EXPERIENCE RATING

Report by the Study Group

1. Introduction

1.1 The group nominated to investigate this subject was,

Booth, G.
Giles, P.
Jessett, R.P.
Karsten, H.P.J.
Rowlandson, W.F.J.
Sanders, M.R.
Trayhorn, M.E.
Co-ordinator, Coe, L.D.

Mr. Giles found it difficult to attend meetings in London, Mr. Trayhorn found himself involved in other non-life activities for the Institute but we welcomed Mr. Cumberworth as his nominee. Mr. Rowlandson retired from his company and thereafter was unable to attend our meetings. We would like to record the help we have received from those who could not stay with us but of course they have no responsibility for this report. It should be recorded that this report is mainly the work of Messrs. Booth, Jessett and Karsten with assistance from Mr. Coe.

1.2 Our first meeting revealed that few of us had much practical exposure to experience rating and it was therefore essential to limit the scope of our studies if anything at all (worthwhile or otherwise) was to be presented in time for September 1977. We therefore decided to ignore no-claims discount in motor insurance for two reasons,

(a) It is well-known and a considerable amount of literature is available already, and

(b) It is more an example of "merit-rating" rather than of experience rating.

1.3 Having eliminated merit-rating we had to consider what we were left with and how to tackle the subject.

By merit-rating we mean the case where the experience of a single risk is allowed to modify the premium. We concern ourselves only with a group of risks even though they may be insured under a single policy.

It was decided to take as a guide line, not necessarily a definition that experience rating means,

" The modification of a rate-book(we prefer to call it a manual) premium to give some recognition to the observed experience being significantly different from the expected."

This guide line still leaves a further sub-division, as to whether the credit or surcharge for experience is to be given at the time of quoting the premium or as a rebate or extra premium after the risk period has expired and the experience corresponding to the risk period has become known. The latter known in the United States as retrospective rating,or more simply retro-rating we refer to only briefly and leave for further development to another occasion.

Thus the study group finally decided to look solely at the problem of modifying the premium to be charged by way of an increase or decrease to the manual premium at the time of quotation.

1.4 The present practice.

Our enquiries suggest that in the U.K. direct market there is at present little or no attempt at a systematic and therefore consistent approach to experience rating.Underwriters certainly examine recent history particularly of loss ratios, but they will argue, and with considerable justification that statistics are not a sufficient guide to assess a premium modification and other considerations must be reviewed. It is our view that in many cases e.g. motor fleets, employer's liability and general liability, this is undoubtedly true. A change of managerial control will have an effect on the experience of a motor fleet. If a new fleet manager is less scrupulous in controlling routes,hours worked and maintenance schedules the ~~the~~ experience must be expected to worsen Similarly a change in the compositon of a fleet, such as a change to a cheaper vehicle for sales representatives, might affect the experience as would also an extension of private use of a company vehicle.

We are agreed that experience rating is never a statistical be all and end all process but what we believe ~~is~~ that a statistical basis can help the underwriter by telling him where his starting point lies, after which he exercises his judgement based on his personal knowledge of the particular risk.

1.5 What is "significant"?

In our guideline we used the words "significantly different from" and we must determine how we can measure significance in this context. We decided to base our researches on the application of credibility theory to the problem and to concentrate in the first place on motor fleet rating. Thus we first set out the theoretical background and later examine the practical problems.

2. Credibility Theory

2.1 Credibility theory is concerned with premium rating for ~~ax~~ risk class that lies within a risk group. Typically there are some ~~xtx~~ data for the risk class, some data for other risk classes within the risk gropp and some data for the risk group as a whole. The problem which credibility theory tackles is the assessment of the best premium rate for each ~~x~~ risk class.

An example of the problems that credibility theory tackles is that of finding the correct premium rate for workmen s compensation insurance for a single contract. Another example concerns fleet rating and is theproblem of finding the correct premium for motor fleet insurance for a particular company.

2.2 Following the notation of Whitney (1918) let x denote the unknown parameter for the risk class and let X denote the unknown paramter for the rest of the risk group. Let p denote the observed ~~xx xxxxi~~ variable for the risk class and let P denote the observed variable for the rest of the risk group. In one example x,X,p,P may all be claim frequencies, in another example x,X,p,P may all be total claim costs per policy.

