Extreme Value Techniques

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GIRO Conference, 15 October 1999

Extreme Value Statistics

- Motivation and Maxima Theory
- Dutch Dike Problem
- Threshold Theory
- Motor XL Rating Problem
- Questions

Extreme Value Statistics

- "Statistical Study of Extreme Levels of a Process"
- Relevant?
  - Catastrophic Events
  - Increasing Insurance Deductibles
  - Estimated/Probable Maximum Loss
  - Excess of Loss Reinsurance
  - Value at Risk/EPD $\Rightarrow$ Capital Allocation

Er…YES!

Hierarchy of Analysis

- Experience Rating - “Burning Cost”
  - Central Limit Theorem
  - Actuaries have ***Added Value***
- Statistical Rating - “Curve Fitting”
  - Lognormal/Pareto (apply threshold?)
  - What Data do we Use?

A Break from “The Norm”

- Central Limit Theorem
  - Class Average I.Q.s
  - Average Observations ~ $N(\mu,\sigma)$
  - Price Insurance by “Expected Claims”
- Extremal Types Theorem or Maxima Theory
  - Class Maximum I.Q.s
  - Maximum Observations ~ $GEV(\mu,\sigma,\xi)$
  - Price Insurance by “Expected Claims Attaching”

Generalized Extreme Value Distribution

$$\Pr(X \leq x) = G(x)$$

$$= \exp \left[- (1 + \xi((x-\mu) / \sigma))^{-1/\xi} \right]$$

EVTs in Action

- Netherlands Flood
- De Haan - “Fighting the Arch-enemy with Mathematics”

Outline of Problem

- Height of sea defences
- 1953 storm surge - sea dike failure
- Need to recommend an appropriate sea defence level
- Flood probability reduced to small “p”
  - 1/10,000 to 1/100,000
- Extrapolation required

Data Analysis

- High tide water levels at 5 stations
  - 60 to 110 years of info
- “Set up” values
  - observed high tide level minus predicted high tide level
  - eliminate set up values below a threshold
  - utilise winter observations only
  - selection to achieve independence
- 15 observations per year
- Exceedance prob for 1953 storm = 1 / 382

Probability Theory

- Distribution function F
- X_1 to X_n are independent observations
- Find x_p so that $F(x_p) = 1 - p$
- Lim F_r $(a_r x+b_r)$
  $= \lim (P ((\max(X_1..X_r)-b_r/)a_r)<x) = G(x)$
- $G(x) = \exp (-(1+ \xi x)^{-1/\xi})$
- Generalized Extreme Value Distribution

Estimation Procedures

- Largest observations per winter
  - iid observations
  - G(x) formula as above
  - maximum likelihood
- Assume all set up levels above threshold L_n follow distribution $1-(1+yx/a(L_n))^{-1/y}$
  - Find y and $a(L_n)$ by maximum likelihood
  - Generalised Pareto distribution
  - Combine with astronomical (predicted) levels to get absolute sea levels

Mr De Haan's short cut

- Lim (U(tx)-U(t))/(U(tz)-U(t)) = $(x^{\xi}-1)/(z^{\xi}-1)$
  - where U = (1/(1-F)) - return period function
  - high quantiles can be expressed in terms of lower quantiles
- Estimation of ξ
  - Lim (U(2t)-U(t))/(U(t)-U(t/2)) = 2^{ξ}

A quick example

- Find ξ and return period for 5m high tide

Sea Level (cm)	Return Period (yrs)
50	4
100	10
200	30

A quick solution

- $(U(2t) - U(t)) / (U(t) - U(t/2)$
  $= (30 - 10) / (10 - 4) = 2^{\xi}$
- Whence $\xi = 1.74$
- $(U(5t) - U(t)) / (U(2t) - U(t))$
  $= (d - 10) / (30- 10) = (5^{\xi}–1) / (2^{\xi}–1)$
- Whence return period = 142 years

Output

- Level with exceedance probability 1/10,000
  - Xp
- Estimate of ξ
- $\xi = 0$
- X_p = 5.1m + sea level (reference level)
- 16 foot walls required!

