EXTREME VALUE TECHNIQUES
PART III: INCREASED LIMITS FACTORS (ILF) PRICING

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Extreme Value Techniques
Part III: Increased Limits Factors (ILF) Pricing

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Abstract. In order to further simplify the "Beta" pricing approach (described in *Extreme Value Techniques - Part I: Pricing High Excess Property and Casualty Layers* and *Extreme Value Techniques - Part II: Value Proposition for Fortune 500 Companies*), an alternative to existing increased limits factors (ILF) rating methodologies (see e.g., *Loss Distributions*, R. V. Hogg and S. A. Klugman, Wiley 1984) is developed in this short note, together with a corresponding pricing tool, the main objective being to make such a new ILF tool consistent with the "Beta" extreme value theory approach to risk quantification while maintaining much of the simplicity of the existing ILF methodologies. The Oil & Petrochemicals industry is used as an example.

Keywords. Extreme value theory, peaks-over-thresholds model, generalized pareto distribution, reference dataset, limited expected value function, mean residual life function, loss elimination ratio, excess ratio for retention, limited expected value comparison test.

Contents.

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1. Introduction

As a basis for increased limits factors (ILF) pricing/rating, Swiss Re International Business (IB) is for example currently using reference layers for the various industries (e.g., the oil and petrochemicals industry, the pharmaceuticals industry, etc.) that have been thoroughly analyzed by both the experienced IB underwriters and the Risk Management department (RM) with respect to: (1) size of risk (number of subsidiaries within and outside of the US), (2) geographical spread of sales (product liability), (3) exposure (premises and products), (4) scope of cover provided (e.g., trigger: claims-made, occurrence; pollution; any extras), (5) attachment point, (6) drop down provided, (7) aggregate limits / number of reinstatements and (8) loss experience / latency exposure. The actual case (client) under consideration is subsequently compared with the reference case(s) in the corresponding industry and the premium of the reference layer adjusted accordingly (e.g., increased by x%). The premium for the targeted layer, e.g., USD 1 million xs 1 million, of excess insurance coverage is then calculated from the adjusted premium of the reference layer, e.g., USD 1 million xs 0 million, with the increased limits factors (ILF, e.g., 20%, 25%, 30%, 40%, etc.) method:

$$P_{\text{target}} = P_{\text{reference}} \left(\frac{\text{target}}{\text{reference}} \right)^{a}$$

$$a = \frac{\log(1+\text{ILF})}{\log(2)}, \ \text{ILF} = 20\%, 25\%, 30\%, 40\%$$

Example: ILF = 25%, a = 0.32193, reference: USD 1 million xs 0 million (1 xs 0).

Target	Premium Factor	Target	Premium Factor
1 xs 1	1.250	1 xs 10	2.164
1 xs 2	1.424	1 xs 20	2.665
1 xs 3	1.563	1 xs 30	3.021
1 xs 4	1.679	1 xs 40	3.305
1 xs 5	1.780	1 xs 50	3.546

2. The "Beta" Insurance Coverage

The purpose of this short note and the corresponding EXCEL-based pricing tool is to make the above approach to ILF pricing/rating consistent with the "Beta" (Extreme Value Theory) approach to pricing/rating excess layers of property and casualty coverage (see List and Zilch [1]):

"Beta" provides multi-year, high-excess, broad form property and comprehensive general liability coverage with meaningful total limits for Fortune 500 clients in the Oil & Petrochemicals industry ("Beta" is also available in other Fortune 500 segments, its program parameters are industry-specific, however).

Coverage is provided at ***optimal layers*** within prescribed minimum and maximum per occurrence attachment points and per occurrence (i.e., each and every loss: E.E.L., see Fig. 1 below) and aggregate (AGG.) limits, split appropriately between property and casualty. These attachment points and limits are derived from the risk profiles and the needs of the insureds

(***Swiss Re's Value Proposition*** for the Oil & Petrochemicals industry, see Geosits, List and Lohner [2]).

The aggregate limits provide "Beta" ***base coverage*** for one year and over three years. Simply stated, if the base coverage is not pierced by a loss, then its full, substantial limits (USD 200M property and 100M casualty) stay in force over the entire three year "Beta" policy term.
Insureds might be concerned they would have no (or only a reduced) coverage if losses were to pierce the base coverage. Therefore, "Beta" includes a ***provision to reinstate*** all or a portion of the base coverage that is exhausted.

Lastly, the "Beta" design includes an ***option*** at the inception of the base coverage ***to extend*** its initial three year high-excess insurance coverage (i.e., the property and casualty base coverage and the provision for a single reinstatement of the base coverage) for an additional three year policy term at a predetermined price.

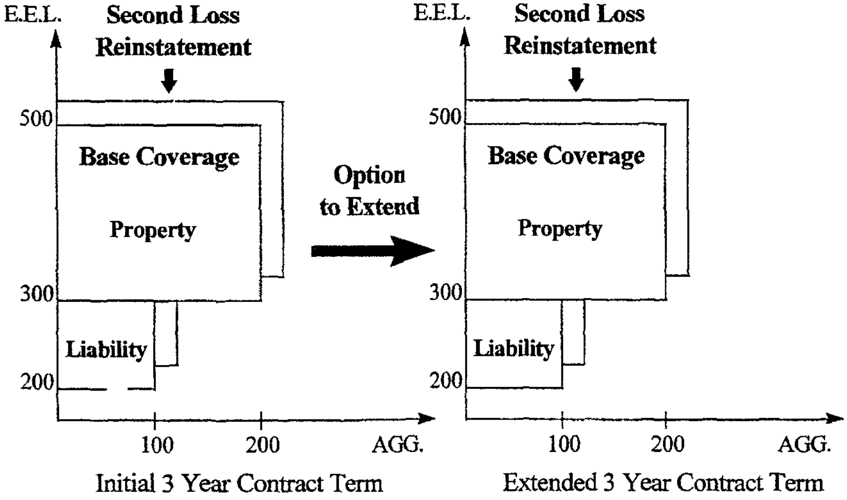

Fig. 1: The "Beta" Insurance Coverage for the Oil & Petrochemicals Industry

From Swiss Re's risk management point of view, optimal layers for "Beta" property and casualty excess coverages are defined as follows:

No annual loss should pierce the chosen property or casualty excess layer more frequently than once every four years (based both on the historical and scenario annual aggregate loss distributions). This translates into a 75% confidence that annual aggregate losses for a given layer of "Beta" coverage will equal zero.[1]

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[1] This optimality criterion is mainly derived from Swiss Re's perception (based upon an extensive Oil & Petrochemicals industry analysis) of a ***"Beta"*** or ***"catastrophic" event***. In the case of "Beta" programs with

The ***risk quantification process*** leading to the above optimal "Beta" layers for multi-year (i.e., three years) high-excess property and casualty Oil & Petrochemicals industry insurance coverage in principle follows standard actuarial tradition - however with some new elements:

The "Beta" implementation team (consisting of Swiss Re and ETH Zurich[2] personnel) has developed and implemented a consistent and stable (with respect to small perturbations in the input data) actuarial modelling approach for "Beta" high-excess property and casualty layers (see Fig. 2 below). This new methodology is based on ***Extreme Value Theory*** (***Peaks-Over-Thresholds Model***[3]) and fits a ***generalized Pareto distribution***[4] to the ***exceedances of a data-specific threshold.***

Once the frequency and severity distribution parameters are determined, ***per claim loss layers are selected and aggregate distributions both within the selected layers and excess of those layers up to the maximum potential individual loss (MPL) in the Oil & Petrochemicals industry (e.g., USD 3 billion for property and USD 4 billion for casualty) calculated.*** This procedure is repeated for sequential layers (usually chosen at the discretion of the underwriter

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combined single limits/deductibles, lower percentiles and thus shorter contract maturities may be preferable from a marketing point of view.

[2] The ETH Zurich "Beta" implementation team was lead by Prof. Dr. Hans Bühlmann, Prof. Dr. Paul Embrechts (Extreme Value Theory) and Prof. Dr. Freddy Delbaen ("Beta" Options).

[3] It has to be noted that ***claims histories are usually incomplete***, i.e., only losses in excess of a so-called ***displacement*** δ are reported. Let therefore (X_i) be an i.i.d. sequence of ground-up losses, (Y_i) be the associated loss amounts in the "Beta" layer $D \le x \le D + L$ and $Z = \sum_{i=1}^{N} Y_i$ the corresponding aggregate loss. Similarly, let $(\mathbb{X}_i)$, $\mathbb{X}_i = X_i 1_{X_i > \delta}$, be the losses greater than the displacement δ and $\mathbb{Z} = \sum_{i=1}^{\mathbb{N}} \mathbb{Y}_i$, $\mathbb{N} = \sum_{i=1}^{N} 1_{X_i > \delta}$, the corresponding "Beta" aggregate loss amount. Some elementary considerations then show that $F_Z \equiv F_{\mathbb{Z}}$ holds for the aggregate loss distributions, provided that $\delta < D$. The ***Peaks-Over-Thresholds Model (Pickands-Balkema-de Haan Theorem)*** on the other hand says that the ***exceedances of a high threshold*** $t < D$ are approximately $G_{\xi,t,\sigma}(x)$ distributed, where $G_{\xi,t,\sigma}(x)$ is the ***generalized Pareto distribution*** with ***shape*** ξ, ***location*** $t \equiv \mu$ and ***scale*** $\sigma > 0$. The threshold $t < D$ is chosen in such a way that in a neighbourhood of t the MLE-estimate of ξ (and therefore the "Beta" premium) remains reasonably stable (see Fig. 2).

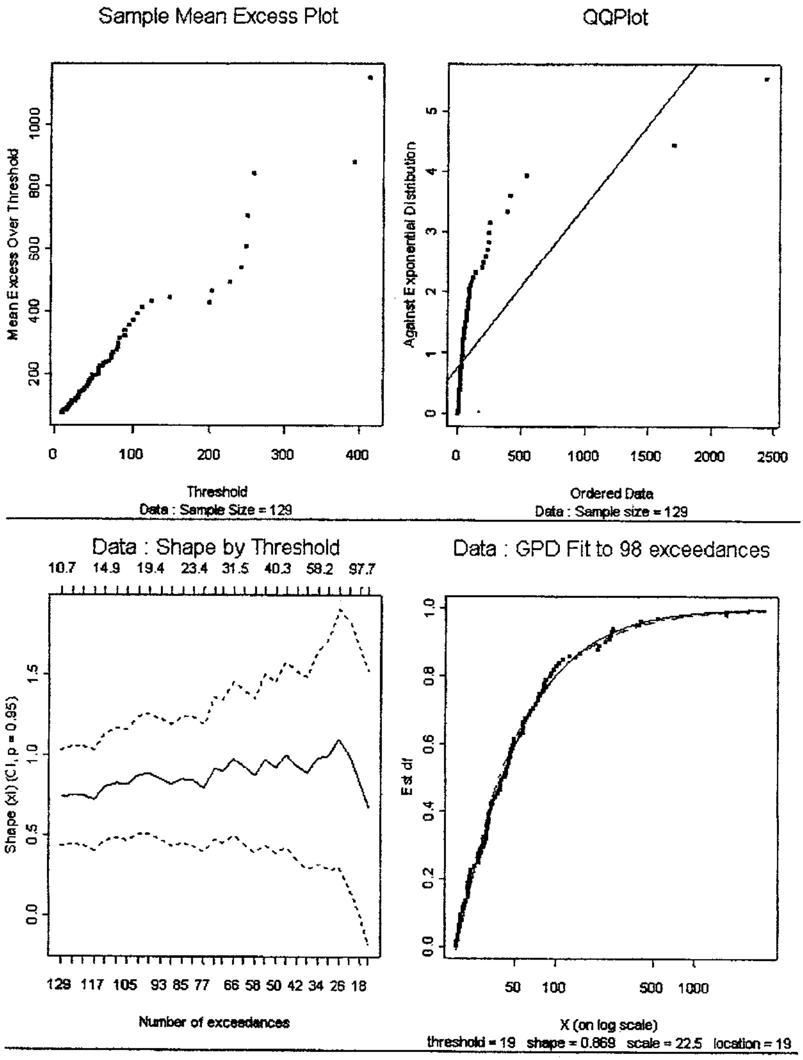
[4] The ***generalized Pareto distribution (GPD)*** is defined by

$$G_{\xi,\mu,\sigma}(x) = \begin{cases} 1 - \left(1 + \xi \dfrac{x-\mu}{\sigma}\right)^{-\frac{1}{\xi}} & \xi \ne 0 \\ 1 - e^{-\frac{x-\mu}{\sigma}} & \xi = 0 \end{cases}$$

where $x \ge \mu$ for $\xi \ge 0$ and $\mu \le x \le \mu - \dfrac{\sigma}{\xi}$ for $\xi < 0$. Compare this with the ***ordinary Pareto distribution (PD)***:

$$F_{\theta,a}(x) = 1 - \left(\frac{a}{x}\right)^{\theta}, \; x > a.$$

to approximate the anticipated "Beta" program structures reflecting the needs of the insureds or the entire Oil & Petrochemicals industry), thus mapping out the "Beta" ***risk potential***. The resulting probabilistic (excess-of-loss) profiles ("Beta" ***risk landscapes*** or ***risk maps***, see Fig. 3 below) can also be used for the securitization[5] of "Beta" portfolio components.