Suppose $\left.\begin{array}{l} p|x \sim N(x, \sigma_1^2) \\ P|X \sim N(X, \sigma_2^2) \end{array}\right\}$ standard distributions, normal and centred on the underlying parameters

$\left.\begin{array}{l} x|X \sim N(X, \sigma_3^2) \\ X \text{ uniform in } [0, L] \end{array}\right\}$ prior probability distributions

Let f denote a ~~most~~ density function.

$$f(x \mid P_p) = \frac{f(x, P, p)}{f(P, p)}$$

The maximum of the posterior distribution (ie of $f(x \mid P, p)$) will be found where $0 = \frac{d}{dx} f(x, P, p)$

$$\begin{aligned} f(x, P, p) &= \int f(P, p \mid X, x)\, f(x, X)\, dX \\ &= \int f(P, p \mid X, x)\, f(x \mid X)\, f(X)\, dX \\ &= \int f(P \mid X)\, f(p \mid x)\, f(x \mid X)\, f(X)\, dX \\ &\propto \int \exp\left\{ -\frac{1}{2\sigma_1^2}(p-x)^2 - \frac{1}{2\sigma_2^2}(P-X)^2 - \frac{1}{2\sigma_3^2}(x-X)^2 \right\} f(X)\, dX \end{aligned}$$

$$\propto \int \exp\left\{ -X^2\left(\frac{1}{2\sigma_2^2} + \frac{1}{2\sigma_3^2}\right) + 2X\left(\frac{P}{2\sigma_2^2} + \frac{x}{2\sigma_3^2}\right) \right\} f(X)\, g(x, p, P)\, dX$$

If $0 = \frac{d}{dx} f(x, P, p)$ then

$$0 = \int \left[\frac{(p-x)}{\sigma_1^2} + \frac{(x-X)}{\sigma_3^2}\right] \exp\left\{ -\left(\frac{1}{2\sigma_2^2} + \frac{1}{2\sigma_3^2}\right)\left(X^2 - 2X\,\frac{\left(\frac{P}{\sigma_2^2} + \frac{x}{\sigma_3^2}\right)}{\left(\frac{1}{\sigma_2^2} + \frac{1}{\sigma_3^2}\right)}\right)\right\} f(X)\, dX$$

The second part of the integrand is identifiable as proportional to ~~the~~ the normal density for $X \sim N\left(\frac{\left(\frac{P}{\sigma_2^2} + \frac{x}{\sigma_3^2}\right)}{\left(\frac{1}{\sigma_2^2} + \frac{1}{\sigma_3^2}\right)}, \frac{1}{\left(\frac{1}{\sigma_2^2} + \frac{1}{\sigma_3^2}\right)} \right)$

Thus $$x\left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_3^2}\right) = \frac{p}{\sigma_1^2} + \frac{1}{\sigma_3^2}\left(\frac{\left(\frac{P}{\sigma_2^2} + \frac{x}{\sigma_3^2}\right)}{\left(\frac{1}{\sigma_2^2} + \frac{1}{\sigma_3^2}\right)} \right)$$

$$x = \frac{P\sigma_1^2 + p(\sigma_2^2 + \sigma_3^2)}{\sigma_1^2 + \sigma_2^2 + \sigma_3^2}$$

This x above is then the best estimate of the underlying risk parameter for the risk class and takes account of data [illegible] (P)

If we define $$z = \frac{(\sigma_2^2 + \sigma_3^2)}{\sigma_1^2 + \sigma_2^2 + \sigma_3^2}$$

then $x = pz + P(1-z)$. z is termed the credibility factor and is the credibility attached to the experience of the risk class.

It can be seen that the above formula for z is intuitively plausible, for example

a) as $\sigma_1 \longrightarrow 0$ $\quad z \longrightarrow 1$

b) as $\sigma_1 \longrightarrow \infty$ $\quad z \longrightarrow 0$

c) as $\sigma_2 \longrightarrow 0$ $\quad z \longrightarrow \sigma_3^2/(\sigma_1^2 + \sigma_3^2) = 1/(1 + \sigma_1^2/\sigma_3^2)$

d) as $\sigma_2^2 + \sigma_3^2 \longrightarrow 0$ $\quad z \longrightarrow 0$

e) as $\sigma_2^2 + \sigma_3^2 \longrightarrow \infty$ $\quad z \longrightarrow 1$

Item (c) above is quite common. The practical problem has then been simplified down to choosing σ_1^2/σ_3^2.

