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# FORECASTING U.K. POPULATION MORTALITY ALLOWING FOR AGE, PERIOD AND COHORT EFFECTS

BY D. O. FORFAR

ABSTRACT

The mortality data (registered deaths and population size) over the years 1961-2007 for the population of England and Wales and for Scotland were obtained from the Office for National Statistics (ONS) and from the Scottish Registrar General. This paper addresses the following questions:

(i) Is there statistical evidence for a cohort effect (i.e. a generation effect separate from the period effect) being present in the data?

(ii) Do both males and females exhibit similar cohort (generation) effects?

(iii) Are period effects (i.e. the improvement in mortality with time) more significant than cohort effects?

(iv) How should one allow, in forecasts of population mortality, for age, period and cohort effects?

(v) Is it sensible to combine male and female mortality experience to determine the period effect and the cohort effect?

(vi) How do the forecasts for the expectation of life at birth, using the Extended-Lee–Carter-Combined (ELCC) model (described in the paper) differ from the (2008 based) Office of National Statistics (ONS) forecasts of the expectation of life at birth?

KEYWORDS

Mortality; Longevity; Expectation of Life; Lee–Carter; Cohort; Period; Mortality Forecasts; ONS

CONTACT ADDRESS

David O. Forfar, M.A., F.F.A, F.S.S., F.I.M.A., C. Math., Consulting Actuary, 24 Kinellan Road, Edinburgh EH12 6ES, U.K. E-mail: doforfar@msn.com

## 1. COMPLETE EXPECTATION OF LIFE AT BIRTH

In the 160 years between 1841 and 2001 there have been sixteen series of life tables for England and Wales, numbered ELT1 to ELT16, covering both male lives and female lives. In 1841, under ELT1, the difference between the complete expectation of life at birth for males and females was 2.0 years but in 2001 this difference had increased to 4.6 years having reached a maximum of 6.2 years in 1971. In the time span of 160 years, the expectation of life has gone from 40.0 years for males (females 42.2) to 76.0 years for males (females 80.6). In the early part of this time span of 160 years, the

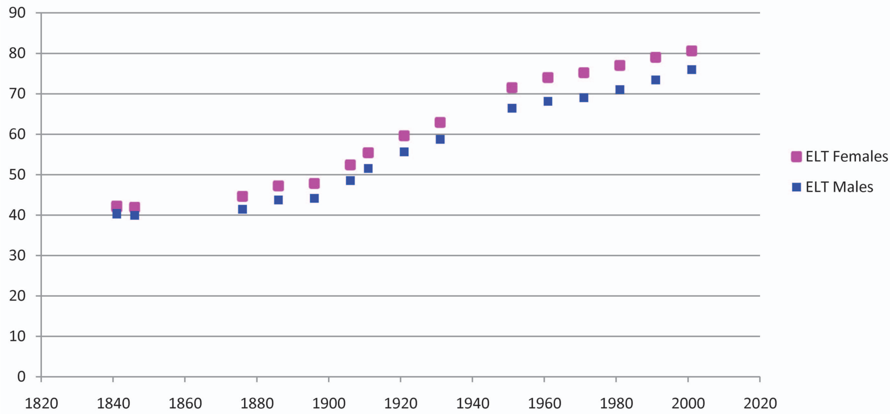


Figure 1. English Life Tables (ELT) 1-16 males and females expectation of life at birth (years) by year of ELT

improvement in mortality was mainly due to the improvement in public health and the elimination (or treatment) of infectious diseases. In more recent years the improvement in the expectation of life has been due to the improvement in mortality at middle or older ages. The improvement in the expectation of life at birth is shown in Figure 1. It is an interesting question as to what the expectation of life at birth will be according to future ELT tables.

## 2. Mortality Model for Forecasting

2.1 The mortality data considered covered the 90 ages (denoted by $x$) from 0 to 89 and the 47 calendar years (denoted by $t$) from 1961 to 2007 for deaths ($D(x, t)$) and population ($E(x, t)$) both males and females, England and Wales (E. & W.) and Scotland. There were thus 4,230 data points for each of the 8 sets of data. The first model investigated was the Lee–Carter model (Lee & Carter, 1992; Lee, 2000) where $a(x)$ and $b(x)$ represent the age effects and $k(t)$ — *Kappa* — represents the period or time effect.

$$\hat{D}(x, t) = E(x, t)\, \mathrm{e}^{a(x)+b(x)k(t)}$$

$$\sum_{x=0}^{x=89} b(x) = 1 \qquad \sum_{t=1}^{t=47} k(t) = 0$$

with the constraints on $b(x)$ and $k(t)$ added in order to make the parameterisation of the model unique.

