. Assume in the first place that the reserve for each policy is invested in assets maturing in the year when the policy will become a claim, either by death or maturity. The alteration in value of a security standing at par, redeemable at par in m years and bearing interest at rate i' which will

follow a change in the rate of interest from i' to i'', is $(i'-i'')\,a''_{\overline{m}|}$, where $a''_{\overline{m}|}$ is calculated at i''. The change in the value of the assets corresponding to the reserves of the valuation group may, therefore, be written

$$\text{Valuation reserve} \times (i'-i'')\,a''_{x+n},$$

where i' was the experience rate of interest before the change occurred and i'' is the new experience rate. For endowment assurances a''_{x+n} will be replaced by $a''_{x+n:\overline{t}|}$, where t is the unexpired term to maturity.

It is improbable that the total of the above will be equal to the change in the value of the assets for the whole fund, but if the ratio of the latter to the former is $(1+r)$ it will be reasonable to treat the change in the value of the assets of each valuation group as

$$\text{Valuation reserve} \times (i'-i'')\,a''_{x+n}\,(1+r).$$

Making the same assumption as before, the reversionary bonus applicable to an individual policy will be

$$[\mathrm{A}_{x+n}(\mathrm{S}+\mathrm{B})-\bar{a}_{x+n}\mathrm{P}_x]\,(i'-i'')\,a''_{x+n}\,(1+r)\frac{1}{\mathrm{A}_{x+n}}$$
$$=\left[(\mathrm{S}+\mathrm{B})-\mathrm{P}_x\frac{\bar{a}_{x+n}}{\mathrm{A}_{x+n}}\right](i'-i'')\,a''_{x+n}\,(1+r).$$

If the method is adopted, appreciation or depreciation will cause violent fluctuations in the bonus scales unless a corresponding alteration is made in the valuation basis. It has usually been stated that, if the contribution method is used, the valuation basis must remain unaltered, but the conditions of the first definition of equity on p. 37 will be fulfilled provided the release or strain resulting from the change in basis is equitably distributed among the individual policies.

For the valuation group the reversionary bonus resulting from the change in the valuation basis will be

$$(\text{Valuation reserve on old basis} - \text{valuation reserve on new basis}) \times \frac{1}{\mathrm{A}'''_{x+n}}$$

and for the individual policy

$$[\{\mathrm{A}_{x+n}(\mathrm{S}+\mathrm{B})-\mathrm{P}_x\bar{a}_{x+n}\}-\{\mathrm{A}'''_{x+n}(\mathrm{S}+\mathrm{B})-\mathrm{P}'''_x\bar{a}'''_{x+n}\}]\frac{1}{\mathrm{A}'''_{x+n}},$$

where A'''_{x+n}, P'''_x and $\bar{a}'''_{x+n}$ are taken on the new valuation basis.

The new valuation basis will be chosen so that the change in the total reserves as nearly as possible offsets the change in the total value of the assets. In order to choose the most suitable basis, use will be made of some approximate method of estimating the reserves required by alternative valuation bases, as, for example, one of the methods suggested by Sir William Elderton and A. H. Rowell in their paper in *J.I.A.* Vol. LVI.

When the valuation rate of interest is altered the opportunity could be taken to change to a mortality table closely representing the mortality of the office, if the table previously in use had become unsatisfactory. This change would eliminate the mortality profit, thus greatly reducing the work involved in the preparation of the bonus scales and avoiding any distortion in the mortality profit for endowment assurances.

Even if the change in the total reserves exactly offsets the change in the total value of the assets, it would be very improbable that the same balance

would be shown for each individual policy. The fluctuations in the bonus scales should not, however, be excessive and, in any case, are unavoidable if equity is to be attained.

The methods outlined above would, of course, be used only when there had been a significant change in the value of the assets, minor fluctuations being dealt with by means of a contingency fund.

The preceding discussion of the modified contribution method has been confined to the case of an office making an annual valuation. The method cannot be applied direct to a quinquennial valuation since, among other objections, the valuation reserves would be poor approximations to the average reserves throughout the quinquennium and the use of average rates of interest or mortality over the quinquennium might not be satisfactory. The difficulties can, however, be removed if an annual valuation is made for internal purposes only (as is no doubt usually done) and bonuses allotted to the valuation groups and bonus scales prepared every year. These annual bonus scales would be combined at the end of the five years to give the bonuses to be allotted to the individual policies for the quinquennium. Bonuses must, however, be allotted to policies going off the books during the quinquennium, in order that the correct bonuses may be written off the valuation groups.

It will be seen that the modified contribution method is better suited to conditions in this country and requires considerably less work than the contribution method as applied in America, particularly if the mortality profit is eliminated as suggested, but that it would involve a serious increase in the valuation work in comparison with that required by a uniform reversionary bonus system. Since this latter system is satisfactory if the second definition of equity given on p. 37 is accepted, it is desirable now to consider whether this definition can be justified.

Second Definition

The position of a without-profit policyholder is similar to that of the holder of debentures in a first-class limited-liability company. All the holders of a particular class of debenture receive the same fixed return upon their nominal holdings, but the actual amounts paid for their holdings by different investors will have varied in accordance with the general level of interest rates at the dates when the purchases were made. Similarly, different rates of premium are paid by different without-profit policyholders to secure the same benefits, the rates of premium charged being determined by the conditions ruling *at the dates the policies were effected.* If the second definition given on p. 37 is accepted, the holders of with-profit policies are in a similar position to the holders of equity shares. As regards the latter, the profits are uniformly distributed in proportion to the nominal shareholdings, irrespective of the actual amounts invested which will have varied in accordance with the rates of interest ruling and the investors' estimates of the prospects of the company at the dates when the investments were made. Similarly, under the second definition, uniform bonuses would be distributed, but the premiums paid would vary and would depend upon the conditions ruling and the bonus prospects at the dates when the policies were effected.

It will be seen that a logical case can be made for the second definition and there is no doubt that its acceptance would remove or simplify many valuation problems.

It would appear best to make the valuation by the bonus reserve method, taking the assets at their market values and using experience rates of interest, mortality and renewal expenses. The miscellaneous profit (or loss), consisting principally of profit (or loss) on the without-profit business, would be calculated and deducted from (or added to) the surplus, and a rate of bonus $k = K$ obtained. The bonus provided by the miscellaneous profit (or loss) would then be added to (or deducted from) k to find the rate of bonus for the valuation period under consideration.

