The Actuarial Profession
making financial sense of the future

EXTREME EVENTS
DERIVING TAIL PROBABILITIES FOR ECONOMIC DISTRIBUTIONS RELEVANT TO INDIVIDUAL CAPITAL ASSESSMENTS

Ralph Frankland and Andrew Smith

Members of the LPEC Benchmarking Stochastic Models Working Party

Extreme Events

A game of two halves:

- Purpose of the Working Party – Context, Need and What we are doing?

- The particular problem with Yield Curves

Extreme Events – Regulatory Background

- The Individual Capital Assessment a requirement of FSA
- Permits the use of internal models
- Should reflect 1in 200 year risks (or stronger if firm's Capital Appetite is stronger)
- Does not require Actuarial certification even for a Life Company

Extreme Events – Regulatory Background

- Solvency II – just around the corner
- Political issues
- Methodologies are not ICA specific
- FSA being kept informed of our work

Extreme Events – Professional Background

- Concern over diversity of models
- Does not encourage confidence, or comparability
- Can we give agree what a “1 in 200 year event” looks like
- How would such agreement be “enacted” in an FSA/BAS environment
- What about non actuaries?

Extreme Events – Specific Considerations

- Equity markets ✓
- Aggregate fixed interest yields ✓
- Yield curves ✓
- Property markets ✗
- Correlations of events ✗
- Credit ✗

Extreme Events – Issues

- Data availability
- Homogeneity (the last 200 years)
- Need to take bold/heroic assumptions
- Potential for new data to change answers
- Conditional/Unconditional probability
- Different definitions of Unconditional probability

Yield Curve Volatility: Swap Rate Annualised Vol + Correlation

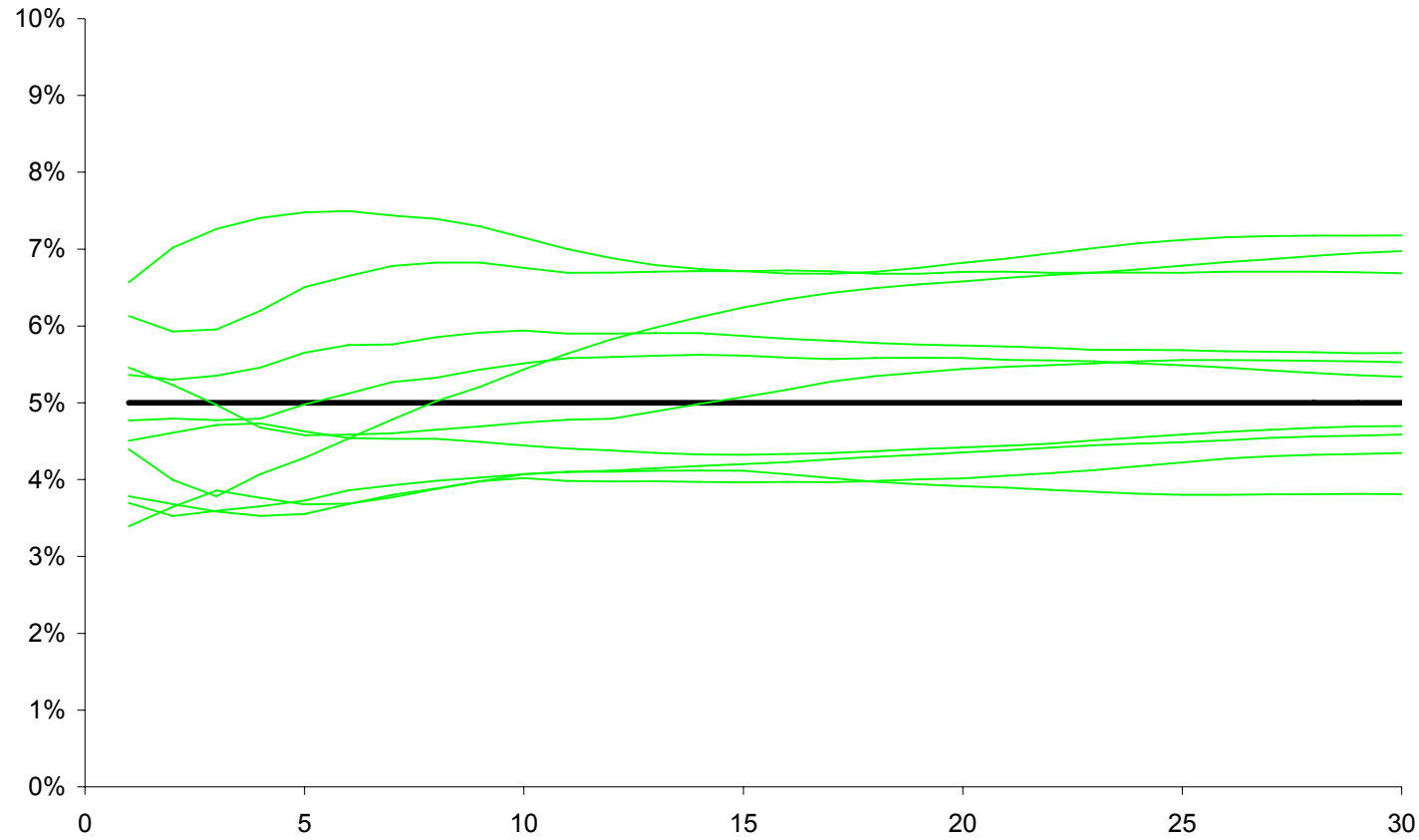
Variance-Covariance of Historic Changes in Swap Data