2.2 The distribution of deaths were assumed to form a Poisson distribution (Brouhns *et. al.*, 2002).

$$\text{probability}\{D(x,t)=D\} = \frac{e^{-\hat{D}(x,t)}\hat{D}(x,t)^{D}}{D!}$$

$$\text{loglikelihood} = \sum_{x,t}\{D(x,t)*\log(\hat{D}(x,t)) - D(x,t)*\log(D(x,t)) - \hat{D}(x,t) + D(x,t)\}.$$

2.3 The Newton–Raphson technique was used to find the values of $a(x)$, $b(x)$ (the age effect) and $k(t)$ (the period or time effect) which maximised the logarithm of the likelihood (loglikelihood) of achieving the number of deaths, $D(x,t)$, that actually occurred.

## 3. Detection of a Specific Cohort (i.e. Generation) Effect

3.1 The Lee–Carter model allows for an age ($x$) effect and a period ($t$) effect (i.e. year or time) but no specific cohort (i.e. generation) effect. The ratio:

$$\frac{\text{actual deaths}}{\text{expected deaths on the Lee–Carter model}}$$

was used to detect whether or not there was a specific cohort effect in the residuals after fitting a Lee–Carter model. In Figures 2-5, the red areas represent a higher number of deaths than the best-fitting Lee–Carter model predicts and the blue areas represent a lower number of deaths. It will be seen that the presence of 45 degree colour bands, of predominantly either red or blue, represent a specific cohort effect in addition to the age/period effects for which the Lee–Carter model has already allowed. This approach is preferred to determine whether a specific cohort effect exists over the approach used by other authors which measures the rate of improvement in mortality — for a given age — with time, which mixes period and cohort effects.

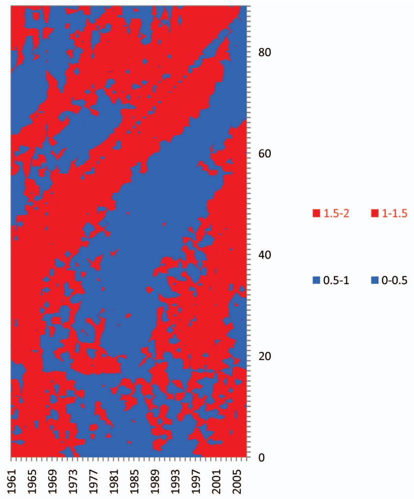


Figure 2. E&W males ratio: Actual-Deaths / Lee–Carter-Expected-Deaths

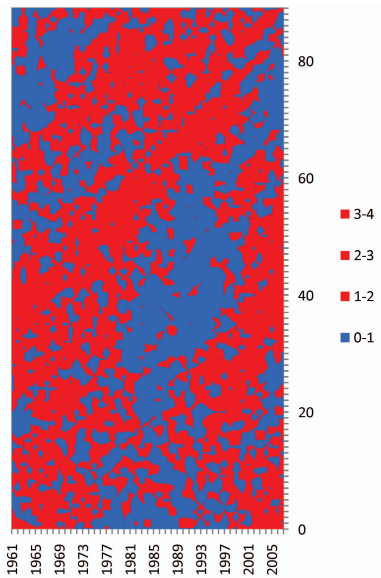


Figure 3. Scotland males ratio: Actual-Deaths / Lee–Carter-Expected-Deaths

3.2 For females the corresponding figures are as follows:

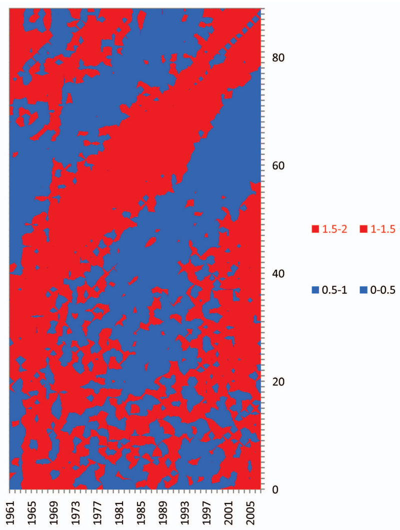


Figure 4. E&W females ratio: Actual-Deaths/Lee–Carter-Expected-Deaths

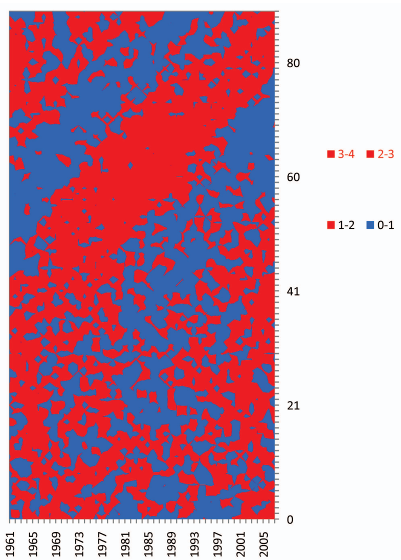


Figure 5. Scotland females ratio: Actual-Deaths/Lee–Carter-Expected-Deaths

3.3 Figures 2-5 illustrate a specific cohort (diagonal) effect. The cohort effect is more pronounced for the mortality data of England and Wales than it is for the corresponding data from Scotland. Thus a cohort effect was added to the basic Lee–Carter (LC) model to make an Extended-Lee–Carter (ELC) model.