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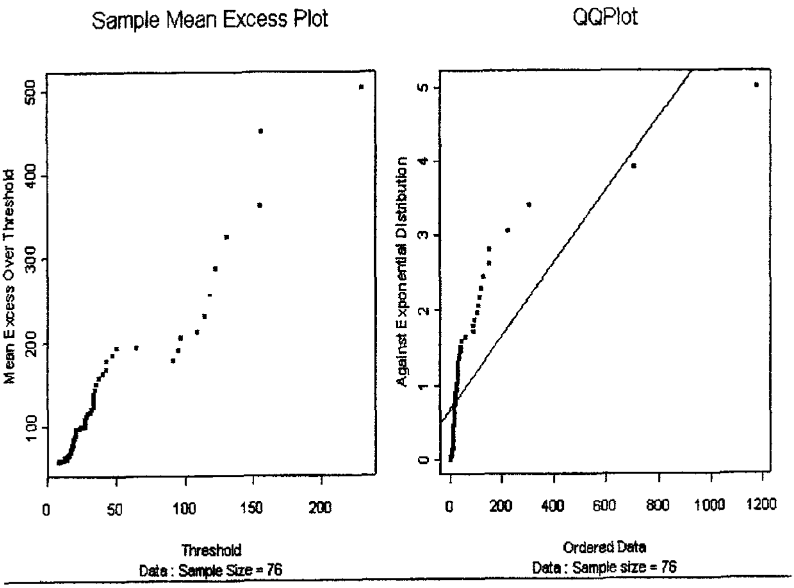
[5] From an actuarial standpoint, securitization is a modern capital markets alternative for traditional retrocession agreements (see also Davis and List [3]).

Fig. 2a: Oil & Petrochemicals Industry Severity Parameters (Property)
Solid Line: GPD, Dotted Line: PD



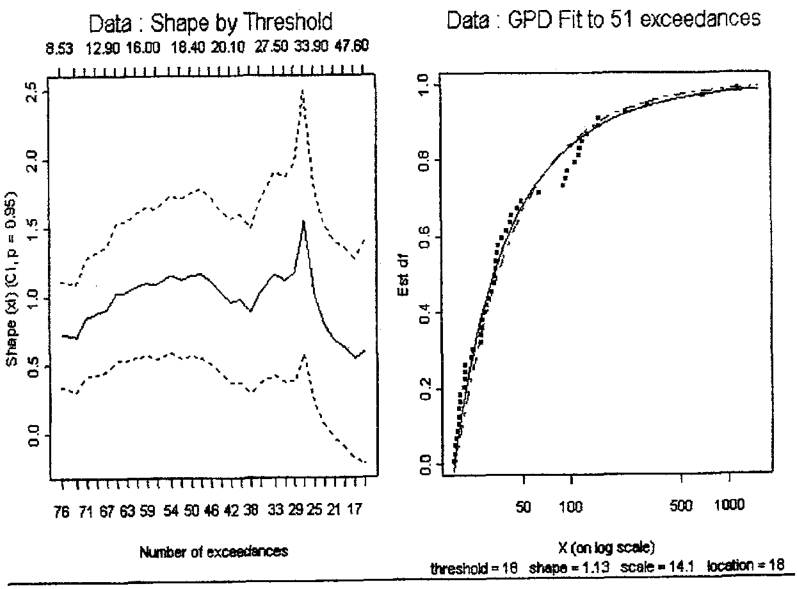
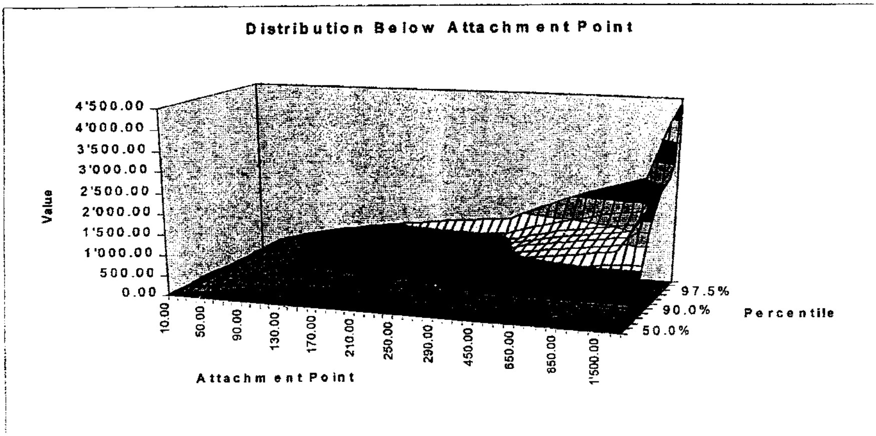


Fig. 2b: Oil & Petrochemicals Industry Severity Parameters (Casualty)
Solid Line: GPD, Dotted Line: PD



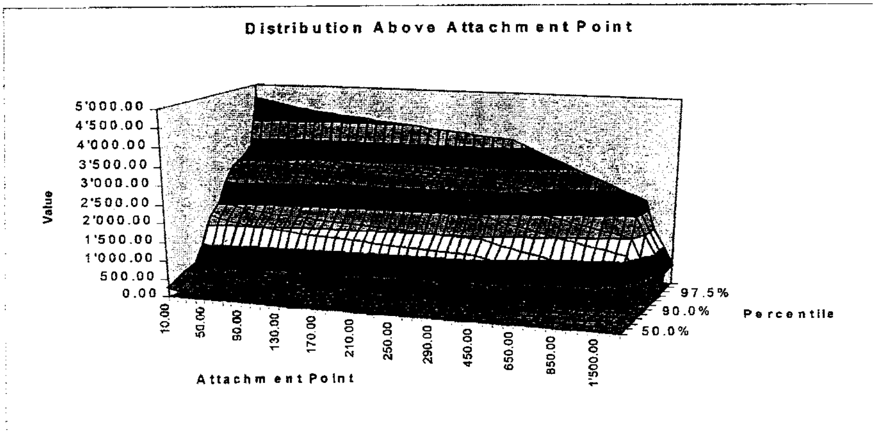


Fig. 3a: Oil & Petrochemicals Industry Risk Landscape[6] (Property)

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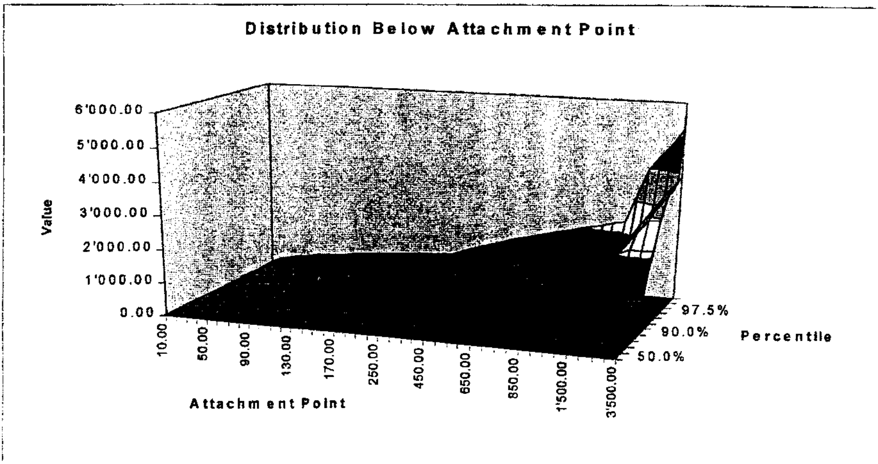
[6] The ***minimum layer width*** can be determined as follows: Consider the ***80th percentile*** in the risk map containing the one year aggregate loss distributions ***below*** the attachment points 10M, 20M, .., 100M, .., etc. (keeping in mind that this percentile indicates the ***expected maximum loss in the fourth year***) and start with the "Beta" attachment point of 300M, i.e., an expected one year aggregate loss of about 535M. Moving to the upper "Beta" E.E.L. coverage point of 500M (= 300M "Beta" attachment point + 200M "Beta" limit), we have an expected annual aggregate loss of about 630M. ***This means that the expected one year aggregate loss in the envisaged "Beta" property layer is about 95M (= 630M - 535M) or, in other words, the "Beta" property coverage (without reinstatement) absorbes two such expected losses on an E.E.L. and a 3 Y AGG. basis.*** This was according to an extensive analysis (carried out during the "Beta" product engineering process) of the risk preferences in the Oil & Petrochemicals industry Fortune 500 segment considered to be sufficient for catastrophic events causing property damage. ***Similarly, on the casualty side, it transpired that a "Beta" layer width of 100M was considered sufficient; the expected one year aggregate loss in the envisaged "Beta" casualty layer (i.e., 100M xs 200M) being 59M (= 371M - 312M),*** see List and Zilch [1] and Geosits, List and Lohner [2].



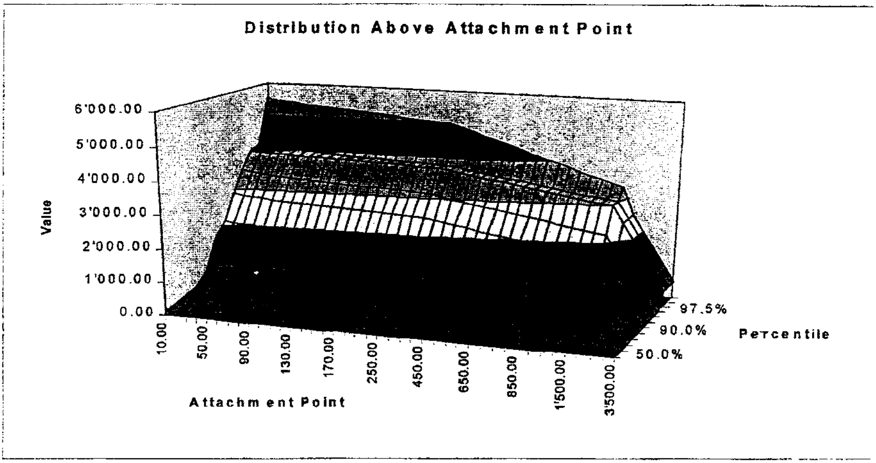


Fig. 3b: Oil & Petrochemicals Industry Risk Landscape[7] (Casualty)

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[7] The determination of standard layers (i.e., ***optimal SIRs and limits***) for "Beta" alternative risk transfer solutions in the Oil & Petrochemicals industry (a similar approach is used in the other "Beta" target industries) is very important for the quantification of Swiss Re's Value Proposition for corporate clients in the Fortune 500 group of companies. The Value Proposition argument itself would be as follows: (1) Optimal layers for "Beta" coverages are characterized by ***efficiency and cost transparency***, a ***high degree of structural flexibility to optimally fit client's asset liability management (ALM) needs*** (see List and Zilch [1], Geosits, List and Lohner [2], Davis and List [3] and Bühlmann, Bochiccio, Junod, List and Zinck [4]), ***significant capacity for property and casualty***, ***long-term stability (Swiss Re capacity)*** and ***high financial security (AAA capital base)***. (2) "Beta" is a genuine alternative risk transfer product that may also include ***sophisticated financial markets components*** (***balance sheet protection***, see Davis and List [3] and Bühlmann, Bochiccio, Junod, List and Zinck [4]) and a new element in the comprehensive range of Swiss Re's (re)insurance coverages and related services for Fortune 500 companies. Note that the "Beta" program also allows for property and casualty layers different from the standard layers (see List and Zilch [1] and Geosits, List and Lohner [2]).

