JIA 120 (1993) 253-309

J.I.A. **120,** II, 253–309

THE EXPECTED RETURN ON UNITED KINGDOM EQUITIES AND AN IMPLICATION FOR PENSION FUND VALUATIONS

BY P. D. JONES, F.I.A.

[Presented to the Institute of Actuaries, 25 January 1993]

ABSTRACT

The paper analyses the relationship between equities and gilt-edged. It shows that the yield ratio (the yield on gilts divided by that on equities) should no longer hold its almost mystical significance of the last decade, being but one component of the expected return on equities, in which the most significant element is the rate of dividend growth relative to the yield on gilt-edged. How equities perform in economic recession is highlighted. The paper reviews the performance of equities over the last 70 years in both nominal and real terms and analyses the historic sources of the equity risk premium, i.e. the excess return of equities over gilt-edged. If the equity risk premium remains at below-average levels in the 1990s, as is possible, constraints may be placed on actuaries' choice of pension fund valuation bases.

KEYWORDS

Dividends; Equities; Expectation; Gilt-Edged Securities; Yields

1. INTRODUCTION

1.1 This paper is in three parts. Section 2 sets out a method of calculating the expected return on equities as a function of the yield on gilt-edged. It shows that the expected return on equities has as a component part, which is usually small, the inverse of the well-known yield ratio, i.e. the yield on gilts divided by the yield on equities. A much more significant component is provided, not surprisingly, by the rate of growth of equity dividends. Data from 1965 are examined.

1.2 Section 3 draws on the methodology of Section 2 to analyse the historical returns on equities since the early 1920s—i.e. over a 70-year period. It shows that until the late 1950s, a high, if volatile, rate of dividend growth and a yield ratio of under one played a significant role in boosting the historical returns on equities to several times those on gilts. Since the 1960s, returns on equities have also been higher than those on gilts, though on a much more modest scale than in the earlier period.

1.3 Section 4 puts forward a personal view of the author. A thrust of the paper is that equities are, by their nature, riskier in nominal terms than gilt-edged securities. Their returns have historically been higher than those on gilt-edged. It follows that actuarial assumptions about expected returns on equities should be higher than those on gilt-edged. In the view of the author (whose expertise lies in investment, not in pensions valuation), this provides a constraint to pension fund valuation assumptions. Perhaps, in current conditions, some pension fund assets are being valued too highly, or alternatively some pension fund liabilities too modestly.

2. The Expected Return on Equities

2.1.1 Since mid 1990, the rate of growth of equity dividends has collapsed from around 15% p.a. to virtually zero (see Figure 1 covering the past 25 years). This situation coincided with the United Kingdom's entry into the European Exchange Rate Mechanism (at an exchange rate which many believe harmed the profitability of British industry) and with an economic recession which turned out to be longer and deeper than virtually anyone expected. Above all, it coincided with a period of rapidly falling inflation (to a low level which, despite the U.K.'s abrupt departure from the ERM, seems to the author to have an air of permanence about it), which has not been matched, point for point, by lower long-term interest rates. Real interest rates (that is to say the excess of the interest rate over the inflation rate) are again equalling historically high levels when the economy is crying out for stimulus.

2.1.2 It is not surprising, therefore, that much attention has been paid to re-evaluating U.K. equities under these new circumstances. On the one hand, the new environment is one of 'low inflation' and many observers have concentrated on this aspect. Comments such as 'lower inflation should permit higher PE multiples' are to be found. On the other hand, 'high real interest rates' are an

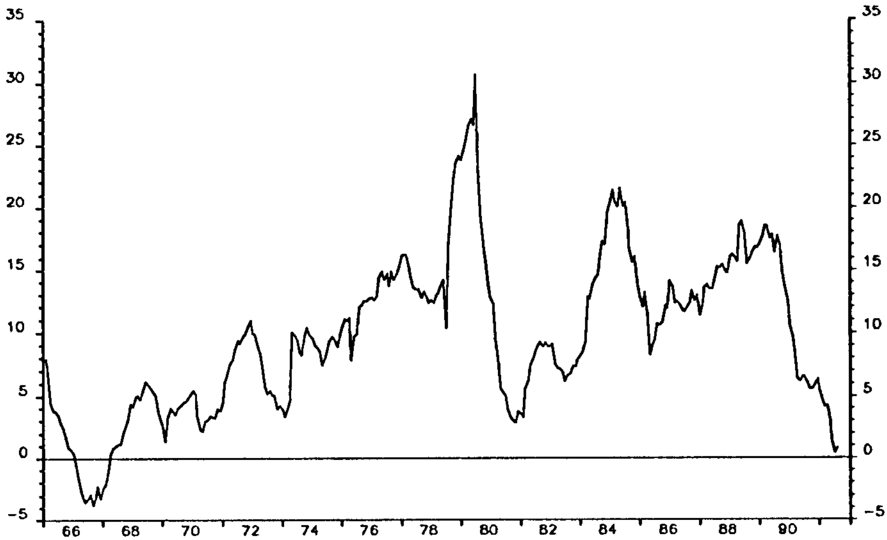

Figure 1. U.K. annual dividend growth %. Source: Datastream (Figures 1 to 7).

equally important, yet negative, factor. It is from this standpoint that the author prefers to analyse the situation, for two reasons. First, high interest rates are clearly harmful to virtually any business enterprise. For example, they raise the burden of debt service and inhibit capital investment financed by borrowings. Second, in themselves, they represent a formidable alternative to other investment media, including equities, property, works of art, gold, etc.

2.1.3 But, above all, classical financial theory has as its central theme, the long-term interest rate—the yield on long-dated gilt-edged, in the case of the U.K. In evaluating U.K. equities (market value approximately £500 bn) relative to 'something else'—it is important that the 'something else' is well researched, readily arbitrageable against equities and large. The U.K. gilt-edged market fits the bill; excluding short-dated stocks, it is currently capitalised at around £100 bn. The long-dated index-linked market, incidentally, is much smaller—amounting to some £15 bn. Index-linked do not feature at all in many investment portfolios. Although this may be controversial, the author regards any evaluation of equities relative to index-linked as subsidiary to the main benchmark proposed.

2.2 *The Yield Ratio*

2.2.1 The traditional tool for evaluating equities relative to gilt-edged is the yield ratio—the yield on gilt-edged divided by that on equities. At earlier times, before 1980, the yield difference was in vogue. The two versions, of course, signal essentially the same message.

2.2.2 Currently the yield ratio is around 2·0—as Figure 2, which covers the 27-year period from 1965, shows. 1965 is the earliest date from which data are easily accessible on a computerised data base. Moreover, although the FT Actuaries All-Share Index dates from 1962, the current gilt indices only started in 1976. In Figure 2, there is used the 20-year Gilt Yield published by the CSO, which is a monthly average figure. Figure 2a shows, separately, the actual equity and gilt yields. The rise in gilt yields from 6% to 17% in the late 1960s/early 1970s is noteworthy.

2.2.3 Figure 2 demonstrates the discontinuity in the yield ratio that occurred in 1967/68. Although this figure does not show it, older practitioners will be aware and Section 3 will confirm that the yield ratio did not rise above one until the late 1950s. The period 1959/67 thus represents an interim period, linking the early period (yield ratio below one) to later times (generally above two).

2.2.4 But first, what about the history of the yield ratio in recent times, say since the 1974 crash? The figure shows that, at around 2, the yield ratio is currently at the low end of its historic range—signifying, on the face of it, that equities are cheap. Apart from two periods of euphoria culminating in the yield ratio rising temporarily above $2\frac{3}{4}$, the range has been between the reflecting barriers of 2 and $2\frac{1}{2}$. However, the yield ratio yardstick has failed in two important respects. First, it does not encapsulate the high variations in the rate of growth of equity dividends seen in Figure 1. Second, equities have been cheap

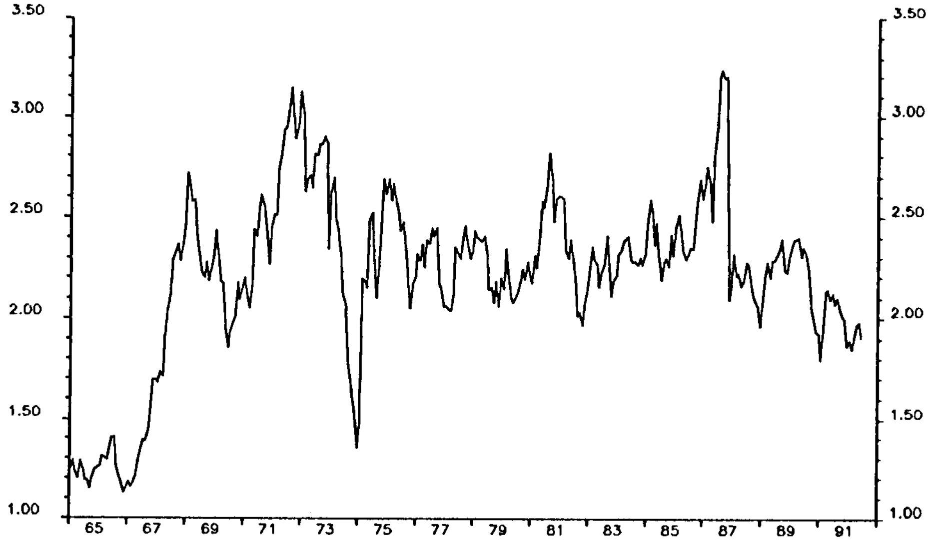

Figure 2. The yield ratio.

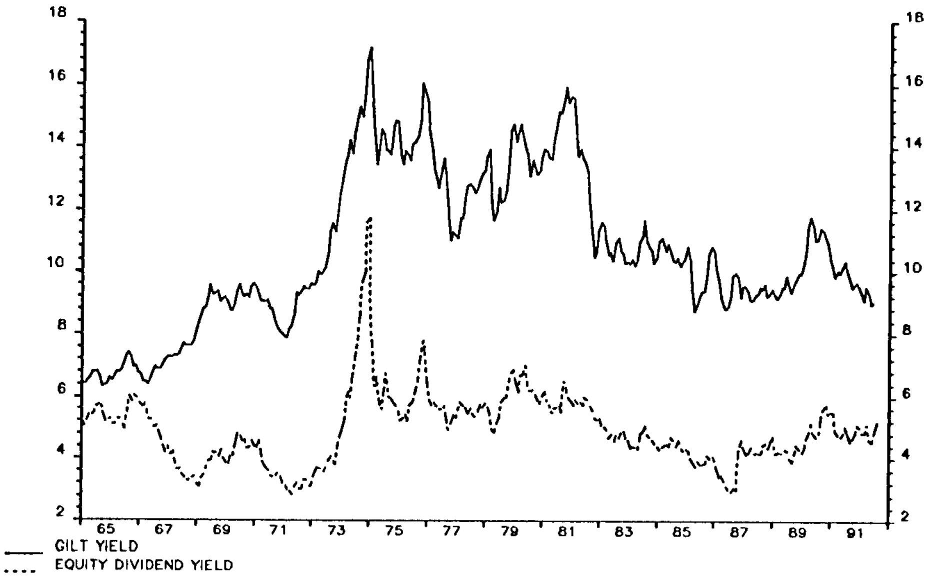

Figure 2a. Equity and gilt yields compared.

Figure 3. Total return on equities v gilts.

against gilts for virtually the entire period since 1976! (See Figure 3, which shows the total return on equities relative to that on gilts since the start of the FT-Actuaries Gilt indices.) The 'cheapness' aspects of this crude yield ratio model have hardly been put to the test. It did, however, correctly indicate the relative dearness of equities in 1981 and 1987, but notice that it did not really do so in mid-1990.

2.3 *The Yield Ratio and Dividend Growth*

2.3.1 The merit of the yield ratio approach lies in its simplicity. In stable environments, simple models generally perform robustly. A more complex model of equities must, however, allow for changes in dividend growth, and it would have to take into account cyclical changes in the risk premium of equities relative to gilts. A simple manipulation of a well-known investment text book relationship—set out in Appendix 1—provides a relationship between: risk premium, yield ratio and dividend growth.

2.3.2 First, we must define our terms. By *risk premium*, we mean the expected excess return of equities over gilt-edged *relative to the gilt-edged rate*. Suppose over a given time interval, the total return (income and capital gain) on equities is 13% and that on gilt-edged is 10%. The end gilt-edged yield was 9%. The excess return of equities over gilts is thus 3%—in money terms. Relative to the gilt rate of 9%, this is 0·33—or 33%. Why divide by the gilt rate? This is because of the principle alluded to earlier, which suggests that the 'risk premiums' should be

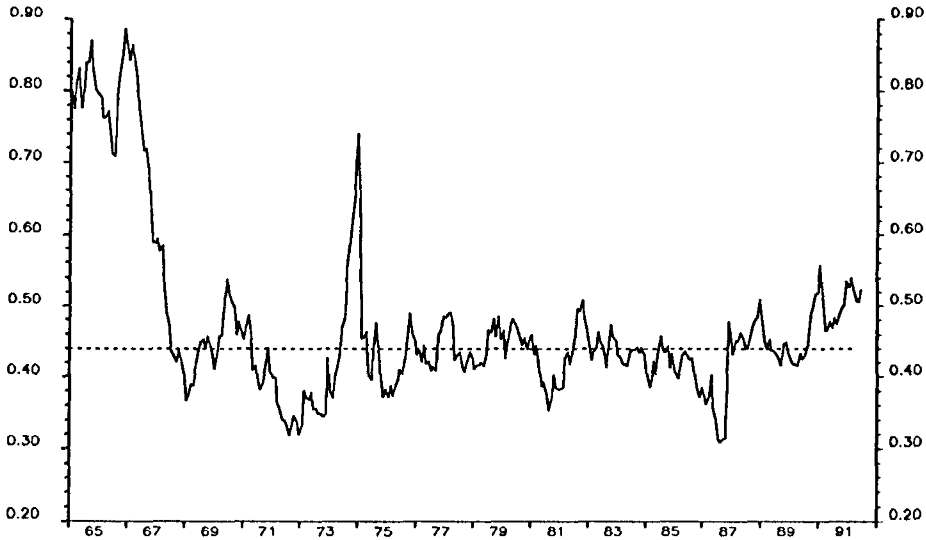

Figure 4. $1/R$ (yield ratio inverted).

'standardised' to facilitate comparisons over different time periods and economic backgrounds. Clearly a risk premium of 3% would be much more valuable at a time of low gilt yields, than at a time of high ones. We denote this 'standardised' risk premium by P.

2.3.3 Second, *yield ratio*. This is expressed here in the form $1/R$, i.e. the inverse of what the market is accustomed to, shown in Figure 2. Figure 4 shows $1/R$ (the 1965-to date version).

2.3.4 Third, the *dividend growth rate*. This is defined by the excess growth of equity dividends over the return on gilt-edged. Again, it is standardised by the gilt yield. So if dividends are growing at 15% and gilt yields are 9%, the excess growth is 6% and the standardised growth rate is $\frac{6}{9}$, i.e. 0·67 or 67%. If dividends are only growing at 9%, the standardised excess dividend growth rate is zero. Currently, of course, with dividends barely growing at all, it is almost minus one. We shall denote this variable by K.

2.3.5 Then the relationship is:

$$P=(1/R)+K.$$

P is a measure of expected return. So the higher P is, the cheaper equities are. So if K is constant, and $1/R$ rises (i.e. if the yield ratio is falling) then equities are, in general, getting cheaper, and of course, if $1/R$ is falling (i.e. the yield ratio is rising) equities are getting dearer. *This accords with the simple model discussed earlier.*

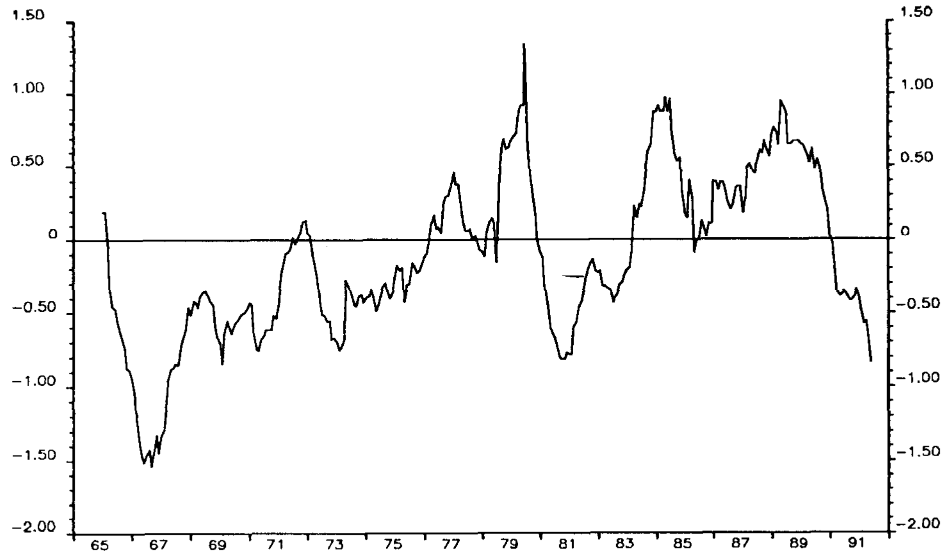

Figure 5. K (excess of equity dividend growth over the gilt yield).

2.3.6 In general, however, K will not be constant—it will fluctuate. Therefore, R (or rather $1/R$) is only a good cheapness/dearness indicator in the short term. *Where K is changing rapidly, $1/R$ becomes almost irrelevant.* In the long run—as every experienced equity practitioner knows—it is the sustainable rate of growth in equity dividends (perhaps over the next economic cycle), which is a component of K, that is important for equities.

2.3.7 The remainder of this section addresses the relationship between K and P, the expected excess return of equities over gilt-edged.

2.3.8 It should be mentioned at this stage that what has been described so far is a simplified version of a more complicated mathematical relationship, which is set out in Appendix 1. We make no apology for using a simplified version: in our view, no different practical decisions would be made if the more complicated form were used. More to the point, this is a subject where an understanding of the relationship between the major economic and financial factors is all important.

2.4 *The 'K' Factor—Excess Dividend Growth over Gilts*

2.4.1 Most readers will at least be vaguely familiar with the history of dividend growth from U.K. equities, shown in Figure 1. Figure 5 shows the K factor, which, it will be remembered, is the standardised excess of the equity dividend growth rate over the gilt rate.

2.4.2 The figure reveals three significant features:

(1) The long-term average is around zero—with the period, say, 1966/77 being

below zero and the subsequent period above—perhaps a secular trend, especially during the Thatcher years. The spike in 1979/80, incidentally, represents the catching-up process in dividends when Mrs Thatcher abolished the dividend controls put in place in the late 1970s by the previous Labour government.

(2) The peak levels of the mid 1980s (K approximately equal to one in 1985 and 1989), seem, to us at least, wholly exceptional. They coincide with the halcyon days of outperformance by U.K. equities—confirmed by a closer look at Figure 3 of the return on equities relative to gilts.

(3) There is a cyclical pattern with K at a low point roughly coinciding with the trough of the economic cycle (e.g. 1966/67, 1974/75, 1980/81 and 1991/92).

2.4.3 Currently, K is approximately minus one, a level not unexpected in view of the severity of the recession. What implications does this have for the risk premium of equities over gilts, that is to say the excess return that we expect to obtain by holding them in preference to gilts?

2.5 *The Expected Excess Return on Equities*

2.5.1 Looking again at the basic relationship of:

$$P = (1/R) + K$$

we can see, in the same figure (Figure 6), K (the solid line) and $1/R$ (the hatched line). The key point is how variable K is and, in contrast, how stable $1/R$. The implication is the not unexpected one that the excess return on equities is made up of two parts—the one, approximately a constant, and the other a variable, which may be plus or minus.

2.5.2 These two items are brought together in Figure 7. This shows the risk premium, P, or expected excess return of equities over gilt-edged. It should be emphasised that it is based on the then published data—the yields at the time, and in particular, the then rate of dividend growth. Obviously markets—especially equity markets—anticipate. So a refinement to our simple model would be to use forecast yields, e.g., in the yield ratio, where one could use the yield on equities say 12 months hence. In more general terms, if the market anticipates, for example, an increase in equity dividends, then this transmits itself into a higher return, first through a lower yield ratio, i.e., a higher value of $1/R$, and only later through an increased value of K.

2.5.3 It is important also to appreciate that Figure 7 is about 'correct' market expectations. It is not about market achievement. To the extent that the market incorrectly evaluates the key future components, e.g. gilt yields, dividend growth, so too will expectations differ from achievement. This is especially the case at a time of rapid change, e.g., K in 1984 or 1991.