## 4. The Extended-Lee–Carter Model for Males and Females Separately

4.1 As the data consisted of 90 ages and 47 calendar years, the Extended-Lee–Carter model (Haberman & Renshaw, 2006; Booth & Tickle, 2008) allowed for the 136 cohorts (*Lambda*, from the 1872 cohort up to the 2007 cohort $\{\lambda(1), \ldots, \lambda(136)\}$) that were present in the data, where $\lambda(1)$, represents the 1872 cohort (generation) ($x = 89$, $t = 1$) and $\lambda(136)$ the 2007 cohort (generation) ($x = 0, t = 47$).

$$\hat{D}(x, t) = E(x, t)\, \mathrm{e}^{a(x)+b(x)k(t)+\lambda(t+89-x)}$$

$$\sum_{x=0}^{x=89} b(x) = 1 \quad \sum_{t=1}^{r=47} k(t) = 0 \quad \sum_{i=1}^{i=136} \lambda(i) = 0.$$

Again Newton–Raphson was used to determine the values of the parameters $a(x)$, $b(x)$, $k(t)$ and $\lambda(i)$ which maximise the loglikelihood.

4.2 Thus the Extended-Lee–Carter model allows for the age, period (time) effects and allows for a specific diagonal or cohort effect, measured by $\lambda$ being a function of $(t + 89 - x)$. The ratio

$$\frac{\text{actual deaths}}{\text{expected deaths on the Extended-Lee–Carter model}}$$

was again used to detect whether or not there was any residual cohort effect remaining in the data. The red areas represent a higher number of deaths than the Extended-Lee–Carter model predicts and the blue areas represent a lower number of deaths. It will be seen that the 45 degree colour bands have practically disappeared, showing the Extended-Lee–Carter model has, for all practical purpose, allowed for the specific cohort effect.

4.3 Using the Extended-Lee–Carter model, the Figures 6-7 for male lives are as follows:

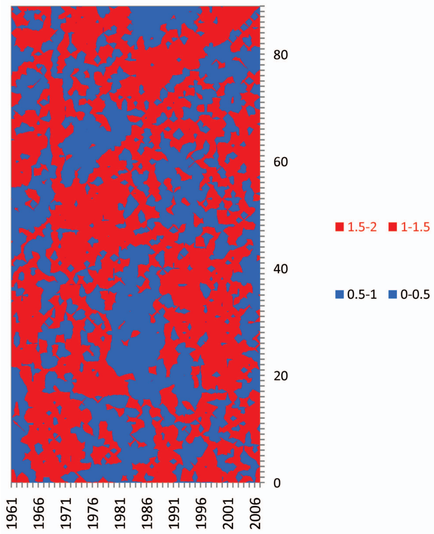


Figure 6. E&W males ratio: Actual-Deaths/Extended-Lee–Carter-Expected-Deaths

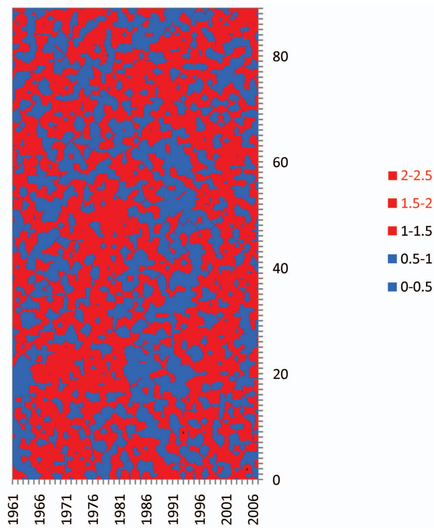


Figure 7. Scotland males ratio: Actual-Deaths/Extended-Lee–Carter-Expected-Deaths

