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EXPERIMENTS IN MORTALITY GRADUATION AND PROJECTION USING A MODIFICATION OF THIELE'S FORMULA

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IN a paper discussed in 1955 (*J.I.A.* 81, 105) the author suggested that certain modifications of Thiele's formula might be suitable for fitting modern mortality. The simplest of these was

$$m_y = B_1 c_1^x + B_2 c_2^{-x^2}, \qquad (1)$$

which might be fitted to male mortality above about age 35. The second modification, which was an attempt to link childhood mortality with adult mortality, was

$$m_y = B_1 c_1^x + B_2 c_2^{-x^2} + A/(y + 1\tfrac{1}{4}), \qquad (2)$$

where y is measured from birth and x from an origin the position of which needs to be determined. These are the formulae which will be used in this paper, and they incorporate respectively the first two and the first three of the components of mortality described in the 1955 paper. A recapitulation of the descriptions of the six components follows, and to avoid giving them any controversial labels it is proposed throughout to refer to them by ordinal numbers.

The first component is represented by a geometric or 'Gompertz' curve.

The second component is in the shape of a normal curve, and it may be recalled that in the reply to the discussion on the earlier paper it was suggested that this curve might be the product of (*a*) a steep geometric curve representing the rate of mortality due to certain (unspecified) 'early killers', and (*b*) the proportion of the population, age by age, predisposed to these killers, which might easily be a bell-shaped curve; this proportion could quite well rise to a maximum at a certain age, after which the new 'predispositions' would be more than outweighed by the deaths, causing the proportion to fall thereafter. And the product of an ascending geometric curve and a normal curve is another normal curve having its maximum at a later point than the first.

The third component appears to be in the shape of a harmonic curve, decreasing throughout life.

The fourth component runs off after the first few years of life.

The fifth component affects the adolescent and younger adult ages and is the cause of the bulge (followed by a trough) which sometimes occurs in the twenties of life; it appears to be of skew cocked-hat shape for male mortality, and is caused largely by accidental or unnatural deaths. Although an attempt was made in the earlier paper to represent this component algebraically so far as concerned the data then being fitted, no particular reasoning could be applied to describe the constants in this term of the mortality formula, and there is no justification for assuming that the same algebraic representation would necessarily be appropriate if other sets of data were examined.

The sixth component is made up of anything caused by temporary disturbances, such as might occur during a war and, at certain ages, for some time after the conclusion of hostilities.

No attempt is made in the present paper to fit any of the last three components algebraically, but it is necessary not to forget them completely.

2. It was suggested briefly in the 1955 paper that the formulae used might have possibilities for the purposes of projection, and certain experiments will now be described in which these possibilities have been examined. The stages of the experiments are:

I. Regraduation of the national male data of England and Wales for the periods 1910–12, 1920–22 and 1930–32 by formulae (1) and (2), excluding those ages affected by the fourth, fifth and sixth components.

Modification of the constants so found, in order to obtain series which are capable of projection, at least for a short period.

Projection to 1951; and comparisons with the curve fitted to the 1951 data in the earlier paper, with the actual deaths in 1951, and with the actual deaths in 1950–52.

II. Graduation of the C.M.I. assured lives' data for the periods 1924–28, 1929–33 and 1934–38 by formula (1), modification of the constants as in I, projection to 1949–52, and comparisons with the 1949–52 experience and with the A 1949–52 Table.

III. Comparisons between national male mortality and assured lives' ultimate mortality; and, if these seem successful, extension of the comparisons to the assured lives' select mortality.

IV. Projections to 1961.

I. EXPERIMENTS WITH NATIONAL MALE DATA OF ENGLAND AND WALES

3. The fitting of the formula was obtained by trial and error, using the data in the same quinary groups as had been used in the construction of E.L.T. Nos. 8–10. Beard has shown (*J.I.A.* 77, 383) that grouping deviations in fives should produce grouped values having a standard deviation $\sqrt{5}$ times the standard deviation of the individual values; i.e. the variance of the grouped deviation is five times that of the individual deviations. If, then, we use King's formula to find five times the Exposed to Risk and Deaths at the central age of each group, we are justified in dividing these by five, regarding the graduation as applying to every fifth age, and testing in the usual way by comparing the deviations with $\sqrt{npq}$ where n is, of course, one-fifth of five times the Exposed to Risk.

Table 1 shows the results of fitting formula (1) to parts of the 1910–12, 1920–22 and 1930–32 data.

4. Logically, the next step should have been to modify these curves in order to incorporate the third term of formula (2). By extending the first two terms back to the childhood ages it appeared that formula (2) could be fitted to the data for 1920–22 and 1930–32 with values of A of approximately ·014 and ·012 respectively, provided it could be accepted that the fourth component continued into the seventh or eighth year of life; but when the 1910–12 data were examined, the exceptionally low values of m_{12} and m_{13} made any such

Table 1. Fitting of formula (1) to national male data for England and Wales for the periods 1910–12, 1920–22 and 1930–32

Age x	Crude m_x	Graduated m_x	Actual deaths	Expected deaths	Deviations (actual minus expected) +	Deviations (actual minus expected) −	Σ deviations +	Σ deviations −	χ^2
				1910–12					
46	·01165	·011549	6,623	6,566	57		57		·50
51	·01598	·016143	7,720	7,796		76		19	·75
56	·02298	·022936	8,817	8,801	16			3	·03
61	·03328	·033222	10,104	10,086	18		15		·03
66	·04835	·049007	11,111	11,263		152		137	2·16
71	·07268	·073185	11,735	11,817		82		219	·61
76	·11142	·109546	9,835	9,670	165			54	3·16
81	·16620	·162461	6,432	6,288	144		90		3·94
86	·23708	·236177	3,321	3,309	12		102		·06
91	·33543	·333882	973	969	4		106		·02
96	·38320	·456942	143	170		27	79		8·12
101	·61228	·604947	12	12	0		79		·01
Totals			76,826	76,747	+416	−337			19·39
				1920–22					
49	·01099	·010824	7,273	7,327		54		54	·40
54	·01647	·016156	8,758	8,594	164		110		3·18
59	·02374	·024074	10,189	10,333		144		34	2·06
64	·03731	·036739	11,937	11,752	185		151		3·02
69	·05637	·057167	12,853	13,035		182		31	2·70
74	·09008	·089472	12,309	12,226	83		52		·62
79	·13849	·138278	9,486	9,471	15		67		·03
84	·20765	·207472	5,202	5,198	4		71		·00
89	·28872	·298900	1,759	1,821		62	9		3·01
94	·40125	·412317	371	382		11		2	·54
Totals			80,137	80,139	+451	−453			15·56
				1930–32					
52	·01303	·012850	8,745	8,625	120		120		1·69
57	·01908	·019287	11,339	11,464		125		5	1·39
62	·02917	·029471	13,612	13,752		140		145	1·47
67	·04674	·046331	16,212	16,068	144			1	1·35
72	·07518	·074688	16,930	16,819	111		110		·79
77	·12005	·120289	14,558	14,587		29	81		·07
82	·18491	·186798	9,004	9,096		92		11	1·14
87	·27376	·273806	3,899	3,900		1		12	·00
92	·38660	·381206	946	933	13		1		·29
97	·49276	·519482	124	131		7		6	·78
Totals			95,369	95,375	+388	−394			8·97