2.5.4 Subject to these caveats, Figure 7 is the acid test of long-term expectations for equities relative to gilts. The zero line implies that the total return on the two types of investment will be the same—if expectations are met. If the line is below zero, gilts are expected to outperform. If it is above zero, equities

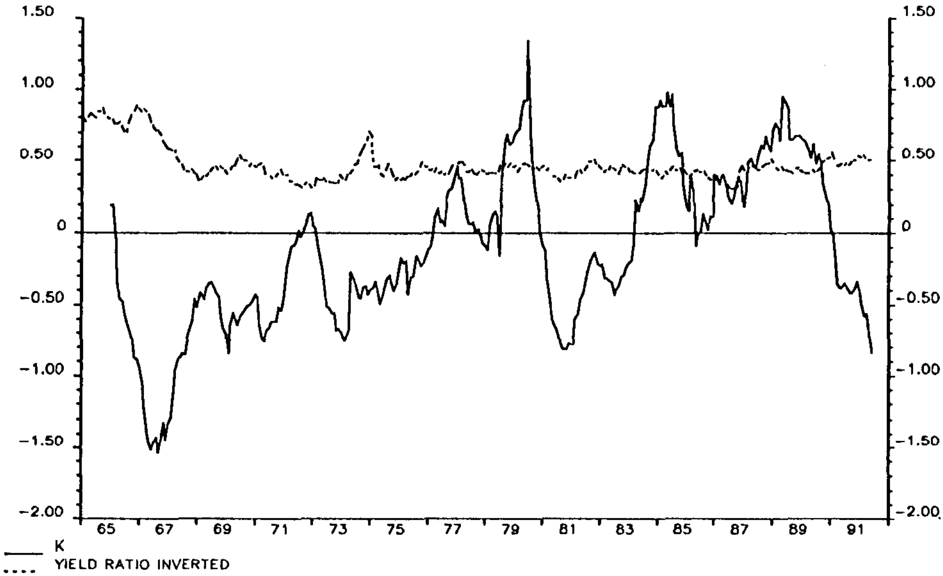

Figure 6. $1/R$ and K compared.

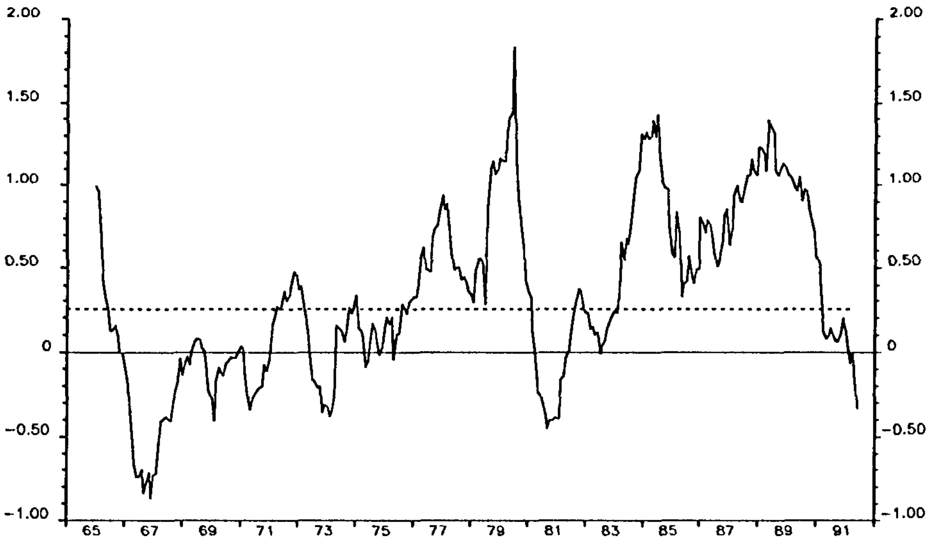

Figure 7. The equity risk premium $(1/R+K)$.

should outperform. Equities, however, are riskier than gilts (for example, the cash flow profile is less beneficial occurring later than a gilt-edged investment of the same total return), so their return should be higher. Arbitrarily, we have nominated a level of 0·25 (of the gilt yield) as being a minimum acceptable level, but the reader will form his or her own views.

2.5.5 The figure clearly demonstrates the attractiveness of equities relative to gilts virtually continuously since 1976. The exceptions were 1981/83 and 1990/92, when, as Figure 3 shows, equities did not do as well. The period of the late 1960s is particularly illuminating—especially 1967/68 when expectations of equities were very low, as the figure shows. In fact equities outperformed gilts—because gilt yields (see Figure 2a) increased from around 7% to almost 10%. Equities did not do well, but gilts did extremely badly (the 1967 devaluation?) So a false indication was given. The message is that it is important to build the correct major changes in expectations into Figure 7!

2.6 *The Expected Return,* P, *in Recession*

2.6.1 It can be observed from Figures 6 and 7 that P is low, often negative, in recession. This can be looked at in two ways. First, in terms of expectations in particular, that the rate of growth of dividend will rise rapidly as the economy recovers—e.g. as it did, in dramatic fashion, in 1984. Alternatively, that at such a stage of the economic cycle, with output in the doldrums, confidence low and public finances in a parlous state, the risk is actually higher in gilt-edged. The assumption must, of course, be made that the economy will eventually recover. In a democracy, that is probably a reasonable assumption, with only the timing and strength of the recovery for debate. In any event, it justifies *temporary* negative values of P.

2.6.2 Figure 8 shows a scatter diagram of P against K, i.e. the line $P=(1/R)+K$. Obviously, the top right quarter of the graph is associated with a booming economy (i.e. high rates of dividend growth), and the lower left with *temporary* recession, but what about stagnation or sluggish growth? What is the right value for P in these circumstances? Well, clearly, P must ultimately become positive, e.g. on some set of forecast economic assumptions.

2.6.3 Consider the point X on the scatter diagram representing an equity market valuation at close to a low point in an economic cycle, with the dividend growth rate at, say, zero. If there was a 'robust' cyclical recovery, with dividends growing, say, to above 5% p.a. and gilt yields falling, say, to 8%, the value of P would move (irregularly, of course) in the direction of the line XY. The rise in K would both allow P to rise above some arbitrary minimum, say 0·25, and permit $(1/R)$ to fall. History could repeat itself and the yield ratio, R, could once again establish itself in its old range of 2 to $2\frac{3}{4}$.

2.6.4 Suppose, on the other hand, that the economic recovery were deferred, apparently indefinitely. The value of P is then more likely to move in the direction XZ, i.e. the value of K would hardly change (the gilt yield might fall) and any required increase in P would have to come directly through $(1/R)$, i.e. R, the yield

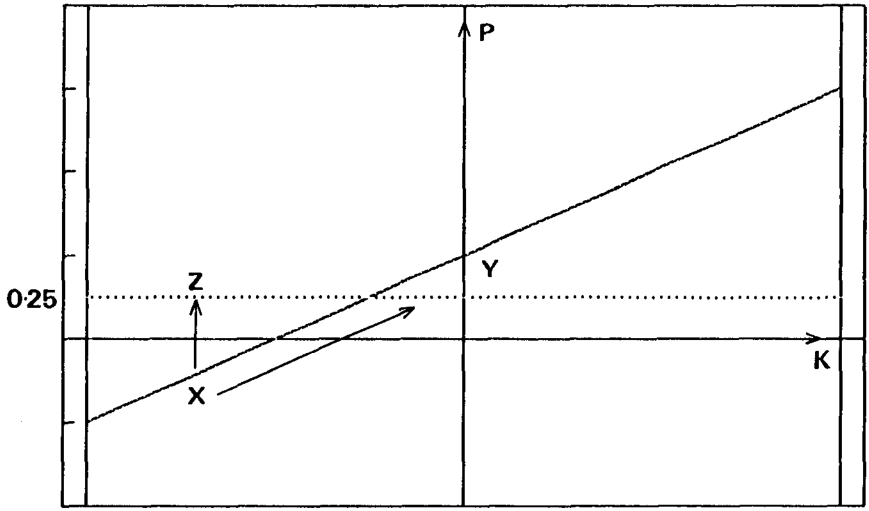

Figure 8. Scatter diagram of P and K.

ratio, would have to fall. In the ultimate—say an early 1930s scenario—it would be to around 1, i.e. gilt and equity yields roughly equal.

2.6.5 The reality, of course, probably lies between these two scenarios. Our earlier arbitrary minimum level of P was set at 0·25, and is also shown in Figure 8. Consider the case, for example, where $K = -0{\cdot}5$ (perhaps long-term dividend growth of 4% and gilt yields of 8%). The formula of:

$$P = (1/R) + K$$

thus gives:

$$0{\cdot}25 = (1/R) - 0{\cdot}5$$

$$\text{i.e. } \frac{1}{R} = 0{\cdot}75.$$

$R = 1{\cdot}33$ and implies a 'correct' yield on equities of 6%, but note the high sensitivity of the relationship. Dividend growth of 5% gives $K = -0{\cdot}38$ and $R = 1{\cdot}6$, implying a correct yield on equities of 5%. We are not here seeking to make forecasts, merely demonstrating that the methodology is available for the reader to make his own preferred calculations.

2.7 *Cyclical Economic Recovery* v *Secular Trend to Little or No Growth*

2.7.1 Perhaps more important than the mere arithmetic of the last section, is the philosophical point. How else can the equity market valuation rate alter to differentiate between, say, the three scenarios: a 'robust' economic recovery (say 3% real growth rate in 1993/94), very sluggish growth (say 1% p.a.) and a deep seated recession—other than through P, the expected risk premium?

2.7.2 The key point for investors and actuaries to ponder is that, at a possibly historic turning point in the growth rate of U.K. equity dividends (has there been a secular change in their share of the national output?) only a sharply lower yield ratio (or sharply higher inverse) can provide the minimum acceptable return for equity shareholders. All the rest is expectation. Whether or not the U.K. economy is seeing a secular change—or merely a prolonged cyclical upswing—is for others to answer.

2.7.3 What the author is clear about is that many investors misunderstand the nature of the yield ratio. For them the level of 2 is synonymous with cheap—as it has been for the past 15 years—and will be again if the economy soon grows at 3% p.a.

2.7.4 However, if there is little or no growth, we suspect that a yield ratio of 2 will turn out to be not a floor, but a ceiling. And since the market will only slowly begin to discern the nature of the upswing, it follows that even if the gloomy scenario is justified, it will only slowly be reflected in equity values. Of course, confidence could 'snap' completely—but the huge success of equities over the last 15 years makes this unlikely. More plausible would be a huge bull market in gilt-edged—not shared by equities. (A minor version of this took place in 1982.)

2.7.5 The purpose of this section has not been to forecast the level of the equity index in the near term. It has been to lay out a methodology which is robust enough to evaluate the worth and risks of equities across a range of different economic scenarios. Section 3 refines the methodology to examine 70 years of historical data.

3. Analysing Equity Returns Since 1923

3.1.1 Section 2, aimed mainly at an investment audience, showed a method of analysing the nominal, i.e., money return on equities—as a function of the return on gilt-edged (the central financial variable in the economy) and of equity dividend growth. It looked at data only from 1965—generally times of economic prosperity. Section 3 draws on the methodology and conclusions of Section 2 to analyse the returns on equities and gilt-edged since the early 1920s (a 70-year period which embraces a range of economic conditions) and comments on an aspect of current pension fund valuation practice. The prime aim of Section 3 is to introduce this topical subject, I hope, in an easily understood fashion—it does not purport to be an exhaustive analysis.

3.2 *Expected and Achieved Risk Premiums*

3.2.1 In Section 2, an expression for the risk premium of equities over gilt-edged was derived. The risk premium, of course, has two facets:

(1) the *achieved* excess return of equities over gilts in the *past*, and
(2) the *expected* excess return in the *future*.

3.2.2 The first is (subject to the quality of the underlying historical data) a matter of statistical fact. It may well not accurately represent, over short time horizons, the expected return that the markets believe that they require, being distorted by random influences, both positive and negative, but to the extent that, on average, expectations (e.g. as to dividend growth) are borne out in practice, the excess returns achieved are an adequate measure of the risk premium that the market requires on equities in the long term. That this is not always true, even over several years, is readily revealed by the experience of the mid 1980s, when the dividend growth rate (and hence achieved excess equity returns) greatly exceeded those that the author believes that actuaries and investors generally were expecting.

3.3 *The Risk Premium on Equities—the Geometric Form*

3.3.1 In Section 2, use was made of the basic equation linking the returns on equities and gilts, as follows (see Appendix 1 for derivation).

An initial investment of 1 in equities grows to:

$$(1+d)(1+k)=(1+g)(1+p)$$

where d = gross yield on equities,
k = rate of growth of gross dividends,
g = yield on gilt-edged, and
p = risk premium or excess return of equities over gilt-edged.

3.3.2 In analysing p, the risk premium, it is essential (as shown in Section 2) to standardise it by the then gilt yield. Hence a relationship was obtained, after some simplification, that:

$$\frac{p}{g}=\frac{d}{g}+\frac{k-g}{g}$$

or $P=(1/R)+K$ (where P = the standardised risk premium, R = the yield on gilt-edged divided by the yield on equities, i.e. the yield ratio and K the standardised excess dividend factor).

3.3.3 This is an arithmetic type of formula and the historical numbers derived there from are easy to understand. For actuarial purposes, especially over long time intervals, it is obviously better to use geometric averages. Hence the geometric equivalent is:

$$\frac{(1+p)}{(1+g)}=\frac{(1+d)}{(1+g)}*\frac{(1+k)}{(1+g)}$$

again making use of the basic equation. Or, adopting simpler notation:

$$\bar{P}=\bar{Y}*\bar{K}$$

where the 'bar' denotes geometric versions of the averages. Note especially that the Y factor is used as a multiplier, i.e. it represents $(1/R)$—merely for ease of understanding what provides the excess return of equities over gilts. Notice also that:

$$(1+p)=\bar{P}(1+g)=\bar{Y}*\bar{K}*(1+g).$$

The most obvious point to note is that if p is to be positive:

$$\bar{Y}*\bar{K}*(1+g)>1.$$

3.3.4 Using data typically found in current actuarial valuations, say $d=5\%$, $g=9\%$ and k at perhaps 5% (assuming some economic recovery), we obtain:

$$\bar{Y}*\bar{K}*(1+g)=(1+p)$$

i.e.

$$\frac{1{\cdot}05}{1{\cdot}09}*\frac{1{\cdot}05}{1{\cdot}09}*1{\cdot}09=1{\cdot}01$$

and thus p is barely positive (around 11% of the gilt yield of 9%), hardly a happy situation for equity values which are already discounting a substantial rise in dividend growth when the economy recovers from recession.

3.3.5 The use of equations with one-year returns to model what is, of course, a continuous process is an approximation; it should not invalidate the broad conclusions drawn in the paper. More detailed investigation of the data might well involve time series analysis, but is beyond the scope of this paper.

3.4 *Methodology for Calculating Equity Returns from Historical Data*

3.4.1 It has been conventional to derive the historical return on equities from the product:

$$(1+d)(1+k).$$

Numerically, of course, that must be approximately true, but this is not the way that the profession should calculate the *expected return* on equities. That is better provided by considering the expected return on gilt-edged and an appropriate risk premium—which must, of course, be positive, if it is to be stable.

3.4.2 Hence the expected return on equities should be derived—and an understanding of its component parts is certainly facilitated thereby—from the equation providing the total return on equities as:

$$\begin{aligned}(1+g)(1+p)&=(1+g)*\bar{P}*(1+g)\\&=(1+g)^2*\bar{Y}*\bar{K}\end{aligned}$$

since:

$$\frac{(1+p)}{(1+g)}=\bar{P}$$

subject to the condition that:

$$\bar{Y}*\bar{K}*(1+g)>1{\cdot}0.$$

3.5 *Historical Sources of Returns on Equities and Gilts*

3.5.1 The source of combined equity *and* gilt data with the longest history (exceeding that of the Actuaries Index which started in 1930) is that published by

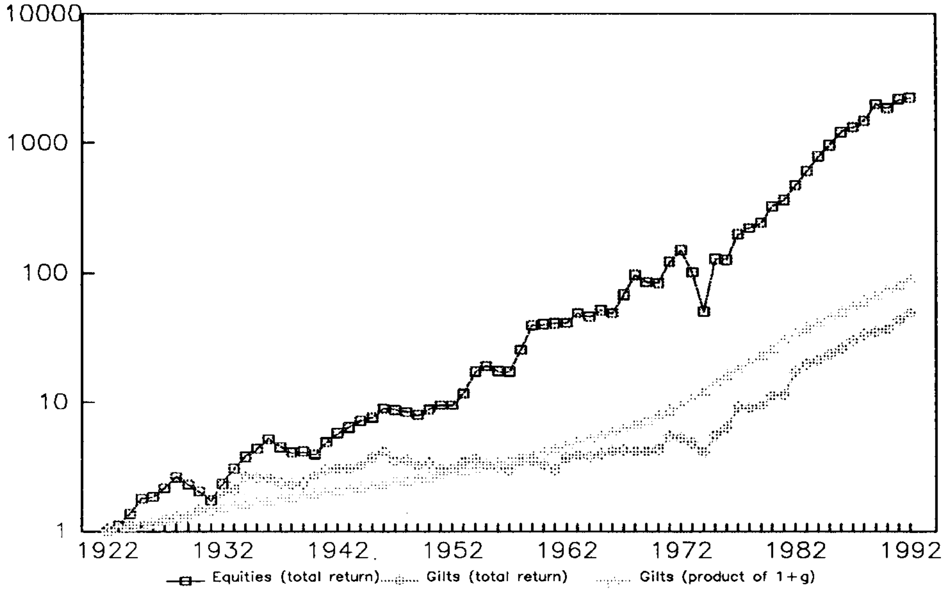

Figure 9. Accumulated total return on equities and gilts.

Barclays de Zoete Wedd, to whom grateful thanks are due for permission to use it. Errors of interpretation are, of course, those of the author. The series was not compiled from outset, a desirable feature of any index series, being first commissioned in 1956. Substantial use was made, prior to 1962, of the old *Investors Chronicle* indices, for both the equity and gilt-edged series. Moreover, the equity data probably suffer from minor imperfections similar to those of the FT-Actuaries Index, in that the dividend series (and hence dividend growth rates) are distorted upwards by the inclusion of high-yielding new issues, principally the privatisation issues in the 1980s.

3.5.2 However, there is no intention to derive numerically precise conclusions from the data—more to provide a historical perspective from the early 1920s and a methodology. The index actually starts in 1919, but the statistics for 1920 and 1921 are volatile (perhaps a reflection of the inflation that followed the First World War and the 'Geddes axe' of 1920). The data for this paper have been extracted from the period beginning in January 1923—and thus cover the 70-year period, ending in mid 1992.

3.5.3 In order to make readers aware of the general profile of the data, they are presented in several ways—in visual form in Figures 9 and 10, in summary form in Table 1, and in graphical form, using arithmetic averages (because these are easier to comprehend); also in tabular form, divided into 7 periods of ten years, using the geometric relationship. The underlying data are displayed in Appendices 2, 3, 4 and 5.

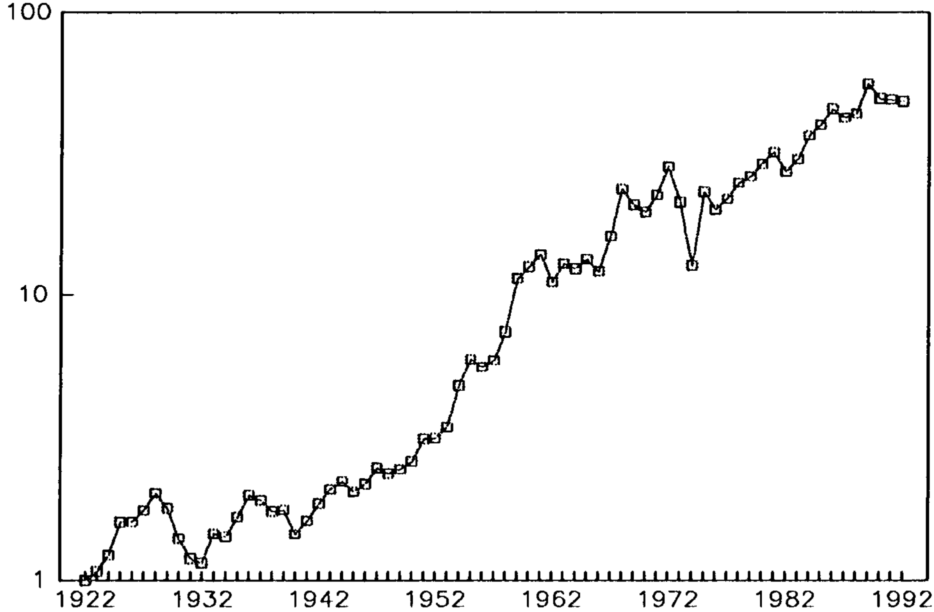

Figure 10. Ratio of total equity to total gilt returns.