4.4 For females the corresponding Figures 8-9 are as follows:

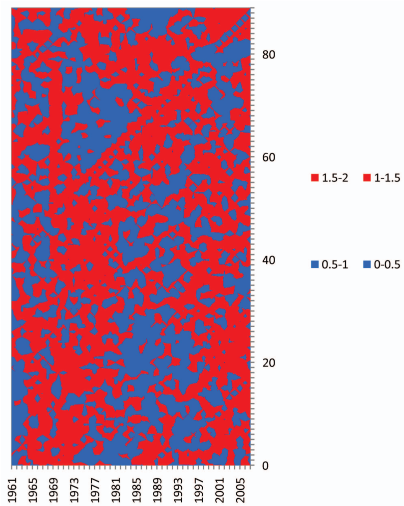


Figure 8. E&W females ratio: Actual-Deaths/Extended-Lee–Carter-Expected-Deaths

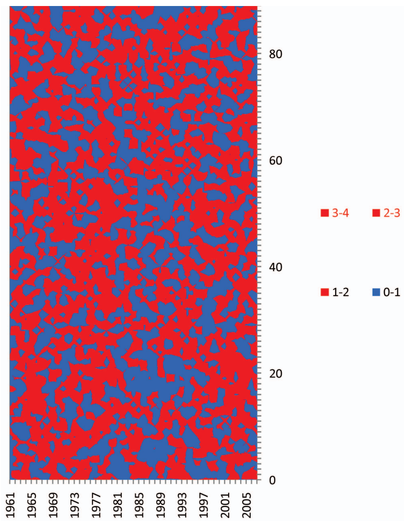


Figure 9. Scotland females ratio: Actual-Deaths/Extended-Lee–Carter-Expected-Deaths

## 5. Statistical Significance

The Lee–Carter model (the null-hypothesis) is nested within the Extended-Lee–Carter model in that all the $\lambda$'s (the cohort effect) are set to zero under the null-hypothesis. The likelihood ratio test between the Lee–Carter model and the Extended-Lee–Carter model shows, in Table 1, a large and statistically significant improvement in the maximum loglikelihood so that the null-hypothesis (the Lee–Carter model) can be rejected in favour of the Extended-Lee–Carter model. The p-value is so small as to be negligible.

Table 1

| | | Loglikelihood | | | |
|---|---|---|---|---|---|
| | Degrees of freedom | E&W males | E&W females | Scotland males | Scotland females |
| Lee–Carter | 225 | −11,616 | −9,517 | −3,185 | −3,265 |
| Extended-Lee–Carter | 360 | −4,481 | −3,404 | −2,215 | −2,202 |
| Likelihood ratio test (twice the improvement in loglikelihood) | 135 | 14,270 | 12,226 | 1,940 | 2,126 |
| p-value | | *** | *** | *** | *** |

*** means that the p-value is negligibly small.

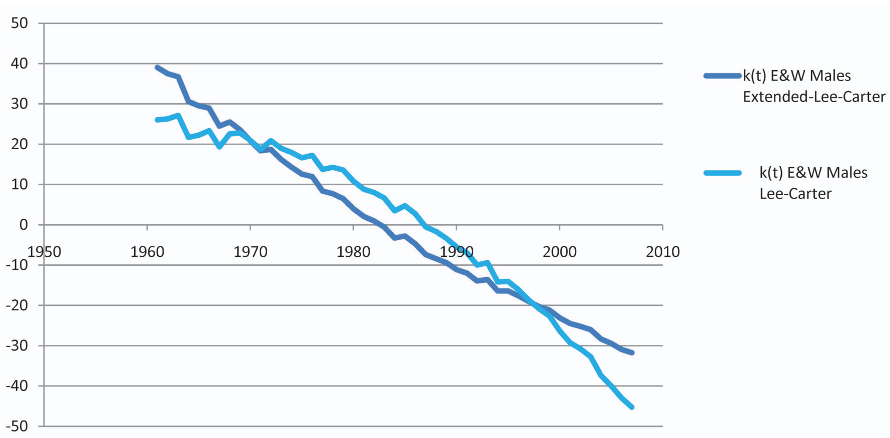


Figure 10. $k^m(t)$ and $k^{Em}(t)$: E&W males: Lee-Carter and Extended-Lee–Carter

## 6. Value of $k^m(t)$ for E&W Males on the Lee-Carter Model and Extended-Lee–Carter Model ($k^{Em}(t)$)

The value of $k^m(t)$ ($k(t)$ for males) under the Lee–Carter model and $k^{Em}(t)$ under the Extended-Lee–Carter model is shown in Figure 10. The value of $k^{Em}(t)$ is more linear than the $k^m(t)$ but still exhibits some residual curvature. In view of the significant effect of the forecasts of $k^m(t)$ and $k^{Em}(t)$ on the future expectation of life at birth, an important question is whether $k^m(t)$ and $k^{Em}(t)$ should be forecast in a linear or quadratic manner.

## 7. Value of $k^f(t)$ for E&W Females on the Lee-Carter Model and the Extended-Lee-Carter Model ($k^{Ef}(t)$)

The value of $k^f(t)$ curves downwards using the Lee–Carter model and $k^{Ef}(t)$ curves upwards using the Extended-Lee–Carter model as shown in Figure 11. In view of the effect on the future expectation of life at birth, an important question is how $k^f(t)$ and $k^{Ef}(t)$ should be forecast.