3. ILF Pricing/Rating

Modelling "Beta" Loss Distributions. Following Hogg and Klugman [6], we first recall some important general actuarial (loss data) modelling concepts and their basic relationships:

(1) ***Empirical Limited Expected Value Function*** for Sample $x_1,..,x_i,..,x_n$

$$E_n(d)=\frac{1}{n}\sum_{i=1}^{n}\min(x_i,d)$$

(2) ***Limited Expected Value Function***

$$E[X;d]=\int_0^d x f_X(x)dx+d[1-F_X(d)]$$

(3) ***Empirical Mean Residual Life (= "Beta" Sample Mean Excess) Function*** for Sample $x_1,..,x_i,..,x_n$

$$e_n(d)=\frac{\sum_{i=1}^{n}\max(x_i-d,0)}{\sum_{i=1}^{n}1_{\{x_i>d\}}}$$

(4) ***Mean Residual Life Function***

$$e(d)=E[X-d|X\geq d]=\int_d^\infty (x-d)\frac{f_X(x)}{P(X\geq d)}dx$$

$$=\frac{\int_d^\infty[1-F_X(x)]dx}{1-F_X(d)},\text{ provided that }\lim_{x\uparrow\infty}\{(d-x)[1-F_X(x)]\}=0$$

(5) ***Loss Elimination Ratio***

$$L(d)=\frac{E[X;d]}{E[X]}$$

(6) ***Excess Ratio for Retention***

$$R(d)=\frac{e(d)}{E[X]}$$

(7) Basic Relationships

$$E[X]=E[X;d]+e(d)[1-F_X(d)]$$

$$L(d)+R(d)[1-F_X(d)]=1$$

(8) ***Limited Expected Value Comparison Test***[8] for Sample $x_1,..,x_i,..,x_n$

$$\frac{E[X;x_i]-E_n(x_i)}{E[X;x_i]}$$

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[8] The limited expected value (LEV) comparison test is an alternative goodness-of-fit test (especially suitable for excess-of-loss data) for the Pareto (PD) and generalized Pareto (GPD) distribution parameters determined with MLE (Extreme Value Theory) techniques.

P() is the given probability measure and $F_x(x) = P(X \leq x)$ the probability distribution function and $f_X(x) = \frac{dF_X(x)}{dx}$ the probability density function of random variable X.

"Beta" Increased Limits Factors (ILF) Pricing. The limited expected value function

$$E[X;d]=\int_0^d xf_X(x)dx+d[1-F_X(d)]$$

is a very useful tool for obtaining pure premiums for "Beta" layers:

1. The loss severity excess of a deductible d is

$$E[X-d|X>d]=\frac{E[X]-E[X;d]}{1-F_X(d)}.$$

If the frequency of a loss (prior to imposing the deductible) is p, then with the deductible, the loss frequency is $p[1-F_X(d)]$. The ***pure premium for excess losses*** is therefore

$$p[1-F_X(d)]\frac{E[X]-E[X;d]}{1-F_X(d)}=p(E[X]-E[X;d])$$

and the ***excess pure premium ratio*** consequently

$$\frac{E[X]-E[X;d]}{E[X]}$$

2. If we now introduce claims inflation at a constant rate r, then the loss frequency after inflation excess of the deductible d is $p\left[1-F_X\left(\frac{d}{1+r}\right)\right]$, the loss severity

$$\frac{(1+r)\left(E[X]-E\left[X;\frac{d}{1+r}\right]\right)}{1-F_X\left(\frac{d}{1+r}\right)}$$

and the pure premium for inflated excess losses consequently

$$p(1+r)\left(E[X]-E\left[X;\frac{d}{1+r}\right]\right)$$

Moving the deductible from d to $d(1+r)$ implies a pure premium for inflated excess losses of

$$p(1+r)(E[X]-E[X;d]).$$

3. Let u be the policy limit of an insurance contract. Then we have a loss severity of $E[X;u]$ and a pure premium of $pE[X;u]$. If $v>u$ is an increased limit, then (in terms of pure premium) the corresponding ***increased limits factor (ILF)*** is

$$\frac{E[X;v]}{E[X;u]}$$

Inflation as above leads to a claims severity of $(1+r)E\left[X;\frac{u}{1+r}\right]$ or of $(1+r)E[X;u]$ with an adjusted limit $u(1+r)$.

4. Finally, let d be the policy deductible and u be the policy limit of a "Beta" contract. Then the loss severity in the "Beta" layer $d\le X\le u$ is

$$\frac{E[X;u]-E[X;d]}{1-F_X(d)}$$

the corresponding loss frequency

$$p[1-F_X(d)].$$

and the ***pure "Beta" premium*** consequently

$$p(E[X;u]-E[X;d]).$$

Claims inflation at a constant rate r changes this to:

$$\frac{(1+r)\left(E\left[X;\frac{u}{1+r}\right]-E\left[X;\frac{d}{1+r}\right]\right)}{1-F_X\left(\frac{d}{1+r}\right)} \text{ (claims severity)}$$

$$p\left[1-F_X\left(\frac{d}{1+r}\right)\right] \text{ (claims frequency)}$$

$$p(1+r)\left(E\left[X;\frac{u}{1+r}\right]-E\left[X;\frac{d}{1+r}\right]\right) \text{ (pure "Beta" premium).}$$

"Beta" Severity of Loss Models. For "Beta" pricing/rating purposes, we consider the following two kinds of loss severity distributions:

1. ***Pareto Distribution (PD)***

$$F_X(x)=1-\left(\frac{a}{x}\right)^{\vartheta},\ x>a$$

$$f_X(x)=\frac{\vartheta a^{\vartheta}}{x^{\vartheta+1}}$$

2. ***Generalized Pareto Distribution (GPD)***

$$F_X(x)=\begin{cases}1-\left(1+\xi\frac{x-\mu}{\sigma}\right)^{-\frac{1}{\xi}} & \xi\neq 0\\ 1-e^{-\frac{x-\mu}{\sigma}} & \xi=0\end{cases}$$

$$\text{(where } x\geq\mu \text{ for } \xi\geq 0 \text{ and } \mu\leq x\leq\mu-\frac{\sigma}{\xi} \text{ for } \xi<0\text{)}$$

$$f_X(x)=\begin{cases}\frac{1}{\sigma}\left(1+\xi\frac{x-\mu}{\sigma}\right)^{-\frac{1+\xi}{\xi}} & \xi\neq 0\\ -\frac{1}{\sigma}e^{\frac{x-\mu}{\sigma}} & \xi=0\end{cases}$$

Using these execess-of-loss distribution models, the ILF techniques presented in this note can be implemented on a notebook computer with reasonable response times for both LEV comparison testing and ILF pricing. The system runs under Windows 3.1, 95, NT 3.51 and