New premiums would be calculated on the valuation basis but with an allowance for initial expenses and loaded to provide a bonus at the rate K used in the valuation. The value of the difference between the full expense loading and the loading for the renewal expenses would meet the initial expenses, so that new policies would not involve any valuation strain and the emerging surplus would not be distorted by a change in the volume of new business. Negative values would only arise should the initial expenses plus the death strain exceed the first premium. This would not occur for the majority of policies, and the slight adjustment necessary to eliminate such negative values would not impose an appreciable strain. Any such strain would be negligible in comparison with that under a net premium valuation.

It will be seen from Table 1 that, if this system of valuation is adopted, it will be possible to have a case II distribution (i.e. mainly long-term investments) and to write up or down the book values of the assets to the new market values following appreciation or depreciation resulting from a change in the general level of interest rates without making any appreciable difference in the rates of bonus emerging from future valuations; but the premiums charged for new policies would have to be altered considerably, since approximately the same bonus would have to be provided at a different rate of interest.

A change in the net rate of interest without appreciation or depreciation will involve a very considerable change in the rates of bonus emerging from future valuations and also an alteration in the premiums charged for new policies. It will be seen from the paragraph 'Method A' on p. 39 and from the bonus for the whole office shown in Table 2 that a reduction in the rate of interest will involve a reduction in the premiums for new with-profit policies and vice versa, since the effect on the bonus for the existing policies is greater than the effect on the bonus supported by the premium scale.

A change in mortality such as that considered in Table 3 would increase the rates of bonus at future valuations but would require little alteration in the premiums for new policies. As already pointed out, however, the exact effect of any change in mortality would depend upon the incidence of the alteration. The effects of any change in the mortality, which, not being considered permanent (e.g. a change resulting from a war, epidemic or claims under a few policies for unusually large sums assured), do not require an alteration in the valuation basis, would be spread forward by making $k = K$. The surplus for the current valuation would, therefore, be much less affected than would be the case under a net premium valuation.

The system would, therefore, largely eliminate fluctuations in the rates of bonus resulting from changes in the general level of gross interest rates or from temporary changes in the mortality experience, but fluctuations would still occur as a result of changes in the net rate of interest without appreciation or depreciation, which would usually be caused by a change in the rate of income tax.

CONCLUSION

If the first definition of equity is accepted and a change occurs in the conditions experienced, equity must be attained by declaring varying rates of bonus. It follows that the uniform reversionary bonus system must be abandoned and the contribution method or some modification of that method adopted.

If the second definition is accepted, a uniform bonus would still be declared after a change in conditions, but the premiums charged for new policies must be altered.

The results of adopting the two definitions are thus diametrically opposed and it is essential that the actuary should be quite clear as to his objective and should pursue it absolutely consistently, and I believe myself that the choice of the definition to be followed is relatively unimportant in comparison with the necessity for consistency in all circumstances. It is difficult, therefore, not to be attracted by the ease of application of the second definition and the relative stability of bonus rates which results from its use.

I should like to record my gratitude to Mr R. E. Beales, F.I.A., for his many helpful criticisms and suggestions. In addition, many of the ideas expressed in this paper have arisen from a study of the remarks made during the discussion on my previous paper and I trust that the various speakers will excuse my omission to make a more detailed acknowledgment of the use I have made of their suggestions.

ABSTRACT OF THE DISCUSSION

Mr W. E. H. Hickox, in opening the discussion, said that abstract words such as 'equity' were always difficult to define, and he thought that such definitions should depend on the purpose for which they were required. The main object of the paper, as he understood it, was to test methods of bonus distribution under changing conditions. For that purpose a scientific definition of equity was wanted and the author's first definition was the right one.

He did not regard the author's attempt to justify the second definition as satisfactory. The analogy between the effecting of a life assurance policy and the purchase of ordinary shares in a limited-liability company did not seem to him to be complete. The purchaser of ordinary shares usually had expert advice to guide him, and the dividends that he received were determined by the directors representing his interests. On the other hand, a life assurance policy was usually effected without any detailed or technical knowledge of bonus prospects, and it was one of the actuary's duties to see that the profits were fairly divided. Even if the analogy were accepted, he thought it would be better made with reference to the premiums paid under a life assurance policy than with reference to the sum assured. But, if the sum assured were adopted as the criterion, the distribution of profits should be in the form of uniform cash bonuses and not uniform reversionary bonuses.

There were three important limitations with regard to the first definition of equity. First, no account was taken of the profits from miscellaneous sources, i.e. the profits from non-participating business or from annuities or from the inheritance of the right to share in an efficient office organization, with the safeguard of contingency funds built up over a long term of years, and a right to interest earned by those funds and by the shareholders' capital. In theory, profits from those sources did not belong to any particular group of policyholders, and actuaries were entitled to a considerable latitude in interpreting the arithmetical results of an investigation such as that under consideration, which could not make scientific allowance for miscellaneous profits. Secondly, equity could be measured only against a standard, which had to be taken as the valuation basis. Therefore, a change in the valuation basis produced a change in the standard of measurement. Thirdly, the author's first definition made no specific reference to the way in which surplus emerged.

It was not clear exactly what was intended by equity as between 'each group of like policies'. Was a 'group of like policies' meant to include all those policies which had entered at the same age and for the same endowment assurance term and had been the same duration in force, or was it meant to consist only of those of them (say, d_{x+t} in number) which would become claims by death or maturity in t years' time? If the first meaning was intended, the actual years in which the surplus emerged were not of great importance, so long as the total surplus eventually allotted to each group of l_x like policies was equitable. In other words, large bonuses could be given in the first year and small bonuses afterwards or a constant rate of bonus could be declared throughout, and in either case there would still be equity if the total bonuses allotted were fair. In fact, methods A and D were the extreme limits of an infinite number of possible methods for fixing the emergence of surplus, all of which had equal virtue if the tests showed reasonable equity. He preferred the second meaning, according to which each batch of d_{x+t} claims emerging in the same year should be regarded as a separate group. Those policies which were just about to become claims would be represented by assets so near to maturity that they would hardly be affected by a change in future conditions or by appreciation or depreciation of investments. If that line of thought were pursued, the only equitable solution was to give the original rate of bonus to policies which were about to become claims, and there were then two possible alternatives: either to make a special bonus declaration for policies becoming claims in the next few years or to make a gradual change in the rate of bonus from its previous rate to its new level.

It seemed to him that there were three totally different ways of matching investments:

(i) The reserves and future premiums of each group of d_{x+t} policies (alike as to duration to death or maturity as well as age and term) to be invested in securities to meet their claims.