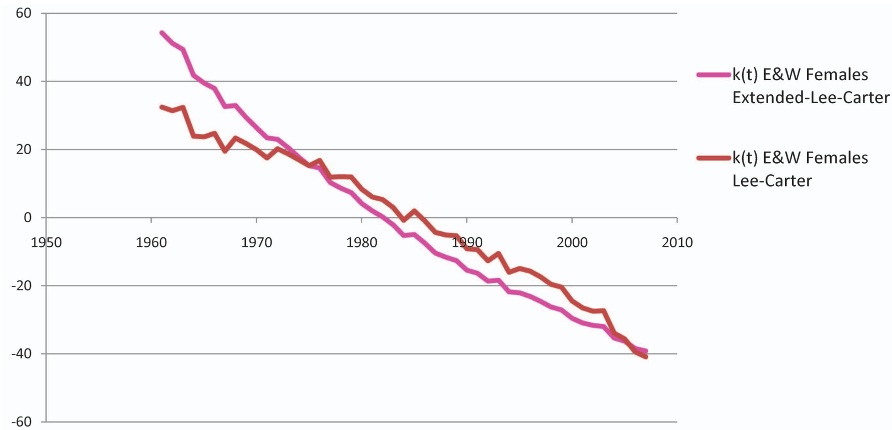


Figure 11. $k^f(t)$ and $k^{Ef}(t)$: E&W females: Lee-Carter and Extended-Lee–Carter

## 8. Value of $k^E(t)$ for E&W Males and Females

Figure 12 shows the values of $k^{Em}(t)$ and $k^{Ef}(t)$ for E&W males and females. The values are very similar.

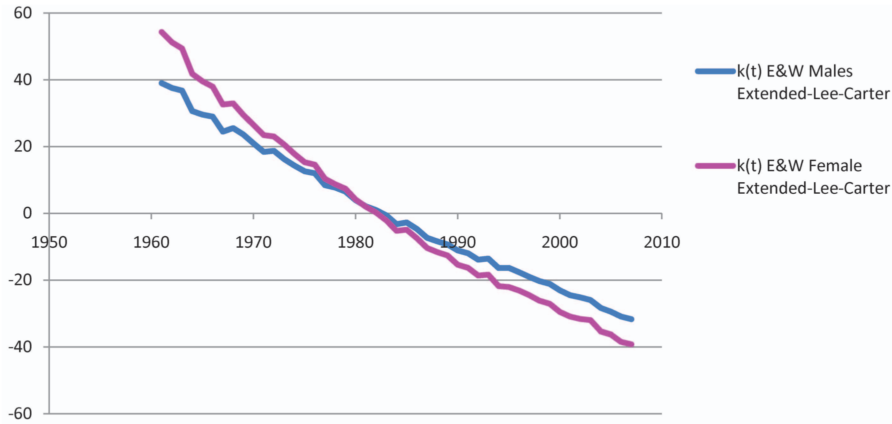


Figure 12. $k^{Em}(t)$ and $k^{Ef}(t)$: E&W males and females: Extended-Lee–Carter

## 9. Forecast of $\lambda(i)$ (*Lambda*) under the Extended-Lee–Carter Model for E&W Males and Females

9.1 As covered in Section 4.1, there were 136 cohorts in the data, $\lambda(i)$ for $i = 1$ to 136, for both males and females. From 1920 onwards, the first difference of $\lambda(i)$ is more or less stationary for males and females, so an ARIMA(0,1,0) model seems not inappropriate for forecasting $\lambda(i)$. For both males and females, the initial values $\lambda(1), \lambda(2), \ldots$ and the end values $\lambda(136)$, $\lambda(135), \ldots$ are based on only one, two, ..., etc. observations of deaths, namely $[D(89,1)$, $D(88,1)$ and $D(89,2), \ldots]$ and $[D(47,0)$, $D(46,0)$ and $D(47,1), \ldots]$, but seem consistent with the other values (which are based on very much more data).

9.2 Furthermore, it is noticeable, as shown in Figure 13, that the $\lambda(i)$ for both males and females exhibit a similar shape suggesting that the cohort effects for both males and females are similar.

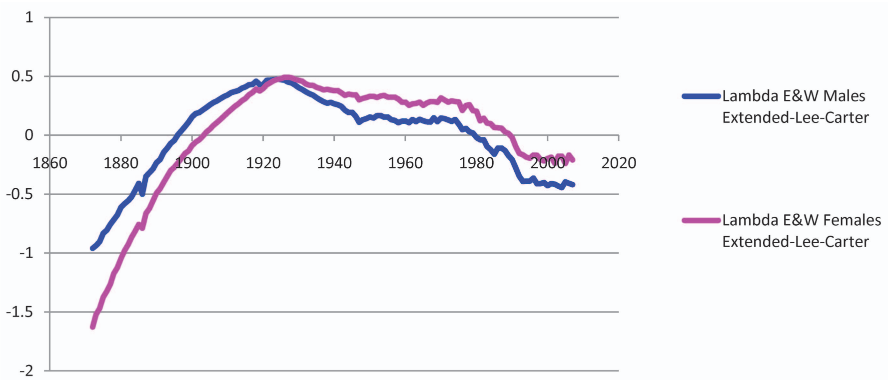


Figure 13. $\lambda^m(i)$ and $\lambda^f(i)$: males and females: Extended-Lee–Carter