Currency	USD		Annualised vol by term		Correlation					
min row	73	06/05/1998	1	0.79%	100%	94%	89%	82%	71%	63%
max row	597	21/05/2008	2	0.96%	94%	100%	99%	94%	84%	77%
hold	1	weeks	3	1.00%	89%	99%	100%	98%	90%	84%
			5	1.02%	82%	94%	98%	100%	96%	91%
			10	0.97%	71%	84%	90%	96%	100%	98%
			20	0.87%	63%	77%	84%	91%	98%	100%

Currency	GBP		Annualised vol by term		Correlation					
min row	25	04/06/1997	1	0.70%	100%	88%	82%	73%	58%	46%
max row	597	21/05/2008	2	0.78%	88%	100%	98%	90%	74%	60%
hold	1	weeks	3	0.79%	82%	98%	100%	96%	82%	68%
			5	0.75%	73%	90%	96%	100%	92%	80%
			10	0.69%	58%	74%	82%	92%	100%	95%
			20	0.65%	46%	60%	68%	80%	95%	100%

Currency	DEM		Annualised vol by term		Correlation					
min row	25	04/06/1997	1	0.54%	100%	94%	89%	81%	66%	54%
max row	597	21/05/2008	2	0.67%	94%	100%	98%	93%	80%	68%
hold	1	weeks	3	0.69%	89%	98%	100%	97%	86%	75%
			5	0.67%	81%	93%	97%	100%	94%	85%
			10	0.61%	66%	80%	86%	94%	100%	96%
			20	0.57%	54%	68%	75%	85%	96%	100%