2.3 Before coming to some examples of how this works out in practice there are some points of understanding to be raised. The first point is that the procedure above assumed normal distributions and obtained a maximum likelihood estimate for the unknown parameter. These assumptions ~~s~~ and choosing ~~thexxxxxx~~ the maximum likelihood estimate make the algebra simple, but they are not necessary. ~~XXXXXXXXXXXXXXXXXXXX~~ Distributions with finite variance will cause no problem and one may seek the expectation rather than ~~txa~~ the maximum likelihood value of the unknown parameter. For example Jewell (1974) has obtained ~~using~~ useful results for the exponential family of distributions. Bailey (1950) has obtained useful results for the Poisson distribution where the underlying prior probability distribution is a gamma distribution.

2.4 The second poiht concerns the necessity for a prior probability distribution. Some authors have purported to do without it, for example Hans Wenger (1973). Essentially their procedure is to obtain estimates of σ_1^2 and σ_3^2 from the data they have. However the use of prior probability distributions is logically more satisfactory.

2.5 The third point concerns methods of experience rating that are not based on statistical models. It is the practice in the U.S.A. to use credibility factors for workmen's compensation insurance. The factors used is normally 1 if the experience is above a certain size, often of the order of 1000 claims. Below this size it is common to use formulae of the form $z = P_{class}/(P_{class} + K)$ where P_{class} is the premium income for the class and K is a constant. The justification for this formula has been attempted by Perryman (1937) but rests mainly on its convenience.

Examples: Suppose n,N represent the number of car years in the experience of the class and the group respectively. Similarly let x,X be the underlying claim frequencies and let p,P be the observed claim frequencies. Suppose P=.14. The problem is to obtain the best estimate of x (for premium rating) from P and p.

This problem may be tackled as follows. Using the same notation as before we may say that

σ_1^2 is estimated by $P(1-P)/n$

σ_2^2 is estimated by $P(1-P)/N$

σ_3^2 is, say, $(\cdot 02)^2$ (variance of prior distribution)

$$\text{Then} \quad z = \frac{(\cdot 02)^2 + \frac{P(1-P)}{N}}{(\cdot 02)^2 + \frac{P(1-P)}{N} + \frac{P(1-P)}{n}}$$

$$\text{As } N \to \infty \qquad z \to \frac{n}{n+300}$$

n	z
33	·1
75	·2

$$x = zp + (1-z)P$$

It can be seen that in this example a credibility type experience rating formula may be useful with fleets of the order of 30.

Sticking to motor fleet insurance a further example may be as follows. Xuppose x,X,p,P represent total claims per policy, n, N represent numbers per policy. Suppose we take P=£40, and estimate

σ_1^2 by £^{2}2000/n

σ_2^2 by £^{2}2000/N

σ_3^2 by £^{2}50

$$\text{Then} \quad z = \frac{\frac{2000}{N} + 50}{\frac{2000}{N} + 50 + \frac{2000}{n}}$$

$$\text{As } N \to \infty \qquad Z \to \frac{n}{n+40}$$

So

n	z
20	·3
40	·5
60	·6

$$x = zp + (1-z)P$$

Bibliography for Section 2

Whitney : The theory of experience rating PCAS 10 1918

Jewell (1974) : Credible means are exact Bayesian for simple exponential families ASTIN 8

Bailey (1950) : Credibility Procedures PCAS 37

Wenger (1973) : A rating method for fire insurance MVSV 73, 1

Perryman (1937) : Experience rating plan credibilities PCAS 24

3 Application of Credibility to Motor Fleet Rating

3.1 It seems preferable that some form of split-plan basis should be used in order to deal with the problem of large claims. This approach has the effect of spreading all claims above a chosen excess point over all insureds so that appropriate allowance is made for the occasional bad year.