Constants:

	B_1	c_1^5	B_2	c_2^{25}	Origin
1910–12	·82599	1·3596	·38095	1·0363	116
1920–22	·51354	1·4203	·19677	1·0568	104
1930–32	·30271	1·4850	·07850	1·1220	92

fit very difficult. It was accordingly decided to proceed to the next stage, of modifying the constants in order to facilitate the task of projection, possibly incorporating the third term at the same stage.

5. An examination of the constants printed at the foot of Table 1 shows that the direction of movement of each constant between 1911 and 1921 was repeated between 1921 and 1931. The progressions give no indication of asymptotic approach to a hypothetical curve of minimum mortality, but this should not matter for short-term projection. The origin seems to have progressed regularly down the ages, though it can scarcely be expected that it can continue for long to retire by as much as 12 years of age in every decade of time. Similarly, it seems unlikely that the constant c_1^5 will advance by about ·06 every decade, although this is more possible if we regard c_1 not as measuring the rate of increase of the first component with advancing age, but rather as measuring the rate of *reduction* of that component from its values at the oldest ages. This leads to consideration of the constant B_1. This is merely the value of the first component at whatever age is chosen as the origin; but the choice of the origin depends upon the critical value of the second component, and is quite independent of the first. This suggests that rather than extrapolating on the actual values found for B_1, which happen to be the values of the first component at different ages for each of the three sets of data, it might be better to extrapolate on the value of $B_1 c_1^x$ at some other age. If we regard c_1 as a rate of reduction it would seem that this age should be a fairly advanced one; but examination of the curves showed that there was a tendency, over the period being examined, for this component to become larger, from decade to decade, at the oldest ages, and the best hope seemed to be to extrapolate on the values somewhere in the sixties or seventies of age, where the first component varied little with time. In general, the values of $B_1 c_1^x$ seemed to be rather low in the curve fitted to the 1920–22 data, as compared with 1910–12 and 1930–32, and it was decided to see whether that curve could be adjusted to give a better progression.

6. Some further trials showed that it was possible to modify the 1920–22 curve by dropping the origin to 99 and raising the values of both c_1 and c_2. The inclusion of the A curve could usually be compensated for at the older ages by small increases in c_1. The adjusted constants were as follows:

	B_1	c_1^5	$B_1 c_1^x$ at age 68	B_2	c_2^{25}	$B_2 c_2^{-x^2}$ at age 68	Origin	A
1910–12	·829027	1·3611	·04298	·378966	1·0362	·01143	116	·0062
1920–22	·439607	1·4500	·04391	·116145	1·0715	·00816	99	·0145
1930–32	·312503	1·4959	·04522	·067143	1·1105	·00601	92	·0120

7. The next stage was to attempt a projection. It was first necessary to choose the projected origin for 1951. The above origins suggest that the tendency for the origin to become earlier in life was slowing down, and on those figures alone an origin at about age 83 might have been selected for 1951. On the other hand, the origin for 1920–22 had been adjusted, and the original values given at the foot of Table 1 suggested a reduction to 68. Had the same formula been used in the nineteen-thirties for projection, it is most unlikely that anyone would have been bold enough to forecast a retarding of 24 years of age in the origin in the space of the forthcoming 20 years, but at

this stage the author must confess to a mild degree of cheating. Remembering that the 1951 fit in the earlier paper had an origin in the sixties, it was decided to take the extrapolated value of 68 for the origin; and for the other constants, to extrapolate boldly on the basis of the 1910–12 and 1930–32 experiences, using the 1920–22 experience merely as a guide when further information was necessary.

Taking, first, the value of c_1^5, the first fits showed an increase of ·1254 from 1910–12 to 1930–32, and the second an increase of ·1348. The boldest projection would, it is thought, give a value of about 1·63.

The projected value of $B_1c_1^x$ at age 68 would seem, from the figures in §6, to be around ·047.

A consideration of the earlier values of B_2 itself, and its rate of decrease from decade to decade, suggests a projected value of ·02 or a little less; on the other hand, a consideration of the values of $B_2c_2^{-x^2}$ at age 68 in 1910–12 and 1930–32 suggests a projected value of about ·003—and 68 is the projected origin. A graphical representation of the three earlier values of $B_2c_2^{-x^2}$ at age 68 suggested an alternative value of ·0045, which was the value decided upon for the projection. The values of c_2^{25} given in §6 are nearly in a straight line, with a projected value of about 1·185.

Finally we require a projected value of A. Our knowledge of the secular improvements in child mortality precludes the assumption that this constant is increasing. In fact, it increased from 1910–12 to 1920–22 but decreased slightly from 1920–22 to 1930–32. It is difficult to tell to what extent these last two experiences were abnormal, but it is thought that the boldest forecast would return this constant to its 1910–12 value of ·0062.

8. Before proceeding to a projection of the fourth and fifth components (possibly graphically), it was decided to compare the above projected values with the 1951 graduation made in the earlier paper. The origin of 67 in the graduation compares with the projected origin of 68, and the value of $B_1c_1^x$ at age 68 was ·0462 compared with a projected value of ·047. However, the value of c_1^5, 1·656, was considerably higher than the projected value, indicating that the projected rates of mortality would be too low at the older ages and too high at the younger ages.

The value of the second component at age 68 is considerably higher than the projected value, while the rate at which this component drops from its maximum (as measured by the constant c_2) is much slacker than forecast. This would accentuate the tendency for the projected rates of mortality to be too low at the ages where the second component is important.

The value of A is lower than the projected value, and this would accentuate the tendency for the projected rates of mortality at the younger ages to be too high.

In the circumstances it seemed to be a waste of time to complete the projection by extrapolation of the fourth and fifth components, knowing that the projected rates at the ages where these have the greatest effect are already much too high. It was therefore decided to leave this possibility until a later stage in the paper, when an attempt will be made to project to 1961.