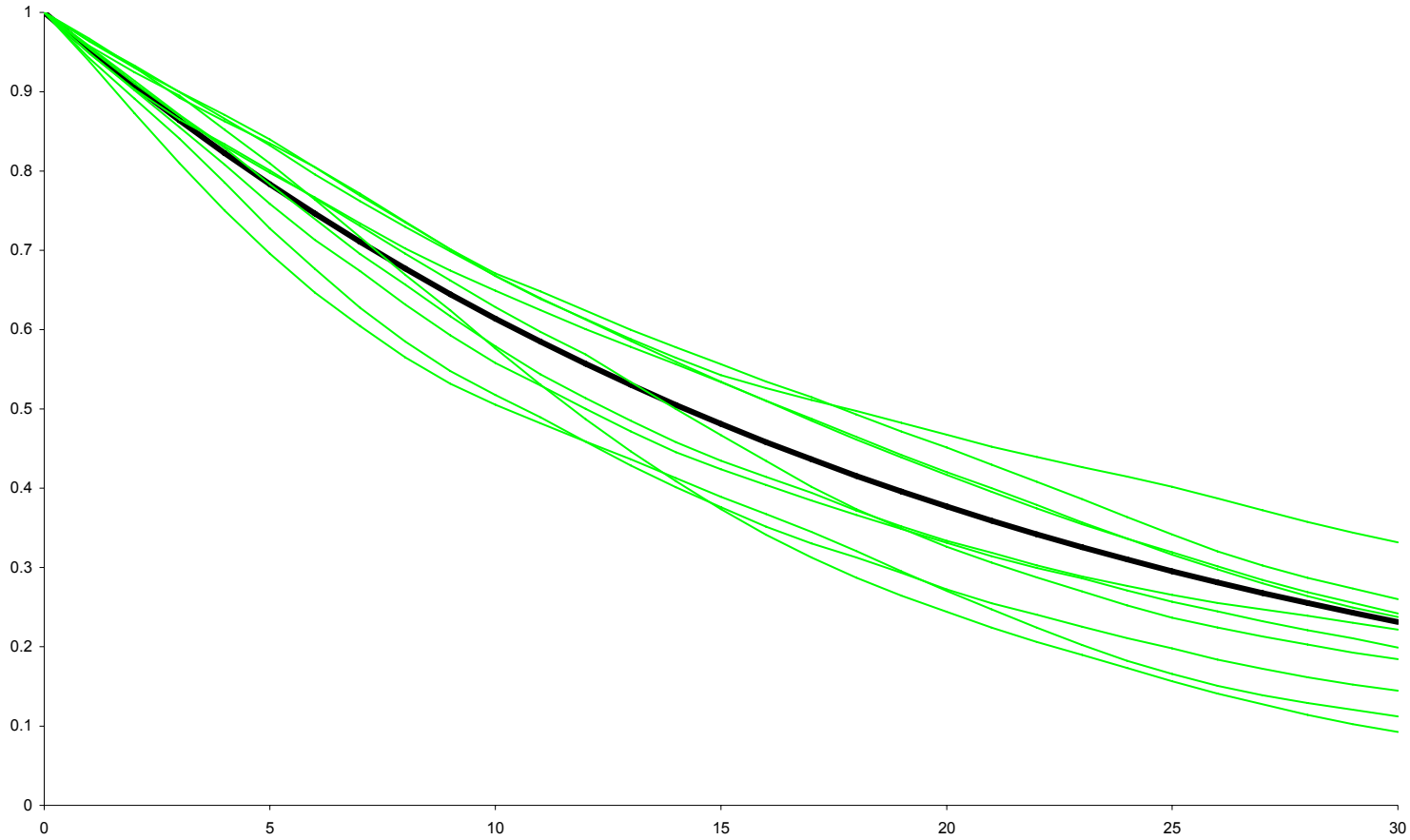
Par Curves: Starting Point (5%) and Possible Stressed Curves