Back to James…

Threshold Theory

- Why just use Maxima Data?
- Exceedance Over Threshold:
  - Generalised Pareto Distribution...
    …but Need to Choose Threshold
- Diagnostic Tool:
  - If GPD holds, the Mean Excess Plot…
    …is Linear for High Values of the Threshold

Generalised Pareto Distribution

$$\Pr(Y \leq y)$$

$$\approx 1 - \Pr(Y > u) \left[1 + \xi(y-u) / \sigma \right]_{+}^{-1/\xi}$$

$$= 1 - \lambda_u \left[1 + \xi(y-u) / \sigma \right]_{+}^{-1/\xi}$$

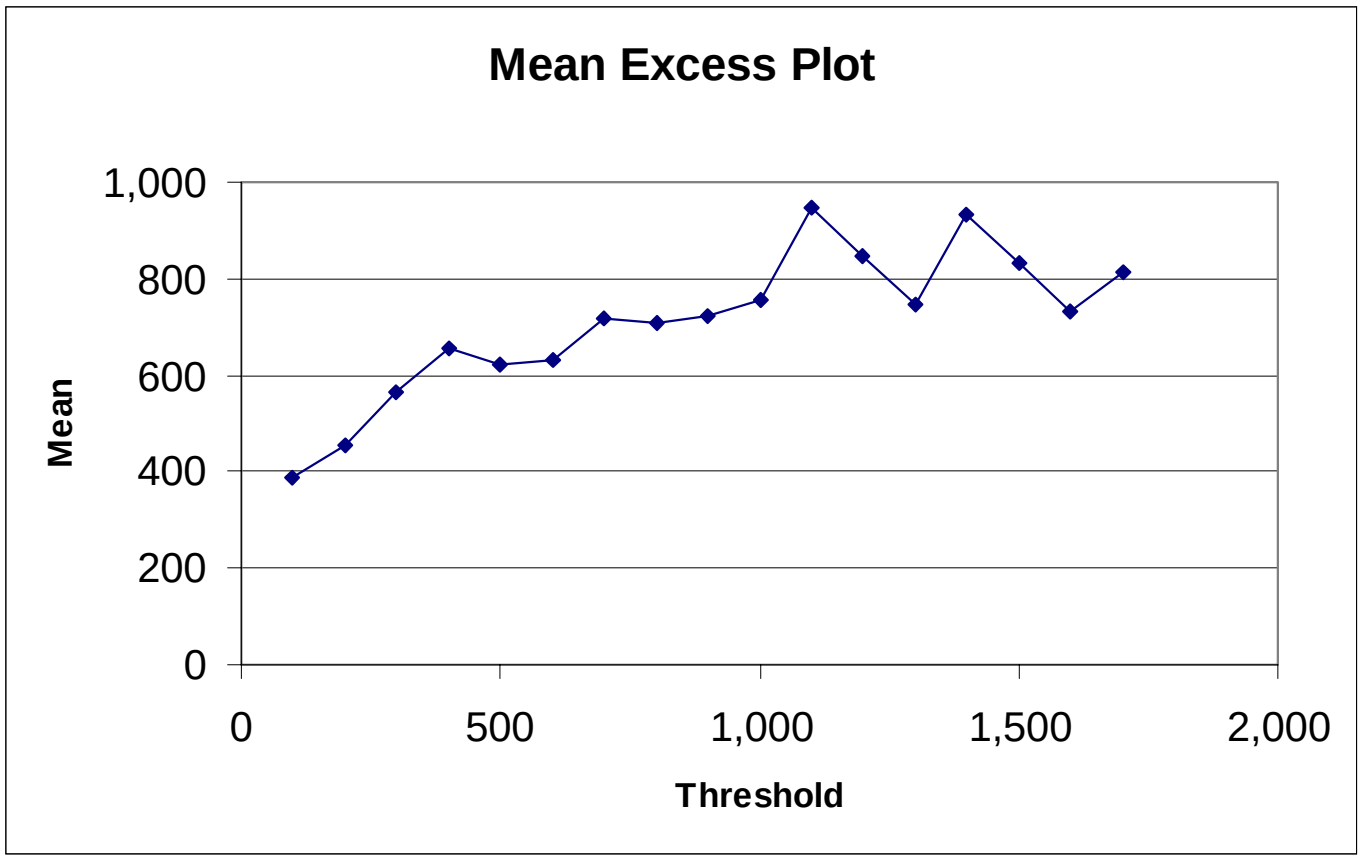
Motor XL Pricing Problem

- Issue - rating high XL layers
- Paucity of data
  - eg £10m xs £10m
  - largest single claim = £9.2m (BIWP 1999)
- Extrapolating from data
- Use data over certain threshold
- Fit curve (GPD)

Analysis

- Claims from 1985 - 1998 (146)
- Paid & Reported Losses
- Data Above Threshold
- Current / Largest Ever
- Revaluation
- Changing Exposure
- Claims by Band & Development of Losses

Excel Output



Problems, Problems, Problems...

- One-off events
  - Ogden, Woolf, Baremo
- Events that haven’t happened
  - Tokyo earthquake,Year 2000, UK flood
- Extreme Events
  - 2*90A, $50Bn US earthquake, US wind
- Man-made Trends
  - building on flood plains, global warming

Extreme Value Statistics

Questions