9. Table 2 (*a*) shows the central rates of mortality at quinary ages from 38 to 93 inclusive, given by the projected values of the constants and by the graduation of the 1951 data. It also shows, as percentages, the ratios which the projected rates bear to the graduated rates, confirming that the projected

rates were on the low side from age 53 onwards, and too high for ages 43 and under. Nevertheless, the values projected for ages 43 to 93 inclusive are, it is submitted, not too bad for a 20-year extrapolation.

Table 2 (*a*). Comparison of projected values of m_x at certain ages with values obtained from the graduation of the national male data for England and Wales for 1951

Age x	Values of m_x according to projection	Values of m_x according to 1951 graduation	Projected m_x expressed as percentage of graduated value (%)
38	·002674	·002474	108·1
43	·004290	·004090	104·9
48	·007082	·007087	99·9
53	·011944	·012334	96·8
58	·020077	·020796	96·5
63	·032727	·033478	97·8
68	·051590	·052246	98·7
73	·080491	·081528	98·7
78	·127234	·12991	97·9
83	·204596	·21143	96·8
88	·332146	·34815	95·4
93	·540930	·57575	94·0

Table 2 (*b*). Comparison, in five-year age groups, of actual deaths in England and Wales (males) in 1951 and 1950–52 with expected deaths according to projected rates of mortality

Age-group	Expected deaths in one year	Actual deaths in 1951	$\frac{\text{Forecast}}{\text{Actual}}$ (%)	Expected deaths in three years	Actual deaths in 1950–52	$\frac{\text{Forecast}}{\text{Actual}}$ (%)
35–39	4,004	3,757	106·6	12,013	11,123	108·0
40–44	6,616	5,956	111·1	19,849	17,345	114·4
45–49	9,949	9,890	100·6	29,848	28,846	103·5
50–54	14,258	14,909	95·6	42,775	42,598	100·4
55–59	19,944	20,260	98·4	59,831	58,610	102·1
60–64	28,283	29,107	97·2	84,849	82,235	103·2
65–69	36,719	37,358	98·3	110,156	105,299	104·6
70–74	42,513	43,312	98·2	127,538	122,023	104·5
75–79	43,109	43,816	98·4	129,327	122,590	105·5
80–84	29,363	30,499	96·3	88,088	85,897	102·6
85–89	14,436	14,526	99·4	43,308	40,781	106·2
90–94	4,005	3,746	106·9	12,016	10,744	111·8
35–94	253,199	257,136	98·5	759,598	728,091	104·3

10. Table 2 (*b*) shows, for five-year age-groups from 35–39 to 90–94, the deaths expected by the projected rates of mortality in 1951 and 1950–52, and also the actual deaths. The expected deaths were calculated by using the projected values of m_x age by age and the central exposed to risk at each age as calculated for the earlier paper.

11. Although Table 2 (*b*) confirms that the projected rates are too low from age 50 upwards as compared with the experience of the one year 1951, they are too high at all ages when compared with the actual experience for 1950–52, which means that for ages 50 to 90 the difference between projected and actual is smaller than the year-to-year variations, a somewhat encouraging result. The percentages given in the last column of Table 2 (*b*) suggest that the true value of A for 1950–52 should be considerably lower than the projected value in order to decrease the mortality rates at the younger ages, and that the true value of B_1 should also be a little lower.

12. The paper was very nearly completed when the Registrar General's Decennial Supplement, England & Wales 1951 Life Tables, was published, and it was thought that some reference and comparisons should be made. The curve fitted in the Government Actuary's Report on Life Tables (referred to in this paper as the G.A.D. formula) resembles formulae (1) and (2) in the inclusion of a 'normal curve' component; this is, however, combined with a logistic curve, with the result that there are two elements each having the effect of slackening off the rate of increase in m_x at the older ages. It was found that a fair fit of the 1950–52 male data from age 40 onwards could be made by means of formula (1) which has, of course, only one 'slackening' element. The main difference between the two fits is that formula (1) only fits from age 40, ages below this being affected by the fifth component, whereas the G.A.D. formula, with its seven parameters, fits down to the twenties. The formula (1) constants are: origin 72; $B_1 = \cdot 0610957$; $c_1^5 = 1\cdot 654$; $B_2 = \cdot 00908$; and $c_2^{25} = 1\cdot 085425$; but an equally good fit could be made with the origin two or three years later in life. Using the same tests as those applied by the Government Actuary there is little to choose between the two formulae over the range of ages in question. To fit formula (2), it was found that the value of A fitted to the 1951 male data in the earlier paper ($\cdot 0044$) when combined with this new fit of the 1950–52 data nearly reproduced the crude values of m_x from ages 2 to 12, and to compensate for the inclusion of the third component it was necessary to drop B_1 very slightly and to raise c_1^5 to $1\cdot 661$. Comparing this with the projected constants obtained in §7, the natural comment is that the author would have done better not to cheat. The projection of the origin to age 68, which was suggested by the earlier fit of the 1951 one-year data, was too bold. It seems that a better projection might have been made if the two components had been extrapolated by using the values at a higher age.

II. EXPERIMENTS WITH C.M.I. ASSURED LIVES' DATA

13. The assured lives' data are already published in three five-year periods 1924–28, 1929–33 and 1934–38. For each of these periods the eight classes were combined, using the data for durations 5 and over. As a preliminary guide, pivotal values of m_x at quinary ages were found by King's method, and in order that the experiments might not become too unwieldy it was decided to keep the data in quinary groups; this follows the precedent set by Beard (*J.I.A.* 77, 383). Table 3 shows the results of fitting formula (1) to those parts of the data which are not affected by the fifth and sixth components, and since the constants seem to offer as good a possibility of projection as they might after modification of the curves (such as was made to the curves originally fitted to the national male data), no such modifications were made in this case. The remarks made in §3 apply to the tests shown in Table 3.