(ii) The reserves and future premiums of each group of l_x policies (alike as to age and original term but not necessarily duration to death or maturity) to be invested in securities to meet their claims.

(iii) The reserves and future premiums of the existing business as a whole, perhaps represented by a model office, to be invested in securities to meet the claims of the business as a whole.

The author's case II was an example of (iii).

The three methods to which he had referred might be called briefly the d_{x+t}, l_x, and Σl_x methods of matching investments.

The author's case I* was unique in that it matched each group of d_{x+t} claims arising in t years' time with its own reserves and future premiums invested to mature at the same time. Duration to date of becoming a claim was a more variable and, he thought, a more important factor than attained age, and case I would have been an excellent tool for investigating equity with regard to claims emerging in successive years. By failing to pursue that line and aggregating together all the l_x policies irrespective of duration to becoming a claim, the peculiar virtue of case I had been lost; for there were many ways of matching investments which preserved equity between a mixed group of policies and were equally justifiable and which yet gave entirely different results.

Case I had its weaknesses as well as its advantages. First, it was assumed that (ignoring bonuses) reserves of $(v^t - \pi a_{\overline{t-1}|})$ were invested in securities maturing in t years' time. That function was negative for some values of t, a result which was absurd and which in practice could be achieved only by 'switching over' investments between existing and new policies and so disturbing the equity of existing policyholders. The second weakness of case I was that it assumed that dividends as they fell due would be automatically reinvested on the same terms as the original investments, whereas in practice dividends could only be reinvested at the current market rate of interest. Nevertheless, he thought that formulae might be devised which would be free from the objections he had mentioned.

He wished to say a few words about unmatched investment policies. Whilst an office might pursue an unmatched investment policy if by so doing it could take advantage of current market conditions, he thought there was a fundamental difference between matched and unmatched investment policies. Matched investment policies confined to existing policyholders the whole benefit or loss from appreciation or depreciation of assets, whereas unmatched investment policies passed some of it on to future policyholders. For example, if interest rates fell an over-investment in long-dated securities might enable an office to maintain the same rate of bonus for the whole office without increasing its premium rates for new entrants, at any rate for a time.

He had perhaps digressed somewhat from the paper, but he wished to stress the point that there were many different assumptions which could be justified and that they must therefore guard against generalizing the results of cases I and II.

Returning to the problem of emergence of surplus, he had criticized methods A and D on the ground that effect was not given to equity as between claims emerging in successive years, but he thought that the methods served a useful purpose in measuring equity as between old and new policies. Method A produced equity between old and new policies if the rates of premium for new entrants remained unaltered, and method D produced equity if the rates of premium for new entrants were adjusted to maintain the original rate of bonus. He thought that in practice the actuary must be at liberty to fix his rates of premium for new entrants on grounds of general policy. He might deem it desirable to pursue a policy of high premium rates and high bonuses or one of low premium rates and low bonuses.

* See Mr Suttie's remarks on pp. 64–5.—Ed. *J.I.A.*

In reviewing the author's conclusions it must be remembered (i) that he had been unable to allow for miscellaneous items of surplus, and (ii) that quite different assumptions with regard to investment policy could be justified. Even so, he believed that the author was correct in his contention that, if a change in conditions was experienced, equity might be possible only by declaring varying rates of bonus. He differed from the author, however, in that he did not think it would generally be found necessary to adopt a contribution system. He believed that the attainment of broad justice, which was all that could be hoped for, could usually be achieved by some simple modification of the uniform reversionary bonus system. In his opinion the practical solution of the problem for offices adopting the uniform reversionary bonus system was to modify it by declaring differential rates of bonus for a limited number of years where careful investigation proved that to be necessary in order to retain broad equity.

Mr H. O. Worger considered that there were two entirely different attitudes of mind with which the problem of equity in bonus distribution could be approached. The first looked backward and, provided certain conditions were fulfilled, paid little heed to the future; it was concerned to distribute a valuation surplus in the light of the ascertainable past. The second disregarded what had happened before the valuation, took its stand at that time, and looked forward to the future. The first attitude led logically to the contribution system of bonus distribution or to some modification of or approximation to it. The second led straight to the bonus reserve valuation and to the author's second definition of equity.

He wondered whether the author was fully prepared to follow his second definition of equity to its logical conclusion. For the purpose of the investigation it had been assumed that all policies were maintained in force until they became claims by death or maturity. In practice, however, a large proportion of policies were not so maintained but were surrendered. Following a valuation which involved an alteration of basis with a consequent adjustment in the rates for new entrants, what attitude should be adopted towards a policyholder who wished to surrender his policy, the policy having been effected at a rate of premium which, in the light of the subsequent changes, was too favourable to him? The difference between the present value of the sum assured with future bonuses and the present value of future premiums less expenses on the altered basis might be far greater than the few premiums he had paid. Were offices prepared to give him on surrender the full value of the excellent bargain that he had made? Again, in the case of a policy effected at what in the light of the altered basis was a specially unfavourable rate of premium, the present value of the assurance benefits might scarcely exceed that of the premiums to be paid less expenses, whereas before the basis was changed the policy might have had a substantial surrender value.

The difficulties he had mentioned were not the only ones. If six different actuaries were supplied with full particulars of the existing premiums, sums assured and bonuses in force, and the premium scale and assets of an office, but no previous valuation results or basis, and if they were asked to recommend the rate of bonus to be declared, assuming that the premium scale for new entrants would be adjusted accordingly, he was sure that six substantially different figures would be given. Equity, according to the author's second definition, coupled with a bonus reserve valuation, rendered the current bonus too much dependent on the opinion of the actuary on factors which could not be estimated closely and about which a wide variation of opinion might reasonably exist.

The difficulties he had mentioned should be contrasted with the simplicity of the retrospective view. Premiums were charged which were based on rates of interest, mortality and expenses known to be safe. After a lapse of time, a valuation was made on the same basis. Since the basis was safe, the interest had been greater and the mortality and expenses less than had been allowed for in the premiums, and consequently there was a surplus. If six actuaries were asked to recommend a bonus scale that would be as equitable as possible, having regard to the surplus which had arisen in respect of each individual policy, he thought there would be very little difference between the six scales recommended. If the retrospective view, with an invariable valuation basis, were adopted, it would be possible to guarantee surrender values and

to base them as well as office premiums, calculations of policy alterations, paid-up policy values, etc., on the valuation basis, and still preserve equity, with a great simplification of all the 'Part II work' which had to be done in an office. He did not like the modification of the contribution method proposed by the author, but there was no doubt that the contribution system could be applied, without the complications of the Sheppard Homans method, by constructing a scale of reversionary bonuses based on specimen policies and applying them to all like policies.