NT 4.0 and is written in the programming languages Turbo Pascal / Delphi. Excel was chosen as the user interface.

```
{ compiler directives }
        {$M 8192,0}           { set local heap to zero }
library HBPRT5;
        { EXCEL interface to the increased limits factors (ILF) pricing module
          Hans-Fredo List / AM, Version 1.0, 28 February 1997 }
uses
        SHELL, GATE, EXCEL, Numbers, Vectors, Matrices, Integral;
const
        { loss history }
                NLO = 1;              { number of losses }
                MFR = 2;              { mean frequency }
                DIS = 3;              { displacement }
        { policy }
                DED = 1;              { deductible }
                LIM = 2;              { limit }
                ILI = 3;              { increased limit }
                INR = 4;              { inflation rate }
        { ILF pricing }
                LER = 1;              { loss elimination ratio }
                ERR = 2;              { excess ratio for retention }
                ILF = 3;              { increased limits factor }
                CSE = 4;              { claims severity }
                CFR = 5;              { claims frequency }
                CPR = 6;              { pure premium }
var
        { EXCEL/Pascal interface areas }
                PTA: APtr;            { LEV comparison test (PD) }
                GTA: APtr;            { LEV comparison test (GPD) }
                PRA: APtr;            { ILF pricing }
        { parameter matrices }
                SA: Boolean;          { successful allocation }
                PP: Matrix;           { PD parameters }
                GP: Matrix;           { GPD parameters }
                SL: Matrix;           { size of losses }
                NL: Matrix;           { number of losses }
                PO: Matrix;           { policy }
        { current exit procedure }
                CEP: Pointer;
{ exit procedure }
        procedure SMDE;
                { shut down the ILF pricing module on exit }
                begin
                        ExitProc := CEP;
                        { matrices }
                                MDisp(PP);
                                MDisp(GP);
                                MDisp(SL);
                                MDisp(NL);
                                MDisp(PO);
                                Matrices.Done;
                        { EXCEL/Pascal interface areas }
                                Dispose(PTA);
                                PTA := Nil;
                                Dispose(GTA);
```

```
                                GTA := Nil;
                                Dispose(PRA);
                                PRA := Nil
                end;
{ read parameters }
        procedure RDDP(EPP, EGP, ESL, ENL, EPO: APtr);
                begin
                        if SA then begin
                                Cast(EPP,PP,Column);
                                Cast(EGP,GP,Column);
                                Cast(ESL,SL,Column);
                                Cast(ENL,NL,Column);
                                Cast(EPO,PO,Column)
                        end
                end;
{ DIS and DEN functions }
        function PDIS(x: RNumber): RNumber;
                { PD distribution function }
                var
                        th, a: RNumber;
                begin
                        if SA then begin
                                Get(th,PP,1,1);
                                Get(a,PP,2,1);
                                if x > RMax(a,0) then
                                        PDIS := 1-Pw(a/x,th)
                                else
                                        PDIS := 0
                        end
                end;
        function GDIS(x: RNumber): RNumber;
                { GPD distribution function }
                var
                        sh, sc, lo: RNumber;
                begin
                        if SA then begin
                                Get(sh,GP,1,1);
                                Get(sc,GP,2,1);
                                Get(lo,GP,3,1);
                                if (sh > 0) and (x > RMax(lo,0)) then
                                        GDIS := 1-Pw(1+sh*((x-lo)/sc),-1/sh)
                                else
                                        GDIS := 0
                        end
                end;
        function PDEN(x: RNumber): RNumber;
                { PD density function }
                var
                        th, a: RNumber;
                begin
                        if SA then begin
                                Get(th,PP,1,1);
                                Get(a,PP,2,1);
                                if x > RMax(a,0) then
                                        PDEN := (th/x)*Pw(a/x,th)
                                else
                                        PDEN := 0
                        end
```

```
        end;
    function GDEN(x: RNumber): RNumber;
        { GPD density function }
        var
            sh, sc, lo: RNumber;
        begin
            if SA then begin
                Get(sh,GP,1,1);
                Get(sc,GP,2,1);
                Get(lo,GP,3,1);
                if (sh > 0) and (x > RMax(lo,0)) then
                    GDEN := (1/sc)*Pw(1+sh*((x-lo)/sc),-(1+sh)/sh)
                else
                    GDEN := 0
            end
        end;
{ LEV and EXP functions }
    function ELEV(d: RNumber): RNumber;
        { empirical LEV function }
        var
            n, i: INumber;
            x, xi, En: RNumber;
        begin
            if SA then begin
                Get(x,NL,NLO,1);
                n := IMin(Trunc(x),Rows(SL));
                En := 0;
                for i := 1 to n do begin
                    Get(xi,SL,i,1);
                    En := En+RMin(xi,d)
                end;
                En := En/n;
                ELEV := En
            end
        end;
    function PIntegrand(x: RNumber): RNumber;
        begin
            PIntegrand := x*PDEN(x)
        end;
    function GIntegrand(x: RNumber): RNumber;
        begin
            GIntegrand := x*GDEN(x)
        end;
    function PLEV(d: RNumber): RNumber;
        { PD LEV function }
        var
            a: RNumber;
        begin
            if SA then begin
                Get(a,PP,2,1);
                if d > RMax(a,0) then
                    PLEV := QRomb(PIntegrand,RMax(a,0),d)+d*(1-PDIS(d))
                else
                    PLEV := 0
            end
        end;
    function GLEV(d: RNumber): RNumber;
        { GPD LEV function }
```

```
        var
            lo: RNumber;
        begin
            if SA then begin
                Get(lo,GP,3,1);
                if d > RMax(lo,0) then
                    GLEV := QRomb(GIntegrand,RMax(lo,0),d)+d*(1-GDIS(d))
                else
                    GLEV := 0
            end
        end;
    function PEXP: RNumber;
        { PD expected value function }
        var
            a: RNumber;
        begin
            if SA then begin
                Get(a,PP,2,1);
                PEXP := QRomo(PIntegrand,RMax(a,0),MaxRN)
            end
        end;
    function GEXP: RNumber;
        { GPD expected value function }
        var
            lo: RNumber;
        begin
            if SA then begin
                Get(lo,GP,3,1);
                GEXP := QRomo(GIntegrand,RMax(lo,0),MaxRN)
            end
        end;
{ LEV comparison tests }
    function PLCT(EPP, EGP, ESL, ENL, EPO: APtr): APtr; export;
        { LEV comparison test (PD);
          EPP: PD parameters,
          EGP: GPD parameters,
          ESL: size of losses,
          ENL: number of losses,
          EPO: policy parameters }
        var
            n, i, j: INumber;
            x, xi, xj, Eni, Ei, ti: RNumber;
            PT: Matrix;
        begin
            { initialization }
                Register;
                MNew(PT,SA);
            { calculations }
                if SA then begin
                    { input quantities }
                        RDDP(EPP,EGP,ESL,ENL,EPO);
                    { output quantities }
                        Get(x,NL,NLO,1);
                        n := IMin(Trunc(x),Rows(SL));
                        Define(PT,n,1,Column);
                    { LEV comparison test }
                        for i := 1 to n do begin
                            Get(xi,SL,i,1);
```

```
                        Eni := 0;
                        for j := 1 to n do begin
                            Get(xj,SL,j,1);
                            Eni := Eni+RMin(xj,xi)
                        end;
                        Eni := Eni/n;
                        Ei := PLEV(xi);
                        ti := (Ei-Eni)/Ei;
                        Put(ti,PT,i,1)
                    end;
                { test results }
                    Map(PT,PTA);
                    MDisp(PT)
            end;
        PLCT := PTA
    end;
function GLCT(EPP, EGP, ESL, ENL, EPO: APtr): APtr; export;
    { LEV comparison test (GPD);
      EPP: PD parameters,
      EGP: GPD parameters,
      ESL: size of losses,
      ENL: number of losses,
      EPO: policy parameters }
    var
        n, i, j: INumber;
        x, xi, xj, Eni, Ei, ti: RNumber;
        GT: Matrix;
    begin
        { initialization }
            Register;
            MNew(GT,SA);
        { calculations }
            if SA then begin
                { input quantities }
                    RDDP(EPP,EGP,ESL,ENL,EPO);
                { output quantities }
                    Get(x,NL,NLO,1);
                    n := IMin(Trunc(x),Rows(SL));
                    Define(GT,n,1,Column);
                { LEV comparison test }
                    for i := 1 to n do begin
                        Get(xi,SL,i,1);
                        Eni := 0;
                        for j := 1 to n do begin
                            Get(xj,SL,j,1);
                            Eni := Eni+RMin(xj,xi)
                        end;
                        Fni := Eni/n;
                        Ei := GLEV(xi);
                        ti := (Ei-Eni)/Ei;
                        Put(ti,GT,i,1)
                    end;
                { test results }
                    Map(GT,GTA);
                    MDisp(GT)
            end;
        GLCT := GTA
    end;
```

```
{ ILF pricing }
    function ILFP(EPP, EGP, ESL, ENL, EPO: APtr): APtr; export;
        { ILF pricing (GPD and PD);
         EPP: PD parameters,
         EGP: GPD parameters,
         ESL: size of losses,
         ENL: number of losses,
         EPO: policy parameters }
        var
            n, p, d0, d, u, v, r: RNumber;
            PER, GER, PRR, GRR, PLF, GLF, PSE, GSE, PFR, GFR, PPR, GPR: RNumber;
            PR: Matrix;
        begin
            { initialization }
                Register;
                MNew(PR,SA);
            { calculations }
                if SA then begin
                    { input quantities }
                        RDDP(EPP,EGP,ESL,ENL,EPO);
                    { output quantities }
                        Define(PR,6,2,Row);
                    { ILF pricing }
                        Get(n,NL,NLO,1);
                        Get(p,NL,MFR,1);
                        Get(d0,NL,DIS,1);
                        Get(d,PO,DED,1);
                        Get(u,PO,LIM,1);
                        Get(v,PO,ILI,1);
                        Get(r,PO,INR,1);
                        PER := PLEV(d)/PEXP;
                        GER := GLEV(d)/GEXP;
                        PRR := (1-PER)/(1-PDIS(d));
                        GRR := (1-GER)/(1-GDIS(d));
                        PLF := PLEV(v)/PLEV(u);
                        GLF := GLEV(v)/GLEV(u);
                        PSE := ((1+r)*(PLEV(u/(1+r))-PLEV(d/(1+r))))/(1-PDIS(d/(1+r)));
                        GSE := ((1+r)*(GLEV(u/(1+r))-GLEV(d/(1+r))))/(1-GDIS(d/(1+r)));
                        PFR := p*((1-PDIS(d/(1+r)))/(1-PDIS(d0/(1+r))));
                        GFR := p*((1-GDIS(d/(1+r)))/(1-GDIS(d0/(1+r))));
                        PPR := PFR*PSE;
                        GPR := GFR*GSE;
                        Put(GER,PR,LER,1);
                        Put(PER,PR,LER,2);
                        Put(GRR,PR,ERR,1);
                        Put(PRR,PR,ERR,2);
                        Put(GLF,PR,ILF,1);
                        Put(PLF,PR,ILF,2);
                        Put(GSE,PR,CSE,1);
                        Put(PSE,PR,CSE,2);
                        Put(GFR,PR,CFR,1);
                        Put(PFR,PR,CFR,2);
                        Put(GPR,PR,CPR,1);
                        Put(PPR,PR,CPR,2);
                    { results }
                        Map(PR,PRA);
                        MDisp(PR)
                end;
```

```
                        ILFP := PRA
                end;
exports
        PLCT            index 1,
        GLCT            index 2,
        ILFP            index 3;
begin
        { access control }
                if (not Pass) or (not Open(MACF)) then
                        Lock;
                ACO := 0;
        { matrices }
                Matrices.Init;
                SA := True;
                MNew(PP,SA);
                MNew(GP,SA);
                MNew(SL,SA);
                MNew(NL,SA);
                MNew(PO,SA);
        { default areas }
                PTA := Nil;
                AIA(PTA);
                GTA := Nil;
                AIA(GTA);
                PRA := Nil;
                AIA(PRA);
        { exit procedure }
                CEP := ExitProc;
                ExitProc := @SMDE
end.

unit Integral;
        { basic integration as in Numerical Recipes in Pascal, p. 129 - 138
          Hans-Fredo List / AM, Version 1.0, 28 February 1997 }
interface
        uses
                Numbers, Vectors, Matrices;
        type
                RFunction = function(x: RNumber): RNumber;
        var
                TrapzdIt: INumber;
                MidinfIt: INumber;
        { Romberg integration }
                function QRomb(F: RFunction; a, b: RNumber): RNumber;
                        { integral of F(x) on the interval [a,b] }
        { improper integrals }
                function QRomo(F: RFunction; a, b: RNumber): RNumber;
                        { integral of F(x) on the interval (a,b), a*b > 0 }
implementation
        ....
begin
        ....
end.
```

For more information on the various modules called by library HBPRT5 and unit Integral, see the papers *Limited Risk Arbitrage Investment Management: Applications in Asset/Liability Management, Optimal Fund Design, Optimal Portfolio Selection and (Re)insurance Claims*

Portfolio Securitization - Part I: The LRA Paradigm and *Limited Risk Arbitrage Investment Management: Applications in Asset/Liability Management, Optimal Fund Design, Optimal Portfolio Selection and (Re)insurance Claims Portfolio Securitization - Part II: LRA Portfolio Management Systems* by M. H. A. Davis and H.-F. List, to appear in AFIR 1999. See also ***Towards a VP-based Client Solution Toolbox*** below.

4. Example "Beta" Layers

In the following paragraphs we shall outline a fairly complete set of reference data for the Oil & Petrochemicals industry with quite realistic Pareto (PD) and generalized Pareto (GPD) severity distribution parameters and excess-of-threshold Poisson or negative binomial frequency parameters. Based on this comprehensive industry analysis, we shall then perform LEV comparison tests for the property and casualty data used in List and Zilch [1] and Geosits, List and Lohner [2] as well as for tanker pollution data. Subsequently, we shall compare our ILF pricing with the EVT approach outlined in List and Zilch [1] using various property and casualty as well as tanker pollution layers. At the end of this note we shall then outline the structure of a sophisticated ***financial/(re)insurance toolbox*** that can be used to design future VP-based client solutions for Fortune 500 companies. Further details on this toolbox can be found in Davis and List [3] and in Bühlmann, Bochiccio, Junod, List and Zinck [4].

Oil & Petrochemicals Industry Reference Datasets. Using the Extreme Value Theory (EVT) techniques outlined in List and Zilch [1], the following is a fairly complete characterization of the Oil & Petrochemicals industry that can be used for pricing high-excess loss layers, for the quantification of Swiss Re's Value Proposition for corporate clients and for the securitization of entire (re)insurance portfolios.

A. Pareto Distributions (PD).

A1. Basic Scenario[9].

Basic Scenario (5% Property, 10% Casualty)								
Property				mean	std		theta	a
BP	Threshold	19.00	Frequency	4,90	3.45	Severity	0.9896	19.1869
EAP	Threshold	21.00	Frequency	5,10	3.34	Severity	0.9755	21.0090
Onshore								
BP	Threshold	15.00	Frequency	3,65	2.96	Severity	0.8607	15.7497
EAP	Threshold	18.00	Frequency	3,65	2.96	Severity	0.8607	18.2322
Offshore								
BP	Threshold	13.00	Frequency	2,00	1.30	Severity	1.0751	14.6972
EAP	Threshold	15.00	Frequency	2,00	1.30	Severity	1.0751	17.0139
Casualty								
BP	Threshold	18.00	Frequency	3.40	3.68	Severity	1.0787	18.3179
EAP	Threshold	24.00	Frequency	3.40	3.68	Severity	1.0787	24.3812

---

[9] The time periods 1997 to 1999 and 2000 to 2002 are called ***"Beta" base period (BP)*** and ***"Beta" extended agreement period (EAP)***, respectively (see List and Zilch [1]).

Basic Scenario (5% Property, 10% Liability)

				mean	std		theta	a
Fire & Explosion								
BP	Threshold	40.00	Frequency	17.05	7.53	Severity	1.4881	40.0243
EAP	Threshold	40.00	Frequency	19.50	8.16	Severity	1.3854	40.1643
Marine								
BP	Threshold	20.00	Frequency	8.50	2.74	Severity	1.1237	20.0110
EAP	Threshold	25.00	Frequency	7.82	2.92	Severity	1.1391	25.2540
Tanker Pollution								
BP	Threshold	0.10	Frequency	3.88	1.87	Severity	0.2765	0.1271
EAP	Threshold	0.20	Frequency	3.76	1.75	Severity	0.2874	0.2170

Basic Scenario (5% Property, 10% Liability)

				mean	std		theta	a
Property Damage								
BP	Threshold	4.00	Frequency	45.57	13.21	Severity	0.8255	4.0055
EAP	Threshold	4.00	Frequency	49.21	13.54	Severity	0.7944	4.0209
Business Interruption								
BP	Threshold	1.00	Frequency	36.57	16.22	Severity	0.5162	1.0197
EAP	Threshold	1.00	Frequency	37.57	16.31	Severity	0.4926	1.0192
Property Damage and Business Interruption								
BP	Threshold	4.00	Frequency	58.86	20.02	Severity	0.7457	4.0203
EAP	Threshold	5.00	Frequency	56.29	18.98	Severity	0.7543	5.0160
Offshore								
BP	Threshold	7.00	Frequency	7.90	5.78	Severity	0.8403	7.0800
EAP	Threshold	8.00	Frequency	8.00	5.74	Severity	0.8349	8.0104
General Liability								
BP	Threshold	1.00	Frequency	39.17	20.79	Severity	0.6900	1.0036
EAP	Threshold	1.00	Frequency	45.83	24.05	Severity	0.6647	1.0007
Product Liability								
BP	Threshold	0.80	Frequency	9.58	5.16	Severity	0.8361	0.8053
EAP	Threshold	1.00	Frequency	10.08	5.42	Severity	0.8315	1.0012
Employer's Liability								
BP	Threshold	0.70	Frequency	7.08	4.56	Severity	1.3982	0.7122
EAP	Threshold	0.90	Frequency	7.17	4.65	Severity	1.3780	0.9301
Automobile Liability								
BP	Threshold	0.70	Frequency	6.25	3.93	Severity	1.1408	0.7133
EAP	Threshold	1.00	Frequency	6.08	3.82	Severity	1.1983	1.0139
Marine Liability								
BP	Threshold	0.40	Frequency	1.92	1.78	Severity	0.4131	0.4286
EAP	Threshold	0.60	Frequency	1.83	1.80	Severity	0.4334	0.7133
All Liability Claims								
BP	Threshold	1.50	Frequency	45.92	24.30	Severity	0.8146	1.5005
EAP	Threshold	1.50	Frequency	53.00	26.61	Severity	0.7522	1.5028

A2. Adjustment Scenario[10].