Table 3. Fitting of formula (1) to C.M.I. assured lives' data, all classes combined, durations 5 and over, for the periods 1924–28, 1929–33 and 1934–38

Age x	Crude m_x	Graduated m_x	Actual deaths	Expected deaths	Deviations (actual minus expected) +	Deviations (actual minus expected) −	Σ deviations +	Σ deviations −	χ^2
				1924–28					
37½	·00332	·00305	515·1	473·7	41·4		41·4		3·63
42½	·00479	·00447	838·8	783·6	55·2		96·6		3·91
47½	·00616	·00661	1,110·6	1,192·1		81·5	15·1		5·61
52½	·00954	·00996	1,482·8	1,548·3		65·5		50·4	2·80
57½	·01553	·01549	1,820·1	1,815·0	5·1			45·3	·01
62½	·02623	·02503	1,986·2	1,895·0	91·2		45·9		4·50
67½	·04169	·04166	2,203·6	2,202·1	1·5		47·4		·00
72½	·06962	·06966	2,561·6	2,562·9		1·3	46·1		·00
77½	·10995	·11325	2,328·9	2,398·7		69·8		23·7	2·29
82½	·16899	·17458	1,498·2	1,547·7		49·5		73·2	1·92
87½	·24805	·25203	622·5	632·5		10·0		83·2	·21
92½	·34491	·34161	143·2	141·9	1·3			81·9	·02
Totals			17,111·6	17,193·5	+195·7	−277·6			24·90
				1929–33					
37½	·00315	·00288	549·5	501·7	47·8		47·8		4·57
42½	·00433	·00432	843·2	840·8	2·4		50·2		·01
47½	·00636	·00649	1,272·1	1,297·3		25·2	25·0		·49
52½	·00954	·00986	1,708·4	1,765·2		56·8		31·8	1·85
57½	·01506	·01535	2,063·7	2,104·0		40·3		72·1	·78
62½	·02536	·02474	2,114·6	2,063·0	51·6			20·5	1·32
67½	·04190	·04121	2,245·7	2,208·9	36·8		16·3		·64
72½	·07047	·06927	2,712·7	2,666·4	46·3		62·6		·86
77½	·11016	·11324	2,586·8	2,659·2		72·4		9·8	2·22
82½	·17387	·17484	1,767·6	1,777·4		9·8		19·6	·07
87½	·24571	·25226	759·0	779·2		20·2		39·8	·70
92½	·40173	·34413	208·5	178·6	29·9			9·9	7·63
Totals			18,831·8	18,841·7	+214·8	−224·7			21·14
				1934–38					
37½	·00275	·00252	606·7	555·1	51·6		51·6		4·81
42½	·00382	·00387	842·7	854·4		11·7	39·9		·16
47½	·00565	·00596	1,277·0	1,347·1		70·1		30·2	3·67
52½	·00912	·00923	1,890·1	1,913·1		23·0		53·2	·28
57½	·01487	·01455	2,407·2	2,356·1	51·1			2·1	1·12
62½	·02408	·02366	2,341·0	2,300·2	40·8		38·7		·74
67½	·03988	·03969	2,223·0	2,212·4	10·6		49·3		·05
72½	·06439	·06679	2,356·7	2,444·3		87·6		38·3	3·36
77½	·10878	·10800	2,474·4	2,456·7	17·7			20·6	·14
82½	·16315	·16345	1,707·5	1,710·7		3·2		23·8	·01
87½	·23824	·23321	813·3	796·1	17·2			6·6	·48
92½	·32805	·32533	209·1	207·4	1·7			4·9	·02
Totals			19,148·7	19,153·6	+190·7	−195·6			14·84

Constants:

	B_1	c_1^5	B_2	c_2^{25}	Origin
1924–28	·28517	1·4600	·15704	1·0735	97½
1929–33	·26024	1·4950	·10438	1·0940	93½
1934–38	·18321	1·5350	·05000	1·1340	87½

Note. The crude values of m_x shown in the above table are the pivotal values found from the crude data by King's method.

It will be seen that, apart from the fact that the graduated values for age $37\frac{1}{2}$ are consistently too low, owing to the likelihood that the rate of mortality in the thirties is affected by the fifth component, the fits are reasonable.

14. An examination of the constants shown at the foot of Table 3 suggests that by 1949–52 the origin might be in the early seventies, say age $72\frac{1}{2}$, while c_1^5 might be between 1·64 and 1·66, say 1·65. The first component $B_1c_1^x$ at age $72\frac{1}{2}$ had values of ·04298 in 1924–28, ·04806 in 1929–33 and ·05065 in 1934–38; in view of the slackening of the increase it might be considered that little or no further increase should be allowed for in the extrapolation, or perhaps even that the value of $B_1c_1^x$ at $72\frac{1}{2}$ might start decreasing, but in fact an extrapolation was made assuming a modest increase to ·053. The second component had values at the same age of ·02667, ·02120 and ·01613 respectively in the three periods under review, indicating reductions of about one-quarter each quinquennium, and applying this reduction rate for extrapolation purposes, a new value of ·007 for 1949–52 was deduced. As before, the constant c_2 gave the greatest difficulty, and it was decided with some diffidence to take the extrapolated value of c_2^{25} as 1·20. This gave values of m_x appreciably above the graduated values according to the A 1949–52 Table, but since the latter table was constructed from data at durations 2 and over, whereas the extrapolation was from data at durations 5 and over, it was thought that it would be useful to examine the data for 1949–52 at durations 5 and over only, for the purpose of comparison with the extrapolated constants.

15. The 1949–52 data at durations 5 and over were first examined in quinary groups, and pivotal values calculated by King's method, as in the case of the earlier data. Several fits were possible from ages $42\frac{1}{2}$ to $87\frac{1}{2}$ inclusive, the best having its origin at $74\frac{1}{2}$ with $B_1 = \cdot 06321$, $c_1^5 = 1\cdot 667$, $B_2 = \cdot 00665$ and $c_2^{25} = 1\cdot 085425$. The grouped data are not shown, because it is proposed to tabulate the individual ages, but it may be said that the grouped deviations are reasonable in size and sign and, for those who like to apply the χ^2 test, that χ^2 has a value of 13·04 for ten age-groups; if adjustments are made by means of the 'All Offices' Variance Ratios shown in the recent Memorandum by the Joint Mortality Investigation Committee on Duplicate Policies (*J.I.A.* **83**, 35), the revised value of χ^2 for the ten age-groups is 9·5.

16. The result of fitting the same curve to the data for individual ages is shown in Table 4. To save space the values of χ^2 are not tabulated, and in any case it has become more fashionable to divide the deviation irrespective of sign by the square root of the actual deaths as a measure of the reasonableness of the size of the deviation. It may be said briefly that after adjusting for duplicates as before, $\chi^2 = 55\cdot 53$ for the 45 values from $39\frac{1}{2}$ to $83\frac{1}{2}$ inclusive, a satisfactory result, or 63·52 for the 48 values from $39\frac{1}{2}$ to $86\frac{1}{2}$ inclusive which is also acceptable. At the ages above $86\frac{1}{2}$ the graduated values are all higher than the crude values. The Joint Mortality Investigation Committee have said in their recent paper on the A 1949–52 Table that 'the understatement of mortality rates in extreme old age is probably appreciable' and the author makes certain comments on this matter in a later section. The ages below $39\frac{1}{2}$ are assumed to be affected by the fifth component. The only unsatisfactory feature of the graduation is the run of nine successive negative deviations in the sixties of age, a feature which is matched by a similar run of 13 negative deviations from $61\frac{1}{2}$ to $73\frac{1}{2}$ inclusive in the official graduation of the A 1949–52 Table—this run being justified in the text of the paper (*J.I.A.* **82**, 3–84).

