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Credit Risk

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The Imperial Hotel, Blackpool

Credit Risk

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Introduction

- Why are we interested in reinsurance credit risk?
 - ICA's
 - Bad Debt Reserves
 - Value for money of reinsurance



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Introduction

- In this presentation we will construct models which address many of the issues concerned with modelling Credit Risk.
- We will apply these models to a simple example company and report the consequences of differing degrees of model sophistication on:
 - The held bad debt reserves and
 - the view of company of the value for money of their reinsurance programme.

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Introduction

- In this presentation we have ignored the issue of the size of the recoveries given that the reinsurer has defaulted.
- For the purposes of our analysis we have assumed a recovery rate of about 50%.
- For this presentation we have used quarterly credit ratings for all P&C insurance companies in the world since 1981. Source: © A.M. Best Company – used by permission.

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Example Company and Reinsurance Protection

- For the purposes of this presentation we will consider a very simple model for a set of losses and a reinsurance programme protecting the company.
- We will assume that the gross premiums are £10m and the aggregate gross losses are distributed as a LogNormal with mean £8 and standard deviation £2.

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Example Company and Reinsurance Protection

- This company is protected by an Aggregate XoL policy (£1m xs £10m) bought from a reinsurer, rated as B++.
- Our company is also protected by a 50% QS policy (after the XoL) provided by the same reinsurer
- The XoL policy was priced by the reinsurer as expected recoveries + 25% of the standard deviation of the recoveries.
- Brokerage on the protection is £45k.
- For simplicity we have assumed that the liabilities are exactly one year in length and there is no investment return possible.

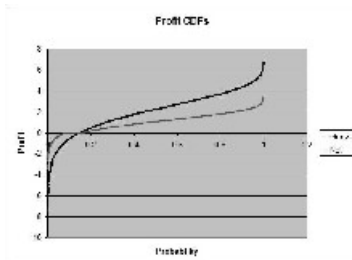
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Example Company and Reinsurance Protection

- The reinsurance premium is thus £185k
- Using a risk measure of VaR(99.5%) as our capital requirements and ignoring default risk we obtain:
 - Gross Capital Requirement: £4.441m
 - Net Capital Requirement: £1.858
- And expected return on capital of:
 - Gross: 45.6%
 - Net: 54.8%

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Example Company and Reinsurance Protection



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Traditional Approach

- The traditional approach to modelling reinsurance default has been to extend the rating categories to include default (which acts as a sink).
- Bad Debt reserves are calculated by applying best estimate "write of factors" derived from historic defaults.
- Transitions are then simulated by sampling the total number of transitions from a multinomial distribution with probabilities of transition either as targeted by the rating agency or fitted to historic data.
- Which reinsurer actually moved rating can then be sampled, without replacement.

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Traditional Approach

- The multinomial has the following probability function:

$$P(\{X_i\} = \{x_i\}) = \frac{M!}{\prod_{i=1}^n x_i!} \prod_{i=1}^n p_i^{x_i}$$

- This can be modelled by simulating from conditional binomials:

$$X_n \sim \text{Bin}(M, p_n)$$

$$X_{n-1} | \{X_i\}_{i=1}^{n-1} \sim \text{Bin}\left(M - \sum_{i=1}^{n-1} X_i, \frac{p_n}{1 - \sum_{i=1}^{n-1} p_i}\right)$$

$$X_i | \{p_k\}_{k=1}^n \sim M - \sum_{k=2}^n X_k$$

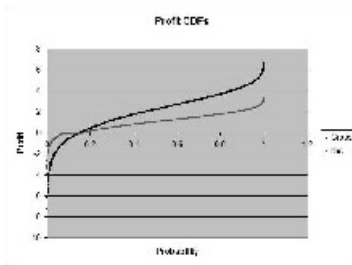
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Traditional Approach

- If we include the results of this analysis in our analysis of our example company we get:
- Bad Debt reserve at start of year: £4,619. ie very small.
- However net capital requirement now rests at £1.927m.
- This reduces the RoC to 47.9%.

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Traditional Approach



The Actual Profit
Model Profit

MultiNomial Parameter Uncertainty

- There is uncertainty surrounding the transition probabilities fitted to the historic data.
- The default probabilities are often based on very small data volumes and thus the probabilities are very uncertain.
- The process of incorporating parameter uncertainty into the multinomial model will also incorporate an element of dependency between the transitions of different companies.