3.5.4 Figure 9 shows the accumulated total return on equities and gilts. An initial investment of £1 in equities at the end of 1922 (shown by the upper line) reached some £2225, in money terms, by 1992 (an annual return of some $11\frac{1}{2}$% p.a.). It represents the accumulation of the total return of both income and capital, at market value. The lower line is the equivalent figure for gilts; for most of the period the gilts chosen were irredeemable gilts. £1 so invested at the end of 1922 reached about £50, at market value, representing an annual return of some $5\frac{3}{4}$% p.a. The intermediate line is the accumulation of the product of $(1+g)$, i.e. of gilt yields; its significance is that it is the deflator of a number of expressions used later in the paper. This annual average return is some $6\frac{1}{2}$%.

3.5.5 Figure 10 shows the accumulated total return on equities divided by that on gilts (at market value). When the line is rising, equities are giving a greater return than gilts. It demonstrates the axiom that equities nearly always achieve superior returns to gilts. Notice, however, that the line is less steep after the mid 1960s than in the 30 years before—because the achieved excess return on equities over gilts was lower.

3.5.6 The figures quoted in Table 1 and throughout the paper are intended to convey orders of magnitude only. Thus, the geometric average return on equities of 11·6% p.a. represents no more, in my view, than expressing the likelihood that the return lay, as far as it ever can be measured, between 11% and 12% p.a.

Table 1. Summary returns from 1923 to 1992

Accumulated total return indexes: (end 1922 = 1·00)	Gilt yield	90
	Equities	2225
1 + geometric average returns: (i.e. 70th roots)		
	Gilt yield	1·067
	Equities	1·116
i.e. equity return given by	$(1+d)*(1+k)$ $=1{\cdot}046*1{\cdot}067$ $=1{\cdot}116$	
Aliter	$=(1+g)^2*\bar{Y}*\bar{K}$ $=1{\cdot}138*0{\cdot}985*0{\cdot}997$ $=1{\cdot}116$	

i.e. $\bar{Y}=0{\cdot}985$ and $\bar{K}=0{\cdot}997$

Notice that $(1+p)=(1+g)*\bar{Y}*\bar{K}=1{\cdot}047$

i.e. roughly a 70% risk premium, related to the average gilt yield of 6·7%

3.6 *Components of Equity Return, in Graphic Form (Arithmetic Version)*

3.6.1 Figure 11 and Appendix 2 show the yield ratio, annually, since 1922. The data clearly break into two distinct periods, with a brief transition in the 1960s. Prior to 1958, equities yielded more than gilts, and hence the yield ratio was less than one. Since 1968 it has, with occasional exceptions including the time of writing, exceeded 2. It reached a peak of over 3 in 1972. In the mid 1960s, during the transitional period, it averaged about $1\frac{1}{4}$.

3.6.2 Figure 12 shows the inverse yield ratio. Remembering that in the arithmetic form, the risk premium on equities is approximately $(1/R)+K$ it follows that if $1/R>1$ and k (the dividend growth rate) was positive, i.e. $K>-1$, then equities had to give a greater return than gilt-edged. That is the significance of the period up to 1958—because $1/R$ was always greater than 1 (usually substantially so). Thereafter equities only gave a greater return if dividend growth was sufficiently high—in particular, sufficiently high in relation to the gilt-edged yield.

3.6.3 Was k sufficiently high prior to 1958? Figure 13 shows $1/R$ (the solid line) and K (the hatched). According to the data, K was hugely volatile—which will have adversely affected the attractions of equities—especially if the components of return were not fully understood—as was probably the case. A particular feature, though, is how K has converged, first, from a range of $+4$ to -5, pre-Second World War, reflecting the extremes of the 1929/30 slump and subsequent recovery, through a period of $+2$ to -2 in the 1950s and 1960s, to a, finally, much tighter $+1$ to -1 of the last decade. No doubt the modern practice of company boards of directors to smooth dividend payments has contributed to this trend. Prior to 1962, the equity index used was small in size, only 30 stocks,

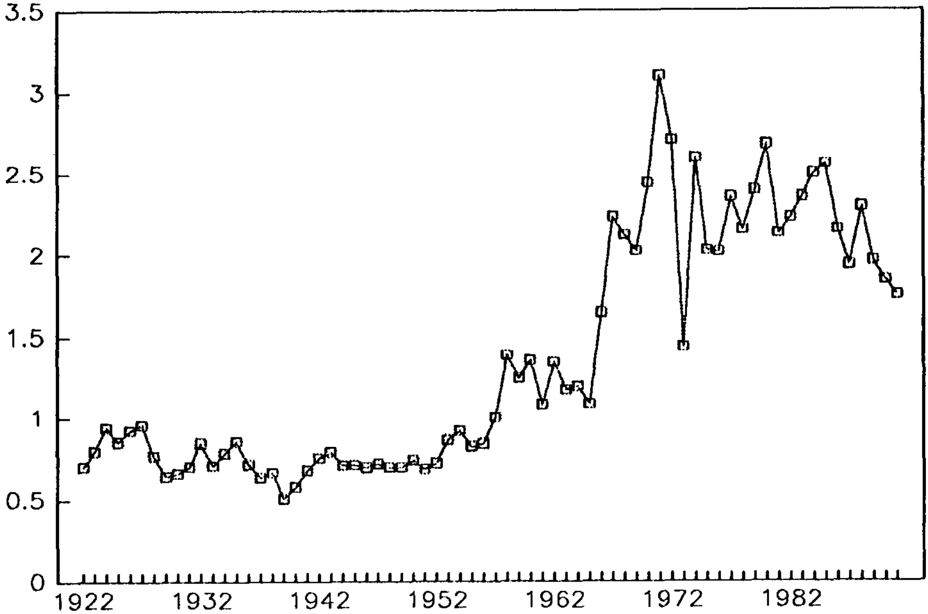

Figure 11. The yield ratio since 1922.

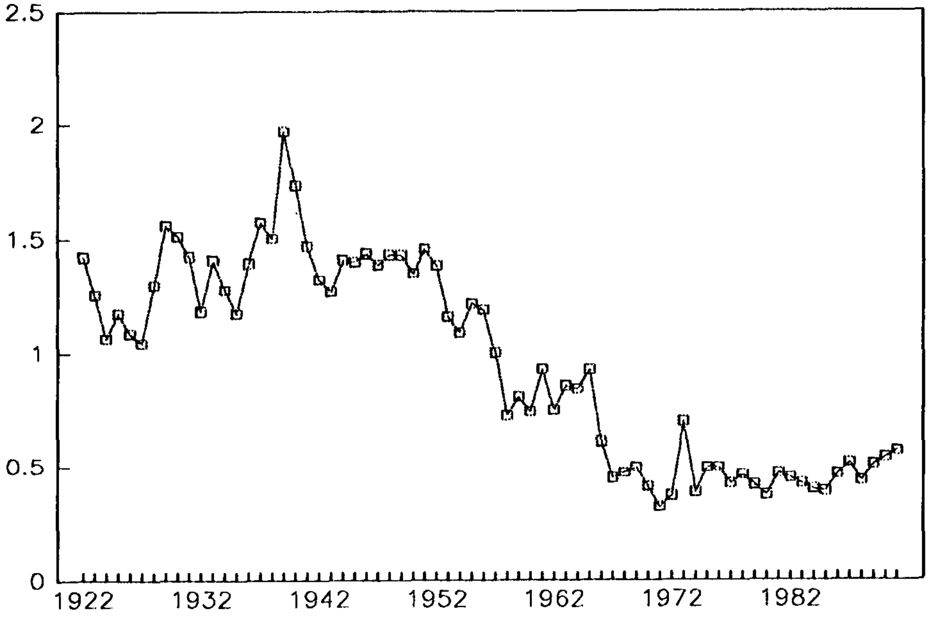

Figure 12. The yield ratio inverted.

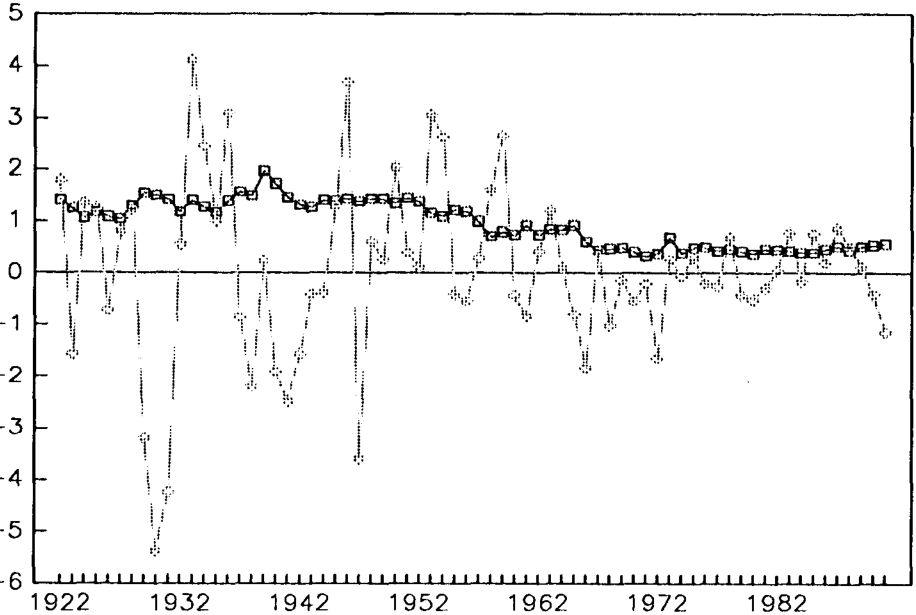

Figure 13. $1/R$ and K.

before being superseded by the FTA All Share Index of 600 plus stocks; this will have also influenced the trends described.

3.6.4 Figure 14 brings together the two separate components of the inverse yield ratio and the K factor into the equity risk premium—the expected excess return on equities over gilts. This only mirrors the actual out-turn to the extent, principally, that dividend growth actually matched that assumed. Over the entire 70-year period it has done so. For some periods, dividend growth has exceeded expectations, e.g. the 1980s. Appendix 4 compares the actual achieved total returns from equities and those from the fitted model; the theory appears adequately to match the practice.

3.6.5 The striking feature of Figure 14 is the huge risk premiums available right up until the mid 1960s—temporarily interrupted, of course, by the 1930s slump and by war. However, the conclusion has to be that equities were misunderstood, neglected, and gave very attractive returns over the long term—risk premiums exceeding 150% of gilt returns. Since the mid-1960s, equities have generally been well researched and have been regarded by virtually all investors as the ideal vehicle for long-term savings, because of the historical ravages of inflation. However, the returns from equities—in terms of risk premiums—have not been anything like as high. Over this period (of 30 years) the risk premium has averaged around 40%.

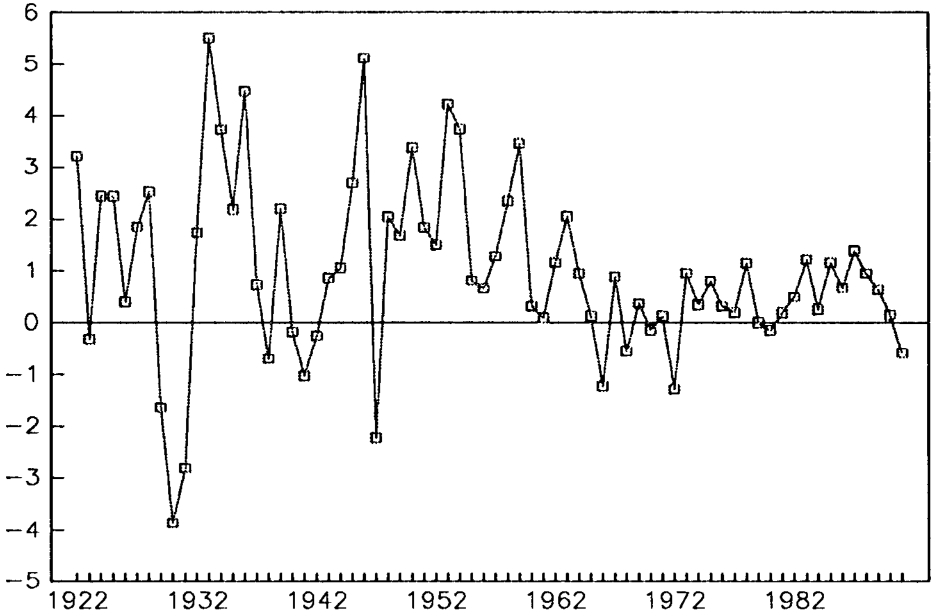

Figure 14. Equity risk premium, $1/R$ plus K.

3.7 *Analysis of the Historic Geometric Returns since 1923 by each of the Seven Decades*

3.7.1 Table 2 shows the differing contributions to the returns since 1923, over each of the 7 decades, and over the entire period. The full data are displayed in Appendix 4. Column (1) shows the average gilt-edged yield and column (2) the contribution that $(1+g)$ makes to $(1+p)$. Column (3), Y, is the inverse of the yield ratio in geometric form, column (4), K, the dividend component.

3.7.2 Column (5) is the product of columns (2), (3) and (4) to produce $(1+p)$ which, as discussed earlier, must over the long term, say a decade, be greater than one. Notice that it has always been positive, the lowest measures being 1·02 in the decades ending in 1972 and 1932—representing risk premiums of respectively 30% and 39%; for example 2·2% divided by a gilt yield of 7·4% in the decade ending in 1972 (see column (7)).

3.7.3 Column (6) is the product of columns (2) and (5) to produce $(1+E)$, i.e. one plus the expected nominal equity return. The average return as shown in Table 1 is $11\frac{1}{2}$%, with a low of 6% to 1932 and a high of 18% to 1982.

3.7.4 Notable low values of $(1+E)$ revealed by Appendix 4 are: 0·97, 0·86 and 0·94 (i.e. equities expected to give a lower return than gilts) at the end of respectively 1930, 1931 and 1932; 0·96 in 1948 and 1973. Notable high values are 1·21 and 1·22 in 1955 and 1956, 1·26 in 1960, 1·36 in 1974 (followed by 1·21 and

Table 2. Analysis of components of money returns in each of 7 decades, 1923 to 1992

Period ending	g (1)	$(1+g)$ (2)	$\bar{Y}=(1+d)/(1+g)$ (3)	$\bar{K}=(1+k)/(1+g)$ (4)	$(1+p)=(1+g)*Y*K$ (5)	$(1+E)=(1+p)(1+g)$ (6)	p/g (7)
1932	4·4	1·044	1·011	0·964	1·017	1·061	0·39
1942	3·1	1·031	1·014	1·007	1·053	1·088	1·71
1952	3·3	1·033	1·012	1·008	1·054	1·087	1·64
1962	4·9	1·049	1·000	1·031	1·082	1·137	1·67
1972	7·4	1·074	0·971	0·980	1·022	1·098	0·30
1982	13·8	1·138	0·934	0·976	1·037	1·182	0·27
1992	10·0	1·100	0·953	1·015	1·064	1·168	0·64
70-year average	6·7	1·067	0·985	0·997	1·047	1·116	0·71

1·28 in 1975 and 1976), 1·33 in 1979 and figures above 1·20 in 1984, 1986 and 1988. The current figure of 1·04 is towards the low end of the historical range.

3.7.5 Column (7) shows the risk premium as a proportion of the gilt yield. The average for the 70-year period is some 70%, with highs of some 170% and a low of 27%. Risk premiums since 1962 have been significantly lower than in earlier periods.

3.7.6 Of particular interest to the profession is the decade ending in 1962, because some members will have spent formative years in business during it. The product of Y and K was at its greatest. It has also one of the highest risk premiums revealed (160% plus). The decade is associated with the 'cult of equity', and with the passing of the Trustee Investment Act. In reality, it was an amazingly good opportunity for equity investment. It is especially important to see such a decade in its proper context, and to contrast it with 1992 when almost all pension funds are committed to virtually 100% equity investment at a time of turmoil in the underlying economic fundamentals. Historically low levels of K are not being neutralised by values of Y anywhere near to one.

3.8 *Real Returns on Equities, i.e. Adjusted for Inflation*

3.8.1 Figure 15 shows (the solid line) the accumulated total return index for equities, deflated by the accumulated inflation index (i.e. the product of $(1+i)$,

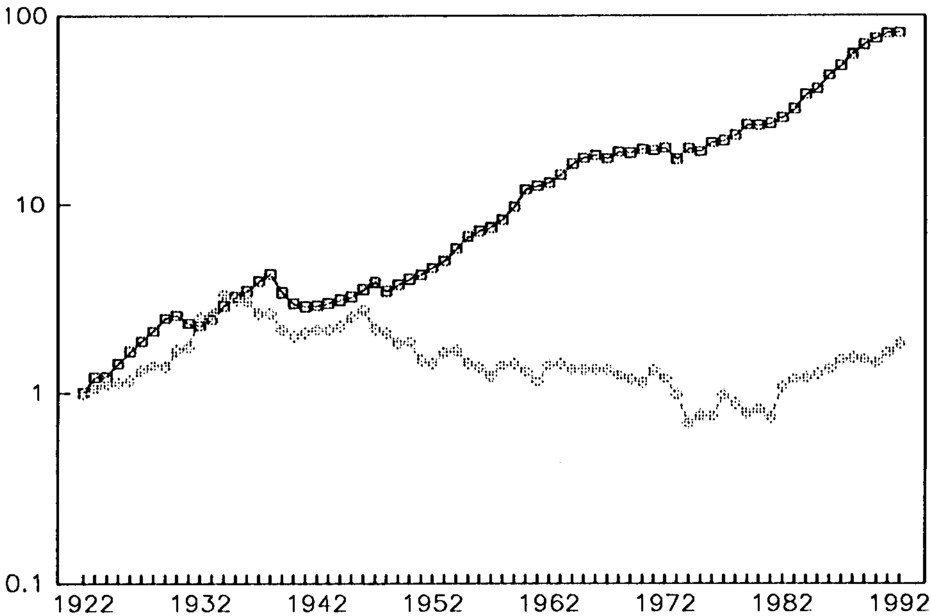

Figure 15. Accumulated real returns on equities and gilts.

Table 3. Analysis of components of real return in each of 7 decades, 1923 to 1992

Period ending	$(1+i)$ (1)	$(1+p)$ (2)	$(1+g)/(1+i)$ (3)	$(1+E)/(1+i)$ (4)	$(1+k)/(1+i)$ (5)	$(1+k)/(1+g)$ (6)
1932	0·98	1·017	1·069	1·086	1·03	0·96
1942	1·06	1·053	0·973	1·026	0·98	1·01
1952	1·04	1·054	0·994	1·046	1·00	1·01
1962	1·02	1·082	1·025	1·109	1·06	1·03
1972	1·05	1·022	1·021	1·043	1·00	0·98
1982	1·14	1·037	1·000	1·038	0·98	0·98
1992	1·05	1·064	1·043	1·108	1·06	1·02
	1·05	1·047	1·017	1·065	1·02	1·00

where i is the inflation rate). The figure thus shows the accumulated real value of equities. The hatched line is the same measure for gilt-edged, valued at market value. Respectively the average annual real returns are some $6\frac{1}{2}\%$ for equities and under 1% for gilts, over the whole 70-year period.

3.8.2 Table 3 shows an analysis of the real returns on equities, by the same 7 decades used earlier. The full data are in Appendix 5. The table shows at column (1) the average value of $(1+i)$, and in column (2) $(1+p)$, the expression for the expected excess return of equities over gilts, taken from Table 2. When this is multiplied by $(1+g)/(1+i)$ representing the expected real return on gilt edged, shown in column (3), the result is a measure (in one plus form) of the expected real return on equities, because from Table 2:

$$(1+E)=(1+p)*(1+g).$$

So:

$$\frac{(1+E)}{(1+i)}=(1+p)*\frac{(1+g)}{(1+i)}.$$

3.8.3 Notice the significant cyclical phenomenon, that the three largest returns on equities (in column (4)) occur at 30-year intervals, i.e. 1932, 1962 and 1992.

3.8.4 Columns (5) and (6) are shown solely to ask a question. Why does current actuarial practice relate k to i—when it is clearly more closely related to g? The mean value of $(1+k)/(1+g)$ is one, and it has a low standard deviation.

