



Institute  
and Faculty  
of Actuaries

# Loss Ratio Uncertainty

Andrew D Smith, University College Dublin  
ansmith@ucd.ie

# Open up the Spreadsheet FitGamma03.xls

- Enter your name into cell C4 (you might need to enable editing)
- You now have your personal set of historic loss ratio data.
- The data depends on the last 3 letters of your name, so if you're a Smith like me, we'll have the same data.



# Your Loss Ratio Data

- These are loss ratios: claims divided by premiums, for 25 consecutive years.
- They might be on an underwriting year basis, or on an accident year basis.
  - It doesn't matter for our purposes.
- Assume all years are fully run off.
- Show of hands for average loss ratio.
- Show of hands for standard deviation of loss ratio.



# Plot a Picture of Historic Loss Ratios against Time

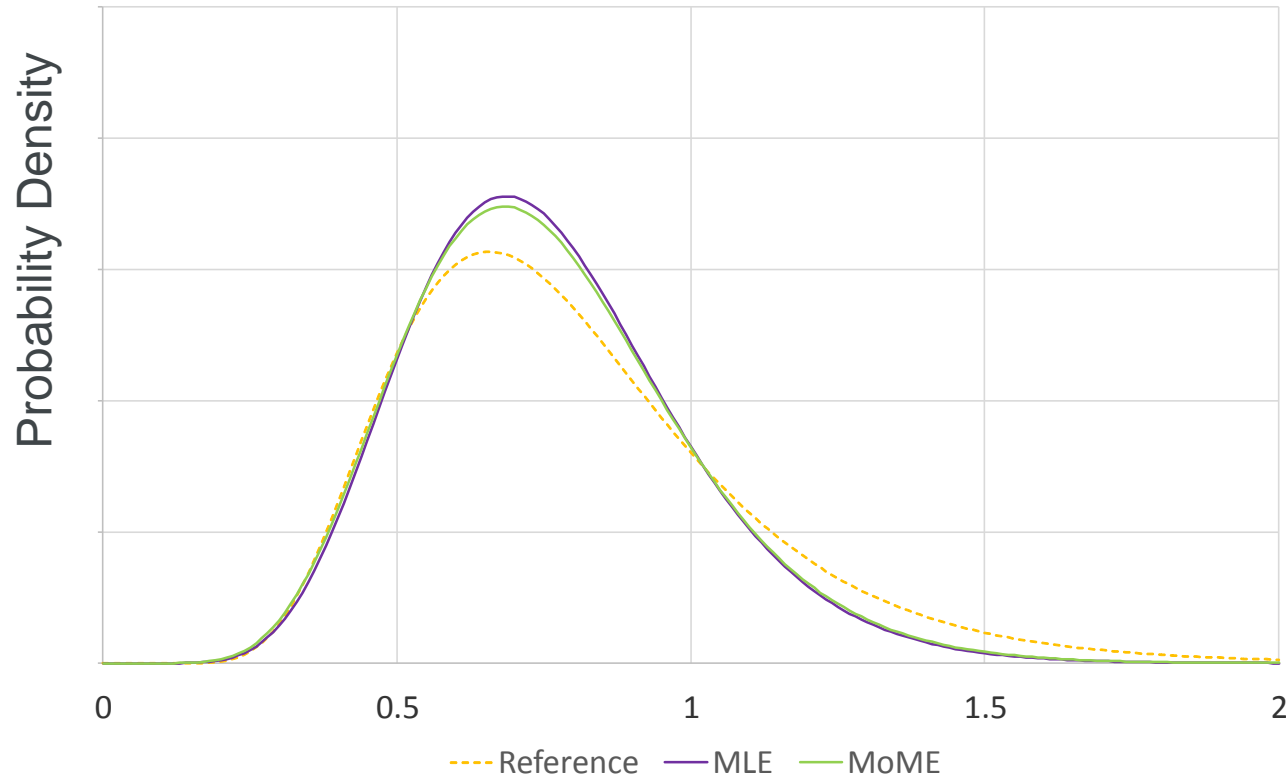
- What do you notice?
- Are they dependent?
- Is there a cycle?
- Can you even tell?



