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GRAUNT'S LIFE TABLE

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Political arithmetic, or demography as we should now call it, was cradled in London where in 1662 a merchant, John Graunt (1620–74), published a remarkable book, *Natural and Political Observations upon the Bills of Mortality*. A copy of this book was included in the Exhibition at the Centenary Assembly of the Institute. The statistics available to Graunt were defective but he handled his limited material with considerable skill. There are few references to Graunt in the *Journal* and we are pleased to publish this article by Dr Glass.—Eds. *J.I.A.*

It is natural, in view of the importance of John Graunt in the history of demography, that considerable attention should have been paid to Graunt's life table. What has attracted particular interest is the arithmetical method by which Graunt arrived at the survivors in his table. Beginning with the London Bills of Mortality, he inferred that 'of 100 quick Conceptions about 36 of them die before they be six years old, and that perhaps but one surviveth 76',* and his survivors at each tenth year from 6 to 76 were given as 64, 40, 25, 16, 10, 6, 3 and 1, l_{80} being 0. It is the intermediate l_x's—from age 16 to age 76—which have constituted the puzzle.

Greenwood† and Willcox have both made interesting suggestions regarding the possible arithmetical method by which these intervening l_x's were obtained. Greenwood, while not claiming to put it forward as Graunt's actual method, showed that a constant ratio of 0·62 would give a fairly good fit. Willcox,‡ on the other hand, has argued that there is no evidence of the use of decimals by Graunt or Petty. This is not, of course, conclusive, and there is ample material to show the use of decimals by Gregory King, writing on similar subjects only a generation after Graunt. What might perhaps tell more against the constant fraction of 0·62 as an explanation is the intrinsic improbability that that particular fraction should have been chosen. In that respect the suggestion of Willcox, that Graunt experimented with fractions of $\frac{5}{8}$ and $\frac{5}{8}$, would seem more plausible. Nevertheless, those fractions, as may be seen from Table 1, do not give an exact fit. And at the same time Willcox's argument that the fraction $\frac{5}{8}$ is Petty's contribution because Petty used it later in his work on Ireland is rather to beg the question—to assume that Petty constructed the life table and then to 'prove' it by showing that the fraction $\frac{5}{8}$ was one used by Petty.§

Ptoukha's hypothesis,|| that Graunt might have used as his constant a decimal

* John Graunt, *Natural and Political Observations*, 1st ed., 1662, p. 69. (I cite throughout this paper the edition by W. F. Willcox, Baltimore, 1939.)

† M. Greenwood, *Graunt and Petty*, *J.R.S.S.* Vol. xci, Part I, p. 82 (1928).

‡ W. F. Willcox, ed. of Graunt, pp. xi–xii, suggests that Petty did the experimenting.

§ In any case, as is shown later, the $\frac{5}{8}$ fraction does not reproduce Petty's results.

|| M. Ptoukha, *John Graunt, fondateur de la démographie*, *Congrès International de la Population*, Paris, 1937, Vol. 2, pp. 71–72.

fraction equal to $\frac{64-1}{100}$, or 0·63, is still more attractive in that, as is shown in Table 1, the results of applying this constant are, when rounded, identical with Graunt's in all cases save one. Ptoukha suggests that in this single case, l_{66}, Graunt might have decided to reduce the 4 to 3 to avoid an otherwise excessive jump in mortality after age 66. But attractive as it seems, I do not believe that Ptoukha's explanation agrees with Graunt's statement, given as part of the introduction to his life table, that 'the numbers following are practically near enough to the truth; for men do not die in exact Proportions, nor in Fractions...'. Though various interpretations of this phrase are possible, I take it as implying that there is not a precise rate of mortality which applies at all ages; and also that Graunt's original calculations gave results in exact whole numbers and not in fractions. If this can be substantiated, it would speak equally against the suggestions of Greenwood and Willcox.

Table 1. l_x's in Graunt's life table

Age (years) (1)	As given by Graunt (2)	Greenwood's constant (3)	Willcox's fractions		Ptoukha's constant 0·63 (6)
			$\frac{5}{8}$ (4)	$\frac{2}{3}$ (5)	
0	100	100	100	100	100
6	64	64	64	64	64
16	40	40	40	43	40
26	25	25	25	28	25
36	16	15	16	19	16
46	10	9	10	13	10
56	6	6	6	8	6
66	3	4	4	6	4
76	1	2	2	4	1
80	0	—	—	—	—
86	—	—	1	3	—

It is possible to approach the problem in a slightly different way by considering first, what arithmetical methods were in use at the time, and secondly, whether Graunt's calculation was in fact done on the l_x's of his table. On the first point, I am not aware of any manuscript material of Graunt, which might help to throw light on his particular methods of work. But there is much manuscript material of Gregory King, a demographer of the same stamp if not perhaps of equal calibre,* dealing only a generation later with problems akin to those tackled by Graunt. King, who was an admirer of Graunt and had read his book with evident care, also tried to estimate age composition, both from local enumerations and by using a life-table approach, and one of his working journals, now in the Public Record Office,† contains many calculations. A common feature of these calculations is the use of 'differencing', not simply as a means of smoothing data, but for the purpose of computing the decrements from cohorts of births. In some cases the aim appears to have been to obtain smoothly declining first differences of deaths at successive ages.