3.8.5 Table 4 shows the real equity returns sorted by order of size. Column (1) is the figure from Table 3 and column (2) the equivalent result, but calculated taking equities at market values; there is no material difference, in aggregate, between the two columns. If the data are divided into two—the high returns (1932, 1962 and 1992) and the rest, it is interesting to note the significant difference in average value of $(1+g)/(1+i)$ i.e. 1·046 and 0·997 in column (3). There is hardly any practical difference in the average values of $(1+p)$, shown in column (4). This might suggest, if the decade to 2002 is to be characterised by high real interest rates, that equities will also give a high real return. A significant component of $(1+p)$, however, is the dividend growth rate; e.g. the factor $(1+k)/(1+i)$, in column (5). Given the high levels of debt in the economy and the low

Table 4. Summary of real returns on equities, in order of size

$\frac{(1+E)}{(1+i)}$ (1)	Real return at Market Value (2)	Year	$\frac{(1+g)}{(1+i)}$ (3)	$(1+p)$ (4)	$\frac{(1+k)}{(1+i)}$ (5)
1·109	1·132	1962	1·025	1·082	1·06
1·108	1·108	1992	1·043	1·064	1·06
1·086	1·114	1932	1·069	1·017	1·03
1·101	1·118	Averages	1·046	1·054	1·05
1·046	1·013	1952	0·994	1·054	1·00
1·043	1·080	1972	1·021	1·022	1·00
1·038	0·985	1982	1·000	1·037	0·98
1·026	1·031	1942	0·973	1·053	0·98
1·038	1·027	Averages	0·997	1·042	0·99

levels of dividend cover, it seems unlikely that $(1+k)/(1+i)$ is going to be much greater than 1·0 for some while. If equities stay around their current valuation basis, it seems quite unlikely that $(1+p)$ can be much greater than 1·0. The prospects for the coming decade thus accord with none of the historical precedents—1932 included (unless negative inflation is experienced).

4. An Implication for Pension Funds: Constraints on Valuation Assumptions about Returns on Equities

4.1.1 Earlier a formula for the total money return on equities was derived as follows:

$$(1+E)=(1+p)*(1+g)$$

subject to the constraint, that over the long term $(1+p)>1$. As has been shown, in the past the minimum value attaching to the value of p has been just over 25% (see Table 2, column (7)). It is normally higher. This suggests that the minimum value of the constraint should be $(1+g/4)$. There is a case for arguing that, if the 1990s are going to be a period of sluggish and often unpredictable economic growth because of the high levels of debt, a constraint of $(1+g/2)$ is more appropriate. The reader must make his own (difficult) decision.

4.1.2 So, we can obtain a set of limits defined by the equations:

$$\begin{aligned}(1+E)&=(1+g)^2 * Y * K\\ &=(1+p)*(1+g)\\ &=(1+g/2)*(1+g) \qquad (1)\end{aligned}$$

or

$$=(1+g/4)*(1+g) \qquad (2)$$

i.e. $\bar{Y} * \bar{K}$ lies between say

$$1/(1+g/2)$$

and say

$$1/(1+3g/4)$$

or, roughly, $(1+d)(1+k)$ lies between $(1+3g/2)$

and

$$(1+5g/4).$$

4.1.3 Table 5 sets out the implications of these constraints. It shows, in the author's view, that the typical actuarial assumptions of $d=5\%$, $g=9\%$ and $k=5\%$ cannot be reconciled, against the current economic backdrop. In particular, it calls into question the reconcilability of future investment returns and the value of existing invested assets; alternatively whether the liabilities are being valued sufficiently conservatively.

Table 5. Implications of the constraint on $(1+p)$

Valuation assumptions taken as given	Constraint $(1+p)>(1+g/4)$	Constraint $(1+p)>(1+g/2)$
	Resulting value of third variable	
$d=5$ $g=9$	$k=6\%$	$k=8\%$
$d=5$ $k=5$	$g=8\frac{1}{4}\%$	$g=7\%$
$k=5$ $g=9$	$d=6\%$	$d=8\%$

4.1.4 There is, of course, a more radical alternative; that because of their special characteristics, in particular the perception (if not necessarily the reality) that equities are a hedge against unexpected inflation and particularly suitable as an investment for pension funds, they will no longer give a significant excess return over gilt-edged. I reject this notion. The cash flow profile of gilts and equities is radically different—and equity dividends can fall instead of rise. Readers will have their own views, but if this is the case, actuaries might just as well notionally re-invest pension fund assets in long-dated gilts and value them on an actuarial basis.

4.2 *Equation for Real Returns on Equities*

4.2.1 Arising out of earlier remarks, there is the implication that we might set, say:

$$\begin{aligned}(1+E) &= (1+p) * (1+g)\\ &= (1+g/4)(1+g)\end{aligned}$$

i.e. that:

$$\frac{(1+E)}{(1+i)} = \frac{(1+5g/4)}{(1+i)}.$$

4.2.2 The implication of this equation is that the higher g is in relation to i, the higher the real return on equities. This relationship (which is based on the equations set out in Appendix 1) clearly does not hold good for all values of $(g-i)$ i.e. the real long rate of interest. In particular at some penally high level of $(g-i)$, which may now be being approached, $(1+E)/(1+i)$ must surely turn down. Further discussion of this matter is beyond the scope of this paper; it does not, in the author's view, invalidate earlier comments.

4.2.3 Incidentally, the use of the simple equation:

$$(1+E)=(1+d)*(1+k)$$

i.e.

$$\frac{(1+E)}{(1+i)}=(1+d)*\frac{(1+k)}{(1+i)}=(1+d),$$

when putting $k=i$ to obtain a real return on equities equal to d, the dividend yield on equities, needs to be checked against the constraints. At current equity yields, there is an implied condition that k is set at not less than two thirds of the gilt yield.

4.3 *Background for Writing the Paper*

4.3.1 It was the experience of the 1980s that prompted the writing of this part of the paper. The author is not a pensions actuary, but has been a trustee of a pension scheme for many years. As an informed trustee, it seemed that the valuation assumptions made for pension valuations in the mid-1980s were not crucial to revealing the underlying financial strength of schemes or the adequacy of the contribution rates set—because surplus was emerging at such a spectacular rate. In particular, if dividends were assumed to grow at 5% p.a. (not untypical, i.e. 15% per inter-valuation period), that amount had often been achieved within one year (see Appendix 2)—about the time the report was presented to the trustees! It only required a belief in modest further dividend growth to take a relaxed view to this item on the trustees' agenda.

4.3.2 Clearly that has changed. The valuation basis once again becomes pivotal to revealing the financial health of any scheme. Yet, there are members of the profession who advocate a relaxation of the actuarial basis—offering the observation that valuations are about the long-term future, not short-term economic difficulties. Whilst that is undoubtedly true, suppose that a valuation time horizon consists of 30 years, say 3 decades, of which one experiences 'average' economic conditions, one favourable and one unfavourable. If there is a widespread belief that the economy is at the start of an unfavourable period, is the profession to ignore it and rely solely on the long-term view, especially, if it might be the case that the favourable decade is actually giving way to the unfavourable? Surely, any trustee should argue (the author certainly would) that the valuation must acknowledge, at least in part, the unfavourable economic background in order to hold the balance between employer and members, valuation by valuation, not decade by decade. A trustee should not expect to be put in a position of asking the employer to remove a deficit which he believes might arise in the near term—he should err on the side that anticipates a modest surplus and expect to release it, if it becomes excessive (as indeed the law now insists).

5. Conclusion

5.1 The paper provides a methodology for evaluating equities relative to gilts. It emphasises the key role played by the rate of growth of equity dividends relative to the gilt yield in the high returns that have been provided by equities in the past. Prior to 1958, equities yielded more than gilts and dividend growth, though erratic, was high in relation to the then gilt yield. Equities handsomely outperformed gilts. Since 1958, equities have yielded less than gilts—but some prodigious rates of dividend growth, especially in the last 10 years have more than compensated. Equities have thus continued to do well—though not, in general, as well as in the period prior to 1958.

5.2 The coming decade opens with equities still yielding less than gilts, but with considerable doubts that equity dividend growth will anywhere near match the yield on gilt-edged. This suggests that the equity risk premium, i.e. the excess return of equities over gilts, will be lower than at any time in the last 7 decades. It raises the question whether some U.K. pension fund assets (still almost entirely invested in equities) are being valued too highly, or, alternatively, the pension fund liabilities too modestly. Pensions actuaries are invited to supplement current valuation practice by also notionally investing the assets in appropriately matched gilt-edged; the reduction in the resulting surplus is a measure of their assumptions as to the future risk premium of equities over gilts.

Acknowledgements

The writing of the paper was, in part, stimulated by the paper 'A Realistic Approach to Pension Funding' by Thornton & Wilson, presented to the Institute in February 1992 (*J.I.A.* **119**, 229). I am grateful to many of my colleagues for their support, encouragement and guidance (especially in relation to Section 2), namely Ian Williams, Peter Coombs, Roger Bootle, Mike Higgins and Ian Shepherdson. Any errors are, of course, my own responsibility.

APPENDIX 1

THE EQUITY RISK PREMIUM, YIELD RATIO AND DIVIDEND GROWTH

Consider the total return in equities over a time interval of say one year, where:

d_t = gross dividend yield at time t

D_t = gross dividend at time t

k_t = rate of growth of gross dividends at time t

and the capital value of equities at time t is given by D_t/d_t.

Then the total return y is given by:

$$1 + y = \frac{\text{End capital} + \text{income}}{\text{Beginning capital}}$$

$$= \frac{\dfrac{D_1}{d_1} + D_1}{D_0/d_0} \quad \text{assuming dividends are paid annually, in arrears}$$

$$= \frac{d_0}{D_0} * \left(\frac{D_1}{d_1} + D_1\right).$$

But:

$$D_1 = D_0(1 + k_1)$$

so:

$$1 + y = d_0 * \frac{D_1}{D_0} * \frac{(1 + d_1)}{d_1}$$

$$= \frac{d_0}{d_1} * (1 + k_1)(1 + d_1).$$

Similarly, the total return z on gilt-edged is given by:

$$1 + z = \frac{\text{End capital} + \text{income}}{\text{Beginning capital}}$$

$$= \frac{\dfrac{C}{g_1} + C}{\dfrac{C}{g_0}} \quad \text{where } C \text{ equals the annual coupon, payable annually in arrear, and } g_t \text{ the gilt yield at time } t.$$

$$= \frac{g_0}{g_1} * (1 + g_1).$$

Now the risk premium p, of equities over gilt-edged may be given by the equation:

$$(1 + y) = (1 + z)(1 + p).$$

More complex models might be postulated—but are beyond the scope of this paper, i.e.

$$\frac{d_0}{d_1} * (1 + k_1)(1 + d_1) = \frac{g_0}{g_1} * (1 + g_1)(1 + p).$$

If the yield ratio r, where:

$$r = \frac{g}{d} \quad \text{remains unchanged,}$$

i.e.

$$\frac{g_1}{d_1} = \frac{g_0}{d_0} \quad \text{i.e. in a 'steady state' situation,}$$

then:

$$(1 + g_1)(1 + p) = (1 + k_1)(1 + d_1)$$

or generally:

$$(1 + g)(1 + p) = (1 + d)(1 + k) \quad \text{'the basic equation'.}$$

So:

$$p(1 + g) = (1 + d)(1 + k) - (1 + g)$$

and:

$$p\frac{(1 + g)}{g} = \frac{d}{g} * (1 + k) + \frac{k - g}{g}$$

which represents the risk premium standardised by the gilt yield. It is a meaningful measure of the excess return that holders of equities will require—relative to the return on gilt-edged. The left side has an extraneous multiplier $(1 + g)$—which is not large—and may, without loss of generality in the use to which we shall put it, be dropped from the left side of the equation. Let us call this standardised equity risk premium P.

Look now at the second item,

first:

$$\frac{d}{g} = \frac{1}{r} = \frac{1}{\text{yield ratio}}.$$

Also, as again $(1 + k)$ is generally, though not always, small, we shall ignore it. We shall denote the adjusted 'r' by R.

The last item, $(k - g)/g$, is the excess of the equity dividend growth *above* the return on gilt-edged—and then standardised by the then gilt yield. Let us denote this by K.

Then:

$$P = \frac{1}{R} + K.$$

This is the form of the equation that is made use of in Section 2. It expresses the risk premium in arithmetic form and has the virtue that it is easy to understand.

Alternatively, the relationship can be expressed in geometric form where it is desirable to analyse the data with greater precision, viz:

$$\frac{(1 + p)}{(1 + g)} = \frac{(1 + d)}{(1 + g)} * \frac{(1 + k)}{(1 + g)}.$$

This form is obtained by re-arranging the basic equation above.

We can denote this by:

$$\bar{P} = \bar{Y} * \bar{K}$$

where:

$$\bar{Y} = \frac{1 + d}{1 + g} \quad \text{and} \quad \bar{K} = \frac{1 + k}{1 + g}.$$

Because the relationship is expressed in a 'one plus' form, it is not in a particularly easy format to understand what is going on inside the data, but it is particularly appropriate for calculating long-term averages—for example for use in actuarial work.

APPENDIX 2
BZW EQUITY-GILT STUDY, BASIC DATA

	Equities				Gilts					
Year	Price index	Income index	Income growth (%)	Equity yield (%)	Price index	Gilt yield (%)	Yield ratio	Inverse yield ratio	'K'	Equity risk premium
1922	100·0	100			100·0					
1923	105·1	113	12·6	6·4	99·8	4·5	0·70	1·42	1·80	3·23
1924	121·5	110	−2·4	5·4	102·6	4·3	0·80	1·26	−1·56	−0·30
1925	152·8	122	10·7	4·8	98·6	4·5	0·94	1·07	1·37	2·43
1926	149·6	134	10·4	5·4	97·0	4·6	0·85	1·17	1·25	2·43
1927	167·4	136	1·3	4·9	99·6	4·5	0·92	1·09	−0·70	0·39
1928	193·4	147	7·9	4·6	100·8	4·4	0·96	1·05	0·81	1·85
1929	160·0	162	10·4	6·1	94·8	4·7	0·77	1·30	1·22	2·52
1930	132·6	147	−9·4	6·7	102·9	4·3	0·64	1·56	−3·20	−1·64
1931	104·9	118	−19·6	6·8	98·0	4·5	0·66	1·51	−5·36	−3·85
1932	135·5	105	−10·7	4·7	132·9	3·3	0·70	1·42	−4·24	−2·81
1933	171·0	111	5·1	3·9	132·8	3·3	0·85	1·18	0·55	1·74
1934	205·0	126	13·8	3·8	165·2	2·7	0·71	1·41	4·12	5·53
1935	225·3	139	10·0	3·7	155·6	2·9	0·78	1·28	2·45	3·72
1936	259·3	147	5·8	3·4	151·5	2·9	0·85	1·17	1·02	2·19
1937	216·1	167	13·5	4·6	133·1	3·3	0·72	1·39	3·09	4·48
1938	184·0	168	0·5	5·5	125·9	3·5	0·64	1·57	−0·85	0·73
1939	178·3	160	−4·3	5·4	122·6	3·6	0·67	1·50	−2·19	−0·69
1940	160·1	167	3·9	6·3	137·8	3·2	0·51	1·97	0·23	2·20
1941	187·0	162	−2·7	5·2	147·9	3·0	0·58	1·73	−1·90	−0·17
1942	211·0	155	−4·4	4·4	147·5	3·0	0·68	1·47	−2·48	−1·01
1943	226·0	152	−1·7	4·1	142·4	3·1	0·76	1·32	−1·56	−0·24
1944	244·9	155	1·8	3·8	146·1	3·0	0·79	1·27	−0·41	0·86
1945	249·7	158	1·7	3·8	163·4	2·7	0·71	1·41	−0·35	1·05
1946	284·4	167	5·7	3·5	176·5	2·5	0·71	1·40	1·29	2·69
1947	266·5	190	14·1	4·3	146·9	3·0	0·70	1·43	3·68	5·12
1948	245·9	175	−8·1	4·3	143·5	3·1	0·72	1·39	−3·60	−2·21
1949	220·7	185	5·7	5·0	126·2	3·5	0·70	1·43	0·62	2·05
1950	233·1	193	4·4	5·0	126·9	3·5	0·70	1·43	0·25	1·68
1951	240·0	216	12·1	5·4	110·3	4·0	0·74	1·35	2·04	3·39
1952	225·9	229	5·8	6·1	105·0	4·2	0·69	1·45	0·39	1·84
1953	266·2	239	4·3	5·4	115·1	3·9	0·72	1·38	0·11	1·50
1954	379·0	276	15·5	4·4	117·7	3·8	0·86	1·16	3·07	4·23
1955	401·0	320	16·0	4·8	101·4	4·4	0·92	1·09	2·64	3·73
1956	345·1	329	2·8	5·7	93·8	4·7	0·82	1·21	−0·40	0·81
1957	321·0	337	2·5	6·3	83·5	5·3	0·84	1·19	−0·53	0·65
1958	452·9	358	6·1	4·8	93·2	4·8	1·00	1·00	0·28	1·28
1959	677·0	405	13·1	3·6	89·6	5·0	1·39	0·72	1·62	2·34
1960	659·2	487	20·5	4·5	78·9	5·6	1·24	0·80	2·66	3·46
1961	639·7	505	3·7	4·8	68·1	6·5	1·35	0·74	−0·43	0·31
1962	611·4	510	0·9	5·0	80·6	5·4	1·08	0·93	−0·83	0·09
1963	701·9	550	7·8	4·1	79·3	5·5	1·34	0·75	0·41	1·16
1964	627·8	623	13·4	5·2	73·0	6·1	1·17	0·85	1·20	2·06

(continued)

APPENDIX 2: (*Continued*)

	Equities				Gilts					
Year	Price index	Income index	Income growth (%)	Equity yield (%)	Price index	Gilt yield (%)	Yield ratio	Inverse yield ratio	'K'	Equity risk premium
1965	670·8	666	6·8	5·2	71·7	6·2	1·19	0·84	0·10	0·94
1966	598·7	675	1·3	5·9	70·3	6·4	1·08	0·92	−0·80	0·12
1967	793·2	636	−5·7	4·2	67·4	6·9	1·64	0·61	−1·82	−1·22
1968	1086·7	705	10·9	3·4	61·2	7·6	2·24	0·45	0·43	0·88
1969	922·9	705	−0·1	4·0	56·5	8·5	2·13	0·47	−1·02	−0·54
1970	865·7	760	7·9	4·6	53·5	9·3	2·02	0·49	−0·15	0·35
1971	1216·0	790	3·8	3·4	63·0	8·3	2·44	0·41	−0·54	−0·13
1972	1435·7	850	7·6	3·1	55·2	9·6	3·10	0·32	−0·20	0·12
1973	934·4	785	−7·6	4·4	45·0	11·9	2·70	0·37	−1·64	−1·27
1974	423·0	953	21·4	11·8	32·6	17·0	1·44	0·69	0·26	0·95
1975	998·6	1087	14·0	5·7	38·8	14·8	2·60	0·39	−0·05	0·33
1976	919·6	1300	19·6	7·4	38·4	15·0	2·03	0·49	0·30	0·80
1977	1371·4	1414	8·8	5·4	50·2	10·9	2·02	0·50	−0·19	0·30
1978	1455·2	1556	10·0	5·6	43·5	13·2	2·36	0·42	−0·24	0·18
1979	1494·2	1940	24·7	6·8	39·5	14·7	2·16	0·46	0·68	1·14
1980	1893·5	2097	8·1	5·8	41·9	13·9	2·40	0·42	−0·42	0·00
1981	2006·5	2261	7·8	5·9	36·8	15·8	2·68	0·37	−0·51	−0·13
1982	2456·8	2440	7·9	5·2	50·2	11·1	2·13	0·47	−0·29	0·18
1983	3013·5	2705	10·9	4·7	52·6	10·5	2·23	0·45	0·03	0·48
1984	3743·5	3217	18·9	4·5	50·8	10·6	2·36	0·42	0·79	1·21
1985	4367·3	3503	8·9	4·2	51·0	10·5	2·50	0·40	−0·15	0·25
1986	5303·2	4152	18·5	4·1	51·2	10·5	2·56	0·39	0·77	1·16
1987	5504·7	4625	11·4	4·4	54·4	9·5	2·16	0·46	0·20	0·66
1988	5923·8	5430	17·4	4·8	54·4	9·3	1·94	0·52	0·87	1·39
1989	7601·0	6242	14·9	4·3	52·2	9·9	2·30	0·44	0·51	0·95
1990	6736·3	6985	11·9	5·4	49·4	10·7	1·96	0·51	0·12	0·63
1991	7483·4	7388	5·8	5·2	53·2	9·5	1·85	0·54	−0·40	0·15
1992	7310·0	7287	−1·4	5·2	55·4	9·1	1·75	0·57	−1·15	−0·58

Source: BZW Equity-Gilt Study, January 1989 edition, subsequent data chain linked from FTA index series.