# Formula for Required Capital

- You are now in charge of an insurer's internal model, responsible for calculating the solvency capital requirement at a 99.5% confidence level.
  - The required capital will be expressed as a fraction of the premium.
  - You have to estimate the relevant fraction.
  - You only have 25 years' data, so fit a distribution and extrapolate to find the 99.5%-ile
- Choose a Gamma distribution and estimate the two parameters by
  - Maximum likelihood estimates (MLE)
  - Method of moments estimates (MoME)
- Gamma distributions have been widely used for loss ratio models, starting with Lloyds RBC model in the 1990s.

# Fitted (Ersatz) Loss Ratio Distribution Comparison



# Comparing the Two Fits

- The mean of a gamma distribution is  $\alpha\beta$  and the standard deviation is  $\alpha^{1/2}\beta$ .
- Who has higher mean loss ratio for MoME? For MLE?
- Who has higher standard deviation for MoME? For MLE?
- If you had to validate the model, you might test the fitted standard deviation against the data. Which method would this test favour?



# Goodness of Fit Tests

- Construction of P-P Plots
- This has:
  - On the X-axis, the fitted probability of a loss ratio  $\leq r$ , for a range of values of  $r$
  - On the Y-axis the observed proportion of observations  $\leq r$ . As we have 25 observations, this goes up in steps of 4%.
- We are hoping for something close to a straight line if the distribution fits well.
- The acceptable deviations from a straight line are governed by the Kolmogorov-Smirnov (KS) distribution. Limits (2-sided 95%) shown in red.





# Did anyone fail the KS Test?

- No?
- That's odd because out of 20 people we'd expect one failure.
- Alternative tests, usually with similar results, look at the area of PP deviations rather than the largest deviation. These give rise to Cramer-von Mises and Anderson-Darling tests.
- We could also look at QQ plots. On the Y-axis we show the observations sorted into increasing order and on the X-axis we show the corresponding quantiles of the fitted distribution. For a good fit, we are looking for a 45 degree line, but (unlike for PP) there is no general distribution for how large deviations could be.

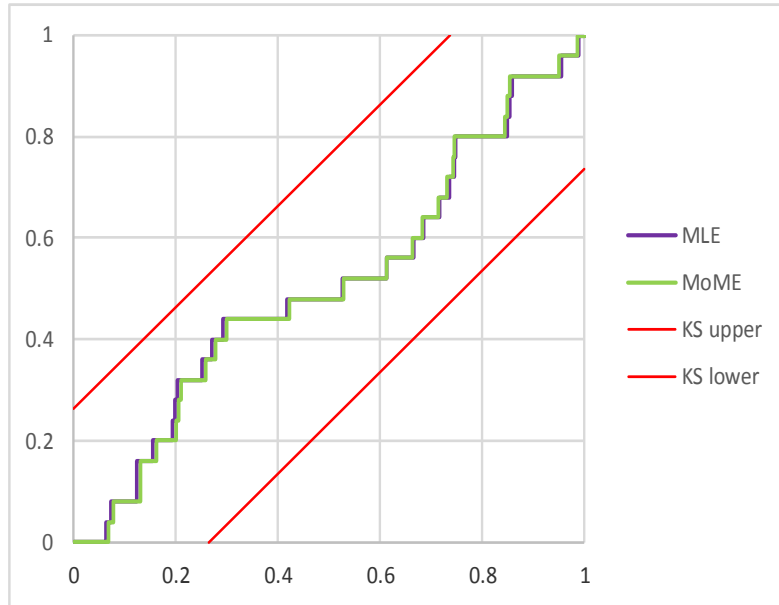


# Rip van Winkel Experiment

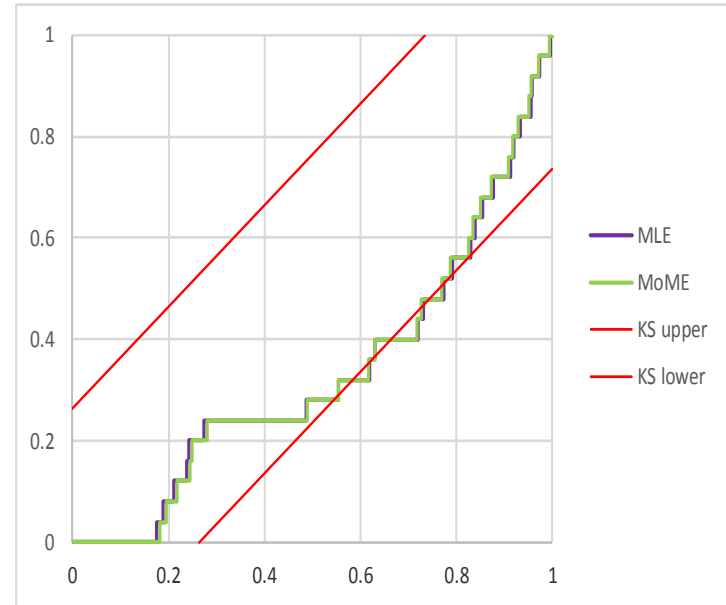
- You fitted your model and then went to sleep for 25 years.
- You have 25 more data points.
- How now would you test your model?
- Back-testing a 1-in-10 event, you would expect 2.5 exceptions in 25 years.
- Raise your hand if you have an exception
  - at the 1-in-100 level,
  - or 1-in-1000?



