The Actuarial Profession
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Excess Volatility Re-visited

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Background (1)

- Excess volatility: at times stock market values deviate substantially from their 'fundamental values'
- Interpreted as evidence for investor irrationality
- Our argument: it's not clear market volatility has been excessive. Changes in market values are not greater than the changes that can be justified by changes in fundamental values
- We're not saying the market is always correctly valued, just that its volatility is not on its own a reliable symptom of irrationality.

Background (2)

The key equation is the dividend growth model:

$$P_t = D_{t+1}/(1 + r_{t+1}) + D_{t+2}/(1 + r_{t+2}) + \dots$$

With constant forecast growth of dividends as at date t, g_t, and constant expected return on equity, r_t,

$$P_t = D_t(1 + g_t)/(r_t - g_t) \qquad (1)$$

Also, for the whole market,

r_t = real risk-free interest rate + expected rate of inflation + inflation risk premium (?) + equity risk premium

Background (3)

- Shiller (1981): assume perfect foresight for dividends and interest rates (extreme rational expectations). Then stock market values vary far more than do the present values of future dividends.
- Campbell & Shiller (1989) are more realistic. They allow forecasts of the real dividend growth rate and real interest rate to vary over time. They use vector autoregression to estimate the forecasts. Conclusion: changes in the estimated forecasts are still insufficient to explain observed volatility.
- Subsequent debate: is the excess volatility thus identified better seen as evidence for changes in sentiment/irrationality, *or* as evidence for (rational) changes in the expected equity premium (Cochrane, 1991)?

Data and forecasts

- Data: UK data, sample period 1921-2008.
- Several sources used
- Simple forecasts, similar in style to the ones used in several papers to estimate the equity risk premium expected in the past, are employed.

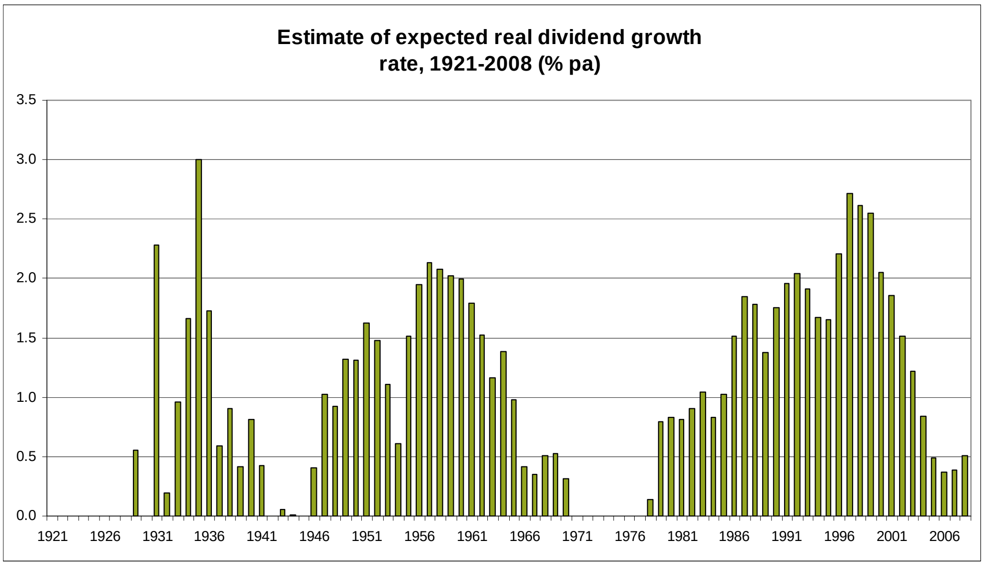
Forecasts (1)

Expected real growth rate of dividends, $g_{real,t}$

The geometric mean real growth rate of dividends during the 25-year period that starts at date *t*-15 and ends at date *t*+9

Constrained to a min of 0% and a max of 3%

Year-by-year changes in $g_{real,t}$ are small: average of the absolute value of the changes is 0.32%.