Table 4. Final graduation by formula (1) of assured lives' data, 1949–52, durations 5 and over, ages 39½ to 86½ inclusive

Age x	First component	Second component	m_x graduated	Central exposed to risk	Actual deaths	Actual minus expected +	Actual minus expected −	$\frac{\lvert A-E\rvert}{\sqrt{A}}$
39½	·0017670	·0001198	·00189	266,657½	513	9·0		·4
40½	·0019571	·0001502	·00211	283,208	601	3·4		·1
41½	·0021678	·0001871	·00235	297,019	651		47·0	1·8
42½	·0024010	·0002316	·00263	305,986½	745		59·7	2·2
43½	·0026594	·0002847	·00294	311,888	899		18·0	·6
44½	·0029456	·0003477	·00329	313,609	1,056	24·2		·7
45½	·0032626	·0004219	·00368	314,896	1,149		9·8	·3
46½	·0036137	·0005086	·00412	315,996	1,276		25·9	·7
47½	·0040025	·0006092	·00461	314,934	1,475	23·2		·6
48½	·0044332	·0007248	·00516	309,593½	1,551		46·5	1·2
49½	·0049103	·0008567	·00577	294,469	1,763	63·9		1·5
50½	·0054387	·0010060	·00644	278,784½	1,935	139·6		3·2
51½	·0060240	·0011736	·00720	265,255½	1,933	23·2		·5
52½	·0066722	·0013602	·00803	251,316½	2,110	91·9		2·0
53½	·0073902	·0015662	·00896	238,330	2,204	68·6		1·5
54½	·0081855	·0017915	·00998	221,080½	2,213	6·6		·1
55½	·0090663	·0020359	·01110	207,768½	2,294		12·2	·3
56½	·0100419	·0022985	·01234	197,697½	2,414		25·6	·5
57½	·0111226	·0025780	·01370	187,899½	2,483		91·2	1·8
58½	·0123195	·0028726	·01519	174,753	2,571		83·5	1·6
59½	·0136452	·0031800	·01683	143,323	2,416	3·9		·1
60½	·0155135	·0034972	·01861	123,502	2,377	78·6		1·6
61½	·0167399	·0038209	·02056	113,695½	2,320		17·6	·4
62½	·0185413	·0041473	·02269	104,747	2,315		61·7	1·3
63½	·0205365	·0044722	·02501	94,497½	2,319		44·4	·9
64½	·0227465	·0047909	·02754	72,427½	1,955		39·7	·9
65½	·0251943	·0050989	·03029	56,881	1,676		46·9	1·1
66½	·0279054	·0053912	·03330	50,399	1,652		26·3	·6
67½	·0309084	·0056630	·03657	46,092½	1,603		82·6	2·1
68½	·0342344	·0059096	·04014	42,394	1,611		90·7	2·3
69½	·0379184	·0061266	·04405	38,502	1,644		52·0	1·3
70½	·0419988	·0063101	·04831	35,510½	1,718	2·5		·1
71½	·0465184	·0064566	·05297	32,892	1,775	32·7		·8
72½	·0515242	·0065633	·05809	30,284	1,778	18·8		·4
73½	·0570688	·0066282	·06370	27,938	1,787	7·3		·2
74½	·0632100	·0066500	·06986	25,604	1,839	50·3		1·2
75½	·0700121	·0066282	·07664	23,503	1,698		103·3	2·5
76½	·0775461	·0065633	·08411	21,628½	1,867	47·8		1·1
77½	·0858909	·0064566	·09235	19,411½	1,873	80·3		1·9
78½	·0951337	·0063101	·10144	16,945½	1,757	38·0		·9
79½	·1053711	·0061266	·11150	14,110	1,561		12·3	·3
80½	·1167101	·0059096	·12262	11,991½	1,515	44·6		1·1
81½	·1292694	·0056630	·13493	10,270½	1,481	95·2		2·5
82½	·1431801	·0053912	·14857	8,570½	1,263		10·3	·3
83½	·1585878	·0050989	·16369	7,207	1,203	23·3		·7
84½	·1756536	·0047909	·18044	5,848½	1,138	82·7		2·5
85½	·1945558	·0044722	·19903	4,689½	902		31·4	1·0
86½	·2154920	·0041473	·21964	3,804	817		18·5	·6
Totals					79,696	+1059·6	−1057·1	52·3

17. Table 5 shows, at quinary ages from $42\frac{1}{2}$ to $87\frac{1}{2}$, the values of m_x produced by the extrapolated constants derived in § 14, by the new graduation of the data for durations 5 and over, and by the official graduation of the A 1949–52 Table. It will be seen that the extrapolated values do not differ from the graduated values of the '5 and over' data by more than 5% at any age.

Table 5. Comparison of values of m_x at certain ages obtained from the constants extrapolated to 1949–52 from the assured lives' data for 1924–38 (durations 5 and over) with the values according to the new graduation by formula (1) of the assured lives' data for 1949–52 (durations 5 and over)

Age x	Assured lives, durations 5 and over. Values of m_x		A 1949–52 m_x	England and Wales males 1950–52. Regraduated m_x
	Extrapolation to 1949–52	Graduation of 1949–52 data		
$42\frac{1}{2}$	·0026	·0026	·0025	·0037
$47\frac{1}{2}$	·0044	·0046	·0045	·0065
$52\frac{1}{2}$	·0076	·0080	·0080	·011
$57\frac{1}{2}$	·013	·014	·013	·019
$62\frac{1}{2}$	·022	·023	·022	·030
$67\frac{1}{2}$	·038	·037	·036	·047
$72\frac{1}{2}$	·060	·058	·059	·074
$77\frac{1}{2}$	·093	·092	·096	·115
$82\frac{1}{2}$	·147	·149	·152	·183
$87\frac{1}{2}$	·239	·243	·235	·297

III. COMPARISONS BETWEEN SELECT, ULTIMATE AND NATIONAL MORTALITY

18. A comparison of the constants for the graduated assured lives' data at durations 5 and over, and the regraduated rates for the national male data for 1950–52 shows, first, that the origin for the national regraduation is $2\frac{1}{2}$ years earlier in life than that for the assured lives' graduation (and it was commented in § 12 that the origin for the national data could well be a few years later), secondly that the two values of c_1 are very similar, and thirdly that the two values of c_2 are identical—although this would probably no longer be true if the origin of one of the graduations were to be changed. After some trials it was found that a reasonable fit of the national data could be made using the same origin and the same value of c_1 as for the assured lives' data, with $B_1 = {\cdot}07680$, $B_2 = {\cdot}01095$ and $c_2^{25} = 1{\cdot}067825$.