APPENDIX 3
ACCUMULATION INDICES

Year	Equities Total return (%)	Equities Accum. index	Gilts Total return (%)	Gilts Accum. index	Product $(1+g)$ accum.	Equities/Gilts $(1+p)$	Equities/Gilts Accum. index
1922		1·00		1·00	1·00		1·00
1923	11·8	1·12	4·3	1·04	1·05	1·07	1·07
1924	21·9	1·36	7·3	1·12	1·09	1·14	1·22
1925	31·7	1·80	0·4	1·12	1·14	1·31	1·60
1926	3·2	1·85	2·9	1·16	1·19	1·00	1·60
1927	17·4	2·18	7·2	1·24	1·24	1·10	1·76
1928	20·8	2·63	5·7	1·31	1·30	1·14	2·01
1929	−12·2	2·31	−1·6	1·29	1·36	0·89	1·79
1930	−11·6	2·04	13·1	1·46	1·42	0·78	1·40
1931	−15·5	1·72	−0·4	1·45	1·48	0·85	1·19
1932	35·2	2·33	40·1	2·04	1·53	0·96	1·14
1933	31·2	3·06	3·2	2·10	1·58	1·27	1·45
1934	24·4	3·80	27·7	2·69	1·63	0·97	1·42
1935	14·0	4·34	−3·1	2·60	1·67	1·18	1·67
1936	19·0	5·16	0·2	2·61	1·72	1·19	1·98
1937	−12·8	4·50	−9·3	2·37	1·78	0·96	1·90
1938	−10·2	4·04	−2·1	2·32	1·84	0·92	1·74
1939	2·1	4·13	0·9	2·34	1·91	1·01	1·76
1940	−4·5	3·94	15·9	2·71	1·97	0·82	1·45
1941	22·9	4·84	10·6	3·00	2·03	1·11	1·61
1942	17·8	5·70	2·7	3·08	2·09	1·15	1·85
1943	11·5	6·36	−0·5	3·07	2·15	1·12	2·07
1944	12·5	7·15	5·7	3·24	2·22	1·06	2·21
1945	5·9	7·57	14·8	3·72	2·28	0·92	2·04
1946	17·9	8·92	10·7	4·12	2·33	1·06	2·17
1947	−2·3	8·72	−14·3	3·53	2·40	1·14	2·47
1948	−3·8	8·39	0·7	3·56	2·48	0·96	2·36
1949	−5·8	7·91	−8·9	3·24	2·56	1·03	2·44
1950	10·9	8·77	4·0	3·37	2·65	1·07	2·60
1951	8·5	9·52	−9·6	3·04	2·76	1·20	3·13
1952	−0·1	9·50	−0·8	3·02	2·88	1·01	3·15
1953	24·2	11·80	14·0	3·44	2·99	1·09	3·43
1954	48·6	17·54	6·1	3·65	3·10	1·40	4·81
1955	10·9	19·45	−10·1	3·28	3·24	1·23	5·93
1956	−9·0	17·70	−3·2	3·18	3·39	0·94	5·57
1957	−1·1	17·50	−6·2	2·98	3·57	1·05	5·87
1958	47·9	25·87	17·0	3·49	3·74	1·26	7·42
1959	54·8	40·06	0·9	3·52	3·93	1·53	11·38
1960	1·8	40·77	−7·0	3·28	4·15	1·09	12·45
1961	1·7	41·46	−8·1	3·01	4·42	1·11	13·77
1962	0·4	41·61	24·7	3·76	4·66	0·80	11·08
1963	19·5	49·73	3·7	3·89	4·91	1·15	12·77
1964	−5·9	46·79	−2·3	3·81	5·21	0·96	12·29
1965	12·4	52·59	4·4	3·97	5·54	1·08	13·24

(continued)

APPENDIX 3: (*Continued*)

Year	Equities Total return (%)	Equities Accum. index	Gilts Total return (%)	Gilts Accum. index	Product $(1+g)$ accum.	Equities/Gilts $(1+p)$	Equities/Gilts Accum. index
1966	−5·5	49·71	4·2	4·14	5·89	0·91	12·01
1967	38·1	68·62	2·6	4·24	6·30	1·35	16·17
1968	41·7	97·22	−2·4	4·14	6·78	1·45	23·46
1969	−11·7	85·86	0·2	4·15	7·35	0·88	20·67
1970	−1·9	84·25	3·6	4·30	8·04	0·95	19·58
1971	45·2	122·37	27·3	5·48	8·70	1·14	22·33
1972	21·7	148·95	−3·8	5·27	9·54	1·27	28·27
1973	−32·1	101·21	−8·9	4·80	10·67	0·75	21·08
1974	−49·4	51·22	−15·2	4·07	12·49	0·60	12·58
1975	149·6	127·81	36·8	5·57	14·34	1·82	22·96
1976	−1·1	126·42	13·7	6·33	16·49	0·87	19·96
1977	57·2	198·70	44·8	9·17	18·28	1·09	21·67
1978	12·1	222·65	−1·8	9·00	20·70	1·14	24·74
1979	9·7	244·17	4·1	9·37	23·74	1·05	26·06
1980	34·1	327·36	20·9	11·33	27·04	1·11	28·89
1981	12·2	367·36	1·8	11·53	31·31	1·10	31·86
1982	28·8	473·20	51·3	17·45	34·78	0·85	27·12
1983	28·4	607·70	15·9	20·21	38·44	1·11	30·06
1984	29·8	788·87	6·8	21·59	42·51	1·22	36·53
1985	21·6	958·98	11·0	23·96	46·97	1·10	40·02
1986	26·4	1212·23	11·0	26·59	51·91	1·14	45·60
1987	8·4	1313·66	16·3	30·92	56·84	0·93	42·49
1988	12·8	1481·53	9·3	33·79	62·12	1·03	43·84
1989	33·8	1982·76	5·5	35·66	68·26	1·27	55·60
1990	−6·6	1852·59	4·6	37·31	75·54	0·89	49·66
1991	16·8	2164·48	18·0	44·04	82·76	0·99	49·15
1992	2·8	2224·67	13·7	50·08	90·32	0·90	44·42

APPENDIX 4

GEOMETRIC VERSION OF THE NOMINAL RETURNS ON EQUITIES

Year	$(1+g)^2$	$\bar{Y}=\frac{(1+d)}{(1+g)}$	Accum. $\bar{Y}$	$\bar{K}=\frac{(1+k)}{(1+g)}$	Accum. $\bar{K}$	$\bar{Y}*\bar{K}$	$(1+E):=Y*\bar{K}*(1+g)^2$	Accum. $(1+E)$	Accum. Index	Actual to Fitted
1922			1·00		1·00			1·00	1·00	1·00
1923	1·09	1·02	1·02	1·08	1·08	1·10	1·20	1·20	1·12	0·93
1924	1·09	1·01	1·03	0·94	1·01	0·95	1·03	1·23	1·36	1·11
1925	1·09	1·00	1·03	1·06	1·07	1·06	1·16	1·43	1·80	1·26
1926	1·09	1·01	1·04	1·06	1·13	1·06	1·16	1·66	1·85	1·11
1927	1·09	1·00	1·04	0·97	1·09	0·97	1·06	1·77	2·18	1·23
1928	1·09	1·00	1·05	1·03	1·13	1·04	1·13	2·00	2·63	1·32
1929	1·10	1·01	1·06	1·05	1·19	1·07	1·17	2·34	2·31	0·99
1930	1·09	1·02	1·08	0·87	1·03	0·89	0·97	2·26	2·04	0·90
1931	1·09	1·02	1·11	0·77	0·80	0·79	0·86	1·94	1·72	0·89
1932	1·07	1·01	1·12	0·86	0·69	0·88	0·94	1·81	2·33	1·29
1933	1·07	1·01	1·13	1·02	0·70	1·02	1·09	1·98	3·06	1·54
1934	1·05	1·01	1·14	1·11	0·78	1·12	1·18	2·34	3·80	1·63
1935	1·06	1·01	1·15	1·07	0·83	1·08	1·14	2·67	4·34	1·62
1936	1·06	1·00	1·16	1·03	0·85	1·03	1·09	2·92	5·16	1·77
1937	1·07	1·01	1·17	1·10	0·94	1·11	1·19	3·47	4·50	1·30
1938	1·07	1·02	1·19	0·97	0·91	0·99	1·06	3·68	4·04	1·10
1939	1·07	1·02	1·21	0·92	0·84	0·94	1·01	3·71	4·13	1·11
1940	1·07	1·03	1·25	1·01	0·85	1·04	1·10	4·10	3·94	0·96
1941	1·06	1·02	1·28	0·94	0·80	0·96	1·02	4·20	4·84	1·15
1942	1·06	1·01	1·29	0·93	0·74	0·94	1·00	4·19	5·70	1·36
1943	1·06	1·01	1·31	0·95	0·71	0·96	1·02	4·28	6·36	1·48
1944	1·06	1·01	1·32	0·99	0·70	1·00	1·06	4·52	7·15	1·58
1945	1·05	1·01	1·33	0·99	0·69	1·00	1·06	4·78	7·57	1·58
1946	1·05	1·01	1·34	1·03	0·71	1·04	1·09	5·23	8·92	1·71
1947	1·06	1·01	1·36	1·11	0·79	1·12	1·19	6·22	8·72	1·40
1948	1·06	1·01	1·38	0·89	0·71	0·90	0·96	5·96	8·39	1·41
1949	1·07	1·01	1·40	1·02	0·72	1·04	1·11	6·62	7·91	1·20
1950	1·07	1·01	1·42	1·01	0·73	1·02	1·10	7·25	8·77	1·21
1951	1·08	1·01	1·44	1·08	0·78	1·09	1·18	8·57	9·52	1·11
1952	1·09	1·02	1·46	1·02	0·80	1·03	1·12	9·63	9·50	0·99
1953	1·08	1·01	1·48	1·00	0·80	1·02	1·10	10·59	11·80	1·11
1954	1·08	1·01	1·49	1·11	0·89	1·12	1·21	12·76	17·54	1·37
1955	1·09	1·00	1·50	1·11	0·99	1·12	1·22	15·52	19·45	1·25
1956	1·10	1·01	1·51	0·98	0·97	0·99	1·09	16·86	17·70	1·05
1957	1·11	1·01	1·53	0·97	0·94	0·98	1·09	18·37	17·50	0·95
1958	1·10	1·00	1·53	1·01	0·96	1·01	1·11	20·43	25·87	1·27
1959	1·10	0·99	1·51	1·08	1·03	1·06	1·17	23·94	40·06	1·67
1960	1·12	0·99	1·49	1·14	1·17	1·13	1·26	30·14	40·77	1·35
1961	1·13	0·98	1·47	0·97	1·14	0·96	1·09	32·76	41·46	1·27
1962	1·11	1·00	1·46	0·96	1·09	0·95	1·06	34·70	41·61	1·20
1963	1·11	0·99	1·44	1·02	1·12	1·01	1·12	38·93	49·73	1·28
1964	1·13	0·99	1·43	1·07	1·20	1·06	1·19	46·46	46·79	1·01
1965	1·13	0·99	1·42	1·01	1·20	1·00	1·12	52·23	52·59	1·01

(continued)

APPENDIX 4: (*Continued*)

Year	$(1+g)^2$	$\bar{Y}=\frac{(1+d)}{(1+g)}$	Accum. $\bar{Y}$	$\bar{K}=\frac{(1+k)}{(1+g)}$	Accum. $\bar{K}$	$\bar{Y}*\bar{K}$	$(1+E):=Y*\bar{K}*(1+g)^2$	Accum. $(1+E)$	Accum. Index	Actual to Fitted
1966	1·13	1·00	1·41	0·95	1·15	0·95	1·07	56·01	49·71	0·89
1967	1·14	0·97	1·37	0·88	1·01	0·86	0·98	55·04	68·62	1·25
1968	1·16	0·96	1·32	1·03	1·04	0·99	1·15	63·11	97·22	1·54
1969	1·18	0·96	1·27	0·92	0·96	0·88	1·04	65·55	85·86	1·31
1970	1·19	0·96	1·21	0·99	0·95	0·95	1·13	74·01	84·25	1·14
1971	1·17	0·95	1·16	0·96	0·91	0·92	1·07	79·45	122·37	1·54
1972	1·20	0·94	1·09	0·98	0·89	0·92	1·11	88·18	148·95	1·69
1973	1·25	0·93	1·01	0·83	0·74	0·77	0·96	85·04	101·21	1·19
1974	1·37	0·96	0·97	1·04	0·76	0·99	1·36	115·41	51·22	0·44
1975	1·32	0·92	0·89	0·99	0·76	0·91	1·21	139·13	127·81	0·92
1976	1·32	0·93	0·83	1·04	0·79	0·97	1·28	178·65	126·42	0·71
1977	1·23	0·95	0·79	0·98	0·77	0·93	1·15	204·90	198·70	0·97
1978	1·28	0·93	0·74	0·97	0·75	0·91	1·16	238·10	222·65	0·94
1979	1·32	0·93	0·69	1·09	0·82	1·01	1·33	317·07	244·17	0·77
1980	1·30	0·93	0·64	0·95	0·78	0·88	1·14	362·58	327·36	0·90
1981	1·34	0·91	0·58	0·93	0·72	0·85	1·14	413·91	367·36	0·89
1982	1·23	0·95	0·55	0·97	0·70	0·92	1·14	469·91	473·20	1·01
1983	1·22	0·95	0·52	1·00	0·70	0·95	1·16	545·44	607·70	1·11
1984	1·22	0·94	0·50	1·08	0·76	1·02	1·24	677·92	788·87	1·16
1985	1·22	0·94	0·47	0·99	0·75	0·93	1·13	769·17	958·98	1·25
1986	1·22	0·94	0·44	1·07	0·80	1·01	1·23	949·14	1212·23	1·28
1987	1·20	0·95	0·42	1·02	0·81	0·97	1·16	1103·81	1313·66	1·19
1988	1·19	0·96	0·40	1·07	0·87	1·03	1·23	1358·04	1481·53	1·09
1989	1·21	0·95	0·38	1·05	0·91	0·99	1·20	1628·17	1982·76	1·22
1990	1·22	0·95	0·36	1·01	0·92	0·96	1·18	1921·06	1852·59	0·96
1991	1·20	0·96	0·35	0·97	0·89	0·93	1·11	2137·00	2164·48	1·01
1992	1·19	0·96	0·34	0·90	0·81	0·87	1·04	2217·67	2224·67	1·00

APPENDIX 5

GEOMETRIC VERSION OF THE REAL RETURN ON EQUITIES

Year	RPI actual	Inflation rate (%)	$(1+i)$	$\frac{(1+E)}{(1+i)}$	Accum.	$\frac{(1+k)}{(1+i)}$	Accum.	$\frac{(1+g)}{(1+i)}$	Accum.
1922	1·00				1·00		1·00		1·00
1923	0·99	−1·5	0·99	1·22	1·22	1·14	1·14	1·06	1·06
1924	1·00	2·0	1·02	1·01	1·23	0·96	1·09	1·02	1·08
1925	0·99	−1·9	0·98	1·18	1·45	1·13	1·23	1·07	1·16
1926	0·99	0·5	1·00	1·16	1·68	1·10	1·36	1·04	1·20
1927	0·94	−5·5	0·94	1·13	1·89	1·07	1·45	1·11	1·33
1928	0·93	−0·5	0·99	1·14	2·15	1·09	1·58	1·05	1·40
1929	0·93	−0·5	0·99	1·18	2·53	1·11	1·75	1·05	1·47
1930	0·86	−7·0	0·93	1·04	2·62	0·97	1·71	1·12	1·65
1931	0·82	−4·7	0·95	0·90	2·36	0·84	1·44	1·10	1·81
1932	0·79	−3·6	0·96	0·97	2·29	0·93	1·33	1·07	1·94
1933	0·80	0·6	1·01	1·09	2·49	1·04	1·39	1·03	1·99
1934	0·80	0·0	1·00	1·18	2·94	1·14	1·58	1·03	2·04
1935	0·82	2·5	1·02	1·11	3·27	1·07	1·70	1·00	2·05
1936	0·84	2·5	1·03	1·07	3·49	1·03	1·76	1·00	2·06
1937	0·89	5·8	1·06	1·12	3·92	1·07	1·88	0·98	2·01
1938	0·86	−2·8	0·97	1·09	4·27	1·03	1·95	1·06	2·14
1939	1·08	24·9	1·25	0·81	3·45	0·77	1·49	0·83	1·77
1940	1·35	25·9	1·26	0·88	3·03	0·83	1·23	0·82	1·45
1941	1·43	5·9	1·06	0·97	2·93	0·92	1·13	0·97	1·41
1942	1·42	−1·1	0·99	1·01	2·95	0·97	1·09	1·04	1·47
1943	1·41	−0·7	0·99	1·03	3·04	0·99	1·08	1·04	1·53
1944	1·43	1·8	1·02	1·04	3·15	1·00	1·08	1·01	1·55
1945	1·46	1·7	1·02	1·04	3·28	1·00	1·08	1·01	1·56
1946	1·47	0·7	1·01	1·09	3·56	1·05	1·14	1·02	1·59
1947	1·61	9·5	1·09	1·09	3·87	1·04	1·18	0·94	1·50
1948	1·71	6·2	1·06	0·90	3·49	0·87	1·02	0·97	1·45
1949	1·76	2·9	1·03	1·08	3·77	1·03	1·05	1·01	1·46
1950	1·80	2·6	1·03	1·07	4·02	1·02	1·07	1·01	1·47
1951	2·02	12·1	1·12	1·05	4·24	1·00	1·07	0·93	1·37
1952	2·08	3·0	1·03	1·09	4·63	1·03	1·10	1·01	1·38
1953	2·09	0·2	1·00	1·10	5·08	1·04	1·14	1·04	1·43
1954	2·15	3·3	1·03	1·17	5·92	1·12	1·28	1·00	1·44
1955	2·26	5·1	1·05	1·16	6·85	1·10	1·41	0·99	1·43
1956	2·31	2·2	1·02	1·06	7·28	1·01	1·42	1·02	1·46
1957	2·41	4·3	1·04	1·04	7·61	0·98	1·40	1·01	1·48
1958	2·45	1·6	1·02	1·09	8·33	1·04	1·46	1·03	1·53
1959	2·45	0·0	1·00	1·17	9·76	1·13	1·65	1·05	1·60
1960	2·50	1·8	1·02	1·24	12·06	1·18	1·95	1·04	1·66
1961	2·59	3·8	1·04	1·05	12·63	1·00	1·95	1·03	1·70
1962	2·65	2·3	1·02	1·04	13·08	0·99	1·92	1·03	1·76
1963	2·70	1·8	1·02	1·10	14·41	1·06	2·03	1·04	1·82
1964	2·83	4·8	1·05	1·14	16·41	1·08	2·20	1·01	1·84
1965	2·96	4·6	1·05	1·07	17·63	1·02	2·25	1·02	1·87
1966	3·07	3·7	1·04	1·03	18·24	0·98	2·20	1·03	1·92
1967	3·15	2·5	1·02	0·96	17·49	0·92	2·02	1·04	2·00

(continued)

APPENDIX 5: (*Continued*)

Year	RPI actual	Inflation rate (%)	$(1+i)$	$\frac{(1+E)}{(1+i)}$	Accum.	$\frac{(1+k)}{(1+i)}$	Accum.	$\frac{(1+g)}{(1+i)}$	Accum.
1968	3·33	5·9	1·06	1·08	18·95	1·05	2·12	1·02	2·03
1969	3·49	4·8	1·05	0·99	18·79	0·95	2·02	1·04	2·11
1970	3·76	7·9	1·08	1·05	19·67	1·00	2·02	1·01	2·14
1971	4·10	9·0	1·09	0·99	19·37	0·95	1·93	0·99	2·12
1972	4·41	7·6	1·08	1·03	19·97	1·00	1·93	1·02	2·16
1973	4·88	10·6	1·11	0·87	17·42	0·84	1·61	1·01	2·19
1974	5·82	19·2	1·19	1·14	19·84	1·02	1·64	0·98	2·15
1975	7·27	24·9	1·25	0·97	19·14	0·91	1·50	0·92	1·97
1976	8·36	15·1	1·15	1·12	21·36	1·04	1·55	1·00	1·97
1977	9·38	12·1	1·12	1·02	21·85	0·97	1·51	0·99	1·95
1978	10·16	8·4	1·08	1·07	23·42	1·02	1·53	1·04	2·04
1979	11·92	17·2	1·17	1·14	26·61	1·06	1·63	0·98	1·99
1980	13·72	15·1	1·15	0·99	26·43	0·94	1·53	0·99	1·97
1981	15·37	12·0	1·12	1·02	26·93	0·96	1·47	1·03	2·04
1982	16·20	5·4	1·05	1·08	29·00	1·02	1·51	1·05	2·15
1983	17·06	5·3	1·05	1·10	31·97	1·05	1·59	1·05	2·25
1984	17·84	4·6	1·05	1·19	37·99	1·14	1·80	1·06	2·38
1985	18·86	5·7	1·06	1·07	40·78	1·03	1·86	1·05	2·49
1986	19·56	3·7	1·04	1·19	48·52	1·14	2·12	1·07	2·65
1987	20·28	3·7	1·04	1·12	54·42	1·07	2·28	1·06	2·80
1988	21·66	6·8	1·07	1·15	62·70	1·10	2·51	1·02	2·87
1989	23·33	7·7	1·08	1·11	69·80	1·07	2·68	1·02	2·93
1990	25·51	9·3	1·09	1·08	75·31	1·02	2·74	1·01	2·96
1991	26·65	4·5	1·04	1·06	80·20	1·01	2·77	1·05	3·11
1992	27·43	2·9	1·03	1·01	80·84	0·96	2·66	1·06	3·29

ABSTRACT OF THE DISCUSSION

Mr P. D. Jones (introducing the paper): Some readers of this paper have the impression that I favour gilt-edged at the expense of equities. Nothing could be further from the truth. Although I have worked for a major gilt-edged house for many years, I have always worked on the equity side. This has given me the opportunity of comparing, on a daily basis, the equity and the gilt-edged markets over several economic cycles, including a couple of major recessions. It is with that background that Section 2 was written. It attempts to show the shortcomings of the yield ratio as a measure of the cheapness or dearness of equities relative to gilt-edged.