As for equity, if any actuaries had to convince a Committee of Inquiry, consisting, say, of a business man, an accountant, a civil servant and two trade union secretaries, that one of the two methods was equitable, he asked which would they choose to defend —the invariable net premium valuation, with a contribution bonus system, or a fund where equity was maintained according to the author's second definition. It should be remembered (to misquote a famous Judge): 'It is not enough that equity must be maintained; it must be manifest to all men that equity is being maintained.' 'All men' included intelligent outsiders, such as those composing the imaginary Committee to which he had referred, as well as an actuary's professional peers.

Mr S. A. Hunt remarked that in the first definition of equity given by the author, i.e. that each group of like policies should receive the bonuses it would have received if it had formed a separate and distinct fund, there was no reference to the way in which the assets should be allocated or indeed to the assets at all. For the purpose of Table 1 the author had assumed two different distributions of the maturity dates of the assets. In a case I distribution it was quite easy to allocate the assets to the particular groups of policies to which they belonged; the sizes and types of other groups of policies which were combined in the same fund made no difference to how the reserves of a particular group of policies were invested. In a case II distribution the position was entirely different. The investment practice which led to such a distribution was not really applicable to any particular group; it was based on the fact that all the different groups of policies were combined together in one fund. Therefore for case II the author had had to make an assumption when considering the equity of distribution of appreciation, and the results shown in the last column of Table 1 were dependent on the assumption made. He did not suggest that the assumption which the author had made was not the most reasonable that could have been made, but it was quite clear that the inequities which the author had produced in the final column of Table 1, for case II, method D, were partly due to the investment policy which had been followed by the company, in that it was possible to allocate to the policies with a short unexpired term assets with a considerably longer term to maturity. The policies with 5 years to run were allocated assets having from 15 to 25 years to maturity, whereas those with 45 years to run were allocated assets having 45 years to maturity.

He questioned whether the investment practice of the office should be the cause of differences between the rates of bonus for the different classes of policies or should increase the differences due to other causes. If it were possible and practicable to maintain a distribution in accordance with case II, any change in the basic rate of interest combined with the corresponding appreciation or depreciation would have no effect on the future bonus prospects. That fact was reflected in the figure of £1. 19*s*. 4*d*. given by the author for method D in case II and repeated at the top of the last column in Table 1. The slight difference between that figure and £2 was due to the fact that it was not practicable to follow exactly the author's case II definition, because the fund would increase for a time before it decreased. It might well be that the company had invested its assets in the manner assumed for the particular purpose of eliminating the effects of fluctuations in interest rates, and that it had been able to do so only because the policies of various groups were combined together in one fund.

He suggested that in the circumstances it was not correct to base an equitable distribution on any allocation of assets to particular groups of policies and that some other method must be found. The point was of some importance when the author came to discuss the modified contribution method and produced a formula for dividing the appreciation under that method. The formula suffered from the disadvantage to which he had referred.

A further point in connexion with the modified contribution method as suggested in the paper was that it was dependent on there being no bonuses surrendered. For endowment assurances it was proposed to use the fixed maturity age method, and a very ingenious way of overcoming the difficulty about mortality profit was employed, but no reference was made to any possible distortion of profit from other sources. If there was any distortion of the loading profit, that could no doubt be corrected by an adjustment in *e*, which the author suggested should be recorded for each policy at entry, but any differences in the reserves for different ages attained for the same duration and for the same original term would have the effect of distorting the interest profit. He thought that with the A 1924–29 table that would not be at all serious, but the author had suggested that the valuation basis with regard to mortality should be adjusted to the experience of the office, and it was quite possible that the errors would then be of more importance. A further error was the fact that the reversion factor used for converting the surplus into reversionary bonus would be incorrect in those cases where the true age differed substantially from the assumed age. No doubt most of the errors could be neglected provided that any policies maturing at very high ages were excluded and dealt with separately.

In discussing his second definition of equity the author had said: 'The system would, therefore, largely eliminate fluctuations in the rates of bonus resulting from changes in the general level of gross interest rates'. That statement was based on a reference by the author that the case II distribution consisted mainly of long-term investments. It should be pointed out, however, that the elimination of fluctuations was dependent not simply on having mainly long-term investments but essentially on maintaining as nearly as possible a case II distribution of assets.

Mr R. W. A. Fowler said that he had always felt that the standard definition of equity was unsatisfactory. In nearly every argument it had been found that absolute equity had to be subordinated to practical considerations and, in justification of departure from the strict ruling, it had generally been said that rough justice had been done —very rough sometimes. It seemed that what had always been needed was a definition of equity which was not impossible of attainment and which could be used as a practical guide in determining questions of distribution of surplus, and he thought that the author's second definition fulfilled that need. Equity in accordance with the standard definition would not be attained over a period of changing conditions if a uniform reversionary bonus method was adopted. A strong case for the adoption of the author's second definition was presented in the paper by means of an analogy between policyholders and shareholders. That analogy would be made more complete, however, if with-profit policyholders were compared not with equity holders only but with joint debenture and equity holders, for the sum assured under a with-profit policy ranked pari passu with the sum assured under a without-profit policy. Thus the portion of the with-profit premium attributable to the sum assured alone would represent the cost of the debentures, whilst the bonus loading would represent the cost of the equity shares. If that view were taken, it was difficult to see why the profit or loss arising from the non-participating business should be treated, as suggested by the author, in a different way from the similar profit or loss arising from the fixed portion of the participating business. Surely greater consistency would be obtained if bonuses were deduced by spreading over the whole duration the effect of any variation of the experience from that assumed in the previous valuation, instead of adjusting only the immediate bonus payable.

Apart from that point, he agreed that, if experience corresponded with that assumed and new premiums were calculated as suggested, a consistent bonus record would be obtained.

Most actuaries would require some form of safety margin in their without-profit premium rates as a buffer against adverse experience. In order to place new with-profit entrants on an equality with existing policyholders, basic premiums should first be calculated on the valuation basis and to those premiums there should be added a small safety margin to produce the without-profit premiums. That safety margin would not

be required for with-profit premiums, as the bonus loading provided the required buffer, and thus the bonus loading would be based on the cost of future bonuses, as brought out by the valuation, subject to a deduction of n times the safety margin allowed in the without-profit rates, where n was the proportion of without-profit business to with-profit business expected in the future. By that means, provided the experience conformed with that assumed in the valuation and the proportion of without-profit business to with-profit business was correctly estimated, the new business would produce the same bonus as that provided for in the valuation. That method conformed with the practice of many offices of loading their with-profit premiums by only a proportion of the cost of the expected bonuses.