Forecasts (2)

Expected real risk-free interest rate, $r_{Freal,t}$, expected rate of inflation, i_t, and expected inflation risk premium, irp_t

1982-2008:

$r_{Freal,t}$ = real yield on 20-year index-linked government bonds (introduced in 1981)

$i_t = 0.8(r_{F,t} - r_{Freal,t})$

$irp_t = 0.2(r_{F,t} - r_{Freal,t})$

where $r_{F,t}$ is the nominal yield on undated government bonds.

Forecasts (3)

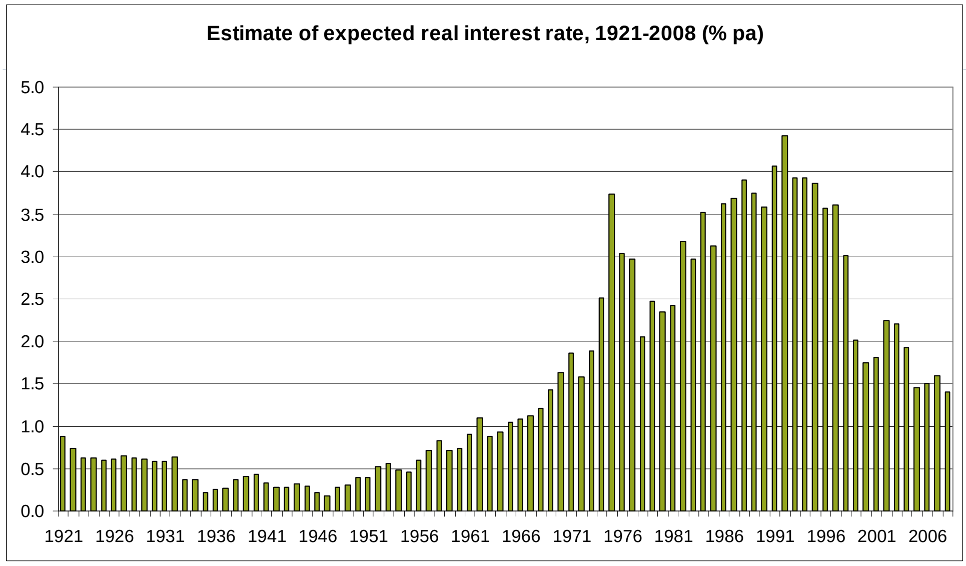
1921-81:

$$r_{Freal,t} = r_{F,t} - (i_t + irp_t)$$

where

$$i_t = 0.323\% + 0.616(\text{nom yield on undated govt bonds})$$
$$irp_t = 0.2i_t/0.8$$

We found that the nominal bond yield provides quite a good forecast of future inflation over the next 10 years. 0.00323 and 0.616 are the coefficients in a regression of geometric mean inflation for years *t* to *t*+9 on the nominal bond yield. $R^2 = 0.30$.



Inferring returns from fundamentals (1)

An explanation of annual market returns in terms of changes in forecast fundamentals

Expected nominal return on equity at date t,

$$r_t = D_t(1 + g_t)/P_t + g_t \qquad \text{(1 re-arranged)}$$

Nominal return due to observed unexpected dividend growth during year starting at date t, $R(\Delta div)_t = R(rgfixed)_t - r_t$

$$= (G_t - g_t)(1 + D_t/P_t) \qquad (2)$$

where $R(rgfixed)_t$ is the return that would have arisen were r_t and g_t not to change, and G_t is the *actual* nominal dividend growth for the year.