## 10. Forecast of a Combined $k^E(t)$ and a Combined $\lambda(i)$ to be used for Both Males and Females

10.1 As noted in Section 8, the figures for $k^{Em}(t)$ and $k^{Ef}(t)$ (for males and females) are very similar, one is therefore tempted to have only one $k^E(t)$, call it $k^{EC}(t)$, for both males and females combined, in order to represent the improvement in public health and medical practice which both males and females have enjoyed to date. Figure 14, of $k^{EC}(t)$, is found to be almost linear so forecasting by an ARIMA(0,1,0) model seems reasonable. As we have 47 years of data we forecast 47 years ahead, i.e. to the year 2054.

10.2 Figure 13 shows a similar pattern for $\lambda(i)$ for both male and female

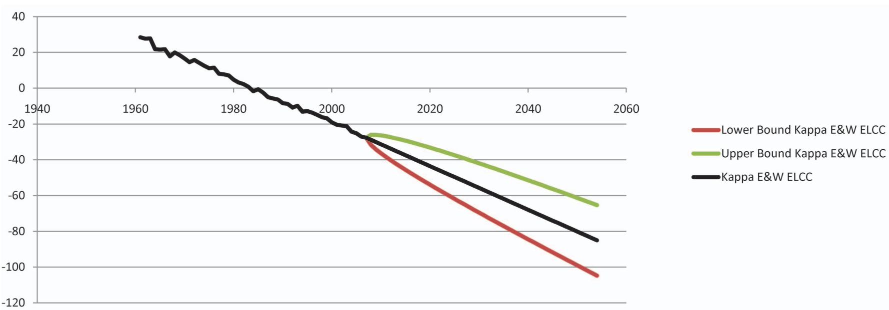


Figure 14. $k^{EC}(t)$ E&W Extended-Lee–Carter-Combined (ELCC)

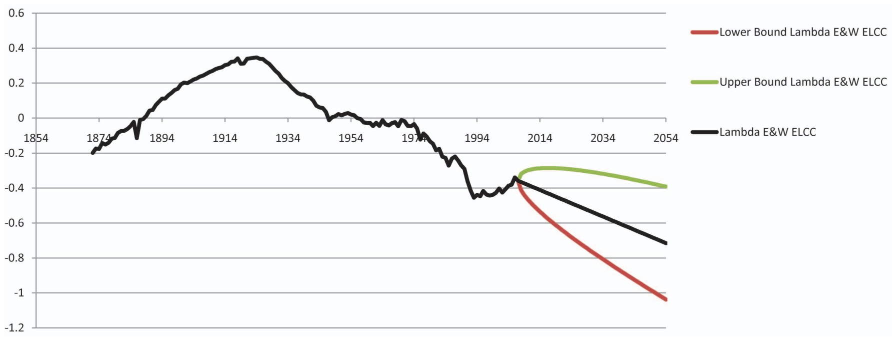


Figure 15. Forecast of a single $\lambda(i)$ for E&W ELCC

lives, so it is tempting to have only one $\lambda(i)$ for both sexes, call it $\lambda^c(i)$, representing one cohort effect. It seems plausible that the health experience for both the male and female generations is similar, e.g. with regard to the understanding by generation (cohort) of the detrimental health effects of smoking, etc.

10.3 The method of forecasting assumed for the specific cohort effect (i.e. the forecast of $\lambda^c(i)$) does not seem to be a critical assumption because, even using the lower bound of $\lambda^c(i)$, does not make more than a difference of 0.2 (91.0 to 91.2 for males) to the period expectation of life at birth for 2054 (see Figure 15).

10.4 To obtain a value of $k^{EC}(t)$ and $\lambda^C(i)$ (i.e. for both males and females combined), the loglikelihood of the occurrence of the male and female deaths is maximised allowing for a different $a(x)$ and $b(x)$ for males and females but the same $k^E(t)$ and $\lambda(i)$ for both males and females. We may call this model the Extended-Lee–Carter-Combined (ELCC) model.

## 11. Forecast of the Expectation of Life at Birth for E&W Males and Females Assuming an Extended-Lee–Carter-Combined (ELCC) Model and Comparison with the 2008 ONS (Office of National Statistics) Forecasts

11.1 The latest ONS/GAD forecasts for England and Wales are the 2008 based forecasts. Figure 16 shows the evolution of the (period) expectation of life at birth for both males and females according to the 2008 based ONS forecasts. It will be seen that the rate of improvement in the ONS expectation of life at birth slows down after the year 2020. Figure 17 shows the corresponding ONS figures for age 65.