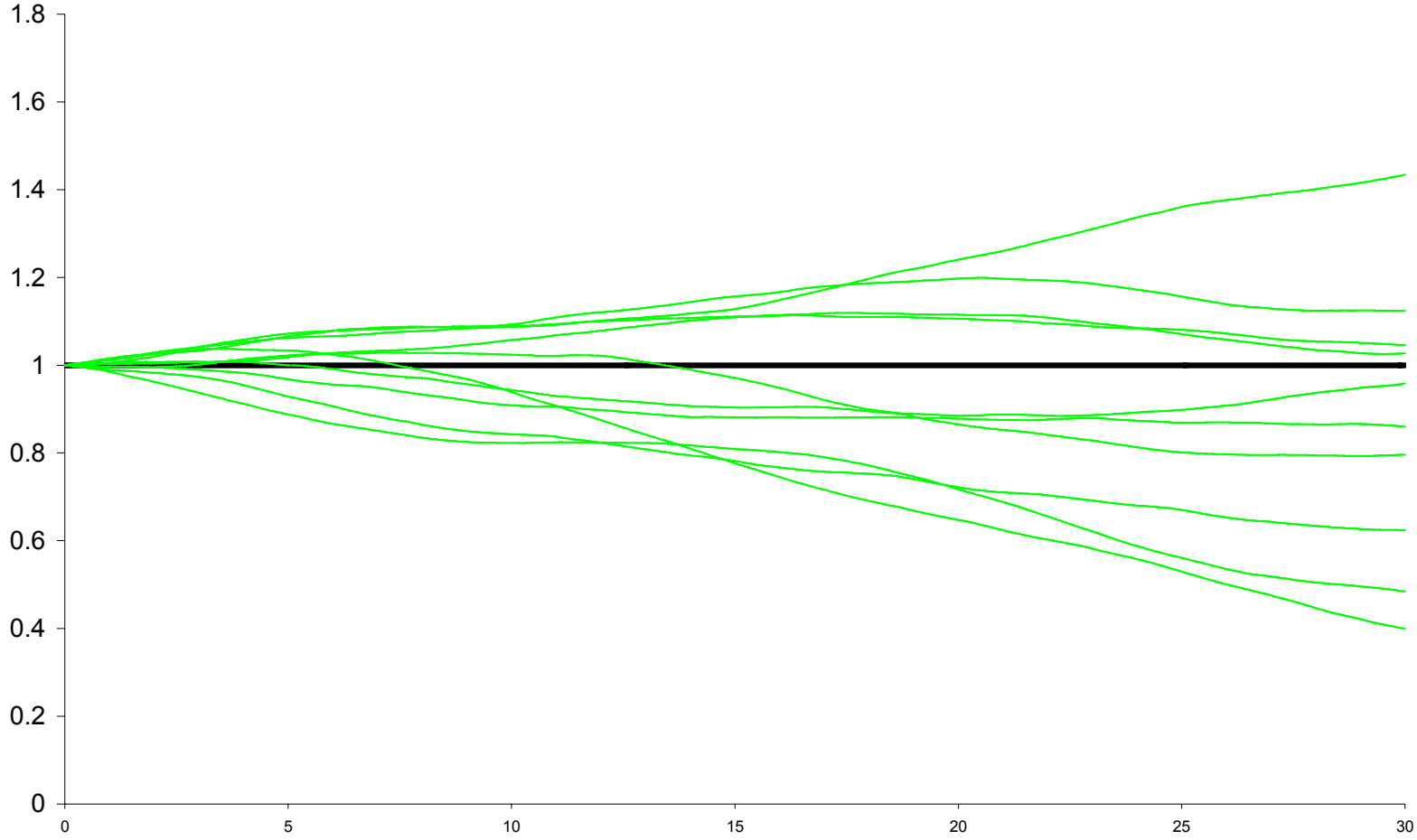
Counting Yield Curve Factors

- Single factor model
  - If we know one point on the yield curve, we can construct all the others
  - We can hedge a 50 year liability with 1-year and 5-year bonds
- Two-factor model
  - If we know two points on the yield curve, we can construct all the others
  - We can hedge a 50 year liability with 1-year, 5-year & 10-year bonds
- Three-factor model
  - If we know three points on the yield curve, we can construct all the others
- How many factors?