The general formula for the premium modification on a split-plan basis is:

$$M = \frac{1}{E}\left[z_p A_p + (1 - z_p)E_p \quad + \quad z_\varepsilon A_\varepsilon + (1 - z_\varepsilon)E_\varepsilon\right]$$

where the subscripts p and ε refer to the primary and excess ranges; z, A and E refer respectively to credibility, total amount of claims incurred, and expected total amount of claims.

Suppose that the primary range accounts for a proportion k of the total amount of claims incurred over the risk group as a whole. We can assume that the expected cost of claims falling in this range is met by kE and exclude the excess range from any experience rating, i.e. assign zero credibility, $z_\varepsilon = 0$

The modification then becomes

$$M = \frac{1}{E}\left(z_p A_p + E_p + E_\varepsilon - z_p E_p\right)$$

and if we put the office premium, $P' = \frac{E}{1 - L}$, where L = loading for expenses and commission, and since $E_p + E_\varepsilon = E$ we get,

$$M = 1 - z_p\left[k - \frac{A_p}{(1 - L)P'}\right]$$

giving a credit of

$$z_p\left[k - \frac{(LR)_p}{(1 - L)}\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (1)$$

where $(LR)_p$ = loss ratio over primary range.

For given k and L we can examine how the credit varies for different loss ratios according to the associated credibility.

3.2 Example: Suppose k = .80 and L = .25

$$\text{Credit} = z_p\left[.8 - \frac{(LR)_p}{.75}\right] \quad \text{from (1)}$$

$$\text{where } z_p = \frac{E}{E_p + K} = \frac{E}{.8E + K} \qquad \ldots\ldots\ldots\ldots \qquad (2)$$

if we assign a maximum credibility of 1.0 we obtain the following table of credits for varying primary loss ratios:

Primary Loss Ratio (%)	Credit (%)
10	67
20	53
30	40
50	13
60 & over	nil.

N.B. For ratios in excess of 60% the method produces a negative credit. In practice it is thought that in these circumstances a loading would be more appropriately an underwriter's judgment.

Clearly the various credits may be proportionately reduced by decreasing the maximum credibility.

The size of the risk class to which maximum credibility is to be assigned needs to be determined,e.g.1,000 car years or 2,000 car years gives, from (2) above, values of K of 200 or 400 in the example where the primary range represents 80 per cent of the total. These values of K result in the following tables of credits as a percentage of premiums.

Table 1
(K = 200)

No. of car years	10	50	100	500	1,000
Credibility	.048	.208	.357	.833	1.0
Loss Ratio (%)					
10	3	14	24	56	67
20	3	11	19	44	53
30	2	8	14	33	40
50	1	3	5	11	13
60 & over	nil	nil	nil	nil	nil

Table 2
(K = 400)

No. of car years	10	50	100	500	1,000	2,000
Credibility	.025	.114	.208	.625	.833	1.0
Loss Ratio (%)						
10	2	8	14	42	56	67
20	1	6	11	33	44	53
30	1	5	8	25	33	40
50	nil	2	3	8	11	13
60 & over	nil	nil	nil	nil	nil	nil

3.3 The above approach has ignored the whole of any claims which exceed a given amount e.g. £5,000. An alternative would be to allot each such claim that value. The credits brought out by this method are identical to those which arise from fixing a higher cut-off point since the result is to alter the value of E_P.

As an example the following tables may be compared with Tables 1 and 2, using the same premium loading.

Table 3
($K = 100$)

No. of car years Credibility	10 .092	50 .345	100 .526	500 .909	1,000 1.0
Loss Ratio (%)					
10	7	26	40	70	76
20	6	22	33	58	63
30	4	17	26	45	50
50	2	8	12	21	23
60	1	3	5	9	10
67.5 & over	nil	nil	nil	nil	nil

Table 4
($K = 200$)

No. of car years Credibility	10 .048	50 .204	100 .345	500 .769	1,000 .909	2,000 1.0
Loss Ratio (%)						
10	4	16	26	59	70	77
20	3	13	22	49	58	63
30	2	10	17	38	45	50
50	1	5	8	18	21	23
60	nil	2	3	8	9	10
67.5 & over	nil	nil	nil	nil	nil	nil

As can be seen the effect is to spread the credits more evenly between fleets of different size and to give greater weight to good experience. For a given primary loss ratio a greater primary range leads to higher credits and the effect of increasing the premium loading is to decrease the credits, as would be expected. All the values in the above tables may be decreased proportionately by assigning a maximum credibility of less than unity.