11.2 The Extended-Lee–Carter-Combined (ELCC) model is used to

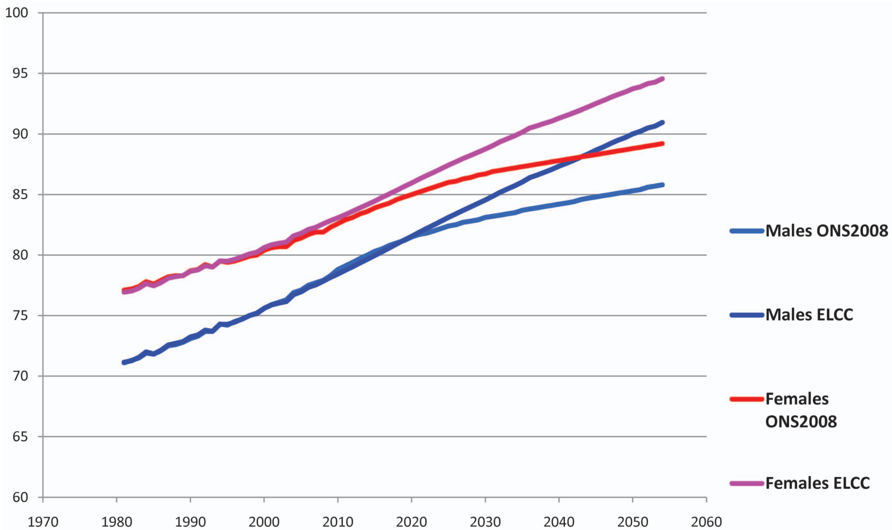


Figure 16. Period expectation of life at birth (years): E&W males and females ONS versus ELCC

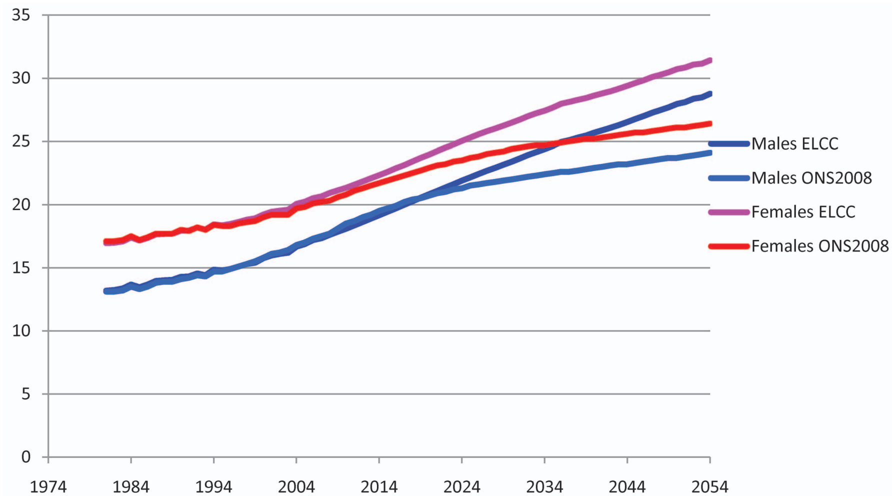


Figure 17. Period expectation of life at age 65 (years): E&W males and females ONS versus ELCC

Table 2. Period expectation of life at birth (years) in year 2054 with 95% bounds in brackets

| | |
|---|---|
| ONS males (2008) | 86 (±4) |
| ELCC males | 91 (±3) |
| | |
| ONS females (2008) | 89 (±3) |
| ELCC females | 95 (±3) |

derive the period expectation of life at birth for E&W males and females and to forecast the future expectation of life at birth — Figure 16 (Oeppen & Vaupel, 2002). Figure 17 shows the ELCC figures for age 65. As many authors have pointed out (e.g. Currie, 2004) forecasts are model-dependent, but the results from the Extended-Lee–Carter-Combined model illustrates that the official ONS forecasts may be an underestimate of the forecast period expectation of life at birth.

11.3 For the year 2054, Table 2 shows a difference of about 5 years in the comparative period expectations of life at birth on the ELCC model and the ONS model, the ELCC model showing a higher figure than the ONS 2008 model.

## 12. Some Evidence that Male and Female Expectations of Life at Birth are Converging

Using the Government Actuary's Department (GAD) yearly data (as opposed to the English Life Table data which is only every ten years), the difference, in 1961, between the period male and female expectations of life at birth was 5.9 years. In the subsequent years, the difference reached a maximum of 6.4 years in 1969, but by 2007 the difference had diminished to 4.1 years. Thus there is some evidence that the gap between the male and female expectations of life at birth are diminishing. On the basis of the above two forecasts (the ONS model and the ELCC model) for the period expectation of life at birth, the gap converges and is eliminated by about 2035 as shown in Figure 18.