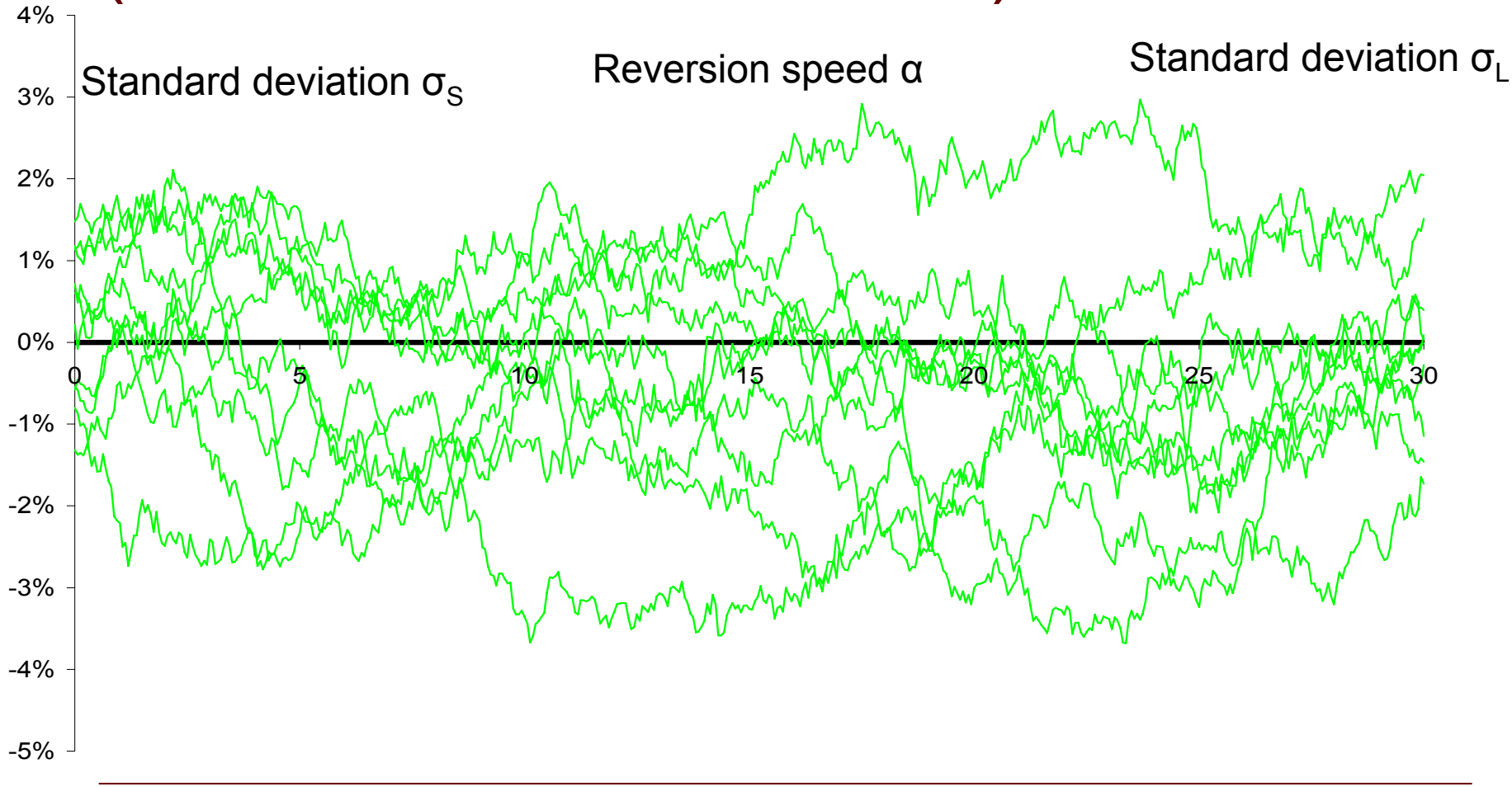
ZCB Price Moves



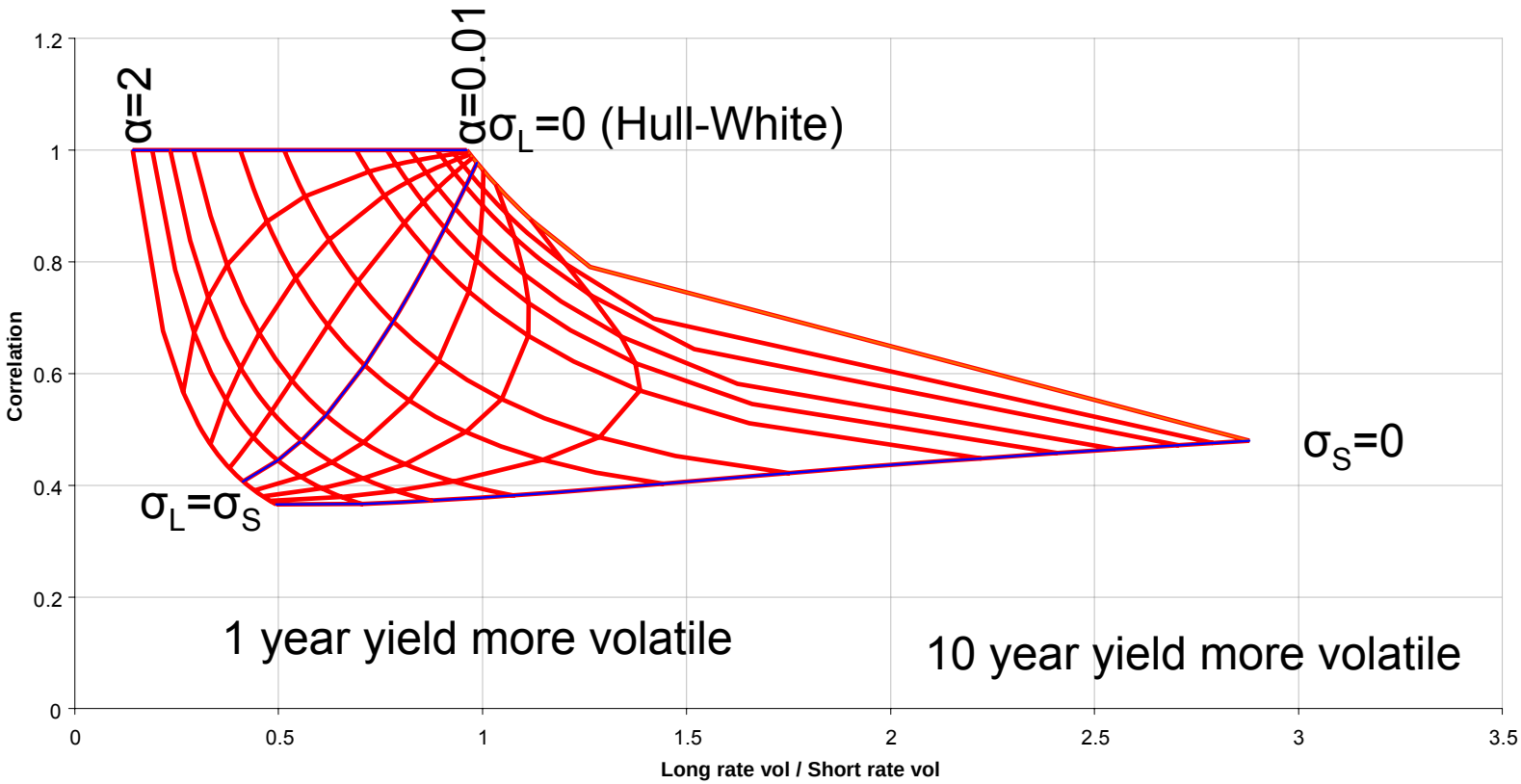
H-function (stressed ZCB / base ZCB) should be differentiable