It was interesting to note that, on the basis of the second definition, with-profit premium rates would require to be increased if interest rates fell, with consequent appreciation, or if income tax fell below 7*s.* 6*d.* in the £, although the expectation of bonus would be little affected by the first change and considerably affected by the second change.

With regard to surrender values, he could not see how the particular definition of equity of bonus distribution to be adopted affected the problem. There had always been the alternative of a prospective or retrospective view. The adoption of the author's second definition did not cause that difficulty at all, and actuaries still had to solve the problem in the way they thought best.

In the case of with-profit policies he was not sure whether all offices allowed for the fact that a certain amount of reserve was held up to provide a uniform reversionary bonus and that some of the bonus loading which had been paid should be returned on surrender. He thought that in most cases it would be possible to give the value of the expected future bonus less the value of the future bonus loading, but it seemed to him that it would probably be better to exclude that item except in the case of limited-premium policies where it was important. Thus, whatever definition of equity was adopted, the actuary would have to determine his surrender-value bases on considerations similar to those that had guided actuaries in the past.

Mr J. L. Anderson said he was glad to contribute to the discussion, because he had submitted a paper on much the same subject to the Faculty of Actuaries about a year previously.

A good deal had been said on the question of the definition of equity. Personally, he thought that it was a mistake to try to reduce equity to a code of rules. He thought that it should be interpreted in the broadest possible way. Equity was defined in *Chambers's Dictionary* as 'the spirit of justice which enables us to interpret laws rightly', the important words being, he considered, 'the spirit of justice'. He could not believe that they would be acting in accordance with that spirit if they escaped responsibility by reducing equity to a rigid code of rules, particularly one such as that given in the second definition in the paper. There must, he thought, be times when the continuance of the same system of bonus that had been used in the past would offend against the spirit of justice.

In short, he took the view that, though the actuary must be prepared to state what he would consider to be an equitable allocation of profits in a particular set of circumstances, it was neither necessary nor desirable that the actuary should put forward any one system as being universally applicable. The author had to some extent admitted the weakness of his position when he stated in his conclusion that 'the choice of the definition ...is relatively unimportant in comparison with the necessity for consistency...'. Personally, he did not regard consistency as a cardinal virtue in the actuarial or in any other sphere.

If the actuary refused to bind himself by any rigid code of rules, his work was, of course, made more difficult, and some of the difficulties arose from the need to distinguish clearly between theoretical and practical considerations. He would suggest, therefore, that problems of allocation of profits should be divided into three stages. At the first stage, which would normally involve a bonus reserve valuation (though not necessarily for publication), the task of the actuary was to decide what he considered to

be an equitable allocation of profits. He agreed with the author that the clearest way of showing the positions of the various groups was to calculate the share of the fund applicable to each group, to add the value of the premiums to be paid, and to deduct the value of sums assured and existing bonuses. The resulting figures were then easily comparable with any scale of bonuses that it might be desired to test.

In allocating the fund between the different groups when there had been appreciation or depreciation, it was necessary to decide how far the experience of any particular group of assets should be related to any particular group of policies. He was very doubtful whether any general solution could be put forward. In the paper which he had read before the Faculty, he dealt with one particular case, that of an office having a considerable proportion of its investments in irredeemable securities at a time when interest rates were tending to fall. The method which he had employed could be used for any distribution of assets, but he did not suggest for a moment that it would necessarily be suitable in all circumstances.

At the second stage, the actuary would have to decide on the actual scale of bonuses to be declared and the scale he hoped would thereafter be maintained assuming conditions remained unchanged. Practical considerations should be deferred to this stage, so that the theoretical allocations at the first stage might be made without bias. Obviously, the system of bonuses previously used would not be lightly thrown over, and he himself would not regard that bonus system as unsatisfactory merely because under its operation a small proportion of policies would receive a rate of bonus substantially higher or lower than they had earned, or because a substantial proportion would receive a rate slightly higher or lower than they had earned.

He was sorry that the author had not included in his tables another method of allocating bonus, whereby the same rate of bonus would be granted to all policies in a given year, the rate varying from year to year. For example, after there had been appreciation the rate would be allowed to fall until it reached a minimum rate, and the reverse would occur when there had been depreciation. The effect of such a system would be to even out some of the inequities shown in Table 1 by method D.

Finally, the third stage involved the setting-up of the actual published valuation basis, whether net premium or otherwise. This he regarded as the least important part of the actuary's work; it was, after all, purely a question of mechanics, and if the actuary had been able to deal with the first two stages adequately the third stage should not present any serious difficulties.

Mr M. E. Ogborn referred to the comparison between the position of holders of participating policies and of holders of ordinary shares, and said that the only criticism which he desired to make was the rather obvious one that when a person bought ordinary shares the purchase price was paid to the vendor, whereas when a person effected a policy in a life assurance office the premiums were paid to the company. He thought that a better comparison was with an investment trust continually open to new members, under which investments were made for life or for a fixed term. If, then, each member paid the market price of the units he bought, received his share of the interest while he was a member, and was paid out on the basis of the market price when he went out, it would be agreed, he thought, that equity was a matter of accounting only. But in the case of a life assurance fund, although that was a picture of the fund in its investment aspect, there were uncertain factors, for example mortality, which made the question of equity more difficult.

Furthermore, the investment was not usually a lump sum paid at the start but annual premiums paid over a period of years computed on terms fixed at the outset, and there was a guarantee against depreciation so that it was not possible to set off all losses on investments by paying out on the basis of the market price. It seemed to him that that was the difficulty with the contribution method of distribution. He did not share the favourable view of it which had been expressed during the discussion, because it seemed to him that the contribution method attempted to reduce equity to a matter of accounting, but in doing so it was not possible to carry that accounting to its logical conclusion. The whole of the surplus was not in fact divisible, because of the uncertain nature of the

factors; and, because the whole of the surplus was not divisible, a large part of the surplus at each valuation had not arisen in the valuation period but had been carried over from previous years. Then again, although perhaps in theory profits and losses on investments might be dealt with by the contribution method, in fact that would mean the abandonment of the guarantee of the basic sum assured. The author had stated on p. 45 'It would seem that the contribution method is completely equitable', but personally he thought that the conditions for its being completely equitable were so impossible of attainment that the method did not approach complete equity in practice. In criticizing it so harshly, he did not mean to say that the results of the contribution method were not equitable, but merely that in practice it was necessary to depart from the strict application of the method by averaging and by other modifications which really did away with the claim of the contribution method to be the only equitable method.