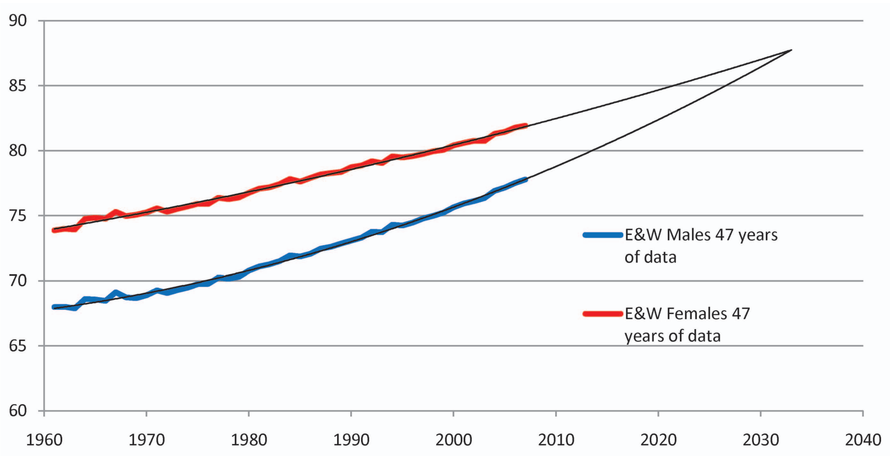


Figure 18. ONS E&W (period) expectation of life at birth for males and females — evidence of convergence?

## 13. Cohort Expectation of Life at Birth

If there is a two-dimensional table of mortality rates by age and calendar year, the period expectation of life at birth corresponds to "*going down a column*" while the cohort expectation of life corresponds to "*going down a diagonal*". As mortality rates are improving over time, the cohort expectation of life at birth is larger than the period of expectation of life at birth. Table 3 shows that, on the basis of the Lee–Carter model, the cohort expectation of life at birth exceeds the period expectation of life at birth by some 12-13% for both males and females and this agrees with the ONS figures (2008 basis). On the Extended-Lee–Carter-Combined (ELCC) model, the period expectation of life reaches 100 in the year 2095 for males and 2079 for females. On the ELCC model, the percentage increase in the cohort expectation (as compared with the period expectation) is 26% (males) and 24% (females) and a cohort expectation of life for the cohort born in 2008 nearly 100 (males) and over 100 (females). That means that, on the basis of the ELCC model, it is true that, as the popular press would have it, "... *to-day's babies will live to 100* ...".

Table 3. Complete expectation of life at birth (either period exp. life or cohort exp. life)

| | Percentage by which cohort exp. life exceeds period exp. life | | | Extended-Lee–Carter-Combined model | |
|---|---|---|---|---|---|
| | ONS | Lee–Carter model | Extended-Lee–Carter-Combined model | Year in which period expectation of life is 100 | Cohort expectation of life for cohort born in 2008 |
| Males | 13% | 13% | 27% | 2095 | 98 |
| Females | 12% | 12% | 23% | 2079 | 101 |

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## References

Booth, H. & Tickle, L. (2008). Mortality modelling and forecasting: a review of methods, *Annals of Actuarial Science*, **3**, 3-43.

Brennan, M. (2008). Mortality projection using extended Lee–Carter to allow for the cohort effect, 2008 MSc Dissertation, Heriot-Watt, University, Edinburgh.

Brouhns, N., Denuit, M. & Vermunt, J.K. (2002). A Poisson log-bilinear regression approach to the construction of projected mortality tables, *Insurance Mathematics and Economics*, **31**, 373-393.

Currie, I.D., Durban, M. & Eilers, P.H.C. (2004). Smoothing and forecasting mortality rates, *Statistical Modelling*, **4**, 278-298.

Emmanuel, A. (2008). Mortality projection using extended Lee–Carter to allow for the cohort effect. 2008 MSc Dissertation, Heriot-Watt, University, Edinburgh

Haberman, S. & Renshaw, A.E (2006). A cohort-based extension to the Lee–Carter model for mortality reduction factors, *Mathematics and Economics*, **38**, 556-570.

Lee, R.D. (2000). The Lee–Carter method for forecasting mortality with various extensions and applications, *North American Actuarial Journal*, **4**, 80-93.

Lee, R.D. & Carter, R.C. (1992). Modelling and forecasting U.S. Mortality, *Journal of the American Statistical Association*, **87**, 659-671.

Oeppen, J. & Vaupel, J.W. (2002). Broken limits to life expectancy, *Science*, **296**, 1029-1031.

Sen, R. (2008). Mortality projection using extended Lee–Carter to allow for the cohort effect, 2008 MSc Dissertation, Heriot-Watt, University, Edinburgh.

Shetty, N. (2008). Mortality projection using extended Lee–Carter to allow for the cohort effect, 2008 MSc Dissertation, Heriot-Watt, University, Edinburgh.