The Actual Profit
Model Profit

MultiNomial Parameter Uncertainty

- If we assume a uniform prior then we obtain the following distributions for the parameter uncertainties, given n observations:

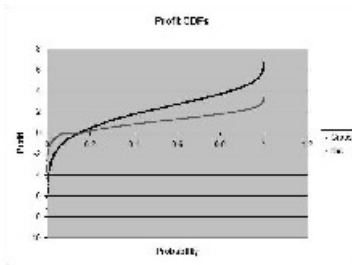
$$P_m \sim \text{Beta}\left(\sum_{j=1}^n x_{mj} + 1, m - 1 + \sum_{j=1}^n x_{kj}\right)$$

$$P_{m>i} \left[\{P_k\}_{k>i} \right] \sim \left(1 - \sum_{k=1}^m P_k\right) \text{Beta}\left(\sum_{j=1}^n x_{ij} + 1, i - 1 + \sum_{j=1}^{i-1} x_{kj}\right)$$

$$P_i \left[\{P_k\}_{k>1} \right] \sim \left(1 - \sum_{k=2}^m P_k\right)$$

The Actual Profit
Model Profit

MultiNomial Parameter Uncertainty



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MultiNomial Parameter Uncertainty

- If we include the results of this analysis in our analysis of our example company we get:
- Bad Debt reserve at start of year: £5,508. ie still very small, but increased from before considering parameter uncertainty.
- Now the net capital requirement rests at £1.943m.
- And this reduces the RoC to 46.3%.

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Momentum

- There is much research to indicate that there is a difference between the transition behavior of companies that have recently moved credit rating.
- A simple approach to modelling this non markov effect is to extend our credit ratings to incorporate a flag stating whether the company has moved credit rating in the previous time period.
 - +1 indicates that it moved up to the current credit rating,
 - 0 indicates that it remained in the current credit rating and
 - -1 indicates that it moved down to the current credit rating.
- All the mathematics previously mentioned still holds.

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Momentum

- The resultant matrix is sparse.
- To illustrate this, if we had only 4 ratings, with the 4th being default then the transition matrix would be as follows:

$$\begin{pmatrix} 0 & P_{11}^{10} & 0 & 0 & 0 & P_{12}^{1-1} & 0 & 0 & P_{13}^{1-1} & P_{14}^{1-1} \\ 0 & P_{11}^{00} & 0 & 0 & 0 & P_{12}^{0-1} & 0 & 0 & P_{13}^{0-1} & P_{14}^{0-1} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ P_{21}^{11} & 0 & 0 & 0 & P_{22}^{10} & 0 & 0 & 0 & P_{23}^{1-1} & P_{24}^{1-1} \\ P_{21}^{01} & 0 & 0 & 0 & P_{22}^{00} & 0 & 0 & 0 & P_{23}^{0-1} & P_{24}^{0-1} \\ P_{21}^{-11} & 0 & 0 & 0 & P_{22}^{-10} & 0 & 0 & 0 & P_{23}^{-1-1} & P_{24}^{-1-1} \\ P_{31}^{11} & 0 & 0 & P_{32}^{11} & 0 & 0 & 0 & P_{33}^{10} & 0 & P_{34}^{1-1} \\ P_{31}^{01} & 0 & 0 & P_{32}^{00} & 0 & 0 & 0 & P_{33}^{0-1} & 0 & P_{34}^{0-1} \\ P_{31}^{-11} & 0 & 0 & P_{32}^{-11} & 0 & 0 & 0 & P_{33}^{-10} & 0 & P_{34}^{-1-1} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

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Momentum

- For our example there are three states that our reinsurer could have. The results are below:

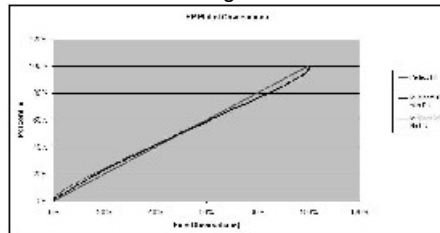
Rating	B++(-1)	B++(0)	B++(+1)
Bad Debt Reserve	£16k	£6k	£4k
Net Capital	£2.172m	£1.974m	£1.906m
RoC	42.1%	46.7%	48.5%

- Note that the RoC for B++(-1) is now smaller than the Gross RoC of 45.6%.