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[10] To make this presentation simple, we only consider the ***basic scenario*** and an ***adjustment scenario*** (see List and Zilch [1] for more details on the ***general classes of "Beta" threat scenarios*** identified).

Adjustment Scenario (10% Property, 20% Casualty)

Property				mean	std		theta	a
BP	Threshold	32.00	Frequency	5.90	3.65	Severity	0.9506	32.3534
EAP	Threshold	40.00	Frequency	6.10	3.70	Severity	0.9220	40.2030
Onshore								
BP	Threshold	30.00	Frequency	3.80	2.78	Severity	0.8295	30.2051
EAP	Threshold	40.00	Frequency	3.80	2.78	Severity	0.8295	40.2030
Offshore								
BP	Threshold	33.00	Frequency	2.20	1.54	Severity	1.2148	33.3544
EAP	Threshold	44.00	Frequency	2.20	1.54	Severity	1.2148	44.3948
Casualty								
BP	Threshold	44.00	Frequency	3.47	3.60	Severity	1.1017	44.0356
EAP	Threshold	70.00	Frequency	3.53	3.68	Severity	1.0569	71.8994

Adjustment Scenario (10% Property, 20% Liability)

Fire & Explosion				mean	std		theta	a
BP	Threshold	65.00	Frequency	18.95	5.57	Severity	1.2435	65.0872
EAP	Threshold	75.00	Frequency	21.18	6.83	Severity	1.1674	75.0256
Marine								
BP	Threshold	35.00	Frequency	9.05	2.98	Severity	1.0022	35.3692
EAP	Threshold	45.00	Frequency	9.36	2.85	Severity	0.9923	45.0154
Tanker Pollution								
BP	Threshold	0.20	Frequency	4.00	1.80	Severity	0.2557	0.2100
EAP	Threshold	0.40	Frequency	3.88	1.76	Severity	0.2606	0.4404

Adjustment Scenario (10% Property, 20% Liability)								
Property Damage				mean	std		theta	a
BP	Threshold	4.00	Frequency	60.64	19.06	Severity	0.7407	4.0140
EAP	Threshold	5.00	Frequency	62.36	19.33	Severity	0.7284	5.0273
Business Interruption								
BP	Threshold	2.00	Frequency	34.93	15.64	Severity	0.5451	2.0127
EAP	Threshold	2.00	Frequency	37.86	16.58	Severity	0.5071	2.0016
Property Damage and Business Interruption								
BP	Threshold	6.00	Frequency	62.43	20.18	Severity	0.7374	6.0415
EAP	Threshold	8.00	Frequency	62.43	20.18	Severity	0.7374	8.0412
Offshore								
BP	Threshold	12.00	Frequency	8.70	6.52	Severity	0.7258	12.1554
EAP	Threshold	17.00	Frequency	8.60	6.51	Severity	0.7480	17.1245
General Liability								
BP	Threshold	2.00	Frequency	40.83	24.55	Severity	0.6728	2.0043
EAP	Threshold	3.00	Frequency	43.75	25.87	Severity	0.6621	3.0493
Product Liability								
BP	Threshold	1.00	Frequency	11.67	7.19	Severity	0.6849	1.0235
EAP	Threshold	2.00	Frequency	11.08	6.69	Severity	0.7240	2.0615
Employer's Liability								
BP	Threshold	1.20	Frequency	7.50	5.68	Severity	1.0683	1.2078
EAP	Threshold	2.20	Frequency	7.42	5.57	Severity	1.1229	2.2074
Automobile Liability								
BP	Threshold	1.00	Frequency	7.08	4.23	Severity	0.8888	1.0378
EAP	Threshold	2.00	Frequency	6.75	4.16	Severity	1.0020	2.1499
Marine Liability								
BP	Threshold	0.80	Frequency	1.83	1.80	Severity	0.4334	1.0750
EAP	Threshold	2.00	Frequency	1.75	1.76	Severity	0.4373	2.1160
All Liability Claims								
BP	Threshold	1.60	Frequency	68.33	39.05	Severity	0.7036	1.6011
EAP	Threshold	3.00	Frequency	65.42	37.53	Severity	0.7148	3.0083

B. Generalized Pareto Distributions (GPD).

This is the ***main distribution model used for excess-of-loss claims*** (see List and Zilch [1] for details).

B1. Basic Scenario.

Basic Scenario (5% Property, 10% Casualty)									
Property				mean	std		shape	scale	location
BP	Threshold	19.00	Frequency	4.90	3.45	Severity	0.8690	22.5000	19.0000
EAP	Threshold	21.00	Frequency	5.10	3.34	Severity	0.8710	25.0000	21.0000
Onshore									
BP	Threshold	15.00	Frequency	3.65	2.96	Severity	0.8430	25.7000	15.0000
EAP	Threshold	18.00	Frequency	3.65	2.96	Severity	0.8790	28.0000	18.0000
Offshore									
BP	Threshold	13.00	Frequency	2.00	1.30	Severity	0.5280	22.0000	13.0000
EAP	Threshold	15.00	Frequency	2.00	1.30	Severity	0.5250	25.5000	15.0000
Casualty									
BP	Threshold	18.00	Frequency	3.40	3.68	Severity	1.1300	14.1000	18.0000
EAP	Threshold	24.00	Frequency	3.40	3.68	Severity	1.1300	18.6000	24.0000

Basic Scenario (5% Property, 10% Liability)									
Fire & Explosion				mean	std		shape	scale	location
BP	Threshold	40.00	Frequency	17.05	7.53	Severity	0.5960	29.0000	40.0000
EAP	Threshold	40.00	Frequency	19.50	8.16	Severity	0.5240	35.2000	40.0000
Marine									
BP	Threshold	20.00	Frequency	8.50	2.74	Severity	0.6450	22.5000	20.0000
EAP	Threshold	25.00	Frequency	7.82	2.92	Severity	0.6220	28.6000	25.0000
Tanker Pollution									
BP	Threshold	0.10	Frequency	3.88	1.87	Severity	2.3148	1.6210	0.1000
EAP	Threshold	0.20	Frequency	3.76	1.75	Severity	2.3118	2.3253	0.2000

Basic Scenario (5% Property, 10% Liability)									
Property Damage				mean	std		shape	scale	location
BP	Threshold	4.00	Frequency	45.57	13.21	Severity	0.8616	6.7768	4.0000
EAP	Threshold	4.00	Frequency	49.21	13.54	Severity	0.8502	7.4703	4.0000
Business Interruption									
BP	Threshold	1.00	Frequency	36.57	16.22	Severity	1.0297	4.5814	1.0000
EAP	Threshold	1.00	Frequency	37.57	16.31	Severity	1.0216	5.2165	1.0000
Property Damage and Business Interruption									
BP	Threshold	4.00	Frequency	58.86	20.02	Severity	0.9112	8.1326	4.0000
EAP	Threshold	5.00	Frequency	56.29	18.98	Severity	0.8952	10.0265	5.0000
Offshore									
BP	Threshold	7.00	Frequency	7.90	5.78	Severity	0.8548	11.7138	7.0000
EAP	Threshold	8.00	Frequency	8.00	5.74	Severity	0.8668	13.1895	8.0000
General Liability									
BP	Threshold	1.00	Frequency	39.17	20.79	Severity	0.9429	2.3502	1.0000
EAP	Threshold	1.00	Frequency	45.83	24.05	Severity	1.0399	2.3425	1.0000
Product Liability									
BP	Threshold	0.80	Frequency	9.58	5.16	Severity	1.2525	0.9149	0.8000
EAP	Threshold	1.00	Frequency	10.08	5.42	Severity	1.2465	1.1531	1.0000
Employer's Liability									
BP	Threshold	0.70	Frequency	7.08	4.56	Severity	0.5079	0.6361	0.7000
EAP	Threshold	0.90	Frequency	7.17	4.65	Severity	0.4673	0.8988	0.9000
Automobile Liability									
BP	Threshold	0.70	Frequency	6.25	3.93	Severity	0.4566	0.9447	0.7000
EAP	Threshold	1.00	Frequency	6.08	3.82	Severity	0.5173	1.1645	1.0000
Marine Liability									
BP	Threshold	0.40	Frequency	1.92	1.78	Severity	1.9624	1.6435	0.4000
EAP	Threshold	0.60	Frequency	1.83	1.80	Severity	1.9322	2.4962	0.6000
All Liability Claims									
BP	Threshold	1.50	Frequency	45.92	24.30	Severity	1.0649	2.1609	1.5000
EAP	Threshold	1.50	Frequency	53.00	26.61	Severity	0.9877	2.7712	1.5000

B2. Adjustment Scenario.

Adjustment Scenario (10% Property, 20% Casualty)									
Property				mean	std		shape	scale	location
BP	Threshold	32.00	Frequency	5.90	3.65	Severity	0.7830	44.5000	32.0000
EAP	Threshold	40.00	Frequency	6.10	3.70	Severity	0.7650	59.3000	40.0000
Onshore									
BP	Threshold	30.00	Frequency	3.80	2.78	Severity	0.7990	53.6000	30.0000
EAP	Threshold	40.00	Frequency	3.80	2.78	Severity	0.8010	71.1000	40.0000
Offshore									
BP	Threshold	33.00	Frequency	2.20	1.54	Severity	0.6890	31.7000	33.0000
EAP	Threshold	44.00	Frequency	2.20	1.54	Severity	0.6930	41.9000	44.0000
Casualty									
BP	Threshold	44.00	Frequency	3.47	3.60	Severity	1.2500	28.1000	44.0000
EAP	Threshold	70.00	Frequency	3.53	3.68	Severity	1.0300	64.1000	70.0000

Adjustment Scenario (10% Property, 20% Liability)									
Fire & Explosion				mean	std		shape	scale	location
BP	Threshold	65.00	Frequency	18.95	5.57	Severity	0.4570	72.2000	65.0000
EAP	Threshold	75.00	Frequency	21.18	6.83	Severity	0.4430	93.8000	75.0000
Marine									
BP	Threshold	35.00	Frequency	9.05	2.98	Severity	0.5560	53.4000	35.0000
EAP	Threshold	45.00	Frequency	9.36	2.85	Severity	0.5720	67.8000	45.0000
Tanker Pollution									
BP	Threshold	0.20	Frequency	4.00	1.80	Severity	2.4526	3.2947	0.2000
EAP	Threshold	0.40	Frequency	3.88	1.76	Severity	2.2916	7.3349	0.4000

Adjustment Scenario (10% Property, 20% Liability)									
Property Damage				mean	std		shape	scale	location
BP	Threshold	4.00	Frequency	60.64	19.06	Severity	0.8933	8.3470	4.0000
EAP	Threshold	5.00	Frequency	62.36	19.33	Severity	0.8901	10.8885	5.0000
Business Interruption									
BP	Threshold	2.00	Frequency	34.93	15.64	Severity	1.0385	7.7504	2.0000
EAP	Threshold	2.00	Frequency	37.86	16.58	Severity	1.0752	9.0206	2.0000
Property Damage and Business Interruption									
BP	Threshold	6.00	Frequency	62.43	20.18	Severity	0.9350	12.2795	6.0000
EAP	Threshold	8.00	Frequency	62.43	20.18	Severity	0.9371	16.2955	8.0000
Offshore									
BP	Threshold	12.00	Frequency	8.70	6.52	Severity	0.6309	32.9695	12.0000
EAP	Threshold	17.00	Frequency	8.60	6.51	Severity	0.6507	42.8175	17.0000
General Liability									
BP	Threshold	2.00	Frequency	40.83	24.55	Severity	0.9951	4.7551	2.0000
EAP	Threshold	3.00	Frequency	43.75	25.87	Severity	1.0209	7.3959	3.0000
Product Liability									
BP	Threshold	1.00	Frequency	11.67	7.19	Severity	1.1106	2.1113	1.0000
EAP	Threshold	2.00	Frequency	11.08	6.69	Severity	1.1514	3.6271	2.0000
Employer's Liability									
BP	Threshold	1.20	Frequency	7.50	5.68	Severity	0.5229	1.6734	1.2000
EAP	Threshold	2.20	Frequency	7.42	5.57	Severity	0.5878	2.6350	2.2000
Automobile Liability									
BP	Threshold	1.00	Frequency	7.08	4.23	Severity	0.2865	2.4522	1.0000
EAP	Threshold	2.00	Frequency	6.75	4.16	Severity	0.3049	4.1517	2.0000
Marine Liability									
BP	Threshold	0.80	Frequency	1.83	1.80	Severity	1.9655	3.7870	0.8000
EAP	Threshold	2.00	Frequency	1.75	1.76	Severity	2.1018	5.9336	2.0000
All Liability Claims									
BP	Threshold	1.60	Frequency	68.33	39.05	Severity	1.0932	3.1196	1.6000
EAP	Threshold	3.00	Frequency	65.42	37.53	Severity	1.0911	5.6706	3.0000

C. Optimal Attachment Points (SIRs).

C1. Basic Scenario.