The yield ratio, which is shown in Figure 2, is widely used in the City and by investing institutions. Its supporters are currently relying on it to justify a view that equities are cheap. The paper suggests an alternative yardstick: the expected return on equities, where the key variable is the excess of the equity dividend growth rate over the gilt-edged yield, that difference being divided by the gilt yield. This yardstick, which is shown in Figure 7, suggests that equities are historically dear relative to gilts. With pension funds so heavily committed to equities, it is important that this profession formulates its own view on the subject.

A central theme of the paper is the need to standardise equity data. "Thou shalt not analyse equity data in isolation," is perhaps the principal point. The yield ratio standardises the equity yield by the gilt yield, and my expected return does the same.

It has been put to me that the gilt yield is the wrong measure of standardisation—surely inflation is the more obvious choice. I disagree. The yield on long-dated gilt-edged is the central economic statistic of the economy. By contrast, inflation is but a concept, and it is a constantly changing one which reflects different social circumstances. Moreover, the buying and selling of investments is not, with the sole exception of index-linked gilts, a function of anticipated real returns. Surely institutional and private investors alike invest on the basis of nominal returns.

Section 3 presents 70 years' investment history in what, I believe, is a new light. In particular, it highlights the significant and seemingly unrepeatable role that the high yield on equities during 1925–55 played. This is shown in Figure 12. Understanding this feature is vital to any analysis of historical real returns on equities, and hence the choice of actuarial valuation bases.

Mr W. J. Smith, F.F.A. (opening the discussion): This paper challenges some of my own ideas, although mainly in respect of the means rather than the end. To me it has two main purposes:

(1) to track the history and workings of the yield ratio, and
(2) to develop a more general approach to assessing the relative merits of gilts and equities in the present investment environment.

The main virtues of the paper are as follows:

(1) It takes a long-term historical perspective, which is always important for this subject.
(2) It is timely. Dividend growth and inflation are at their lowest for around 25 years, gilt holdings are at their lowest percentage of assets in modern times for many institutional investors, and the Government has a massive funding requirement ahead of it.
(3) The yield ratio does have a significant place in many investors' priorities, and this is always worthy of examination.

It was also good to see a brief discussion of the role of democracy in investment risk; a subject surprisingly ignored given its importance (in particular, the impact that it can have on the future value of money).

My principal concern with the paper is that so much of the analysis was conducted in nominal terms, with inflation not treated explicitly until Section 3.8. Until then the author addressed only two dimensions of the three-dimensional problem of equities, gilts and inflation. Placing inflation more centrally in the analysis of returns resolves a number of the questions posed by the author later in the paper, and returns the preservation of spending power, which is the rationale for long-term savings, to the heart of the debate.

When Section 1 points out that equities are riskier than gilts in nominal terms two thoughts arise:

(1) What risk is meant? If we mean volatility, then certainly I agree, and we would expect a lower-yielding, longer-duration instrument, such as equities.
(2) If we rebased risk relative to inflation, then that volatility is likely to be reduced, but that acknowledges a different form of risk than volatility.

Section 2 addresses the components of historical returns from equities and the value, or otherwise, of the yield ratio in predicting this. It touches only briefly on the United Kingdom's ERM membership, but this was a period which threw into relief some of the issues addressed in the paper. It was ERM membership which catalysed the equity-gilt debate in the U.K., although this is a debate which continues outside the ERM. To see why it did so, if we think of gilts as money lent today against a claim on future money, while equities are a claim on real economic assets, the two main enemies of the value of future money are *devaluation* and *inflation*. They are also two long-term characteristics of the U.K. financial system, and the two factors which the ERM was specifically established to combat. Joining the ERM removed, for that period, the opportunity for devaluation and inflation as a means out of recession, which is the traditional U.K. response. We have had some evidence of that in the last few months. It achieved this by maintaining high real interest rates and punishing risk, if we relate risk to the value of returns deferred to the future, rather than in the present. As a result, gilts substantially outperformed equities throughout the U.K.'s period of membership.

Now that the U.K. is out of the ERM, and while long-term rates are little changed and still high in real terms at the long end, the yield curve is positive. This has had a direct impact on the performance of equities relative to gilts, though not yet on inflation. Low inflation certainly permits higher price/earnings multiples. The logic is the same as the author uses to standardise yields in the paper—3% growth is worth more when inflation is low than high. This does not really depend on whether interest rates are high or low in real terms—they are held constant for the purpose of that exercise. Earnings growth is depressed by high real rates, and one usual effect of the measures taken to cut inflation is to cause earnings to fall—so, good for the P/E and bad for the E (earnings) was the message of the ERM.

It is not yet controversial, as the author conjectures in § 2.1.3, to treat index-linked as subsidiary to the main benchmark, but it may become so, if the Government uses them extensively in its funding over the next couple of years—which high real long-term interest rates may encourage it to do.

Considering the yield ratio and its role in the investment community, its highest profile has really been as an indicator of sentiment, which may be irrational, rather than value, which we like to think of as rational. In Figure 2 extremes of sentiment below 2 and above 3 are measured. The yield ratio spent most of the period after the 1967 devaluation between 2 and 3, which was more due to a reappraisal of gilts, where the yield basis moved substantially, rather than, as the author argues, a reappraisal of equities. However, this underlines the point that the yield ratio tends to work best in stable conditions. The stable number is not necessarily a good value indicator; as the author notes, equities gave a substantially greater return than gilts over much of this period. It could be argued that the yield ratio is not below 2 now, since dividend cover is so low. In part this is due to the earnings depression that we normally see in a recession. It should be noted that, in the past, dividends have not usually grown in real terms with cover below 2·3 times. It we adjusted the present yield in the market to a cover of about 2·3, the yield ratio would be 2·5 or slightly higher.

Sections 2.3 and 2.4 are important in developing the ideas in the paper. The first one standardises by using ratios where others might traditionally use differences. Thus, if a stock moves from a 9% yield to 6%, the price increase is 50%. If it moves from a 6% yield to 3%, the price increase is 100%. The author's approach isolates the elements of the yield ratio and dividend growth in arriving at a risk premium. Division by the gilt-edged rate allows the effect of a substantial yield shift in gilt-edged, such as occurred in 1967 and 1982, to be examined separately. The excess of dividend growth, K, is bound to be highly volatile, but I was less taken with the stability of $1/R$ mentioned in Section 2.5. This arises, in part, because it is an inverse, locked between zero and one most of the time. If you look at this measure against itself, it changed by around 60% between 1987 and 1991.

Section 2.6 addresses the behaviour of P, the risk premium, through a recession. The author suggests that this can make the risk higher in gilt-edged than in equity. He is viewing 'risk', in this context, as the future value of money rather than volatility. He invokes democracy as a contributory factor, and one cannot doubt that democratic forces contributed to the devaluations of 1967 and 1992. By removing the threat of devaluation, albeit temporarily, the U.K.'s membership of the ERM showed how important devaluation can be in determining the relative merits of gilts and equities.

Section 3 is an analysis of historic returns, and discusses the risk premium both historically and in terms of expectations. I subjected Figure 10, which shows equity returns relative to gilts, to some political analysis. The 1951–64 and 1979–92 Conservative Governments saw steady outperformance by equities against gilts on the scale of the graph, with the middle period, dominated by Labour Governments, showing a more volatile trend. The key reason that the excess return of equities over gilts was lower after the mid-1960s, mentioned in §3.5.5, was a reappraisal of gilts, whose yield rose through 6%, later into double figures, while that of equities stayed quite stable. This is shown to better effect in Figure 11. The running yield on equities has been very stable over long periods of time, and, in 50 of the 70 years covered in Appendix 2, it has been between 4% and 6%.

The author points out, in Section 3.6, the change in composition of equities used over the period covered by the paper. This is an important point. Today capital goods comprise only around 14% of the All-Share Index market capitalisation, similar to the weighting of utilities, which had no presence 10 years ago.

The author, in Section 3.8, turns to inflation. Of the 6·5% p.a. real return from equities over the whole period, I estimate that around 1·8% is attributable to real dividend growth—slightly less than real GDP over the period, and so the components of total return from equities in real terms have been the running yield plus that small real dividend growth. This, in turn, gives one of the answers to the question the author poses about the current actuarial practice of relating k to i, inflation, rather than to g, the gilt yield.

I agree that the outlook for dividend growth in the near future is poor. High real interest rates and low dividend cover certainly imply that. I have looked at some of the contracts in the market to see what the expectations are. In the futures market, what is priced into the short to medium term is dividend growth of around 4–5%. This is the same range as the inflation which is implied by the gap between conventional and index-linked gilts—so the market would appear to have priced underlying dividend growth and inflation fairly closely in line at the present time.

The author has taken as his subject the yield ratio, and highlighted a number of its limitations. I would have preferred greater analysis of the risk of devaluation and unanticipated inflation, which have been very important elements of return in the past. The ERM has highlighted why they can be such surprising and important investment factors, but the author is certainly right to highlight the present combination of high gilt yields and low equity dividend growth.

Mr D. P. Hager: The paper raises two important issues:

(1) security valuation for pension fund actuarial purposes, and
(2) the debate that we have seen raging since devaluation on whether pension funds should increase their amount of gilt-edged securities and decrease their dependence on equities.

I do not agree with the author when he suggests that most investment decisions are made in nominal terms. I standardise in index-linked gilt terms, because it is easier to think in real rates of return than in nominal rates.

Pension fund actuaries must use credible valuation bases which their clients understand. In §4.1.3 a typical U.K. valuation basis is discussed. Based on the data presented in the paper, this typical basis appears to lack credibility. For example, in 1990, when equity market values declined some 6%, clients became uneasy about their schemes' funding. Nevertheless, many of the actuarial valuations carried out using the discounted income method produced surpluses. This was because dividends had increased over 1990 by round about 12%, and so actuarial asset values increased accordingly. Some trustees have problems with this concept. They believed that things were getting rather bleak in the 1990/91 period, and yet some actuaries were tending to show an improvement in actuarial terms, and appeared to be having no regard to the substantial increase in the pay-out ratios and, therefore, drops in the retention ratios for corporates.

Many actuaries do not want to use market values for the assets, and make appropriate adjustments on the liability side of the equation. If we did start using this method, we might find that pension fund trustees could relate to the assets and the overall valuation more easily.

I agree with §4.3.2., that the profession should not just rely on the long-term view, and must find ways of relating valuation bases to current and expected economic reality. We must place greater emphasis on trends, on solvency, on funding and on accounting figures over the same time spans that corporate management can deal with, and even a decade is quite some way.

In the next decade the rate of growth of dividends is likely to be less than we have seen during the 1980s. Companies need to rebuild their retention ratios. This may result in reduced equity returns and, therefore, in some investors having a preference for other asset types. I do not agree with the comments in §3.8.5, that the prospects for the next decade accord with none of the historical precedents. That might have been a reasonable belief in the summer of 1992, when membership of the ERM and the economic straitjacket that then appeared to be a permanent feature made it possible that bonds may be a suitable long-term investment for a final salary pension fund. History has told us that, whenever we think we ought to give up the fundamental belief in equities, circumstances do not stay favourable for bonds for long. Devaluation and departure from the ERM have fundamentally changed the outlook for bonds against equities. It is, therefore, unfortunate that the debate has got going since the ERM departure. Inflationary pressures will probably come back. Real interest rates may fall and the supply of gilts will be abundant. So will the next decade be any different? Has the economic reality for the U.K. economy really changed? It will probably be equities, index-linked gilts and property that provide the excess returns over conventional gilts in the next decade. I think that trustees should think carefully before switching to conventional bonds.

Mr T. G. Arthur (in a written contribution that was read to the meeting): I agree with the author's conclusions about the current over-valuation of the equity market. I have reached them by a different route, first highlighted by J. R. Hemsted in his paper, 'One-year Returns and the Degree of Risk' (*J.I.A.* **95,** 19). The yield ratio is indeed inadequate, as is the yield gap or any other indicator which ignores return on capital. As Hemsted showed, equality of return between an equity and a bond occurs when the equity's return on capital equals the yield gap divided by the retention ratio.

I do not find the concept of risk premium at all helpful; it can be determined only with hindsight, and seems circular, in that the risk premium at any time is a function of itself at another time.

The simplest way to see equity over-valuation is to appreciate that, over more than 30 years, earnings growth has *not* kept pace with price inflation, despite retention ratios averaging much more than those currently obtained. Over 60 years the figure is only marginally positive. The restoration of earnings growth to match price inflation may well need a one-off dividend cut of around a third, putting yields at around 3%, and still providing an overall return of only 3% real. This process overvalues current equity prices relative to index-linked bonds by perhaps 25%, without any allowance for risk!

The lack of real earnings growth is not surprising. I have shown elsewhere that fundamental economics dictate that aggregate and indiscriminate equity investment does not, and cannot, capture economic growth.

Mr A. F. Wilson: The first matter of concern is the concept of a yield ratio. I could never understand why it worked, nor find anybody who could give me the fundamentals as to why it should work. I regret that I do not see them in the paper. I have felt, for many years, that somehow dividends might be expected to keep pace roughly with inflation, and that the yield on equities might be independent of inflation. The gilt yield might be expected to be related to the rate of inflation expected in the near future. Yet the yield ratio is totally independent of inflation. The author seems to concur, because, in Section 2.2, he simply says that it works. However, in Section 2.7 he says, but maybe not for the future.

In the paper we look at different measures of inflation, which is confusing. The yield on gilt-edged, g, presumably is geared in some way to the expectations of what inflation will be over the next 10–15 years. The rate of growth of gross dividends, k, on the other hand, is likely to be much more closely related to inflation 3 years ago, as shown in the paper 'A Realistic Approach to Pension Funding', by P. N. Thornton & A. F. Wilson (*J.I.A.* **119,** 229). Why should it be assumed that inflation over the next 10 years is going to equal that of 3 years ago? Maybe on average it is, but, if I am trying to make an investment decision for the next 3–5 years, I would be foolish to assume that averages are going to work in that way.

My point comes across clearly in the question asked in § 3.8.4: "Why does current actuarial practice relate k to i—when it is clearly more closely related to g?" The author asks this from his perspective, which is that of g being the important criterion. Looking at it as a pension fund actuary— and maybe this is the reason why I applaud the idea of 80–90% in equities—I cannot see what relevance g has. If I am forecasting what liabilities I have, and what I should be using as assets to match them over the next 20, 30, or 40 years, inflation is much more important, indeed fundamental. Is not g very much a result of supply and demand? I suggest that the rate of inflation expected in the future is based very much on what inflation has been in the past. Is this not why it was so low right the way through the 1920s, 1930s, and 1940s? Inflation was expected to disappear, probably after some deflation, following the two World Wars. It was only when it was found that that was not going to happen, in the 1950s, that the yield gap emerged, not because equities suddenly changed their value, but because gilts changed theirs.

I disagree with the idea that there is not a close relationship between k and i. I do not necessarily think it is a worse relationship because it comes to 1·02 rather than 1·00. I am not convinced that there is a significant difference between the standard deviations. Given the rounding in the figures, it is very difficult to tell.

I think that somewhere there is an inconsistency within the data of Appendix 2. A figure for income growth of −7·6% in 1973 is given, and of +21·4% in 1974. However, looking at Figure 1, at no stage during that period is dividend growth negative, nor does it ever approach the 21% which is mentioned in the Appendix. I suspect that what has happened here is that the income index and the price index have been obtained separately, and that the relationship between the two is not as good as it should be in getting the income growth over very short periods. Over the longer term it is quite adequate.

Now, g is a gilt yield. In pension fund valuations we do not assume that our valuation yield is the gilt yield. I suggest that the parameters given are consistent with a gilt yield of between 8¼% and 7%, so I see no inconsistency.

Mr W. J. Bishop: Rather than discuss the specific analysis in the paper, I have considered an alternative approach. The methodological base for this comes from Brian Reading of Lombard Street Research, although any projections and conclusions are mine, not his. Historic equity returns over 6 decades up to 1991 are separated into 5 components:

(1) inflation (represented by the GDP deflator),
(2) real GDP growth,
(3) changes in the ratio of corporate earnings to GDP,
(4) changes in the ratio of dividends to earnings (which is prominent in the paper), and
(5) changes in the ratio of equity prices to dividends.

The most significant feature to emerge from this analysis is that the 3 latter components were, for the only time, all positive in the 1981/91 decade, and that their combined contribution to the equity investment return was significantly greater than in any other decade since 1931.

The projection of investment returns over longer periods, such as a decade, is a notably hazardous exercise, owing to the rapidly expanding funnel of doubt. Nevertheless, it may still be of interest to suggest a possible outcome for the 1991/2001 period. Before sterling left the ERM in September 1992, I might have projected the GDP deflator at 3–4% p.a., and GDP growth at around 2% p.a. Following the change, I would be inclined to raise the projections for the deflator to 4–5% and for growth to 2–3%, giving nominal GDP growth of some 6–8%. The corporate earnings' share of GDP was certainly high, cyclically, at the end of the 1980s, and on some measures high also on a secular basis. It would thus be hard to project much positive contribution from this source. Even more clearly, as can be seen from the paper, dividend payout seems likely to be negative, not positive, in the next decade's return. Also, it could be that fairly low inflation and lower real yields would bring gilt yields down to 7–8%, allowing equity yields, on a ratio of, say, 2, to drop to 3½–4%, giving an additonal return over the decade of 2–3% p.a. starting from end-1991 levels. Nevertheless, while I have tried not to be unduly pessimistic, adding up the components gives a total return of somewhere in the 8–11% p.a. range, marginally different from end-1991 gilt yields and probably insufficient clearly to justify preferring equities.

Obviously pension fund asset distributions should, and will, continue to be influenced by the very-long-term record of equity returns, substantially exceeding fixed-interest returns. However, the industry, as a whole, has pushed the boat out in the all-equity direction, to an extent that could seem excessive as we go through the present decade. This would be particularly so if my economic assumptions proved too optimistic. This conclusion, although arrived at by a different route, seems consistent with that implied by the paper.

Mr J. Plymen: Mr Wilson prefers that the inflation factor be brought into the comparison. I am surprised that inflation is brought in as a secondary factor, as the difference between the gilt yield and the yield on index-linked.

I agree with the author in the use of the index statistics to determine policy. This is a reflection of the original objective of the indices, as set out in the paper by C. M. Douglas, 'The Statistical Groundwork of Investment Policy' (*T.F.A.* **12,** 173). The indices were required so that they could be used for determining investment policy.