19. It can now be said that the result of recent experiences is that the first component of the rate of mortality is, for assured lives' ultimate data, about 82% of the same component of the national male rate; while the second component is slightly over 60% of the second component of the national male rate at the origin, the percentage becoming smaller as we move away from the origin because the assured lives' curve has a steeper value for c_2. For assistance in comparison, a column has been added to Table 5 showing the values of m_x at certain ages given by the regraduation of the national male data. On the other hand, if the curve for the assured lives' 1929–33 data is compared with that for the 1930–32 national male data it will be seen that, whilst the origins

and the values of c_1 were similar for the two sets of data, the assured lives' first component was only about 74% of the national but the assured lives' second component was higher than the national at all ages. The Joint Mortality Investigation Committee has recently published a note on a comparison of trends of population mortality and assured lives' mortality (*J.I.A.* **83**, 153), and one of the conclusions was that the improvements have been more marked in the national male data than in the assured lives' data up to age 54, and conversely at higher ages. So far as the first component is concerned, the improvement in national male mortality appears to have left less scope for divergence between assured lives' mortality and national mortality—so far as such divergence is caused by permanent selection—with the result that the comparative percentage has increased, between 1929–33 and 1949–52, from 74% to 82%; on the other hand, there appears to have been some change in the character of the second component of assured lives' mortality, in so far as the 1929–33 experience was that this component was higher than in the case of the national male mortality (suggesting that assured lives after the first five years of duration included, as a class, a greater proportion of predisposed lives), whereas the 1949–52 experience is that the permanent forms of selection are now able to eliminate nearly two-fifths of such lives. As it is at the higher ages that the second component has its greatest effect, this could account for the fact that the marked improvement in assured lives' mortality at the older ages is not matched either by the younger ages or by the national mortality.

20. It was with these thoughts in mind that it was decided to extend the experiments to the select data for 1949–52. For duration 0 it was found possible to fit a first component of 59·1% of the ultimate first component (or nearly 49% of the national first component) with c_1^5 still 1·667 and no second component at all. For duration 1 the first component is 67% of the ultimate (or about 55% of the national) again with the same value for c_1^5, and a small second component has appeared with its maximum value (or B_2) ·00152 at an origin of $56\frac{1}{2}$, with $c_2^{25}=1{\cdot}38038$. For durations 2 to 4 combined the first component is up to 78% of the ultimate (or about 64% of the national) and c_1^5 is still 1·667; while the second component is a little higher than for duration 1 with its origin one year of age higher, the details being $B_2=\cdot 00182$, $c_2^{25}=1{\cdot}22180$, and origin $57\frac{1}{2}$. The values of m_x resulting from these graduations, together with the actual and expected deaths, are shown in Table 6, but the table would become somewhat unwieldy if an attempt were made to show the tests of the graduations. It is sufficient to say that by any test the three graduations are acceptable from age $38\frac{1}{2}$ upwards, apart from the fact that some of the duration 1 deviations are rather large—as measured either by χ^2 or by $|A-E|/\sqrt{A}$; these are necessary features of any graduation of the duration 1 data, including the official graduation of the A 1949–52 Table. They have to be on the large side from age $40\frac{1}{2}$ to $44\frac{1}{2}$ inclusive, but since these are of alternate signs they are harmless; while a very large positive deviation at age $59\frac{1}{2}$ is essential to avoid excessive overstatement of the mortality at the older ages.

21. The comparison of the select, ultimate, and national data may be summarized by saying that, according to the latest experience, permanent selection accounts for nearly 20% of the first component of the national male mortality while temporary selection accounts for more than another 30%;

Table 6. Regraduation of assured lives' data, 1949–52, durations 0, 1, and 2 to 4, ages $38\frac{1}{2}$ to $83\frac{1}{2}$ inclusive