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Modeling the Difference of the Value

Dependency

- Question: How good is the model we have constructed in modelling historic transitions?



The Acton of Penetration
Modeling the Difference of the Value

Dependency

- Answer: Our approach thus far is not sufficiently dispersed.
- History tells us that transition is more dispersed than the multinomial model.
- In other words companies are not independent of each other when they default.
- This makes intuitive sense in that companies have common exposures both in terms of lines on the same policies and to common events.

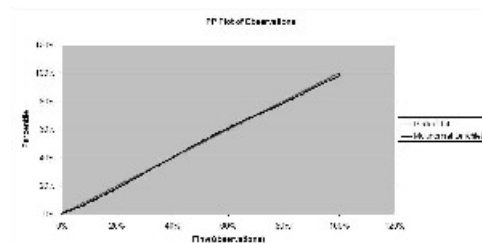
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Dependency

- A simple method of taking this into account is to extend the modelling we performed to cater for the parameter uncertainty to allow for extra dispersion, beyond that due to the parameter uncertainty.
- This is the Multinomial Dirichlet distribution.

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Modeling the Risk of the Future

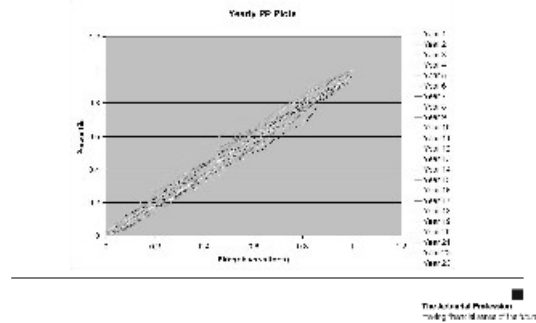
Dependency



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Dependency

- Separately by year this gives us the following plots:



Dependency

- Below is the table of results for this model compared to the results for the multinomial model with parameter uncertainty:

	Rating	B++(-1)	B++(0)	B++(+1)
Multinomial with Parameter Uncertainty	Bad Debt Reserve	£16k	£6k	£4k
	Net Capital	£2.172m	£1.974m	£1.906m
	RoC	42.1%	46.7%	48.5%
Multinomial Dirichlet	Bad Debt Reserve	£16k	£6k	£4k
	Net Capital	£2.193m	£1.993m	£1.916m
	RoC	41.6%	46.2%	48.2%

- As expected all the RoC's decreased.
- Note that the RoC for B++(-1) has now decreased even further below the Gross RoC of 45.6%.

Dependency between default and gross experience

- Intuitively we would expect there to be some dependence between market profitability and reinsurance transition rates.
- When the market is loss making we would expect reinsurers to be under more stress and thus have larger downward transition probabilities.
- This results in there being a greater risk of default when companies most need their programmes.
- Our proxy for the market combined ratio is that for the US market.

Dependency between default and gross experience

- Thus far we have considered the issue of transitions from each credit rating separately.
- We would expect there to be some dependency between transitions from different credit ratings.
- As with the dispersion issue for companies with the same original rating, this is due to the fact that companies have common exposures.
- We have not gone into any detail as to the nature of this relationship in this presentation.

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Conclusion and Comments

- The incorporation of an understanding of the uncertainties surrounding reinsurance default can materially alter both:
 - The perception of the value for money and effectiveness of reinsurance protections.
 - The level of appropriate bad debt reserves.

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Conclusion and Comments

- Issues with modelling credit default
 - There is a range of default probabilities within a single credit rating.
 - Many operations are either not rated by all or by any agencies .
 - It is difficult to extract reinsurer history as opposed to direct writers.
 - There is some evidence to say that rating agencies used to be too slow to change their ratings. The appropriateness of historic transitions can thus be called into doubt.

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Conclusion and Comments

- Issues with modelling credit default
 - Despite the large number of observations, some measurements have low statistical significance.
 - It is very difficult to get data on the level of recovery given reinsurer default.
 - Models based solely on the credit rating/history of the reinsurer will always be more limited than models using more information. Examples would include surplus asset models such as the Merton model.
 - Some of the apparent correlation within credit rating may be due to near replicate entries, such as parent-subsidary.

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Questions and Discussion

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