4. Application to Liability Insurance

Before extending the approach used for motor fleet rating to liability rating it is necessary to compare the two claim distributions.

The results of one company in 1974 (as developed to 1976) were as follows,

Motor (including private and fleet)

x (£'000)	F(x)		
1.0	0.0218	Proportion below £1,000	0.9782
5.0	0.0027	Mean claim	145
10.0	0.0009	S.D. of amount of 1 claim	650
15.0	0.0004	Coefficient of variation (C.V.)	4.4
20.0	0.0002	Skewness	26.7

Employers Liability

1.0	0.237	Proportion below £1,000	0.763
5.0	0.039	Mean claim	1035
10.0	0.014	S.D. of amount of 1 claim	2438
15.0	0.006	Coefficient of variation (C.V.)	2.4
20.0	0.004	Skewness	7.4

Because of the high proportion of very small motor claims (97.8%) the motor experience shows a higher C.V. and skewness but these measures tend to disguise the differing shapes of the two distributions. For example, in motor the claims over £5,000 represented 18% of the total claim outgo, whereas in E.L. over £5,000 represents 38% of total claims.

Either a much higher excess point must be chosen for E.L. to use the simplified system applied to motor or a split plan approach would seem more appropriate. We were fortified in coming to this conclusion by noting that in the United States the National Council on Compensation Insurance use a split plan basis for workers' compensation insurance rating. At the time of writing this report the study group have not been able to formalise an approach to experience rating as applied to liability business as written in the U.K. Hopefully further information will be available in September at the seminar.

5. Retro-rating

Retro-rating consists of a method of giving affect to the experience after the period of insurance has expired and the actual experience is "known". In the simplest terms it can be said that the insured pays the burning cost subject to a maximum and minimum and pays certain costs of the insurer.

The retro-premium can be written as follows,
(Basic expense factor)(Manual preium)
+ (Excess loss factor)(Manual premium)(Claims cost factor)
+ (Actual losses incurred)(Claims cost factor).
The first line represents the charge for overhead expenses and cost of writing the business.
The second line represents the part of the manual premium for the claims in excess of the maximum loaded for claims handling charges.
The third line represents the cost of handling the claims which fall within the "burning cost" range.
Thus in total the premium covers overhead expenses, costs of handling all claims and the risk cost if the losses exceed the maximum.

In practice the maximum and minimum are defined as ratios to the manual premium, i.e. loss ratios. As an example a policy could provide that the insured will bear all losses up to a ratio of 80% but if losses are lessthan 40% he pays 40% and of course in addition he pays a premium.

From the insurer's point of view he receives his costs and profit loading and an "insurance charge". The insurance charge is the risk cost of losses above the maximum ratio less the relief for losses below the minimum.
Credibility factors can be introduced either into the losses and expenses or into the losses only.
Algebraically the premium can be written,
$R = V_r + D_r + (1+J)E(1-z) + (1+J)Iz + (1+J)zL.$
Where,

V_r is the provision for expenses other than claim expenses in the premium R.
D_r is the provision for profit and contingencies in the premium R.
J is the loading for claims handling costs.
E is the expected losses.
I is the insurance charge.
L is the actual losses.
z is a credibilty factor.

If the method became a common practice the office would prepare tables which would enable the calculation to be made quite simply from the chosen maximum and minimum loss ratios.

For furthe details the reader is referred to a paper by Carlson in P.C.A.S. Vol. 28 1941.

6. Summary.

In the U.K. market today the traditional basis of underwriting still operates. In most lines of business a tariff rating system no longer applies so that insurers are able to apply individual ratings in a highly competitive market. Also in recent years most insurers have improved their statistical input and have available the means to use methods having a more reliable statistical base. It seems certain that in the future more use will be made of statistical methods and it is important therefore that these methods should be theoretically sound. In other countries, notably the United States methods have been developed, and what is more important have been proved for use in our business. Nowhere is this so clearly demonstrable as in the use of credibility in rate-making and particularly in experience rating. The U.K. at present lags behind and hopefully we can influence future developments.