Figure 14, the equity risk premium, shows the dramatic impact on the equity market of the 'cult of the equity'. Until 1962 equities were not in much demand by the institutions, and the risk premium fluctuated violently. In 1962 the cult of the equity involved an enormous change in the supply and demand for equities. Institutions that had hardly any equities before began pouring money into the equity market, pushing down the expected yield. The equity risk premium, which was averaging around 2 with considerable variance, settled down to a lower level, averaging around $\frac{1}{4}$–$\frac{1}{2}$ with much less variance. The suggestion of the author that the equity risk premium ought to be about 25% above the gilt yield is much too high. It does not reflect current practice.

In my paper 'The Actuarial Background to Investment Policy' (*T.F.A.* **40,** 445), I suggested that the evidence from the indices was that equity dividends just about kept up with inflation over the long term. There was no evidence, over a long term, of dividends doing any better than inflation. Appendices 2–5 reveal a fascinating story. In Appendix 5 we have $(1+k)/(1+i)$, that is the dividend growth rate divided by the inflation. From 1922 to 1938 there was a deflationary period. When there is deflation, equity dividends are almost certain to gain on the cost of living. This is shown here. The accumulative factor, meaning the dividend growth gain on the inflation, went from 1 to 1·95 over 1922–38. In 1939 there was a burst of inflation, and then the accumulative factor went down from 1·95 in 1939 to 1·13 in 1941. That means that, over that period when there was inflation, the dividends lagged the inflation. From 1942 to 1969 there was a long period of modest annual inflation, 2%, 3% or 5%, and the accumulative factor of 1·09 in 1942 increased to 2·02 in 1967.

From 1967 there was again an inflationary period, and the accumulative factor fell from 2·02 in 1967 to 1·47 in 1981. Thereafter inflation levelled out, and dividends gained on inflation. Although the rapid growth of dividends in the last few years is obviously due to the improving economy, some of it can be due to the reduction of inflation.

At the moment I believe that, for long-term planning, the difference between the gilt yield and the index-linked yield can be used as a proxy for the long-term inflation, currently about 4·9%. The dividend yield is 4·1%, which gives an expected yield on equities of 9·0%. The yield on gilts is about 8·7%. My theory is that, with inflation modest and possibly declining, we will have dividends doing better than inflation. An expected yield of 9·1% against 8·7% is not bad. I do not think that equities are necessarily much riskier than gilts on these terms.

Mr T. S. Shucksmith: I agree that many pension funds are too heavily invested in equities. Equities may not be a perfect hedge against inflation, and I am concerned that many pensions actuaries may be over-valuing them. Some actuaries are valuing equities on dividend yields that have only been seen about twice in the last 70 years for the purposes of a bulk transfer payment, the terms of which are not negotiable. I am also concerned that some pensions actuaries may be overestimating future expected returns on equities and underestimating the risks. If actuaries allow for high assumed returns on equities, and fail to differentiate between expected and guaranteed cash flows in the calculation of individual transfer payments (cash equivalents), then the situation moves from one of financial play between corporate wheeler dealers to the realms of iniquity.

The paper leaves me dissatisfied in a number of respects. The author points out in § 2.3.8., that an understanding of the relationship between major economic and financial factors is all important, but he reveals very little of this understanding in the paper. He does not put forward a coherent economic theory or model for the equity risk premium and for his straightforward statistical analysis. There are, however, some good paragraphs, and these include § 3.8.5.

It is not clear in the paper whether the author thought of the yield ratio as a measure of the relative cheapness of equities. He did not bring in earnings and P/E ratios, although investor cognoscenti think in terms of buying earnings and earnings growth per share rather than simple dividends. I cannot see the logic of the author's approach in standardising equity returns and underlying growth by the long-term gilt yield, and, in particular, making that standardisation by division. At any point in time, expectations of inflation are a weighty element in both expected dividend growth and in the long-gilt yield, and would, from time to time, be very significant components in both the numerator and denominator of the author's equity risk premium formula. I see no reason for the expected excess return on equities or the equity risk premium to be higher, as an absolute amount, when expected inflation is high, or, conversely, to be lower when expected inflation is low. Neither can I share the author's conviction that equities will, and must necessarily, earn more than long gilts. Investment returns depend on the balance of supply and demand for each investment. Markets cannot demand a risk premium, nor for that matter can investors. Sometimes buyers appear to have the upper hand and sometimes sellers. Pension funds now account for a very significant proportion of equity market capitalisation, and this proportion is still rising. If enough dominant investors ignore the risk and look upon equities as super-high-yielding indexed gilts, then it is possible for them to drive down the income yield to such an extent that the investment return will be less than on other traditionally less risky investments. We may be entering a decade or longer in which the control of inflation is being taken as a much more serious priority than at any time since the Second World War. We may have a situation where monetary policy is having to restrain the otherwise inflationary effect of general fiscal laxity. These factors may lead to historically high real returns on long gilts, and to low dividend growth, both absolutely and relative to inflation.

Mr D. J. Parsons: I have never yet seen an actuarial valuation which identified specifically the expected long-term return on U.K. gilts as being the investment return statistic. This possible misunderstanding of the author affects his conclusions and concerns expressed in Section 4 about pension scheme valuation assumptions. I found it hard to cope with the 'non standard' notation which he used. For most of us g is dividend growth and i is investment return. It seems that in § 3.7.4 it even confused him. It does not show actuaries in a good light when, having identified a 30-year cycle, the author insists on averaging his data and statistics over a period of 70 years. It can be positively misleading.

The relationship between dividend growth and the return on gilt-edged securities is interesting, and, perhaps, should form part of the decision-making process for the valuation assumptions. It is difficult to make predictions about what levels gilts will be yielding over the lifetime of a pension fund, whereas an estimate of inflation in the long term has proved reasonably reliable over the last few decades. The author appears to agree.

Paragraph 3.8.5 and Table 4 are of the greatest interest. If history repeats itself, the next 20 years or so will be extremely unkind to pension schemes. According to my own research, the last 2 years have not been too good, so there might be only another 18 years to go.

I am not convinced by the final sentence of § 3.8.5; consistency with history *can* be achieved over the next decade. The author expects a significant positive real return on gilts in the long term. I think that this is unsustainable. If inflation falls, gilt yields, when looked at in the long term, will also fall, and if gilt yields stay high, inflation will rise. In either case the historical consistency will be achieved. The problem, as always with investment markets and perceptions, is that, as soon as you find an important trend or a rule, the market reacts in such a way that it no longer works.

Mr N. D. Freethy: Actuaries are the acknowledged wizards when it comes to analysing and forecasting mortality trends, but when it comes to economic forecasts, although we are arguably as well qualified as any to make them, we are surely equally well qualified to get the short-term forecasts wrong, particularly on the timing of vital turning points. Like Mr Shucksmith, I could not help noticing that the paper is virtually silent on the historic relevance of trends in earnings and earnings yields. Such statistics are, indeed, notoriously incomplete or unrepresentative—and you have only to point to examples like Japan and creative accounting to see why. However, if reliable figures for earnings are difficult to come by, the trends are surely more easily discernable. The *Financial Times*, for example, has published daily P/E ratios (or earnings yields) alongside dividend yields for decades, so the trends of earnings yield v. dividend yield, for example, can certainly be compared in some meaningful form.

The importance of all this becomes more obvious when, if I read him right, one of the author's reasons for suspecting the current value of U.K. equities is due to the proportion of earnings paid out by way of dividends having increased virtually ever since the ending of dividend restraint. Earnings have been decelerating, the recession is still with us—put these things together and the conclusion is inescapable. The outlook for dividends, and hence share prices, looks fairly bleak. Or is it? Supposing, instead, the recession clears, earnings leap up, and dividends resume their upward march. This may be considered unlikely by most, but the probability is not zero, certainly not 2 years out. This is where actuaries should be careful not to fall into the trap of being branded as economic forecasters, because as, inevitably in the long run, we eventually get it wrong, this could well achieve more publicity for us than the many things which, as a profession, we do rather well.

However, the author is shrewd and sensible enough not to conclude that we should rush out and sell all our U.K. equities, but, if we accept for the moment his conclusion that they are being valued too highly, the implication is that holdings should be reduced to the bottom or even below the normal range. So what do we do with the rest of the money; invest in property? Running yields on property are very much higher than equities, and prices are at a very different point in the cycle. The difficulty is in finding a suitable vehicle, particularly for small funds, but this may be a suitable home for a proportion of the money.

This still leaves much money to be invested. Are fixed-interest gilts suitable? For final salary schemes they offer absolutely no inflation proofing, and if my alternative prospect of the recession clearing does come about, the danger will be a pick up in inflation.

What about index-linked gilts? Their history is very short and all one way, in that their real yield has steadily increased since they were first issued; so the benefits of historical analysis are not strictly relevant. Their characteristics are often misunderstood by large numbers of long-term fund investment managers. This is reflected in the derisory percentage (about 3%) held in that sector, with the one house with 10% apparently holding a hostage to fortune. The point is that index-linked stocks are affected as much by the future course of long-term interest rates as they are by the market's view of future inflation. If inflation returns, their merits are obvious. If inflation stays low, most likely interest rates will fall, and index-linked stocks will also benefit. Given these characteristics, the real yield on index-linked gilts, approaching 4% at present, looks arguably much less unattractive than the outlook for U.K. equities as expounded in the paper.

The author has made the point that the market in index-linked gilts is small, but I am sure that the Government would not be averse to feeding the market if the demand were there, not least on the cynical view that the inflation-linked tab for servicing these stocks would be picked up, not by the present Government, but by its successors. Admittedly, this might inhibit performance in the short term, but would create a more viable outlet for the majority (as opposed to the minority) of U.K. pension funds.

We must beware of a temptation to be lured into forecasts such as "U.K. equities will fall". Instead, our approach should be to attach percentages to the probability that, if they do fall, the investment strategy to cover it should be this, and to the alternative possibility that, if they do not, the investment strategy should be something else; and then go somewhere in between.

Professor A. D. Wilkie: The paper raises many interesting questions, although it does not answer them, at least to my satisfaction.

The author dismisses the yield on index-linked stocks as of no importance. I do not agree. It now takes the place of Consols in the 19th century, the basic risk-free long-term security. It does not matter whether all investors hold the stock. All investors are free to hold it, and the quantities in issue, although smaller than shares or conventional fixed-interest stock, are not so trivial that the yield is distorted by scarcity.

The yield gap of the past has been re-created by the yield gap between shares and index-linked. It was notable that that gap became negative during the summer of 1987, indicating the then over-valuation of shares.

Relative to index-linked, the yield on conventional fixed-interest should be expected to equal the yield on index-linked plus some estimate for future inflation, plus or minus a premium. Those that think in real terms would want a premium for conventional fixed-interest. Those that think in money terms would want a premium for index-linked. It is not obvious what the relative weight of money in these compartments is, and so it is not obvious which way the premium should go. Nor is it easy to estimate the premium, since this also requires knowledge of the market's estimate of future inflation. Relating ordinary shares to index-linked, the dividend yield on shares can be expressed as being equal to the yield on index-linked, plus the market's assessment of future real dividend growth, which may be positive, negative or zero, plus a premium, which in this case would probably be positive, since the risks of shares in real terms are surely greater than the risks on index-linked. By putting together my two relationships the author's one is obtained. However, he assumes that the premium on shares relative to fixed-interest should be relatively large, whereas my formulation shows that the premium could be of any size, and could possibly be negative.

I, therefore, find nothing inconsistent in the author's hypothesis, in § 3.3.4, of a 5% initial yield on shares, 5% dividend growth, 9% return on fixed-interest and a 1% premium. Nor would I find figures of 5, 5 and 10 or 4·5, 4·5 and 9 inconsistent. I, therefore, do not find the author's implications in Table 5 convincing at all. The author confuses the issue by using the previous year's growth of dividends as an estimate of long-term future growth. It is no more appropriate than to use the latest annual increase in the Retail Prices Index as an estimate of long-term future inflation. In both cases it is sensible to look at the whole history of price inflation or of dividend growth, and to determine an estimate of the long-term future from some reasonably long-term period in the past. An exponentially weighted moving average is a handy way of giving rather more weight to recent experience and rather less weight to past experience, letting the past decay at the rate you choose. However, if you are estimating over 20 years ahead, I would like to look at least 20 years back, and possibly 40; certainly not just 1 year.

The author has not really done any serious statistical analysis of the data presented in the paper. The BZW index is not the only source of data. The FT-A indices plus the old Actuaries Index take us back reliably at monthly intervals to 1930, and there are some monthly data going back beyond that, in Douglas's paper that Mr Plymen has already referred to (*T.F.A.* **12,** 173). I have used this in an analysis I have carried out. The point of that paper is to show the extent to which the dividend yield at any time is a good predictor of share price performance, or the total return on shares, whichever you want to use, over a number of months ahead. It is not a very good predictor for the return over 1 month, but it turns out to be a very good predictor over about 6–7 years. The correlation coefficient between share price performance over 6½ years—that is, share price performance in money terms—and dividend yield at the beginning of the period is as high as 0·8. This suggests that similar investigations into the relation with the return on fixed-interest would be useful. It is not surprising to find that the return on fixed-interest is positively correlated with the initial yield when the fixed-interest stock is purchased, to the extent of a correlation coefficient exceeding 0·9 over the ensuing 25 years.

It is, therefore, not surprising to find that the relative share price performance, that is the difference in performance between shares and fixed-interest, is positively correlated with the excess of share yield over Consols yield. I describe it as an 'excess', even though it has often been negative, but, in effect, this is the same as the author's inverse yield ratio. However, as well as investigating over the whole of the period from 1923 to 1992, it would be desirable to split the time up into sub-periods, and it would also be appropriate to look at the multiple regression of share price or share price relative on both the share yield and the Consols yield at the beginning of the period. I have not carried out these investigations fully, but it certainly looks as if there is quite strong positive correlation between the excess total return on shares and the author's inverse yield ratio. However, it would be sensible to quantify this better. Turning it into log terms, so that the inverse yield ratio is the log of the share yield minus the log of the Consols yield, I would want to use multiple regression to see whether a one-for-one ratio was appropriate, or whether some other weights might give a better predictor. It would then be appropriate to have a look at how this related to the stochastic investment model I have proposed in the past, because it would be possible, either by analysis or by simulation, to discover the correlations implied over particular periods by my model.

I am grateful to the author for having suggested these lines of research, and I hope that either I or someone else will have time to consider them fully and report back to you in due course.

Professor G. T. Pepper, C.B.E.: I question the significance of one of the assumptions in the paper. In Figure 6, $1/R$, the inverted yield ratio, which is basically the market's valuation basis, is stable compared with K, which is the annual rate of growth of dividends compared with the gilt yield. I am puzzled about the stability of the inverted yield ratio graph compared with the graph of K. In Appendix 1, the author introduces an assumption that the yield ratio $r = g/d$ remains unchanged; that is that the ratio of the rate of interest in the gilt-edged market to the dividend yield in the equity market remains unchanged. It is not clear how far this assumption runs through the paper and how important it is to the analysis itself, but it is questionable.

Mr I. J. Kenna: The question which arises from the paper is: "What dividend growth?" Real dividend growth is shown as $(1+k)/(1+i)$ in Appendix 5, which shows that it has fallen continuously over the past 5 years, until it reached -4% in 1992. This is the longest period of continuous fall since 1922. I proved in *The Actuary* (**I**, 5, 34) for November 1990, that, under otherwise stable conditions, real dividend growth rates tend to fall continuously. In practice this has not happened in the past, because of special circumstances. The tendency of real dividend growth rates to fall is now clearly appearing in practice as well as in theory. More, and greater, negative real growth rates can be anticipated. Companies are, naturally, doing their best to avoid this by economy measures. At best their efforts can succeed only temporarily.

The Government's scope for intervention is limited by the free market which has been developing since 1979. If the last 5 years are representative, negative real dividend growth will equate to negative nominal dividend growth. Equity prices, on the other hand, are discounting high positive dividend growth. Shares are overvalued.

Mr G. D. Cox (a visitor): I am an economist. The author appears to be reinventing the wheel in Section 2. He has put his theory together piecemeal. The basic textbook model about the Stock Market is the dividend discount model. It has 4 terms, and what is in the paper is not so different from it. However, in §3.3.2 the use of g is confused and it can be removed from all 3 terms.

The dividend discount model states that the expected growth of dividends into the indefinite future is equal to the discount rate plus the risk factor minus the equity market yield. If you do that calculation on tomorrow's data in the *Financial Times* you will find what the market is expecting in terms of the growth of dividends over the indefinite future. Interestingly, and uniquely in this country, this can be done by using the index-linked gilt or the conventional gilt as the discount rate. If conventional gilts are used the result is the market's expectation of the growth of nominal dividends. If the index-linked gilt and a lower discount rate are used the result is the expectation of the growth of *real* dividends.

The expected nominal and real dividend growth rates can be expressed approximately in 2 equations with a 3rd, involving expected prices, creating a closed 'triangular system'.

$$(\Delta D_e)_R = y_{IL} - y_{EQ} + R_{EQ/IL}$$

$$\Delta D_e = y_G - y_{EQ} + R_{EQ/G}$$

$$\Delta P_e = y_G - y_{IL} - (R_{EQ/G} - R_{EQ/IL})$$

where ΔD_e = expected growth rate of nominal dividends (p.a.)

$(\Delta D_e)_R = \Delta D_e$ in real terms (p.a.)

y_G = gilt yield (p.a.)

y_{IL} = index-linked gilt yield (p.a.)

$R_{EQ/IL}$ = risk factor between equities and index-linked gilts (p.a.)

$R_{EQ/G}$ = risk factor between equities and gilts (p.a.)

ΔP_e = expected inflation (p.a.).

Figure 7, the equity risk premium, depends on the accuracies of people's expectations. The author draws our attention to the current indication (1992/93), which is supposed to be negative for the holding of equities. However, I would draw your attention to the position of this model during 1987. It was giving no warning that you should get out of equities. In that October was the worst incident in the last 20 years. Any model has to have an indicator that such happenings might occur.

Professionals in the investment market do not use the yield ratio; it is recognised that it does break down, it is biased by changes in inflation, and recently its behaviour has changed—2 is no longer the ceiling.

As far as holding more gilts is concerned, although this argument rages on, I assure you, through simple tautologous arithmetic, that the industry will be holding more gilts in the next few years. The Government has to sell up to £50 billion. There is little chance of foreign interest in gilts, and of share issues through rights in the equity markets coming out at levels to prevent the ratios of gilts to equities rising, so, perhaps, we should all relax to some extent.

The Bank of England assure me that they only issue as much index-linked as they do now to satisfy demand. If demand increased they would issue more. Because of the benefit for matching liabilities, index-linked holders are willing to accept a lower overall rate of return. Therefore, the interest cost of issuing index-linked gilts for the Government is less than conventional gilts.

I give a note of caution on dividend cover and earnings. The level of earnings and the dividend cover statistics over time are completely biased by the historic cost basis of accounting, and the figures need to be adjusted to get a real picture.

Mr N. Ryan (a visitor): I have investigated the remarks made in § 2.3.8, to the effect that you can keep r constant throughout the period, when it, quite clearly, is not constant. Professor Pepper raised this issue. If you rewrite the basic equation you can divide the premium into parts that are dependent on r and those that are dependent on K. It is clear that in times of high values of starting values of r, for instance now when it is 2, that the analysis is approximately right to within a small order error, but that when the values of r are quite low, as in the early part of the period, then this proposition does not follow at all, and that the importance of K declines proportionately. The ratio is about the square root of 5 for peculiar reasons. Analysis of the tables provided by the author shows that, if you split the 70-year period rather crudely into two parts, the difference between the performance during the time when the regime was effectively low values of r, and the high values of r in the last 35 years, the results come out differently.

The yield ratio averaged 0·75, approximately, in the first half period. The arithmetic mean of the second half is just under 2. The standard deviations are about 0·1 in the first half, and over 0·5 in the second half. Comparison of the two variance ratios on the yield ratio between the first and second half gives a proportion of more than 2. The crude estimate suggested approximately the square root of 5, about $2\frac{1}{4}$. Comparison of the inverse yield ratios produces about $2\frac{1}{2}$ as the proportion between the variance ratios in the first and second half periods, which strongly implies that they are different regimes and need to be analysed separately.

Before this paper was published there were good reasons for many pension funds reducing their investment in equities. After the publication of the paper there are good reasons for many pension funds having high proportions in equities. The reasons are not to do with how you measure them, but with what you hope the long term will bring.

Mr G. M. Lindey, F.F.A.: I was interested in the assertion of the author, in § 2.1.1, that he considers the current low rate of inflation "to have the air of permanence about it". I quote from Professor

Wilkie's paper 'Indexing Long-Term Financial Contracts' (*J.I.A.* **108,** 299), in which he proved that "no single time series model obviously describes the progress of prices over a long period". He went on to say, "Rather than seek for better ways of forecasting the future, it is better to accept the uncertainty and plan in the context of an unpredictable inflation rate". I think it is a bold statement to say that low inflation is permanent.