Age x	Duration 0			Duration 1			Durations 2 to 4 combined		
	$m_{[x]}$	Actual deaths	Expected deaths	$m_{[x-1]+1}$	Actual deaths	Expected deaths	m_x	Actual deaths	Expected deaths
$38\frac{1}{2}$	·00094	35	30·8	·00109	33	36·5	·00135	143	130·9
$39\frac{1}{2}$	·00104	24	34·7	·00122	40	40·6	·00151	152	142·7
$40\frac{1}{2}$	·00116	27	37·0	·00137	57	46·8	·00171	178	158·7
$41\frac{1}{2}$	·00128	40	37·7	·00154	38	49·9	·00192	185	179·2
$42\frac{1}{2}$	·00142	35	39·5	·00173	61	51·1	·00217	158	199·7
$43\frac{1}{2}$	·00157	37	41·2	·00195	38	54·0	·00245	193	214·8
$44\frac{1}{2}$	·00174	47	45·6	·00221	71	57·1	·00277	224	222·1
$45\frac{1}{2}$	·00193	40	44·0	·00251	75	65·2	·00312	237	228·4
$46\frac{1}{2}$	·00214	42	41·7	·00284	64	63·6	·00351	253	247·1
$47\frac{1}{2}$	·00237	45	42·5	·00322	57	61·3	·00394	245	259·4
$48\frac{1}{2}$	·00262	42	43·6	·00364	65	63·1	·00441	254	263·6
$49\frac{1}{2}$	·00290	63	50·9	·00410	69	65·3	·00492	286	250·5
$50\frac{1}{2}$	·00321	40	41·1	·00460	68	75·3	·00547	253	242·6
$51\frac{1}{2}$	·00356	40	33·9	·00514	50	61·9	·00606	219	251·7
$52\frac{1}{2}$	·00394	25	32·3	·00571	53	51·7	·00669	233	249·0
$53\frac{1}{2}$	·00437	36	31·3	·00630	53	49·3	·00737	237	235·3
$54\frac{1}{2}$	·00484	53	34·7	·00693	43	47·8	·00808	225	205·6
$55\frac{1}{2}$	·00536	29	29·5	·00757	65	52·6	·00883	203	186·6
$56\frac{1}{2}$	·00593	22	23·3	·00825	44	43·9	·00964	176	188·2
$57\frac{1}{2}$	·00657	13	20·2	·00895	33	34·9	·01050	169	184·4
$58\frac{1}{2}$	·00728	22	19·7	·00970	27	29·9	·01141	166	169·7
$59\frac{1}{2}$	·00806	20	22·8	·01050	47	27·9	·01241	134	139·0
$60\frac{1}{2}$	·00893	19	17·7	·01136	31	31·4	·01348	125	118·7
$61\frac{1}{2}$	·00989	18	13·1	·01232	15	24·1	·01466	123	114·4
$62\frac{1}{2}$	·01096	13	11·8	·01338	8	16·9	·01595	125	107·4
$63\frac{1}{2}$	·01214	8	11·0	·01457	17	14·8	·01738	93	94·6
$64\frac{1}{2}$	·01344	9	13·7	·01591	5	13·7	·01897	74	72·0
$65\frac{1}{2}$	·01489	10	10·5	·01742	21	16·9	·02074	61	60·0
$66\frac{1}{2}$	·01649	4	6·7	·01912	7	12·9	·02272	62	58·9
$67\frac{1}{2}$	·01827	4	4·2	·02103	8	7·9	·02493	61	55·5
$68\frac{1}{2}$	·02023	0	3·8	·02317	1	4·7	·02739	51	46·9
$69\frac{1}{2}$	·02241	2	3·3	·02558	5	4·7	·03015	21	30·9
$70\frac{1}{2}$	·02482	2	1·9	·02826	7	3·3	·03323	22	20·3
$71\frac{1}{2}$	·02749	1	1·2	·03125	1	2·2	·03666	14	14·4
$72\frac{1}{2}$	·03045	1	1·0	·03458	0	1·0	·04049	10	11·5
$73\frac{1}{2}$	·03373	0	·9	·03827	0	1·1	·04475	5	7·7
$74\frac{1}{2}$	·03736	1	·4	·04237	2	·7	·04948	5	4·3
$75\frac{1}{2}$	·04138	2	·4	·04692	0	·6	·05475	2	2·7
$76\frac{1}{2}$	·04583	0	·2	·05196	0	·3	·06059	3	1·8
$77\frac{1}{2}$	·05076	1	·3	·05755	0	·2	·06707	1	1·0
$78\frac{1}{2}$	·05622	0	·5	·06374	0	·3	·07426	1	·3
$79\frac{1}{2}$	·06227	0	·3	·07060	0	·4	·08223	0	·4
$80\frac{1}{2}$							·09106	0	·3
$81\frac{1}{2}$							·10085	1	·3
$82\frac{1}{2}$							·11169	0	·3
$83\frac{1}{2}$							·12371	0	·1
Totals		872	880·9		1279	1287·8		5383	5373·9

and that permanent selection accounts for about 40% of the second component of the national male mortality, the whole of the remainder being eliminated during the first year of assurance by temporary selection. It is impossible to trace the precise course of the reversion from duration 0 mortality to ultimate mortality without knowing more about individual durations over 4, and even if such detailed data were available any possible conclusions might be masked by spurious selection.

ADVANCED AGES

22. In recent attempts to fit curves to the national and the assured lives' data it has been found difficult to avoid negative deviations (i.e. expected deaths exceeding actual) in the late eighties and the nineties of age. In the official fit of the national male data for 1950–52, the use of the G.A.D. formula—which has two 'slackening-off' elements—has helped, but the deviations over age 87 are not published and the fact that the crude rates of mortality actually decrease after age 94 suggests that the figures might have something more to show if investigated in a different way. The Government Actuary's Report mentions, on page 6, the irregularities caused by using the mid-1951 populations as an estimate of the exposed to risk (in each of the years 1950, 1951 and 1952) and, on page 8, mis-statements of age at the oldest ages. It is not possible to do anything about the mis-statements, but something can be done about the exposed to risk which might cause some improvement, although it is admitted that even then the 'exposed' cannot be expected to be perfect. The Registrar General's Statistical Reviews give particulars of deaths, in age-groups, according to month of occurrence, and it is thought that one of the main inaccuracies in the use of the mid-1951 population as the exposed to risk is caused by the exceptionally heavy deaths which occurred in the first quarter of 1951. Taking the age-group 85 and over, whereas the male deaths in the first half of 1950 were 53·8% of the deaths in the whole year, and in the first half of 1952 53·9% of the deaths in the whole of that year, the 1951 deaths were divided into 62·5% in the first half and 37·5% in the second half. The abnormality of 1951 would be still more marked if analysed into quarters. If, now, these percentages are applied to the actual deaths in the years in question for each age over 85, we may say that a second approximation to the exposed to risk for the three years, ignoring migration and mis-statements, might be given by the expression

$$\left(P_{x+1}+\tfrac{3}{8}\theta_{x+1}^{51(1)}+\tfrac{1}{8}\theta_{x+1}^{50(2)}+\tfrac{1}{8}\theta_{x}^{51(1)}+\tfrac{3}{8}\theta_{x}^{50(2)}\right)+P_x$$
$$+\left(P_{x-1}-\tfrac{3}{8}\theta_{x-1}^{51(2)}-\tfrac{1}{8}\theta_{x-1}^{52(1)}-\tfrac{1}{8}\theta_{x}^{51(2)}-\tfrac{3}{8}\theta_{x}^{52(1)}\right),$$

where the P terms refer to the mid-1951 populations, the θ terms to the deaths in the year shown as an index, the (1) or (2) in the index indicating the first or second half of the year. The use of this expression gives the following corrected crude values of m_x, and it will be seen not only that the corrected values increase for one more year of life than the uncorrected but also that they progress more regularly. The corrected values are all less than the uncorrected, and possibly this would also be so if the exposed to risk were similarly adjusted at the earlier ages. This is all rather conjectural, and scarcely justifies a further regraduation based on the incomplete data of month of death given in broad age-groups only, but it is perhaps permissible to ask whether, in future, a better estimate of the exposed to risk over three years might not be made by the use of the census populations at three consecutive years of age adjusted by the relevant deaths.