There were some interesting points in Tables 2 and 5. Looking at Table 2, it struck him very forcibly that the rates of bonus for the existing policies decreased with duration. That was the very reverse of the situation that was feared some years ago; the fear then was that the new business would not support the same bonus as the existing fund: The problem posed by Table 2 was that the new premiums supported a higher bonus than the fund. Why did the bonus decrease with duration? The explanation given by the author on p. 44 did not really go deep enough to answer that particular question. If the method of distribution of bonus was such that the bonus distribution at any time amounted merely to the interest surplus earned in the period there would be no difficulty; the difficulty arose because with the simple and compound reversionary systems the bonus depended upon an estimate of the future as well as on the actual experience of the past, and if the conditions changed the bonus was affected. In fact, too much bonus had been distributed and the decrease in the rate of future bonus was necessary to correct that position: too much had been given previously, and therefore less must be given thereafter.

A point of detail concerning those Tables was that the author had assumed an annual distribution of profits, so that he had only one year's bonus to deal with at the time of the valuation. If he had assumed a 3-year or a 5-year period of distribution of profits, he would have made the problem easier, though he would, of course, have made the paper less interesting. It should be borne in mind, however, that a longer period between each distribution would give some margin.

Another point concerning those two Tables was that the rate of decrease in Table 2, the simple bonus table, was considerably more than the rate of decrease in Table 5, which was concerned with the compound bonus. Why should that be the case? He thought the explanation was that the compound reversionary bonus depended upon a heaping-up of surplus; it implied a larger element of deferment, and consequently was nearer to the position where the bonus at any time merely represented the interest surplus earned in the period.

That led him to the thought that there might be some other system of distribution which would obviate the necessity of worrying about the rate of interest. As a matter of fact, the original method of distribution of bonus, the first one ever used, included a large element of deferment, because at each valuation one-third of the surplus was carried forward for the purpose of general security. There was a large element of deferment, and the bonuses allotted reflected that fact and were given according to the whole duration of the policy at the time of the distribution, so that a policy 20 years in force received double the bonus that was allotted to a policy 10 years in force. If that method of distribution of bonus were still in operation and if premiums were charged on the basis of 3 % interest loaded for a bonus of 20*s.* per cent, the assumption of an abrupt change to 2½ % interest would mean that new business would only support a bonus of 15*s.* per cent, and so would existing business with a very slight adjustment for duration. In one sense, therefore, actuaries might be said to have brought the problem on themselves by departing from the principles of their predecessors!

He had spent some time on Tables 2 and 5 because he thought that they were fundamental to the situation which had arisen out of the war, with the increase in the rate of income tax and the decrease in the rate of interest. The new business would, in the

assumed conditions, support a higher bonus than the existing fund. The life assurance funds had, however, a certain amount of appreciation and the true picture lay somewhere between that shown in Table 1 and that shown in Tables 2 and 5. There was, he supposed, one obvious solution, which was to take credit for sufficient appreciation to bring the bonus up to the rate which the new business would support.

At the bottom of p. 37, the author had said: 'Since a level profits loading is charged for the bonus the obviously equitable method of distribution would be as a level cash bonus...'. A level cash bonus was, he thought, not equitable, for the reason that at any valuation the whole of the surplus was not in fact divisible; part of the surplus had arisen from previous periods. The reversionary bonus systems were adopted because the whole of the surplus at a valuation could not be divided, and the element of deferment present in a reversionary bonus system was necessary to equity.

Of the two definitions of equity given on p. 37, the first seemed to him to ignore the mutual principle and the second turned equity into a form of contract rather different from his own conception of equity. He had looked on equity as 'the application of principles of justice to correct or to supplement law'; not as a fixed contract, but as the correction in the interests of justice of the strict operation of a contract. He did not think that it was possible to limit the consideration of equity either to the past or to the future as had been suggested. The problem was a threefold problem. It arose in the past, but it was necessary to take account also of the realities of the present and the prospects of the future. It arose in the past, because the problem dealt with an actual fund, with actual policies and actual premiums, and with actual statements made in the past, some wise and some, no doubt, unwise. There were actual decisions which had been taken as to previous divisions of surplus, actual factors which had affected the fund. Account had to be taken of all those things in considering what was equitable, and there could not, he thought, be any one completely equitable solution: equity depended on a consideration of all the factors at the time. He did not think that it was possible, therefore, to have an exact definition of equity.

Mr B. F. Taylor said that in his view the most valuable part of the paper was p. 37, where attention had been drawn to the question of equity and two definitions had been given. He was surprised that an obvious third had not been mentioned, which was that equity was attained if the policyholder received as much bonus as he had been promised by the inspector when he completed his proposal. It was easy to see the amusing side of that, but it was worth considering whether there was anything serious in it. It was well known how difficult it was to explain the technicalities to a policyholder who found it impossible to understand just what was meant by mortality profit, loading profit and so on, or just why his bonus was not as much as his neighbour's.

In arriving at the bonus to be allocated, the policy was placed with others in a group, according to the method adopted; but why should the policy be placed in a group, and, if it was to be, on what logical or equitable basis should the allocation be made? Was the group to be the class of policy, the valuation group, the group of policyholders who joined the company in a particular year or quinquennium, or the one policyholder himself? He submitted that there was just as much logic or equity in using any one basis as in using any other. The policyholder had taken out a with-profit policy with a particular company, and that and the size of the bonus he received was all he knew. Why should the policyholder be penalized because he happened to be placed with a group of suicidal gentlemen or people suffering from heart trouble, while his neighbour was placed with a group of perfectly respectable and healthy people?

There were two points that should be noted in the definitions on p. 37. The first definition said: 'each group of like policies'—which was what he had referred to—'should receive the bonuses it would have received if it had formed a separate and distinct fund'. That was worth noting, in view of what appeared later with regard to the distribution of assets. In the second definition there were the words 'uniform bonuses', but was that an essential condition of the definition? Perhaps it was not clear what the author meant by 'uniform'. In that same definition there were also the words 'the premiums charged to new entrants correspond to their expectation of bonuses'. What

precisely might that be? The bonuses were not guaranteed and future conditions were unknown when the policy was effected. Was the 'expectation', then, the figure that the actuary thought right or that the inspector believed in?