I was taken aback by the author's opening remark that inflation is but a concept. Here is another concept—lunch! Try telling a pensioner, who has been on a fixed income for many years and cannot afford his lunch, that what he is suffering from is an actuarial concept and not hunger, and see where it gets you!

Risk-free proxies vary from investor to investor. As the marginal investor shifts from one category to another—for example, from life companies to final salary pension schemes and then, perhaps, to money purchase pension schemes, it is perfectly conceivable that the risk-free proxy may well change. I think that shifting marginal preference has to be investigated further.

Risk premium is correctly defined in the paper as the expected excess return. It is a forward-looking concept, and to look back at historic out-performance and equate that with what the risk premium was in the past is quite misleading.

I agree with Mr Cox that an appropriate forecasting tool is the dividend discount model, because it seems the best fit for the actuarial profession. The dividend discount model, in whatever form you use it, is actually an estimate of the internal rate of return of any form of investment, and the DDR of the equity market is the one we should be looking at now.

An area for investigation is a triangular one. We have looked at the relationship between the dividend discount rate, or the forecast rate of return of equities, against conventional gilts, and possibly against indexed gilts. We should be looking at equities against conventional fixed-interest; equities against index-linked gilts; and index-linked gilts against conventional fixed-interest. We should see if the 3 legs of that triangle confirm each other. We have been working on this and we have 2 of the legs, but the 3rd leg, the confirmation leg, so far has confounded us. We will keep on trying, and I hope that the profession will do so as well.

Mr P. G. Scott (closing the discussion): The paper highlights the recent experience of the excess returns in the 1980s, and links that excess return to the excess dividend growth that was received over that period. This is a fundamental issue when trying to look forward to the sort of excess returns we may obtain over the decades to come.

A number of speakers have asked the question: why does the yield ratio work? Clearly that question can be asked of any model, whether it is the model in this paper, or the dividend discount model which has been discussed by a number of speakers. It is important to remember that the yield ratio will work well for predicting equity values when gilt yields are stable. Consideration of current inflation and the increasing supply of gilts indicates that gilt yields are not stable at the present time. It is important to remember that when using the model in the paper.

A number of speakers considered ERM membership. Have we gone through a period of change? Is it now over? Now that we are out of the ERM, are we now back to economic conditions that we understand fully? I agree with the view that we need to analyse the issue and to plan for uncertain inflation. However, we should not rule out the prospects of a low inflation environment. After all, we are still in Europe; and the capacity for this country to run completely divergent economic policies, compared with our partners in Europe, is limited if we are going to continue to benefit from open borders. Certainly we should plan within our investment scenarios for the possibility of lower inflation.

It is interesting that many economists at the present time are suggesting that exit from the ERM may add, say, 1% to the long-term rate of inflation, leading to a current range of 4–5%, which is certainly not heady in the light of the experience that we have had.

The author asks the question as to whether current valuation bases for valuing equities overvalue equities or understate liabilities? It is difficult to draw significant conclusions as far as the current valuation yields are concerned. It is interesting that a number of speakers picked up on the question as to whether the issue is actually about the asset allocation to equities rather than the valuation criteria used.

It is interesting to study the period 1982-92. At the end of 1982 the average U.K. final salary pension fund had 57% in equity assets. The data suggest that, at the end of 1992, that 57% had risen to 86% and, in addition to that 86% in equities, there was a further 7% in overseas bonds.

If we look back to 1982, what were we thinking about at that time when advising pension funds? Were we thinking in terms of a much more depressed economic outlook? Were we looking at pension funds which were more mature, which had larger surpluses than we have today? I think, when looking at the analysis of where we have come from, that we have pushed the boat out too far as far as equity investments are concerned.

The great strength of this paper is that it raises the fundamental question once again: what should be the relationship between assets and liabilities for pension funds, and then how should we value them?

The President (Mr L. J. Martin): We have had before us this evening a most topical paper; two-thirds investments and one-third pensions! The author is to be congratulated on having produced this paper so quickly, it having been triggered by the paper by Thornton & Wilson (*J.I.A.* **119,** 229), which was two-thirds pensions and one-third investments!

I have a couple of comments. These relate to the relationships between actuaries and economists as regards pension fund valuation bases. I have heard the discussions over many years between all those concerned in deciding on what the economic parameters should be for a valuation. In the past it has seemed to me that economists have been telling actuaries that we have been too conservative. Are times changing? We shall see.

As pension funds become more mature, we must be particularly careful in changing valuation assumptions. Just to touch the valuation assumptions, certainly as they relate to the valuation of equity assets, makes a large difference to the answer. At the moment, pension funds are of the order of £300 billion. In maturity, if we go on funding as we are—and there is no reason why we should not—that figure, in real terms, may increase to something of the order of £500 billion. If pension funds had all started 20 years earlier than they actually did, funds might now be up to that kind of figure. Touch the valuation bases of mature funds, and the answers will change a lot. This paper helps us to concentrate our minds on what those long-term assumptions might be. Careful as we go!

It has been a most timely paper—and a most interesting discussion. Mr Jones can be well pleased in seeing the number of people that there are here this evening. The hall is completely full, and the discussion could have gone on a long time. Many congratulations and thank you, Peter.

Mr P. D. Jones (replying): The opener and a number of other speakers have raised the point about democracy, about politics and, in particular, about unintended or unexpected inflation. I accept that. There is a corollary. It means that you can drive equities to such a high level—that is, on the insurance principle—that it follows that gilt-edged give you a higher return. If that is what investment managers want—to invest in equities, and, perhaps, actually to underperform the bond market, as they have done for the last $2\frac{1}{2}$ years—then so be it, and that will justify very small, even negative, risk premiums.

Incidentally, the data in Figure 1 and in the Appendices are different. One is the All-Share Index and the other is the BZW data.

Mr Bishop's remarks were valuable in being broad brush and going to the important main point of this discussion. Dividend growth, at the end of the day, is closely related to the rate of growth of nominal GNP. If we line up 99 economists, we will get a huge variation in their forecasts of nominal GNP in the 1990s. There are those forecasting very low inflation, and those forecasting high inflation. He seemed to be saying that the central forecast of nominal GNP growth in the 1990s was about 7%. That, in my view, would probably justify the equity market at around this level; that is that the risk premium would be significantly positive. However, if you start talking about 5%—with real 2% and inflation of 3%, or perhaps the other way round—then I suspect that you do not justify current equity levels; the risk premium would be inadequate—certainly by comparison with historical precedent.

Mr Freethy raised the issue of earnings. I would have looked at earnings; but there just is not a long enough history. We do not have earnings per share on the All-Share Index; we only have it on the Industrial Index or the 500. In the long term, of course, if dividend cover is constant, the analysis should run parallel.

Professor Wilkie and I will have to agree to disagree. If I had a great deal of index-linked data on a market that I regarded as viable—and I regard that word as very important—then I would willingly look at it, but that is for the future. One of the reasons I wrote the paper, as indeed the President alluded to, was that actuaries look at very long historical trends. That is what I have done, and sadly we do not have it for index-linked gilts.

I do not deny the charge made by Mr Cox of my re-inventing the wheel. I have only re-presented the data with which the profession is familiar. It should be standardised, and that is a central theme of Section 3. Otherwise, the importance of the yield ratio in the earlier period cannot be grasped. Several speakers referred to the fact that we appear to have two populations, that is pre-1958 and post-1958, and I think that is right.

Mr Lindey spoke about unexpected inflation. I may have been unnecessarily provocative in making my remark the way I did about inflation. I am aware that pensioners have to eat lunch just as I do, but the fact remains that the data of the gilt market are of greater quality than the inflation data.

The paper has stimulated us all to look again at the relationship between assets and liabilities in a valuation balance sheet. At this time, I am sure that is very important.

WRITTEN CONTRIBUTORS

Mr N. Ryan: 1 In § 2.3.8 the author remarks: "what has been described so far is a simplified version of a more complicated mathematical relationship . . . in our view, no different practical decisions would be made if the more complicated form were used." This note examines that statement, qualifies it in certain circumstances, and comments on the author's conclusion.

2 We take the second formula from page 282 (Appendix 1):

$$\frac{d_0}{d_1} * (1 + k_1)(1 + d_1) = \frac{g_0}{g_1} * (1 + g_1)(1 + p)$$

and use the yield ratio r, $r = g/d$, but without assuming, as the author does, that it remains constant, and rewrite the formula as:

$$p = \frac{r_1}{r_0} * \frac{(1 + k_1)(1 + d_1)}{(1 + g_1)} - 1.$$

Now, r_1, k_1, d_1 and g_1 are taken at time t_1, i.e. they are functions of t, and we will accordingly recast the formula as:

$$p = \frac{r}{r_0} * \frac{(1 + k(t))(1 + d(t))}{(1 + g(t))} - 1.$$

Consider the variation of p, both with respect to r, and with respect to t:

$$\frac{\partial p}{\partial r} = \frac{1}{r_0} * \frac{(1 + k)(1 + d)}{(1 + g)}$$

$$= \frac{1}{r_0} * \Phi(t;k,d,g) \qquad (1)$$

where $\Phi = \Phi(t;k,d,g) = (1 + k(t))(1 + d(t))/(1 + g(t))$ is a function of t, which depends on k, d and g, and encapsulates the non-r effects, i.e. K-effects in the author's terminology.

3 Considering p as two-dimensional, we are led to the general surface element, which for ordinary surfaces in 3-space is:

$$d_\sigma = \left[1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2\right] dx \cdot dy.$$

This expression has the dimensions of a variance; so to make it into a standard deviation, and at the same time translate it into the language of premium surfaces, we write the line element as:

$$d_\lambda = \sqrt{\left[1 + \left(\frac{\partial p}{\partial r}\right)^2 + \left(\frac{\partial p}{\partial t}\right)^2\right]}.\mu$$

in which μ is a vector differential. For our purposes, the important part, which gives the 'size' effects, is the *kernel*, J;

$$J = \sqrt{\left[1 + \left(\frac{\partial p}{\partial r}\right)^2 + \left(\frac{\partial p}{\partial t}\right)^2\right]}. \tag{2}$$

From (1):

$$J = \sqrt{\left[1 + \left\{\frac{\Phi(t)}{r_0}\right\}^2 + \left(\frac{\partial p}{\partial t}\right)^2\right]}.$$

4 Now, $\Phi \approx 1{\cdot}05 - 1{\cdot}10$ (most of the time), i.e., $\Phi = 1 + \varepsilon$, where ε is a small positive number. So when r_0 (the 'r at start' value) is large, say ≈ 2, as it is today, therefore $(\Phi(t)/r_0)^2$ is quite small, and we may write:

$$J_{(2)} = \sqrt{\left[1 + \left(\frac{\partial p}{\partial t}\right)^2\right]} + \varepsilon_1 \tag{3}$$

where ε_1 is another small number, but not necessarily equal to ε, and the subscript (2) indicates the region in which r_0 is to be found, i.e., the 'regime' of r.

When r_0 is small, however, say $\approx \frac{1}{2}$, as it has been in the past, we have:

$$J_{(1/2)} = \sqrt{\left[5 + \left(\frac{\partial p}{\partial t}\right)^2\right]} + \varepsilon_2$$

$$= \sqrt{5} \cdot \sqrt{\left[1 + \left\{\left(\frac{1}{\sqrt{5}}\right) \cdot \left(\frac{\partial p}{\partial t}\right)\right\}^2\right]} + \varepsilon_2 \tag{4}$$

(ε_2 not necessarily $= \varepsilon_1$).

5 Although ε_1 and ε_2 introduce uncertainties, these will mainly be a worry if we were trying to predict *absolute* values; for *relative* comparisons they will tend to cancel out.

Broadly, then, we compare the

$$\text{high-}r \text{ regime, } J \approx \sqrt{\left[1 + \left(\frac{\partial p}{\partial t}\right)^2\right]},$$

with the

$$\text{low-}r \text{ regime, } J \approx \sqrt{5} \cdot \sqrt{\left[1 + \left\{\left(\frac{1}{\sqrt{5}}\right) \cdot \left(\frac{\partial p}{\partial t}\right)\right\}^2\right]}.$$

The qualitative result is that:

(A) in the high-r regime:
  (A1) the variation due to $\partial p/\partial r$ is unimportant, while
  (A2) the variation due to $\partial p/\partial t$ (which involves K) is what counts—this supports the author's thesis;

(B) but in a low-r regime:
  (B1) the total variation is greater because of the $\sqrt{5}$ factor in front, or 'the p-surface is more crinkly', while
  (B2) the variation due to K is *relatively less important*, because of the $(1/\sqrt{5})$ factor multiplying

$\partial p/\partial t$, and the variation due to r correspondingly more important—which opposes the author's thesis, or at least limits it.

6 The broad conclusion is that in a high-r regime the author is right, and K-variation is the thing to watch out for.

In low-r regimes, however, this is by no means the case.

One vital question is, are we at a transition? The above analysis says nothing about $r \approx 1$ regimes, but it is suspected that they are unstable; nor is there anything to show which way an $r \approx 1$ regime will go next.

7 In aid of these thoughts some relatively crude statistics may be adduced. Just by looking at Figures 11 and 12, one gains the impression of the period splitting into two halves, both as to the value of r (the regime), and also as to its volatility. Since the data cover 70 years, we have therefore roughly divided them into 2 halves:

1st half: 1923–57,

and

2nd half: 1958–92.

For the 2 halves we have calculated the means and standard deviations of the yield ratio and the inverse yield ratio, as set out in Table 1.

Table 1

		Yield Ratio	Inverse Yield Ratio
Regime 1 (1923–1957)			
Mean	M_1	0·751627	1·355069
Standard Deviation	S_1	0·100267	0·188461
Ratio	$R_1 = S_1/M_1$	0·133400	0·139078
Regime 2 (1958–1992)			
Mean	M_2	1·958423	0·559503
Standard Deviation	S_2	0·541335	0·183543
Ratio	$R_2 = S_2/M_2$	0·276414	0·328046
Relativity	R_2/R_1	2·072061	2·358708

The 2 numbers for relativity may be compared with the number suggested earlier, $\sqrt{5} = 2{\cdot}23607$. The discrepancy may be accounted for, partly by our having ignored the difference between ε_1 and ε_2, and partly by the fact that in the 1st half, or low-r regime, r was not equal to $\frac{1}{2}$: it averaged about $\frac{3}{4}$, although it should be remembered that r_0 is a starting value, not an average. Overall, these rather rough and ready statistics give us some comfort that the earlier analysis is on the right lines.

8 What are the implications for pension funds? We will not comment on the valuation question, but rather on the implied consequences for policy. In the first place there are many funds which, even before the publication of this paper, ought to have had a lower exposure to equities; and there are many others which ought to have a high exposure even post-this paper, and even if the author's is speculation about a future low-inflationary era is correct.

Any consideration of detailed policy for a specific fund must start from a knowledge of the detailed liabilities, and this is not changed in the light of the paper. But equally, future investment decisions will take place before a background of the current economic landscape—so, if parts of that landscape, such as the p-surface, become more crinkly than in the past, policymakers should take that into account.

This paper has been useful in calling attention to this question, and perhaps pointing our gaze towards the problems of whether the p-landscape is evolving, and if so in which direction.

Mr P. N. Thornton: I note that this paper, in part, was sparked off by the paper by A. F. Wilson and myself (*J.I.A*, **119**, 229), and I welcome the way the paper seeks to cross the bridge between actuaries specialising in investment work and those specialising in pensions. I would like to see more of a bridge between pensions and investment.

However, I am not convinced that an analysis of the relationship between equity returns and fixed-interest returns is necessarily helpful to the assessing of long-term prospects for equities. This is particularly so for funds which have a heavily equity orientated investment strategy or, as is becoming more common, a specific long-term benchmark for the proportions invested in equities and fixed-interest, amongst other asset classes.

The author suggests that the future prospects for equity returns are somewhat lower than has been the case in the recent past and, in particular, that growth prospects are less good. I would not disagree with this, but I would disagree with his conclusion that, as a consequence, pension fund assets are being valued too highly in actuarial valuations or, alternatively, that pension fund liabilities are being valued too conservatively (by which I assume he means that too *low* a value is being placed on them). This might have been the case had actuaries fully reflected the high rates of return on equities experienced in past years in funding assumptions, but I do not believe that this is the case.

I would not dissent from the suggestion that, ideally, the long-term assumptions would be adjusted according to the part of the economic cycle one happens to be in. The only difficulty is judging what part of the cycle one is in at any particular valuation date.

Our own paper suggested that there should be deliberate margins for caution built in relative to 'best estimate' assumptions. I think this leads to a more orderly progression of pension scheme finances than where long-term assumptions are constantly moved around, as they were in the 1970s and 1980s, in response to shifting views of future prospects, which is what the paper seems to be advocating.

The author subsequently wrote: Three significant topics arose during the discussion, which I believe are worthy of further comment.

The first is the apparent view of some members of the profession that equities are the sole asset class suitable for pension funds—not because they are a hedge against an expected rate of inflation—but because they are a hedge against unexpected inflation. The argument, of course, applies only to unexpectedly *high* inflation. This argument can clearly be used to justify the purchase of equities with a zero (or even negative) risk premium over gilt-edged, on the insurance principle. Personally, and speaking as an investment practitioner, I reject this argument, but recognise others' line of thought. The corollary, of course, is that equities would dramatically under-perform gilt-edged if inflation were unexpectedly *low*—as I believe it may be during the 1990s, because of the pervasive influence within the U.K. and world economies of debt deflation. Whilst I agree with the opener that the risk in equities relative to inflation may be lower than that in gilt-edged at times of volatile inflation, I do not believe the argument would hold good when inflation turns out to be low and stable. If this argument that equities should be held as a hedge against unexpected inflation were to hold sway, it could constitute a discontinuity with the past—when, as Section 3.5 states, equities have nearly always achieved superior returns to gilt-edged. Mr Lindey offered some thoughts on this discontinuity, and I agree that it may well reflect the present dominance of the pension funds in equity investment.

Second, Professor Pepper quite correctly queried the assumption in Appendix I, that the yield ratio, r, remains in a 'steady state'. My simple response is to draw an analogy with the use of the gross redemption yield in the gilt-edged market. That calculation assumes that the interest payments are reinvested at the gross redemption yield, i.e. at the 'answer'. Any investor who believes that he will re-invest at yields significantly different from the gross redemption yield is, of course, making a forecast of future yields and should invest or disinvest accordingly. He will not need to make use of a gross redemption yield calculation—though that does not invalidate the technique. I would adduce a similar argument in support of my approach. If the yield ratio is 2—as it is at the time of writing—a belief by an investor that it will shortly become either $1\frac{3}{4}$ or $2\frac{1}{4}$ immediately throws up a valuation anomaly between the two markets. It does not, in my view, invalidate the technique described in the paper.

Fortunately, Mr Ryan, in a contribution which breaks new ground in my experience, lends tacit support—with the yield ratio at its current level. If I understand him correctly, he believes that there are two 'semi-stable' regimes, one where the yield ratio is significantly greater than 1 and one where it is significantly less than 1. This accords, not only with the outcome of the last 70 years (as Mr Ryan observes), but with my own 'gut feeling' as an investor. Moreover, by manipulation of the algebra in Appendix I, putting $r_0 = 1$, it can be seen that:

$$P = \frac{r(1+k)(1+d)}{(1+g)} - 1$$

$$\simeq r(1+k) - 1.$$

Just by observation, it can be seen that, if k is small, a likely precondition of r being approximately equal to one, then p is approximately equal to $(r-1)$, i.e. very small itself. Surely an unstable environment, especially for equities?

Mr Ryan and Professor Wilkie both brought to the discussion greater mathematical and statistical skills than I possess. It demonstrates one of the profession's seemingly insoluble problems of combining advanced mathematical skills with practical insights (in this case, investment skills). However, more work must clearly be done in this area.

Lastly, I do not understand the remark of Mr Wilson, Mr Parsons and others that the valuation yield of a pension fund is not the gilt yield. I accept that it will not be the actual gilt yield at the valuation date, but would expect it to bear a relationship to the actuary's expectation of the yield at which fixed-interest investments can be made over the life of the fund. If it is not, then I believe it should be. Mr Hager, at least in presentational terms, seemed to share this view. Not to relate valuation assumptions (in aggregate) to a plausible set of economic assumptions for the future seems to be, at the least, discourteous to the client, perhaps even arrogant and at worst dangerous. Neither is a luxury that the profession should indulge in.