Basic Scenario (5% Property, 10% Casualty)		
Property		
BP	Opt. Attachment Point	300.00
EAP	Opt. Attachment Point	350.00
Onshore		
BP	Opt. Attachment Point	250.00
EAP	Opt. Attachment Point	290.00
Offshore		
BP	Opt. Attachment Point	90.00
EAP	Opt. Attachment Point	110.00
Casualty		
BP	Opt. Attachment Point	250.00
EAP	Opt. Attachment Point	300.00

Basic Scenario (5% Property, 10% Liability)		
Fire & Explosion		
BP	Opt. Attachment Point	550.00
EAP	Opt. Attachment Point	600.00
Marine		
BP	Opt. Attachment Point	300.00
EAP	Opt. Attachment Point	350.00
Tanker Pollution		
BP	Opt. Attachment Point	300.00
EAP	Opt. Attachment Point	400.00

Basic Scenario (5% Property, 10% Liability)		
Property Damage		
BP	Opt. Attachment Point	650.00
EAP	Opt. Attachment Point	700.00
Business Interruption		
BP	Opt. Attachment Point	650.00
EAP	Opt. Attachment Point	750.00
Property Damage and Business Interruption		
BP	Opt. Attachment Point	1500.00
EAP	Opt. Attachment Point	1500.00
Offshore		
BP	Opt. Attachment Point	230.00
EAP	Opt. Attachment Point	270.00
General Liability		
BP	Opt. Attachment Point	300.00
EAP	Opt. Attachment Point	450.00
Product Liability		
BP	Opt. Attachment Point	60.00
EAP	Opt. Attachment Point	80.00
Employer's Liability		
BP	Opt. Attachment Point	10.00
EAP	Opt. Attachment Point	10.00
Automobile Liability		
BP	Opt. Attachment Point	10.00
EAP	Opt. Attachment Point	10.00
Marine Liability		
BP	Opt. Attachment Point	40.00
EAP	Opt. Attachment Point	50.00
All Liability Claims		
BP	Opt. Attachment Point	450.00
EAP	Opt. Attachment Point	500.00

C2. Adjustment Scenario.

Adjustment Scenario (10% Property, 20% Casualty)		
Property		
BP	Opt. Attachment Point	600.00
EAP	Opt. Attachment Point	800.00
Onshore		
BP	Opt. Attachment Point	500.00
EAP	Opt. Attachment Point	700.00
Offshore		
BP	Opt. Attachment Point	180.00
EAP	Opt. Attachment Point	240.00
Casualty		
BP	Opt. Attachment Point	550.00
EAP	Opt. Attachment Point	850.00

Adjustment Scenario (10% Property, 20% Liability)		
Fire & Explosion		
BP	Opt. Attachment Point	1000.00
EAP	Opt. Attachment Point	1500.00
Marine		
BP	Opt. Attachment Point	600.00
EAP	Opt. Attachment Point	800.00
Tanker Pollution		
BP	Opt. Attachment Point	900.00
EAP	Opt. Attachment Point	1500.00

Adjustment Scenario (10% Property, 20% Liability)		
Property Damage		
BP	Opt. Attachment Point	1500.00
EAP	Opt. Attachment Point	1500.00
Business Interruption		
BP	Opt. Attachment Point	1500.00
EAP	Opt. Attachment Point	2000.00
Property Damage and Business Interruption		
BP	Opt. Attachment Point	2500.00
EAP	Opt. Attachment Point	> 2500.00
Offshore		
BP	Opt. Attachment Point	450.00
EAP	Opt. Attachment Point	600.00
General Liability		
BP	Opt. Attachment Point	700.00
EAP	Opt. Attachment Point	1500.00
Product Liability		
BP	Opt. Attachment Point	120.00
EAP	Opt. Attachment Point	250.00
Employer's Liability		
BP	Opt. Attachment Point	20.00
EAP	Opt. Attachment Point	30.00
Automobile Liability		
BP	Opt. Attachment Point	20.00
EAP	Opt. Attachment Point	30.00
Marine Liability		
BP	Opt. Attachment Point	80.00
EAP	Opt. Attachment Point	130.00
All Liability Claims		
BP	Opt. Attachment Point	1500.00
EAP	Opt. Attachment Point	2000.00

Property and Casualty Layers. Recall from List and Zilch [1] and Geosits, List and Lohner [2] that the ***"Beta" standard layers***

USD 200M xs 300M Property
USD 100M xs 200M Casualty

implement Swiss Re's Value Proposition for Fortune 500 clients in the Oil & Petrochemicals industry: the associated "Beta" risk maps (see Fig. 3 above) indicate the ***optimal self-insured retentions (SIRs, = optimal "Beta" attachment points)*** for such corporates. The EVT techniques outlined in List and Zilch [1] then lead to the following pure premiums (one year premium, base period, basic scenario):

	GPD	PD
USD 200M xs 300M Property	42.2000	49.5500
USD 100M xs 200M Casualty	24.8000	20.5920

The ILF techniques developed in this note confirm these premium indications and also show a very good fit (as measured by the LEV comparison test) of both the Pareto distribution (PD) and the generalized Pareto distribution (GPD) to the loss data:

A. Property.

Policy	
Deductible:	300.0000
Limit:	500.0000
Increased Limit:	700.0000
Inflation Rate:	0.00%

ILF Pricing	**GPD**	**PD**
Loss Elimination Ratio:	**0.4174**	**0.2142**
Excess Ratio for Retention:	**10.0253**	**11.9395**
Increased Limits Factor:	**1.0661**	**1.0808**
Claims Severity:	**148.2741**	**153.6096**
Claims Frequency:	**0.2848**	**0.3225**
Pure Premium:	**42.2243**	**49.5355**

Scale:	1'000'000	**No. of Losses:**	129
		Mean Frequency:	4.9000
		Threshold:	19.0000
		Period:	BP

Limited Expected Value Comparison Test

Generalized Pareto (GPD)		**Pareto (PD)**	
Shape:	0.8690	Theta:	0.9896
Scale:	22.5000	a:	19.1869
Location:	19.0000		

Year of Loss	**Size of Loss**	**LEV Comparison Test**	**Year of Loss**	**Size of Loss**	**LEV Comparison Test**

[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	###############
[illegible]	[illegible]	###############
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	###############
[illegible]	[illegible]	###############
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	###############
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	0.1150
1975	[illegible]	[illegible]
1975	25.8069	0.1037
[illegible]	24.0715	0.0946
[illegible]	21.2488	0.0767
1975	19.5721	0.0605
[illegible]	17.4967	###############
1976	40.2896	[illegible]
1976	36.1357	0.1290
1976	32.8741	0.1215
1976	23.6098	0.0918
1976	23.4489	0.0908
1976	19.7631	0.0640
1976	19.6402	0.0629
1976	15.6121	###############
1976	12.5143	###############
1976	12.4675	###############
1976	11.3412	###############
1977	12.3307	###############
1977	20.5515	0.0692
1977	19.1869	0.0588
1977	12.3975	###############
1977	[illegible]	[illegible]
1977	101.5678	0.1777
1978	[illegible]	0.1527
1978	[illegible]	0.1227
1978	18.3714	###############
1978	[illegible]	###############
1978	14.5082	###############
1978	[illegible]	###############
1978	[illegible]	###############
1979	[illegible]	[illegible]
1979	[illegible]	[illegible]
1979	[illegible]	###############
1979	[illegible]	###############
1980	[illegible]	[illegible]
1980	[illegible]	[illegible]
1980	[illegible]	[illegible]
1980	[illegible]	[illegible]
1980	[illegible]	[illegible]
1980	[illegible]	[illegible]
1980	[illegible]	[illegible]
1980	[illegible]	0.1409

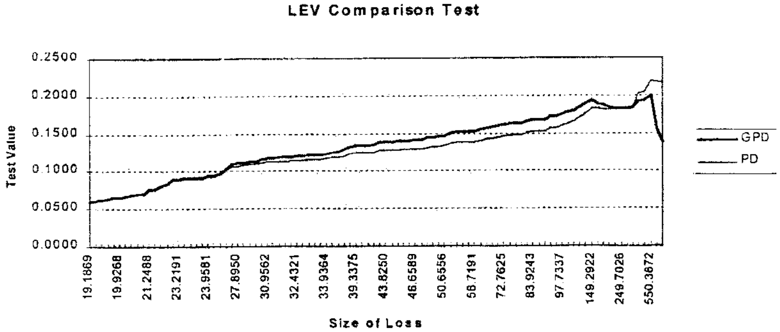
1972	[illegible]	0.0930
[illegible]	[illegible]	###############
[illegible]	[illegible]	###############
[illegible]	[illegible]	0.1383
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	###############
[illegible]	[illegible]	###############
[illegible]	[illegible]	0.1820
[illegible]	[illegible]	0.1317
[illegible]	[illegible]	###############
1975	49.7026	0.1810
1975	[illegible]	0.1807
1975	[illegible]	0.1137
1975	28.7865	0.1091
1975	28.7402	0.1090
1975	25.8069	0.1017
1975	24.0715	0.0936
1975	21.2488	0.0768
1975	19.5721	0.0606
1975	17.4967	###############
1976	40.2896	0.1253
1976	36.1357	0.1210
1976	32.8741	0.1152
1976	23.6098	0.0910
1976	23.4489	0.0901
1976	19.7631	0.0642
1976	19.6402	0.0631
1976	15.6121	###############
1976	12.5143	###############
1976	12.4675	###############
1976	11.3412	###############
1977	12.3307	###############
1977	20.5515	0.0694
1977	19.1869	0.0588
1977	12.3975	###############
1977	[illegible]	0.1132
1977	101.5678	0.1630
1978	[illegible]	0.1384
1978	[illegible]	0.1161
1978	18.3714	###############
1978	[illegible]	###############
1978	14.5082	###############
1978	11.6294	###############
1978	[illegible]	###############
1979	[illegible]	0.1662
1979	[illegible]	0.1074
1979	[illegible]	###############
1979	[illegible]	###############
1980	[illegible]	0.1831
1980	[illegible]	0.1568
1980	[illegible]	0.1528
1980	[illegible]	0.1518
1980	[illegible]	0.1434
1980	[illegible]	0.1403
1980	[illegible]	0.1303
1980	[illegible]	0.1292