# Example P-P Plots: In-sample and Out-of-sample



In-sample



Out-of-sample



# Goodness of Fit Tests and the Lilliefors Effect

Context	Effect	$H_0$	Test size if unadjusted KS bounds used
Kolmogorov-Smirnov		Data is a random sample from the stated distribution.	5%
In-sample fit	Lilliefors	Data is a random sample from some gamma distribution.	< 5% As fitted mean = sample mean
Out-of-sample fit (Rip van Winkel)	Reverse Lilliefors	Past and future data are random samples from a common gamma distribution.	> 5%

- An abnormally low failure rate (the Lilliefors effect) is welcome for generating green lights in validation reports
- Should validators be concerned if failure rate < 5% ?

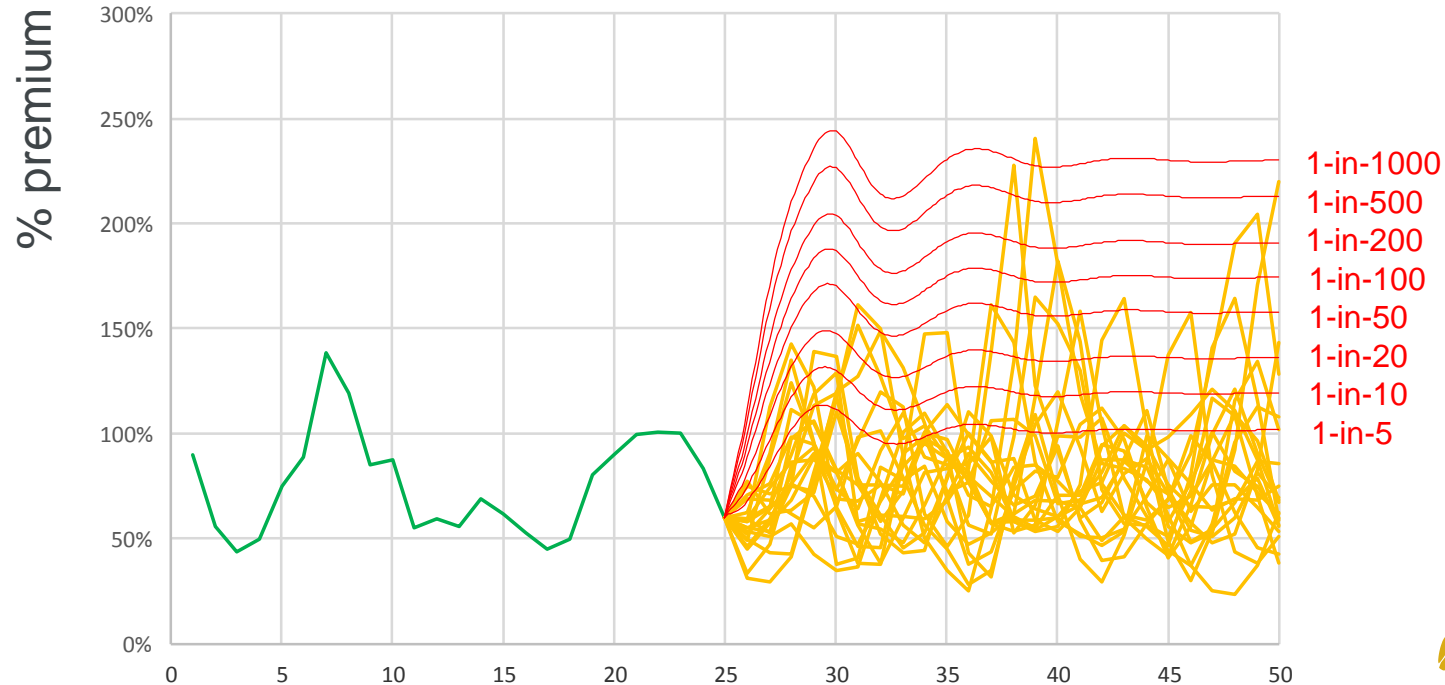
# The Bit we Can't Do in Real Life

- Given real data, we don't know the generating process, so we can't tell for sure how accurate the fitted model is.
- This loss ratio example used generated data from a *reference model*,
  - which in fact was the exponential of a normal AR2 time series.
- The models you have fitted – IID Gamma – are *ersatz models*,