Inferring returns from fundamentals (2)

The return that is due to a change in r_t or g_t is the actual return, R_t, less the return justified by the actual growth in dividends:

$$R\{\Delta[(r - g)/(1 - g)]_t\} = R_t - R(rgfixed)_t$$

where

$$\Delta[(r - g)/(1 - g)]_t = (r_{t+1} - g_{t+1})/(1 + g_{t+1}) - (r_t - g_t)/(1 + g_t)$$

We now decompose the return that is due to a change in r_t or g_t …

Inferring returns from fundamentals (3)

We show that

$R\{\Delta[(r - g)/(1 - g)]_t\} = R(\Delta r_{Freal})_t + R(\Delta irp)_t + R(\Delta erp)_t - R(\Delta g_{Freal})_t$ (3)

where $R(\Delta r_{Freal})_t = [r_{Freal,t}/(1 + g_t) - r_{Freal,t+1}/(1 + g_{t+1})] \times (1 + G_t) \div (r_{t+1} - g_{t+1})/(1 + g_{t+1})$

and analogously for the other variables.

erp_t is the expected equity risk premium as at date t.

It is the difference between the expected return on equity and the expected return on the risk-free asset:

$$erp_t = r_t - r_{F,t} = r_t - (r_{Freal,t} + i_t + irp_t)$$

Inferring returns from fundamentals (4)

Since we have estimates for all the other variables, the equity risk premium is calculated as the residual. This ensures that:

the expected return + (1)
the return due to unexpected dividend growth + (2)
the return due to a change in r_t or g_t (3)
is always exactly equal to the actual return for the year.

When we assume a fixed equity premium, we have estimates of what the returns *would have been*, given estimates in the other forecast variables.

Results: expected equity premium

- Inferred arithmetic mean expected premium for full sample period (1921-2008) is 3.3%.
- The ex post mean premium is 4.9%
- For the 50 years 1950-99, our mean expected premium is 3.0%, compared with an ex post premium of 9.1%.
- These results agree with those of a number of other recent studies (e.g. Blanchard, 1993; Fama & French, 2002).

Have changes in the expected equity premium contributed to market volatility?

The expected premium has certainly varied.

Min = -0.2%; max = 7.0%; std dev = 1.8%

Std dev of return due to changes in erp_t is 22.7%, close to std dev of actual returns, which is 23.9%

But changes in the expected premium do *not* contribute to volatility, because they often dampen down the return that would have arisen had the premium not changed:

$R_t - R(\Delta erp)_t$. Correlation coefficient for series $R_t - R(\Delta erp)_t$ and $R(\Delta erp)_t$ is -0.32.

Why does the expected premium vary so much?

Two possibilities:

1. Our estimates of $r_{Freal,t}$ and $g_{real,t}$ are not variable enough, in which case changes in the inferred premium will be overstated.

2. The returns for some years could have an irrational component, so the apparent changes in the premium we measure are a symptom of irrationality.

Our main point

Observed volatility can be explained without assuming either changes in the expected premium or irrational pricing

- We now assume the expected premium is *fixed* at 3.3%.
- We calculate simulated fixed-premium market returns: the returns that would have arisen given the observed changes in the forecasts of the other variables.
- Standard deviation of fixed-premium returns is 28.1%, or 22.9% assuming there's no inflation risk premium. Std dev of actual returns of 23.9%.

Return on equity with expected premium fixed at 3.3%

	Return %	***Std dev %***
Simulated return	13.4	*28.1*
Of which, return due to		
Expected div growth	9.7	*3.4*
Unexpected div growth	1.0	*7.5*
Change in $r_t - g_t$ (= change in div yield)	2.7	*26.6*
Of which, return due to change in		
Expected real interest rate	0.7	*7.1*
Inflation risk premium	0.1	*3.3*
Expected real growth rate of dividends	1.9	*22.3*
Actual return	12.6	*23.9*

What happens if you alter the forecasts to make them less variable?

The range of possible volatilities that result from reasonable sets of estimates is broad, and the actual market volatility is well within the range.

Conclusion

- We offer a simple and transparent framework for analysing how changes in forecast fundamentals affect market values
- What were the forecasts of ‘the market’ at past dates? No one knows. But in our view the level of ‘rational volatility’ cannot be measured very precisely
- If you think market forecasts changed like our forecasts, you’ll think there was no excess volatility (... though you might then say that the past forecasts themselves were excessively volatile!).