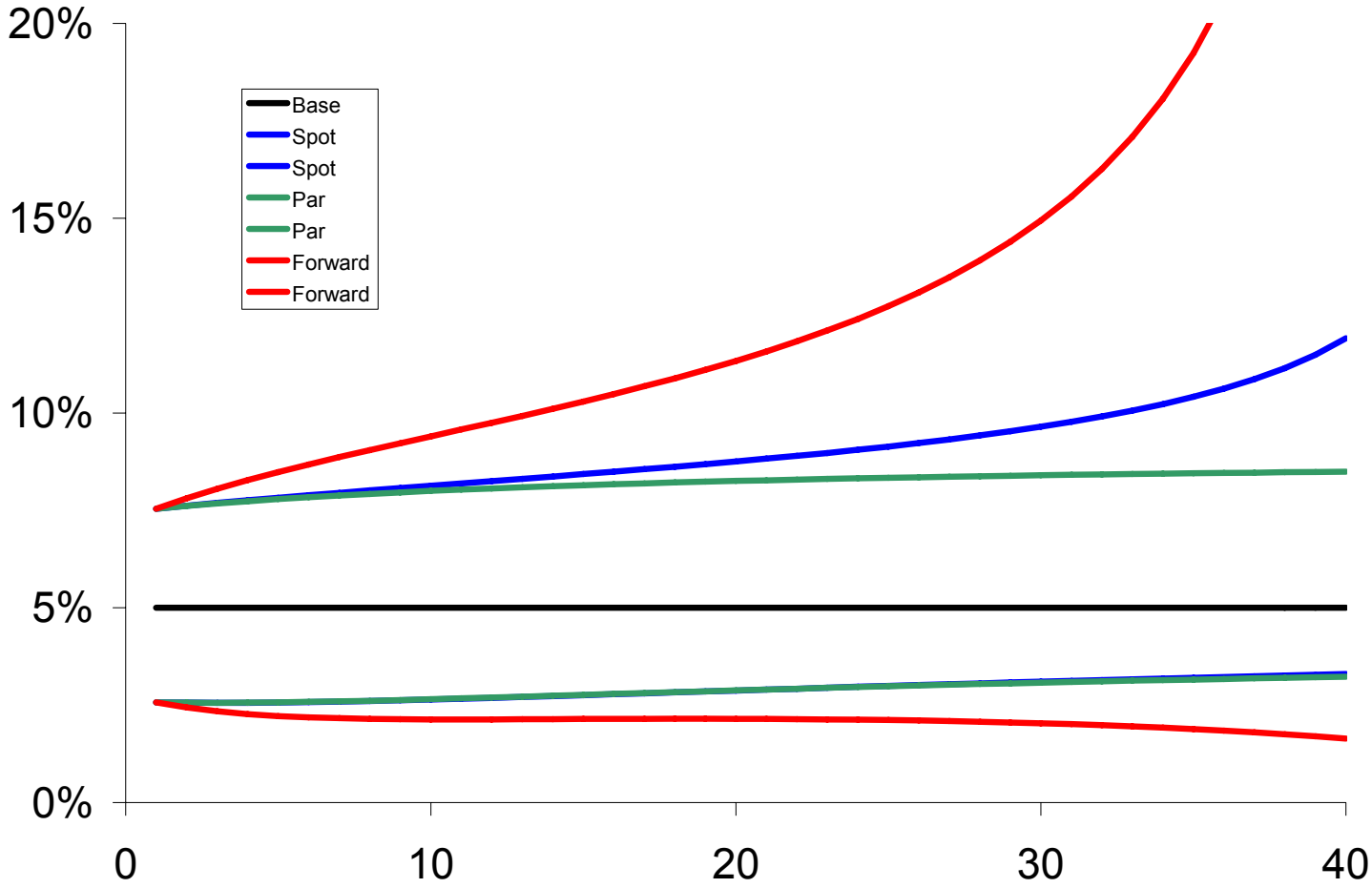
Ornstein Ühlenbeck Processes (Gradient of H – Function)