Turning to the conclusion of the paper, with which he agreed substantially, he noted that the author had said that if the second definition was accepted, the premiums charged for new policies would have to be altered if a change in conditions were to occur. Taking triennial and quinquennial valuations it might be said that at no two successive valuations had conditions been alike during the last 15 years. He had in mind the conversion of the 5% War Loan, the financial crisis in 1931, the alterations due to the introduction of the A 1924–29 mortality tables, the extra mortality due to the war just concluded, the low rates of interest during and since the war, the increase in new business combined with the reduction of rates of premium at the time of the introduction of the new mortality tables, and the reduction in new business during the war with a consequential decrease in expense ratios. All those things had happened within a comparatively short time, and, if similar frequent changes were to continue, each scale of premium rates would require separate consideration when a bonus was to be declared. Thus there would be an increasing number of different scales of premium rates, and it might be difficult with a uniform bonus to avoid injustice.

On p. 41 a reduction in the rate of interest earned from $3\frac{1}{2}$% to 3% was referred to as equivalent to an increase of 2*s.* 6*d.* in the £ in the rate of income tax. That meant that the fund must previously have been earning a gross rate of interest of 4% and that the tax had increased from 2*s.* 6*d.* to 5*s.*, but there was no mention of 4% in the paper. Again, presumably the valuation rate of interest was reduced to 3%, but that was not explicitly stated. There were several similar occasions in the paper, which he thought made it more difficult to follow.

A point which seemed to have been glossed over was the question of expenses. How did the author arrive at a standard rate for the expenses, and how did he allow for the incidence of new business? As far as he himself had been able to observe, there was nothing like constancy in the expense ratio of most offices. A paper had been read some years ago which had attempted to set out an analysis of expenses, but many assumptions had had to be introduced, and he did not think that the results suggested that it was an easy matter to deal with. It might be found, for example, that the expenses, particularly bearing in mind commission, varied appreciably from group to group.

The author had referred only to whole-life and endowment assurances. Personally, he would like to see in every paper which dealt with valuations or model offices or surpluses a reference to whole-life assurances by limited payments. They always seemed to be omitted, and he sometimes wondered whether it was because they were a little too difficult to deal with. He would particularly refer to the contribution method in that connexion.

Like a previous speaker, he was amazed to find that the author had calculated the group mortality profit from endowment assurances by means of an assumed maturity age. Though that assumption might be good enough for purposes of valuation, it might lead to serious error in the calculation of mortality profit. It also gave rise to an error in the loading, i.e. in the difference between the office and pure premiums. There was no reason to suppose that those two errors would counterbalance in the case of a particular group. With regard to mortality, the author had glossed over the error by suggesting that any distortion was unlikely to be serious unless a very unsuitable mortality table was used, but that did not affect the question as between groups. On the other hand, earlier in the same section the author had introduced q', and it would be of interest to know how q' would be obtained, especially as the valuation table was assumed to be so close to the actual experience.

With regard to the calculation of mortality profit, the author had suggested that it did not matter very much whether the initial or final reserve was used, but actually it made a large percentage difference if the initial reserve was used with q in calculating the expected strain, and the error was usually greatest where the amount was smallest. He himself would have thought that an easier way to deal with the matter would be to work out the expected death strain, using a prepared table of factors for each group. The

factor was of the form $A_0(1+i)-A_1$ (for complicated types of policies there might be another term to bring in), and the factor was to be multiplied by $S+S.P/d$. If the factors were calculated per 1000 sum assured and the figures $S+S.P/d$ scaled down correspondingly, the work could be done very rapidly and would give as accurate results as any other method he knew.

He found difficulty in understanding exactly what e was intended to represent in the calculation of the loading profit, and he wondered whether the author had actually tried the method in a practical case. He felt that the difficulties had been understated by the author, and that if equity was aimed at, its true conception, and not some second approximation to rough justice, should be constantly borne in mind.

Mr A. T. Haynes wished to defend the case II distribution of assets, which he did not think had been very fairly dealt with in the paper. An actuary who attempted to set up a case II distribution with the idea in mind which lay behind the definition on p. 39 would be rather surprised to find anything in the last column of Table 1 except the figure of £2 % running down from the top to the bottom. After all, the assets had been arranged in order of redemption date so that, no matter what appreciation or depreciation might result purely from a change in the level of interest rates, the proceeds available for each policy should remain unaffected. That being so, it was disturbing to find that some policies were to receive far less, and some far more, than the basic bonus of £2 %.

At the top of the last column of Table 1, the group figure of £1. 19*s*. 4*d*. % appeared in place of the figure of £2 % that would have been expected. The reason for that could be seen by studying the case II distribution of assets on p. 39; that distribution had clearly been distorted by the author's five-yearly groupings. The author had set the first seven years' premium and interest income against claims which included only five years' maturities, leading to the emergence of surplus cash in the early years. If the author had constructed his model office on an annual basis, it would have represented a stationary fund which, if closed, would have immediately suffered an annual reduction. The corresponding assets in the case II distribution redeemable in each year from the first year onwards would then have been equivalent to the particular year's decrement in the fund. Such a 'perfected' case II distribution, with the element of distortion removed, would have produced a bonus of £2 % for the whole office in place of the author's figure of £1. 19*s*. 4*d*. %.

On the other hand, the very considerable element of distortion in the separate groups in the last column of Table 1 was due to the method adopted by the author in the allocation of assets; the allocation seemed quite unfair to the assumptions which underlay the asset distribution. Those assumptions implied that the office would be viewed as a whole and not in groups. Maturities and death claims in the early years would be paid very largely out of premium and interest income and only in so far as the income was insufficient for the purpose would claims be met out of assets, which in the 'perfected' case II distribution he had suggested would be redeemable in each year to exactly the required extent. If the author had followed out that basic assumption instead of making another assumption for the purpose of allocating assets, he thought it would have been found, as in fact theory demanded, that every group in the last column of Table 1 would rightly have received a bonus of £2 %.

That line of argument, of course, involved a somewhat different approach from that adopted and applied to each group in the early part of the paper, but he personally preferred an approach that married all the groups and allowed them to protect each other; surely that was the principle of life assurance. If the whole were to be subdivided into a number of small groups, was there any logical reason for not dealing with each individual policy? He felt that it was possible to go too far in searching for equity by way of subdivision. If a change in the general level of interest rates led to appreciation or depreciation of such an amount as would permit a continuance of the basic bonus of £2 % over the business as a whole, why should a departure be contemplated from the practice of distributing uniform bonuses to all groups? In those circumstances he preferred the conclusions based upon the author's second definition of equity, although he would not necessarily accept that definition unconditionally.