23. In the assured lives' data at advanced ages, similar features are exhibited for different reasons. The paper by the Joint Mortality Investigation Committee on the A 1949–52 table suggested it was due to the

England and Wales, crude values of m_x for 1950–52

Age x	Uncorrected value	Corrected value
86	·261	·256
87	·277	·270
88	·282	·281
89	·325	·311
90	·342	·338
91	·364	·347
92	·401	·387
93	·435	·422
94	·496	·449
95	·486	·471
96	·441	·435
97	·457	·413
98	·376	·375

inclusion of paid-up policies under which contact with the lives assured had been lost, and this undoubtedly is one of the causes. It is thought there may be two other causes. One is that a small proportion of female lives are still included in the data, and with the lower mortality of females it may be expected that there would be a higher proportion of them at the older ages. The other is the late notification of deaths; this is relatively unimportant at the younger ages where, in any case, the deaths are only a small proportion of the exposed, and where possibly there is a tendency for less delay in notification. But at old ages where the true rate of mortality may be approaching 50% or even more, the inclusion of a policy on the books for, say, a year after death would cause a large proportionate distortion in the exposed to risk. Although the mortality at these very high ages is financially unimportant, it may not be so unimportant statistically, and there might be some advantage in carrying out a special investigation into mortality over age 85 (say) by a card system. The total exposures at these ages in 1949–52 were only slightly in excess of 100, so little work would be involved.

IV. EXTRAPOLATION TO 1961

24. It may be interesting, before concluding, to attempt an extrapolation to the period around 1961, for both national and assured lives' mortality. Considering first the constants affecting the first and second components, the following are those fitted to the data for the periods reviewed and those suggested for the 1961 period:

National male data

	c_1^5	c_2^{25}	Origin ι	$B_1c_1^x$ at age 72	$B_2c_2^{-x^2}$ at age 72
1910–12	1·361	1·036	116	·0550	·0243
1920–22	1·450	1·072	99	·0591	·0155
1930–32	1·496	1·111	92	·0624	·0126
1950–52	1·667	1·068	74½	·0595	·0108
1960–62 (extrapolated)	1·692	1·068	72	·0580	·0100

Assured lives' data (durations 5 and over)

	c_1^5	c_2^{25}	Origin	$B_1c_1^x$ at age 72½	$B_2c_2^{-x^2}$ age 72½
1924–28	1·460	1·073	97½	·0430	·0267
1929–33	1·495	1·094	93½	·0481	·0212
1934–38	1·535	1·134	87½	·0507	·0161
1949–52	1·667	1·085	74½	·0515	·0066
Period around 1961 (extrapolated)	1·692	1·085	72	·0507	·0060

25. This leaves the miscellaneous components which affect the younger ages. An examination of the effect of the fifth and sixth components showed that their effect on the ages between the 'teens' and 40 increased from 1910–12 to 1920–22, possibly owing to an increase in accidental death-rates. 1920–22 showed a persistence in the thirties of age, possibly a result of the 1914–18 war. It was decided just to use the figures for 1930–32 and 1950–52, but to discard the fact that the 1930–32 figures extended into the forties (doubtless also owing to the war). It was also decided to lump the third component in with the fifth and the sixth for the purpose of extrapolating at these ages, and to make no attempt to extrapolate for the constant A. By deducting the first and second components at certain ages from the crude national pivotal values of m_x the following estimates of the combined effects of the third, fifth and sixth components were obtained:

Combined effect of third, fifth and sixth components on m_x (national male data)

Age	1930–32	1950–52
17	·00119	·00069
22	·00168	·00099
27	·00120	·00083
32	·00077	·00059
37	·00070	·00033

These two sets of figures are so far apart in time, and give so little indication of what movement might be expected, that it is thought that little better can be done than to assume the 1950–52 figures will be repeated. In any case, there may be little justification for assuming there may be any regular change in what is largely due to unnatural causes.

26. Similar figures for the assured lives' data for 1949–52 are:

Combined effect of third, fifth and sixth components on m_x. assured lives, 1949–52, durations 5 and over

Age	1949–52
22½	·00082
27½	·00061
32½	·00036
37½	·00017

It was again assumed that these values would be repeated. It is interesting to note that the differences in the effect of these miscellaneous components upon the national male rates and the assured lives' rates respectively are approximately equal to the values of the national third component (the A term), suggesting that one of the results of selection may be the complete removal of this component.

27. The extrapolated values of m_x at quinary ages are:

Values of m_x projected to the period around 1961

Age	National (males)	Assured lives (durations 5 and over)
17	·00087	
22	·00130	·00106
27	·00139	·00106
32	·00160	·00113
37	·00219	·00151
42	·00341	·00237
47	·00611	·00425
52	·01057	·00750
57	·01750	·01281
62	·02795	·02113
67	·04364	·03396
72	·06800	·05410
77	·10750	·08692
82	·17374	·14203
87	·28648	·23587
92	·47886	·39586

CONCLUDING REMARKS

28. The original purpose of this paper, as stated in §2, was to describe certain experiments designed to examine the possibility that formulae (1) and (2) might be useful for projection. It is submitted that the experiments demonstrate

(1) that formula (1) is suitable for fitting both assured lives' and national male mortality, subject to the inclusion of an adjustment below age 40 to allow for the fifth component, and the inclusion of the third term of formula (2) if it is desired to extend the curve to include the childhood ages;
(2) that the formula is useful in indicating the differences between national male and assured lives' mortality;
(3) that it is also useful in indicating the difference between select and ultimate rates of mortality;
(4) that it may assist in following secular trends;
and (5) that it has possibilities for extrapolation.

29. On the extrapolation possibilities, it may be said that, like all forms of projection, short-term extrapolation is easier than long-term; and that the main difficulty in tracing the movements of the constants over the last forty years has been the retarding of the origin with the advance of time.

30. The fact that extrapolation has been possible in some degree is suggestive of the possibility that the year of experience may be a much more important factor than year of birth. It seems reasonable to suppose that the

rates of mortality in any year, due to either the first or the second component, are largely affected by the extent to which medical science has progressed, and this is quite independent of the year of birth; and there is no reason to suppose that the proportion of predisposed lives susceptible to the second component is in any way affected by the year of birth, except in so far as such predispositions have existed ever since birth. It seems likely that the third component—and only the third component—which decreases throughout life from a high level in the early years of life, may be affected by pre-natal care, by the quality of attention at birth, and by care during the early years of infancy, and may accordingly be correlated to the year of birth. And it has been seen that the third component has apparently no effect at all on assured lives' mortality, and little effect on national male mortality after the years of adolescence. This is, of course, conjectural but it has been mentioned because it seems to the author that there may be some danger in attaching too much importance to the year of birth as a factor affecting mortality; and that tracing generation mortality may prove to be a will-o'-the-wisp.