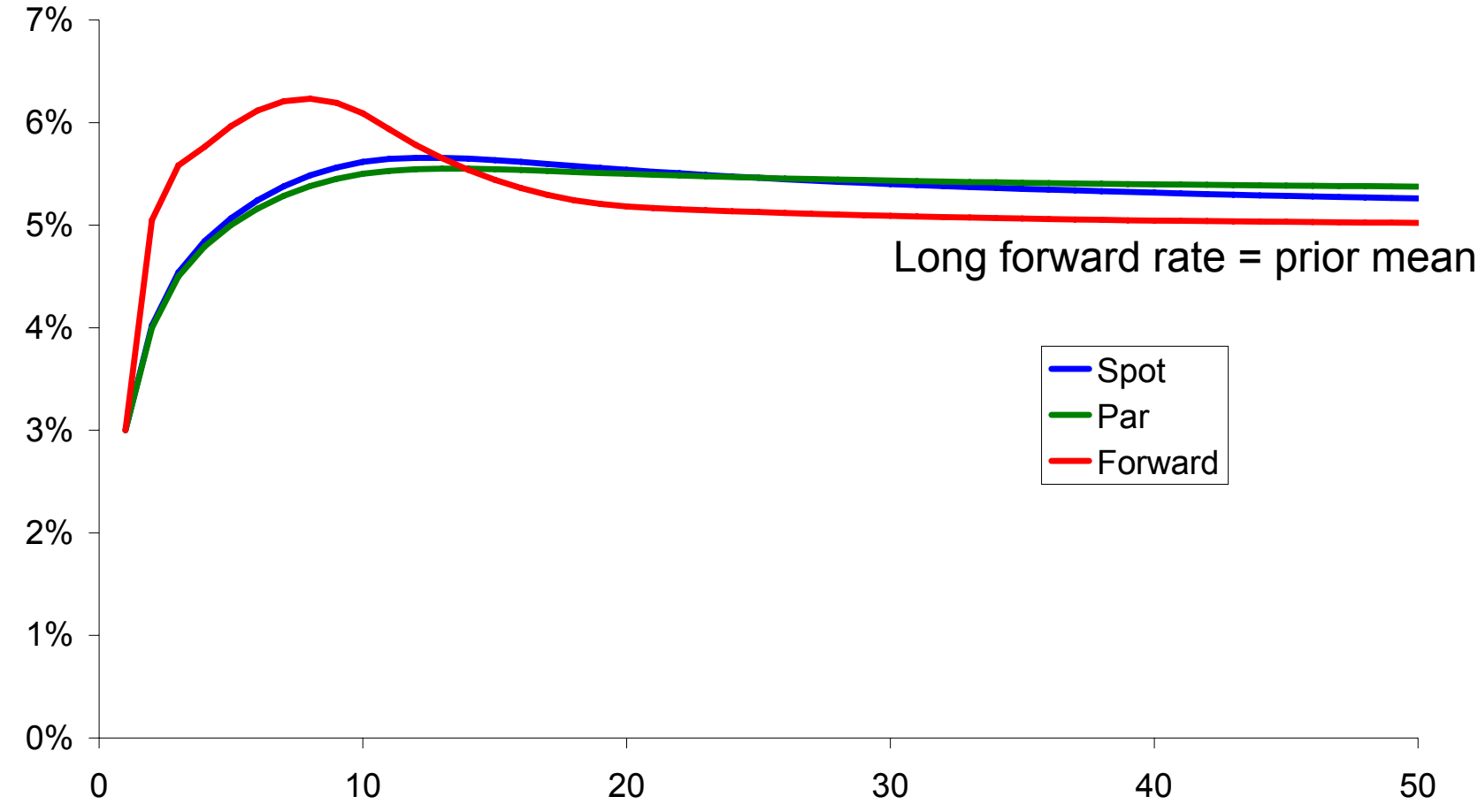
Calibrating the IOU Model to 1 & 10 year Volatility + Correlation