Mr C. E. Kingham, in closing the discussion, said he felt that a 'prefabricated' speech would be out of place. The walls had gone, and he was left with a concrete slab, which was that they had been far too theoretical and had to be more practical. He had learned that by an injunction from a predecessor who was worried about equity between teetotallers and non-abstainers. For years that predecessor used to deal with the matter in detail by building up separate funds; how it was done on such small figures he did not know, but the teetotallers always got more bonus and everybody seemed to be happy except the actuary, who realized the difficulties. Having decided in about the year 1900 that some change ought to be made, the actuary in question said that his contribution system 'was theoretically very fine, but in practice had not been found to work well and produced great fluctuations'. 'The largest and most progressive and best-managed offices', he said, 'have therefore abandoned this system and have generally adopted a compound reversionary system for all ages and durations. This system is easily understood by the assured.' That, apparently, was the criterion of equity. He rather liked the concluding remark: 'It must be borne in mind that these rates of bonus are not brought out by any particular set of calculations, but, like all practical systems of bonus distribution, are rather the result of carefully considered compromise between various conflicting elements.'

He felt, after listening to the discussion, that he had not much patience with a policy involving 'matched investments' and treatment of appreciation as surplus available for distribution, and he did not feel that it was wise to keep on tinkering with premium rates.

He had expected the discussion to follow the lines of that on the author's previous paper, where the practical aspect was emphasized rather more. He had had an uneasy feeling that they were going to be invited to return to the contribution method of distributing profits. He preferred the author's second definition of equity, with the emphasis placed on equity to generations of policyholders rather than on individual equity, but most actuaries had been born into a world of compound or simple uniform bonuses and would probably remain there.

The President (Mr R. C. Simmonds), in proposing a vote of thanks to the author, said that all those present would have been asking themselves, as they had often done before, 'What is equity?' Much had been said about it in the discussion, but many of the suggestions seemed to be mutually destructive or at least merely to reflect different aspects of the matter; there had not been, as it were, a whole view. He felt that Mr Taylor (though, from his tone, he had seemed to say it with some scorn) was in fact approaching the truth when he had compared equity with a second approximation to rough justice. If in that matter they could reach a second approximation, they would not have done badly. There had been a broad survey of numerous facts and each of those present would have to return to his own problem and to try to do his best with it in the light that he had—the trouble with bonus distribution was that the light was so imperfect and the view so short.

Mr T. R. Suttie, in reply, said that he admired Mr Anderson's idea of having no rigid definition of equity, and the phrase 'the spirit of justice' was very attractive, but when it came to deciding on an actual bonus distribution it did not seem to him to give concrete guidance. He was sorry that it should be felt that he had not given sufficient credit to the case II distribution of assets, which Mr Haynes favoured. He thought that he had; he liked it himself, and on p. 51 he had pronounced in its favour. He agreed that the particular model office which he had chosen in order to reduce the work had distorted the results and changed the bonus from £2 % to £1. 19*s.* 4*d.* %, but that was a difficulty which would usually arise in practice, because most offices were expanding and it would seldom be possible to make a distribution of investments which would give the exact figure of £2 %.

Mr Suttie subsequently wrote as follows:

To avoid any possible confusion, I should like to point out that the distribution of assets to which Mr Hickox referred as case I was that adopted in my paper appearing

in *J.I.A.* Vol. LXXII, p. 203. The distribution described in the present paper as case I provides that the reserves of each group of like policies shall be invested in assets maturing in the year in which the policies will become payable if they remain in force until maturity, and it was my intention that like policies should be those with a common year of entry, age at entry, and original term, irrespective of the unexpired duration to the date of becoming a claim. If this last factor is taken into account and the argument in the paragraph headed 'First Definition' at the foot of p. 37 is accepted, only a level cash bonus (adjusted to allow for initial expenses and commission) can be considered equitable when the experience conditions coincide with the assumptions made in calculating the premiums.

I agree with Mr Hickox that, if only one change in experience conditions occurs, equity could probably be attained by some simple modification of the uniform reversionary bonus system, but it seems doubtful whether this would be satisfactory if more than one change should occur within a comparatively short period.

Mr Hunt is wrong in suggesting that the modified contribution method is entirely dependent on there being no bonuses surrendered. If such surrenders have occurred, the total bonus to be added to each valuation group can still be calculated exactly as described in the paper, but the bonus shown in the scale for an individual policy must be modified in any case where previous bonuses have been surrendered.

I agree with Mr Ogborn's explanation of the decreasing rates of bonus shown in Tables 2 and 5, but I should like to point out that I did not, as he appears to think, attempt to explain this decrease, since the reason seemed to be sufficiently obvious and was not strictly relevant to the purpose of the paper.

It is true that by historical accident a considerable element of deferment was introduced into bonus distributions and that this has been perpetuated by the use of the uniform reversionary bonus system, but it does not seem that this element of deferment is essential. If the premiums include an adequate margin of bonus loading, the security of the office could be sufficiently safeguarded by this margin in future premiums and the whole surplus arising in each valuation period could be treated as being divisible. In these circumstances a level cash bonus would be the natural method of distributing profits.

Mr Taylor's difficulty regarding the change in the rate of interest referred to on p. 41 has arisen from his omission of the word 'approximately'. As he says, net rates of interest of $3\frac{1}{2}\%$ and 3% correspond to a gross rate of interest of 4% and income tax rates of 2*s.* 6*d.* and 5*s.* in the £ respectively, but they are also equivalent to a gross rate of $4\frac{1}{2}\%$ and income tax rates of 4*s.* 5*d.* and 6*s.* 8*d.* in the £ respectively, a difference of 2*s.* 3*d.*, which is sufficiently close to 2*s.* 6*d.* for the purpose of the paper, viz. a comparison of the effects on the rates of bonus of alterations in the rates of interest and expenses following a given change in the rate of income tax.

Similarly, his query as to the method of obtaining q' when the valuation table closely represents the experience mortality is answered in the last paragraph on p. 49, where it is stated that the change to the new valuation table would eliminate the mortality profit, i.e. in these circumstances q' would be equal to q.

An analysis of the expenses is of course difficult, but presumably it must be attempted in order to arrive at the expense loading to be included in the premiums for new policies. If it were found that the expenses did not vary between different groups of policies there would be no need to record e and its place could be taken by a suitable percentage of the premiums, or more probably by a combination of percentages of the sums assured and premiums.

Nothing in Mr Anderson's remarks has altered my conviction that the bonus distributions made during the past 30 years would have been more satisfactory if a greater effort had been made to follow consistently some clear conception of equity.