1980	44.7391	0.1398
1980	42.8378	0.1380
1980	39.6057	0.1341
1980	24.5325	0.0975
1980	22.5191	0.0896
1980	22.1589	0.0892
1980	16.7284	##############
1980	16.1147	##############
1980	14.6972	##############
1981	55.2491	0.1510
1981	22.4521	0.1205
1981	19.9268	0.0654
1981	19.9131	0.0653
1981	16.1175	##############
1981	15.1250	##############
1982	201.0951	0.1884
1982	81.3885	0.1661
1982	46.6589	0.1418
1982	18.8229	##############
1982	18.1484	##############
1982	16.8802	##############
1983	83.9243	0.1672
1983	72.7625	0.1619
1983	39.3375	0.1337
1983	32.6808	0.1210
1983	29.9075	0.1157
1983	23.4316	0.0907
1984	251.4295	0.1826
1984	125.0842	0.1889
1984	76.4709	0.1635
1984	34.4508	0.1253
1984	28.2358	0.1116
1984	14.8633	##############
1984	13.1091	##############
1985	111.6914	0.1842
1985	92.7532	0.1725
1985	64.1743	0.1575
1985	58.7191	0.1531
1985	58.1402	0.1526
1985	49.9301	0.1457
1985	45.7062	0.1407
1985	39.5062	0.1339
1985	23.5386	0.0916
1985	22.3148	0.0843
1986	43.8548	0.1389
1986	38.4978	0.1321
1986	20.0597	0.0666
1987	250.9872	0.2003
1987	222.8360	0.1844
1987	[illegible]	0.1605
1988	213.9419	0.1804
1988	174.8795	0.1932
1988	97.7337	0.1755
1988	49.5184	0.1459
1988	33.9264	0.1240
1989	706.4610	0.1354

1980	44.7391	0.1284
1980	42.8378	0.1273
1980	39.6057	0.1246
1980	24.5325	0.0961
1980	22.5191	0.0890
1980	22.1589	0.0887
1980	16.7284	##############
1980	16.1147	##############
1980	14.6972	##############
1981	55.2491	0.1372
1981	22.4521	0.1145
1981	19.9268	0.0656
1981	19.9131	0.0655
1981	16.1175	##############
1981	15.1250	##############
1982	201.0951	0.1824
1982	81.3885	0.1506
1982	46.6589	0.1299
1982	18.8229	##############
1982	18.1484	##############
1982	16.8802	##############
1983	83.9243	0.1518
1983	72.7625	0.1465
1983	39.3375	0.1243
1983	32.6808	0.1149
1983	29.9075	0.1112
1983	23.4316	0.0901
1984	251.4295	0.1813
1984	125.0842	0.1760
1984	76.4709	0.1481
1984	34.4508	0.1182
1984	28.2358	0.1081
1984	14.8633	##############
1984	13.1091	##############
1985	111.6914	0.1704
1985	92.7532	0.1574
1985	64.1743	0.1426
1985	58.7191	0.1388
1985	58.1402	0.1384
1985	49.9301	0.1329
1985	45.7062	0.1291
1985	39.5062	0.1245
1985	23.5386	0.0906
1985	22.3148	0.0841
1986	43.8548	0.1278
1986	38.4978	0.1237
1986	20.0597	0.0668
1987	250.9872	0.2202
1987	222.8360	0.1810
1987	[illegible]	0.1453
1988	213.9419	0.2168
1988	174.8795	0.2048
1988	97.7337	0.1606
1988	49.5184	0.1325
1988	33.9264	0.1172
1989	1706.4610	0.2174

1989	395.5887	0.1926
1989	258.9508	0.1837
1989	175.2968	0.1629
1989	174.7880	0.1627
1989	41.8250	0.1389
1989	31.1848	0.1180
1989	21.2858	0.0770
1989	20.5380	0.0708
1990	127.6284	0.1099
1990	15.7497	################
1991	50.6556	0.1466
1991	34.9383	0.1264
1991	33.0232	0.1218
1991	30.9562	0.1177
1991	21.8101	0.0809
1991	20.4733	0.0702
1991	16.1901	################

1989	395.5887	0.2028
1989	258.9508	0.1831
1989	175.2968	0.1474
1989	174.7880	0.1472
1989	41.8250	0.1278
1989	31.1848	0.1127
1989	21.2858	0.0771
1989	20.5380	0.0711
1990	127.6284	0.1068
1990	15.7497	################
1991	50.6556	0.1336
1991	34.9383	0.1190
1991	33.0232	0.1155
1991	30.9562	0.1125
1991	21.8101	0.0809
1991	20.4733	0.0705
1991	16.1901	################



B. Casualty.

Policy	
Deductible:	200.0000
Limit:	300.0000
Increased Limit:	400.0000
Inflation Rate:	0.00%

ILF Pricing	**GPD**	**PD**
Loss Elimination Ratio:	**0.1033**	**0.3194**
Excess Ratio for Retention:	**10.1900**	**8.9688**
Increased Limits Factor:	**1.0814**	**1.0650**
Claims Severity:	**82.6205**	**79.7911**
Claims Frequency:	**0.2992**	**0.2580**
Pure Premium:	**24.7203**	**20.5862**

Scale:	1'000'000	**No. of Losses:**	76
		Mean Frequency:	3.4000
		Threshold:	18.0000
		Period:	BP

Limited Expected Value Comparison Test

Generalized Pareto (GPD)

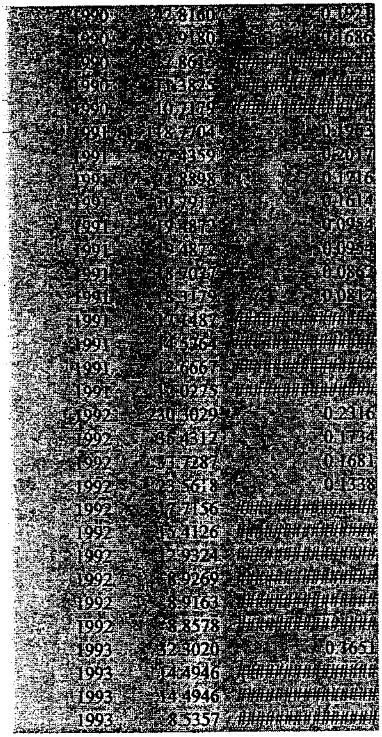
Shape: 1.1300
Scale: 14.1000
Location: 18.0000

Year of Loss	Size of Loss	LEV Comparison Test
1979	40.3650	0.1870
1983	157.0645	0.2076
1984	109.3680	0.1978
1985	47.6413	0.2013
1985	37.5227	0.1798
1985	27.1866	0.1498
1985	21.5007	0.1140
1985	21.1521	0.1137
1985	19.8644	0.0999
1985	19.1601	0.0916
1986	28.7166	0.1532
1986	19.0597	0.0904
1986	16.3544	################
1986	16.3292	################
1986	15.6921	################
1987	131.2434	0.1994
1987	29.8122	0.1586
1987	27.6752	0.1515
1987	21.3984	0.1160
1987	17.6893	################
1987	17.4040	################
1987	17.1187	################
1987	15.9775	################
1987	14.2656	################
1988	711.4636	0.2633
1988	312.7638	0.2466
1988	155.6245	0.2070
1988	114.7368	0.1965
1988	92.0779	0.2048
1988	64.7969	0.2158
1988	34.0221	0.1689
1988	33.7187	0.1681
1988	27.5066	0.1509
1988	27.2343	0.1500
1988	20.0989	0.1027
1988	19.7124	0.0981
1988	18.4467	0.0829
1989	178.9738	0.2795
1989	95.4969	0.2025
1989	30.6581	0.2059
1989	24.6453	0.1409
1989	21.5281	0.1172
1989	12.1246	################
1990	122.6111	0.1969
1990	43.3005	0.1931

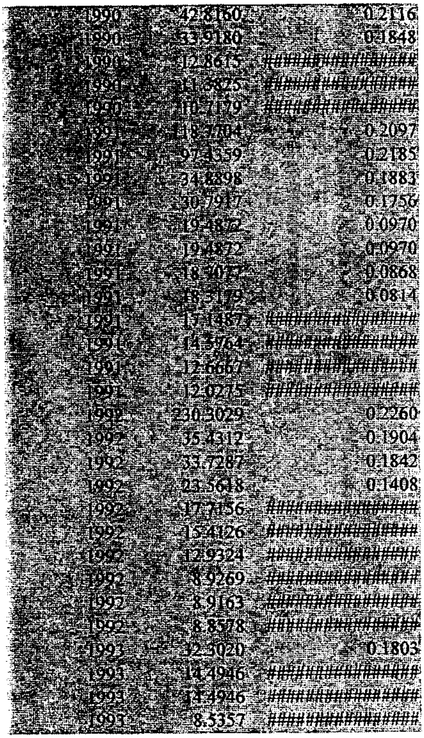
Pareto (PD)

Theta: 1.0787
a: 18.3179

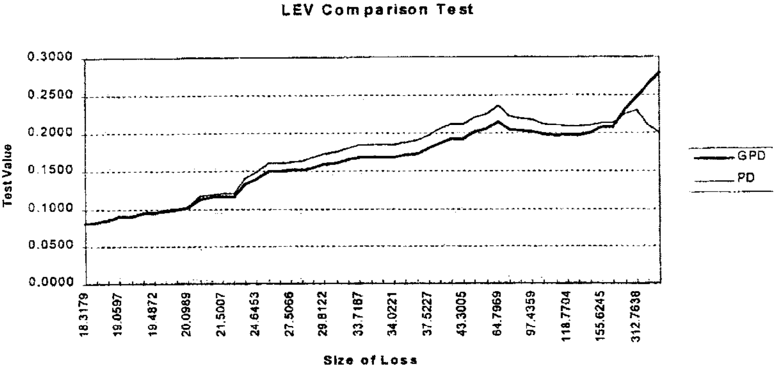
Year of Loss	Size of Loss	LEV Comparison Test
1979	40.3650	0.2058
1983	157.0645	0.2140
1984	109.3680	0.2127
1985	47.6413	0.2215
1985	37.5227	0.1977
1985	27.1866	0.1609
1985	21.5007	0.1213
1985	21.1521	0.1176
1985	19.8644	0.1020
1985	19.1601	0.0928
1986	28.7166	0.1677
1986	19.0597	0.0915
1986	16.3544	################
1986	16.3292	################
1986	15.6921	################
1987	131.2434	0.2105
1987	29.8122	0.1720
1987	27.6752	0.1630
1987	21.3984	0.1202
1987	17.6893	################
1987	17.4040	################
1987	17.1187	################
1987	15.9775	################
1987	14.2656	################
1988	711.4636	0.2102
1988	312.7638	0.2300
1988	155.6245	0.2136
1988	114.7368	0.2106
1988	92.0779	0.2224
1988	64.7969	0.2362
1988	34.0221	0.1852
1988	33.7187	0.1842
1988	27.3066	0.1623
1988	27.2343	0.1611
1988	20.0989	0.1051
1988	19.7124	0.1000
1988	18.4467	0.0832
1989	178.9738	0.2007
1989	95.4969	0.2197
1989	30.6581	0.2264
1989	24.6453	0.1484
1989	21.5281	0.1216
1989	12.1246	################
1990	122.6111	0.2095
1990	43.3005	0.2127



1990	42.8160	0.1921
1990	33.9180	0.1686
1990	12.8615	################
1990	11.3825	################
1990	10.7179	################
1991	118.7704	0.1963
1991	97.4359	0.2017
1991	34.8898	0.1716
1991	30.7917	0.1614
1991	19.4872	0.0954
1991	19.4872	0.0954
1991	18.7077	0.0863
1991	18.3179	0.0812
1991	17.1487	################
1991	14.5764	################
1991	12.6667	################
1991	12.0275	################
1992	230.3029	0.2316
1992	35.4312	0.1734
1992	33.7287	0.1681
1992	23.5618	0.1338
1992	17.7156	################
1992	15.4126	################
1992	12.9324	################
1992	8.9269	################
1992	8.9163	################
1992	8.8578	################
1993	32.3020	0.1651
1993	14.4946	################
1993	14.4946	################
1993	8.5357	################



1990	42.8160	0.2116
1990	33.9180	0.1848
1990	12.8615	################
1990	11.3825	################
1990	10.7179	################
1991	118.7704	0.2097
1991	97.4359	0.2185
1991	34.8898	0.1883
1991	30.7917	0.1756
1991	19.4872	0.0970
1991	19.4872	0.0970
1991	18.7077	0.0868
1991	18.3179	0.0814
1991	17.1487	################
1991	14.3764	################
1991	12.6667	################
1991	12.0275	################
1992	230.3029	0.2260
1992	35.4312	0.1904
1992	33.7287	0.1842
1992	23.5618	0.1408
1992	17.7156	################
1992	15.4126	################
1992	12.9324	################
1992	8.9269	################
1992	8.9163	################
1992	8.8578	################
1993	32.3020	0.1803
1993	14.4946	################
1993	14.4946	################
1993	8.5357	################


LEV Comparison Test

Tanker Pollution Layers. Applying the same techniques to a tanker pollution layer