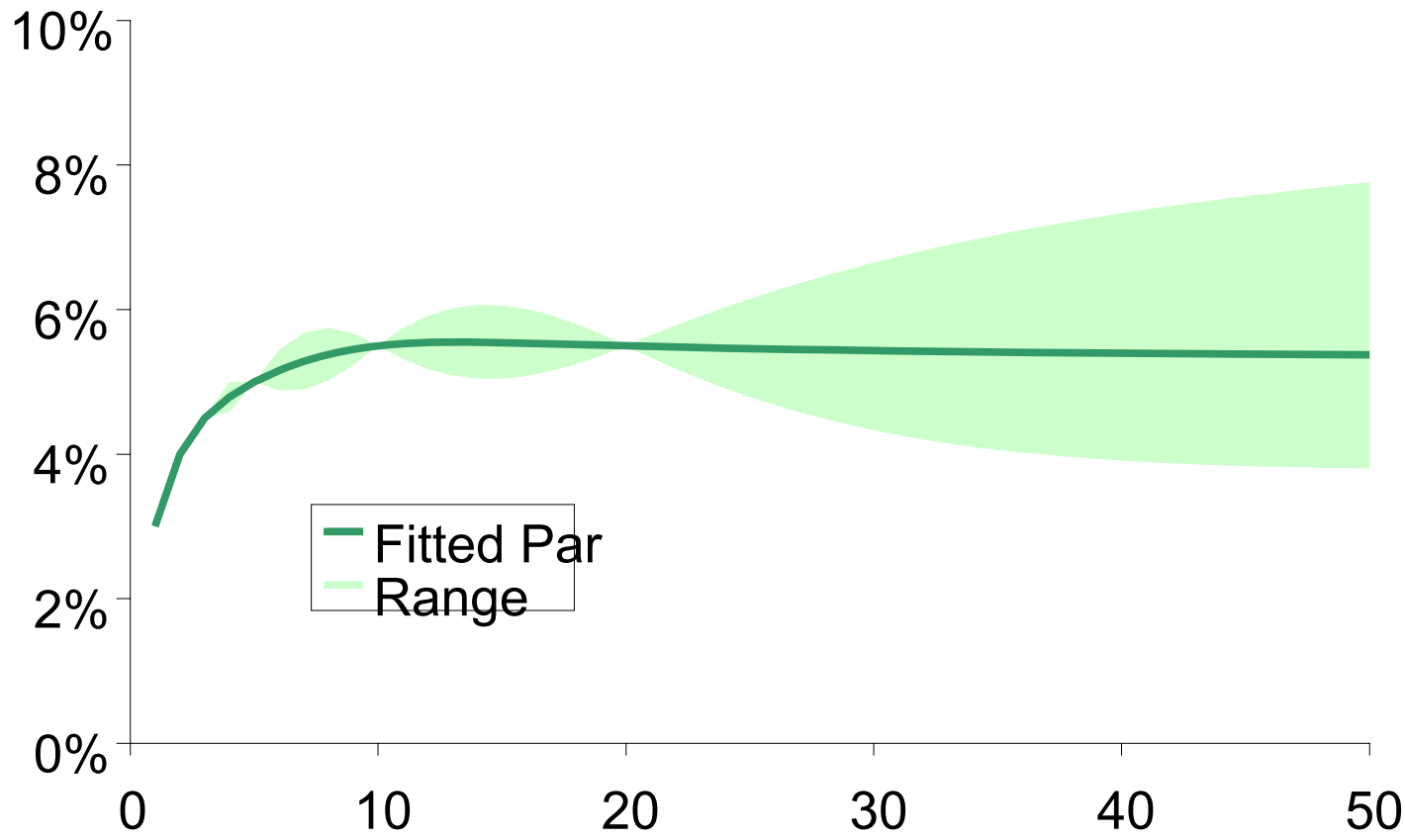
Stresses for Spot, Par and Forward Rates (0.5%-ile to 99.5%-ile)



Conditional Mean Distribution



Quantifying Interpolation / Extrapolation Error in Par Yields



Continuing Work

- Equity fat tail models
- Interest rate dimension reduction
- Quantifying estimation error