  - ie substitutes for the reference model
- Our experiment has parameter error and model error.

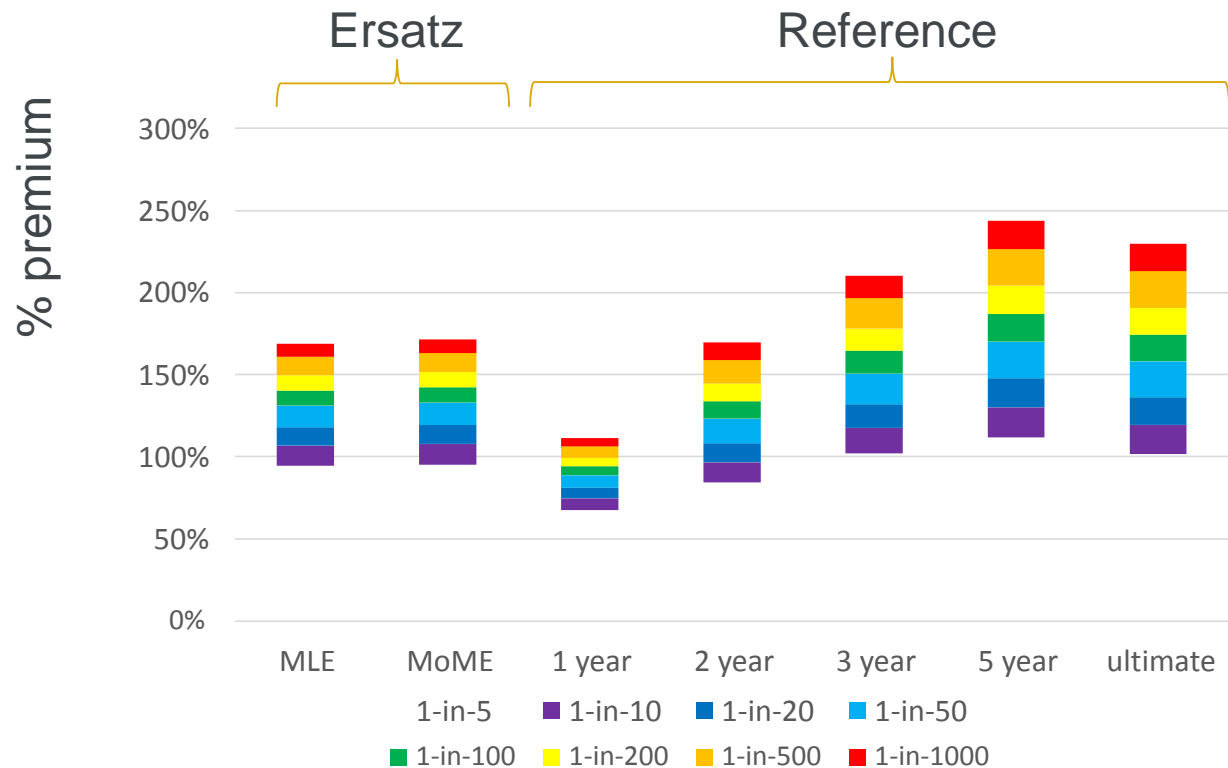


# Twenty Possible Futures (from the Reference Model)

Loss Ratios Past and Future, with Conditional Percentiles



# Comparing Ersatz to Reference Loss Ratio Percentiles



# Comparing Ersatz to Reference Models

- There are several differences:
  - The ersatz models have the same percentiles for all future years, while the reference model has a term structure across future time horizons.
  - The reference model has lognormal rather than gamma distributions, so the tails of the fitted distribution are inevitably too thin.
  - However, at 99.5% the ersatz percentile is sometimes too low and sometimes too high
- Was it foolish to fit a mis-specified model?
  - Yes: if you want to apply statistical criteria such as unbiasedness, consistency and efficiency. These make sense only if the fitted model is well-specified.
  - No: because mis-specification is a fact of life and we need to understand its consequences.