USD 200M xs 300M Tanker Pollution

confirms the above findings:

A. EVT Techniques.

	GPD	PD
USD 200M xs 300M Tanker Pollution	50.3000	84.0100

B. ILF Techniques.

Policy	
Deductible:	300.0000
Limit:	500.0000
Increased Limit:	700.0000
Inflation Rate:	0.00%

ILF Pricing	**GPD**	**PD**
Loss Elimination Ratio:	**0.0226**	**0.0090**
Excess Ratio for Retention:	**13.4102**	**8.4860**
Increased Limits Factor:	**1.2151**	**1.2758**
Claims Severity:	**177.8277**	**185.3958**
Claims Frequency:	**0.2828**	**0.4531**
Pure Premium:	**50.2908**	**84.0047**

Scale:	1'000'000	**No. of Losses:**	81
		Mean Frequency:	3.8800
		Threshold:	0.1000
		Period:	BP

Limited Expected Value Comparison Test

Generalized Pareto (GPD)		**Pareto (PD)**	
Shape:	2.3148	Theta:	0.2765
Scale:	1.6210	a:	0.1271
Location:	0.1000		

Year of Loss	**Size of Loss**	**LEV Comparison Test**
1979	88.3231	0.1065
1979	8.8662	0.0861
1979	33.1564	0.1582
1979	10.8166	0.1902
1980	[illegible]	0.1241
1980	5.0677	-0.1231
1980	2.5342	0.1720
1980	1.1536	0.1641
1980	1.0801	0.0650
1981	[illegible]	0.0350
1981	0.4052	0.1770
1981	[illegible]	0.0150
1982	2.3428	0.0480
1982	[illegible]	0.1619
1982	0.1903	0.0333
1983	0.0762	################
1983	1.5974	0.1998

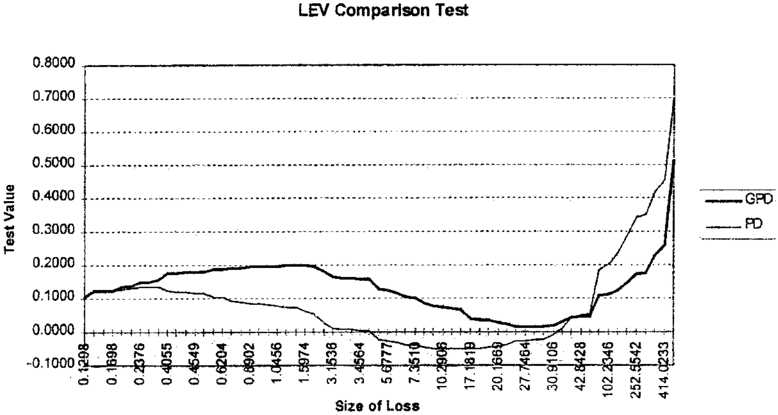
Year of Loss	**Size of Loss**	**LEV Comparison Test**
1979	88.3231	0.1855
1979	8.8662	-0.0452
1979	33.1564	0.0037
1979	10.8166	0.1057
1980	[illegible]	0.1226
1980	5.0677	-0.0252
1980	2.5342	0.3436
1980	1.1536	0.0099
1980	1.0801	0.0482
1981	[illegible]	0.0480
1981	0.4052	0.1232
1981	[illegible]	-0.0223
1982	2.3428	0.0518
1982	[illegible]	0.0075
1982	0.1903	0.1275
1983	0.0762	################
1983	1.5974	0.0624

1983	17.3979	0.0358
1984	1.0218	0.1964
1984	0.6204	0.1903
1984	3.5195	0.1574
1985	0.1298	0.1050
1985	0.8902	0.1945
1985	28.8630	0.0152
1985	1.2221	0.1991
1985	6.1354	0.1165
1986	7.0606	0.1046
1987	0.0643	################
1987	10.2906	0.0734
1987	10.8589	0.0701
1987	2.5965	0.1798
1987	54.9950	0.0289
1987	1.2040	0.1989
1987	43.3267	0.0475
1988	0.0503	################
1988	7.3510	0.1013
1988	0.1698	0.1243
1988	102.2346	0.1121
1989	0.0173	################
1989	0.0502	################
1989	0.4549	0.1796
1989	0.0369	################
1989	0.1993	0.1369
1989	0.8178	0.1938
1989	0.0960	################
1990	0.1271	0.1034
1990	1.0456	0.1967
1990	0.2376	0.1499
1990	5.4499	0.1270
1990	0.0268	################
1990	0.8021	0.1937
1990	20.1669	0.0280
1990	0.9219	0.1949
1991	3.2867	0.1609
1991	0.0571	################
1991	26.7794	0.0160
1991	1557.8141	0.5100
1991	1.8274	0.1971
1992	0.0525	################
1992	0.0823	################
1992	357.0632	0.2267
1993	0.2710	0.1593
1993	124.7963	0.1242
1993	0.4562	0.1798
1993	171.1819	0.0369
1993	0.4888	0.1786
1993	171.6517	0.1467
1993	22.8451	0.0223
1994	264.3469	[illegible]
1994	0.0481	################
1994	9.8748	0.0266
1994	0.0593	################
1994	0.0713	################

1983	17.3979	-0.0491
1984	1.0218	-0.0791
1984	0.6204	0.1054
1984	3.5195	0.0030
1985	0.1298	0.1068
1985	0.8902	0.0852
1985	28.8630	-0.0212
1985	1.2221	0.0728
1985	6.1354	-0.0297
1986	7.0606	-0.0370
1987	0.0643	################
1987	10.2906	-0.0498
1987	10.8589	-0.0497
1987	2.5965	0.0287
1987	54.9950	0.0132
1987	1.2040	0.0733
1987	43.3267	0.0545
1988	0.0503	################
1988	7.3510	-0.0386
1988	0.1698	0.1227
1988	102.2346	0.2055
1989	0.0173	################
1989	0.0502	################
1989	0.4549	0.1172
1989	0.0369	################
1989	0.1993	0.1291
1989	0.8178	0.0898
1989	0.0960	################
1990	0.1271	################
1990	1.0456	0.0782
1990	0.2376	0.1327
1990	5.4499	-0.0222
1990	0.0268	################
1990	0.8021	0.0910
1990	20.1669	-0.0437
1990	0.9219	0.0836
1991	3.2867	0.0064
1991	0.0571	################
1991	26.7794	-0.0282
1991	1557.8141	0.7015
1991	1.8274	0.0551
1992	0.0525	################
1992	0.0823	################
1992	357.0632	0.4160
1993	0.2710	0.1342
1993	124.7963	0.2363
1993	0.4562	0.1172
1993	171.1819	-0.0490
1993	0.4888	0.1189
1993	171.6517	0.2863
1993	22.8451	-0.0372
1994	264.3469	0.3517
1994	0.0481	################
1994	9.8748	-0.0490
1994	0.0593	################
1994	0.0713	################

1994	0.4495	0.1792
1995	0.1630	0.1215
1995	414.0233	0.2594
1995	40.6274	0.0409
1995	0.0375	################
1995	0.2404	0.1507
1995	30.9106	0.0195
1995	27.7464	0.0150

1994	0.4495	0.1177
1995	0.1630	0.1211
1995	414.0233	0.4523
1995	40.6274	0.0411
1995	0.0375	################
1995	0.2404	0.1329
1995	30.9106	-0.0096
1995	27.7464	-0.0256



Towards a VP-based Client Solution Toolbox. List and Zilch [1], Geosits, List and Lohner [2] and this note have outlined a consistent set of state-of-the art techniques and tools for modelling excess-of-loss claims data. These tools are available in the form of a corresponding ***toolbox called EVT (Extreme Value Techniques)*** that runs under Windows 3.1, 95, NT 3.51 and NT 4.0:

EXTREME VALUE TECHNIQUES (EVT)

- ***Claims Data Handling***
- ***Claims Data Analysis***
    - Frequency Statistics
    - Severity Statistics
- ***Excess-of-Loss Claims Modelling***
    - Pareto (PD)
    - Generalized Pareto (GPD)
- ***Advanced Scenario Techniques***
    - Parameter Uncertainty
    - Simulation
- ***Multi-year, Multiline Contract Pricing***
    - Extreme Value Theory (EVT)
    - Increased Limits Factors (ILF)
    - Coverage Futures and Options
- ***Risk-adjusted Capital (RAC)***
    - Calculation
    - Optimization
- ***Value Proposition (VP)***
    - Optimal Coverage Structures
    - Value Quantification

Fig. 4: Extreme Value Techniques (EVT) Toolbox

Modern VP-based client solutions for Fortune 500 companies often require sophisticated financial engineering, too. Davis and List [3] and Bühlmann, Bochiccio, Junod, List and Zinck [4] present the corresponding stochastic models and applications (for excess-of-loss claims on the liability side and interest rates, foreign currencies, stocks and stock indices, etc. on the asset side). Moreover, a sophisticated ***financial/(re)insurance toolbox*** for the design of such alternative risk transfer solutions is outlined: ***EVT handels the liability side*** while ***an extended form of the Rubinstein implied tree model is used for the asset side*** (with asset cashflows potentially contingent on loss events on the liability side) of such transactions.

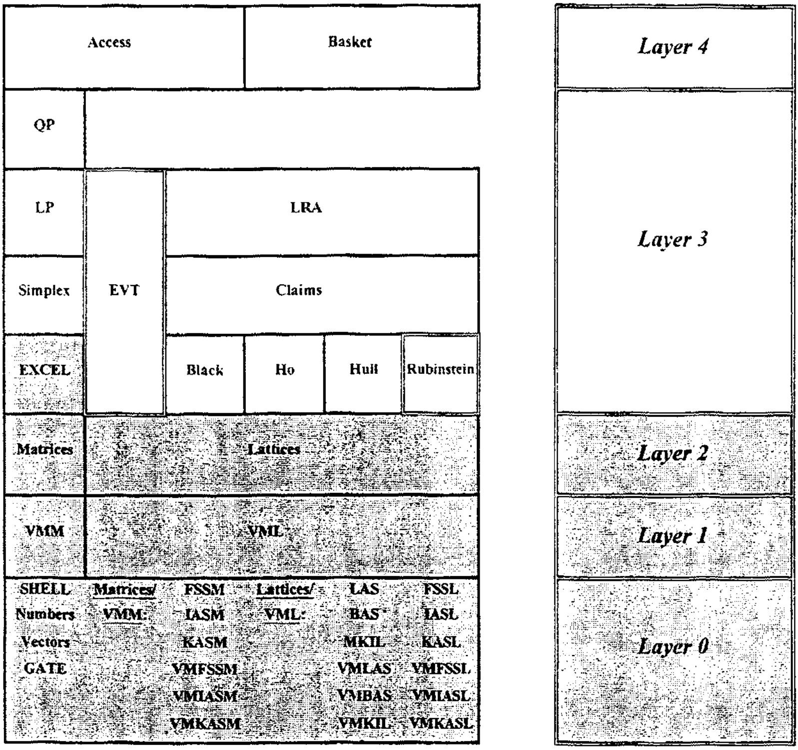


Fig. 5: Financial/(Re)insurance Toolbox

For more information on the various modules of this toolbox, see also the papers *Limited Risk Arbitrage Investment Management: Applications in Asset/Liability Management, Optimal Fund Design, Optimal Portfolio Selection and (Re)insurance Claims Portfolio Securitization - Part I: The LRA Paradigm* and *Limited Risk Arbitrage Investment Management: Applications in Asset/Liability Management, Optimal Fund Design, Optimal Portfolio*

Selection and (Re)insurance Claims Portfolio Securitization - Part II: LRA Portfolio Management Systems by M. H. A. Davis and H.-F. List, to appear in AFIR 1999.

5. References

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[3] M. H. A. Davis and H.-F. List, *Risk/Arbitrage Strategies: A New Concept for Asset/Liability Management, Optimal Fund Design and Optimal Portfolio Selection in a Dynamic, Continuous-Time Framework - Part V: A Guide to Efficient Numerical Implementations*, to appear in AFIR 1998

[4] N. Bühlmann, V. Bochiccio, S. Junod, H.-F. List and S. Zinck, *Extreme Value Techniques - Part IV: Fin Re Pricing*, to appear in ASTIN 1998

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[7] *Practical Risk Theory for Actuaries*, C. D. Daykin, T. Pentikaeinen and M. Pesonen, Chapman & Hall 1994