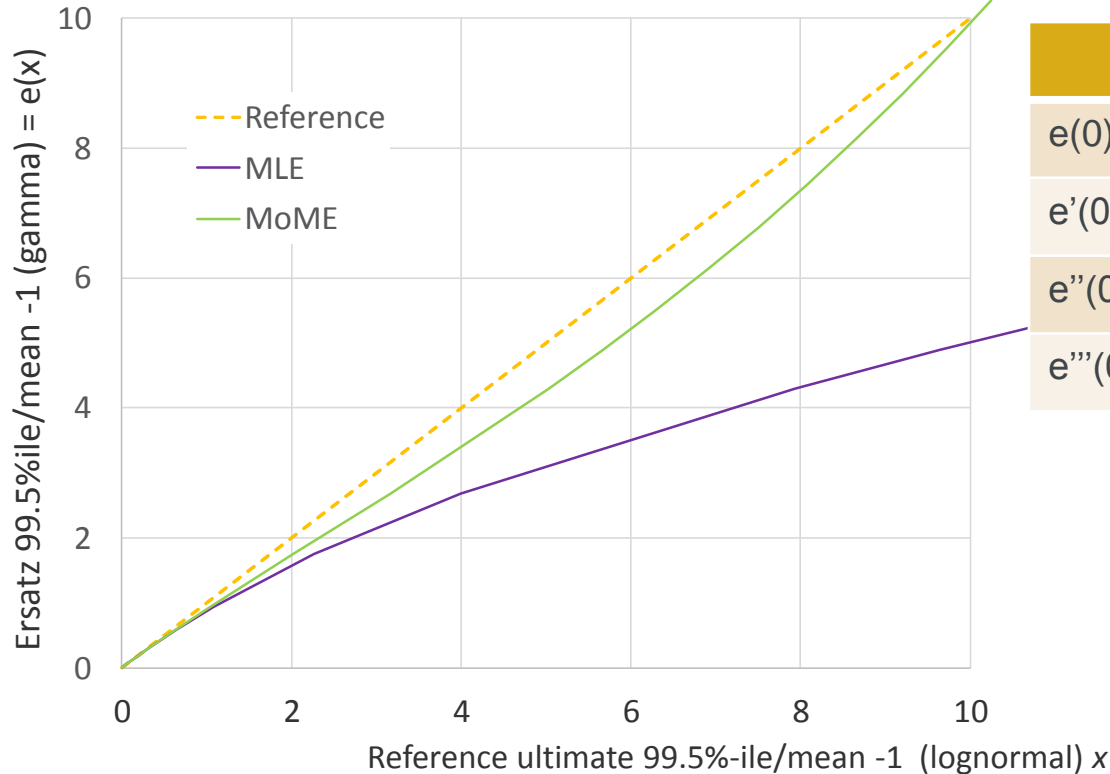


# Randomising the Reference Data

- Change TRUE to FALSE in the front sheet.
  - How many exceptions in-sample?
  - How many exceptions out-of-sample?
  - How many in each future year?
- The out-of-sample exception rate is an example of an ersatz model test, because we can perform the test even if the model is mis-specified.
  - We might say a method takes account of model and parameter error if it passes an exceptions test.
- Never forget that the model you are fitting is bound to be wrong, but it just might be an acceptable substitute for the right model.



# Theory: What happens for Very Large Data Samples?



	Ref	MLE	MoME
$e(0)$	0.000	0.000	0.000
$e'(0)$	1.000	1.000	1.000
$e''(0)$	0.000	-0.283	-0.283
$e'''(0)$	0.000	0.092	0.359



# Conclusions

- Take care over  $H_0$  in goodness of fit tests for fitted (ersatz) models.
  - Don't forget how Lilliefors effects might invalidate your confidence bounds.
- Test ersatz methodologies on randomly generated data.
- Consider asymptotic results for large samples.
- A mis-specified model can still be fit for purpose.
- MoME may be a better estimator than MLE.
  - Depending on how you define your objectives.
- Be explicit about what a good percentile estimate means
  - This involves choices.



# Questions

# Comments

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