* I am prepared to argue on behalf of King as an outstanding demographer, but this is obviously not the place to do so.

† In Treasury bundle T. 64/302.

Since this technique requires persistence and care rather than any specialized skill or romantic imagination, there is no reason why Graunt should not have used the same method.

Secondly, while Greenwood, Willcox and Ptoukha have obtained their possible fractions by working from the survivors in the life table, Graunt's primary emphasis, in describing—if that is the term appropriate to the somewhat inexplicit account given—his method, was on the numbers dying in the successive decades. Though saying that he sought 'six mean proportional numbers between 64, the remainder, living at six years, and the one, which survives 76', the numbers he then put forward were the numbers of deaths. It would thus be appropriate to study the deaths rather than the survivors in looking for Graunt's arithmetical technique. When this is done, a regularity of approach becomes apparent, and this is shown in Table 2.

Table 2. Graunt's data on deaths. Ratio to deaths in first 6 years (i.e. 36) expressed in vulgar fractions

Decades, etc. (1)	Absolute numbers (2)	In twelfths (3)	First differences of numerator, taken positively (4)	In thirty-sixths (5)	First differences of numerator, taken positively (6)
Up to 6 years	36	$\frac{12}{12}$	4	$\frac{36}{36}$	12
6–16	24	$\frac{8}{12}$	3	$\frac{24}{36}$	9
16–26	15	$\frac{5}{12}$	2	$\frac{15}{36}$	6
26–36	9	$\frac{3}{12}$	1	$\frac{9}{36}$	3
36–46	6	$\frac{2}{12}$	—	$\frac{6}{36}$	2
46–56	4	—	—	$\frac{4}{36}$	1
56–66	3	—	—	$\frac{3}{36}$	1
66–76	2	—	—	$\frac{2}{36}$	1
76–	1	—	—	$\frac{1}{36}$	—

In column (3), the ratios of deaths in successive decades to deaths in the first six years of life are expressed in terms of twelfths. It is then seen, in column (4), that the first differences of the numbers dying—once the deaths in the age-group 6–16 years are taken at $\frac{2}{3}$ of the deaths in the first six years of life—decline consistently by one unit for each decade up to age 46. This yields a total of 90 deaths, leaving 10 still to be allocated, one of which must, by hypothesis, occur after age 76. Clearly some deaths would have to be allocated to each decade, and a simple method of doing this, and still retaining as far as possible the scheme of consistently diminishing first differences, would be to regard the deaths after age 46 as fractions of 36 (the number prescribed for the first six years of life), or in other words as whole numbers. Unit declining first differences could then be taken up to age 66, after which the first differences would have to be constant in order to ensure, given the hypothesis of one survivor after age 76, that the total deaths at all ages amount to 100, equal to the radix of the life table.

This explanation of the arithmetical process may seem far fetched. But there are two points of support. First, the method yields—without any fractions or rounding of fractions—the precise values given in Graunt's life table. Secondly, it helps to explain the age statistics given by Petty in his account of Ireland. In *The Political Anatomy of Ireland** Petty put forward the following estimate:

'People in all	1100 M.
Of above 6 years old	704
16	462
26	297
36	198
46	132
56	88
66	77'

Petty used these figures in the same incorrect way as did Graunt—that is, he used the l_x columns as if they were T_x columns. But the figures are clearly not obtained simply by multiplying Graunt's l_x's by 11, for otherwise, as Hull points out, the results for the respective ages would be 704, 440, 275, 176, 110, 66 and 33. Nor does the constant fraction hypothesis reproduce Petty's results. Willcox believed that Petty, having assumed a total population of 1·1 millions, then also assumed that two-thirds were above age 6, two-thirds of the remainder above age 16, and so on. But this does not reproduce the results. In the first place it is clear that the number at age 6 is exactly equivalent to Graunt's survivors at that age—in other words, Petty assumes that 36% of people born die in the first six years of life. Taking a radix of 1100, that gives him exactly 704 at age 6. Thereafter the hypothesis of a constant survival fraction of $\frac{2}{3}$ could only be supported by accepting three mistakes in computation, of which two are improbable. Taking $\frac{2}{3}$ of 704 would yield approximately 469 (the question of rounding fractions again arises) and it is conceivable that, having written this, the figure was then read as 462. But $\frac{2}{3}$ of 462 would yield 308, and this would need to be written as 297 in order that the $\frac{2}{3}$ hypothesis should apply to the survivors at ages 36, 46 and 56 (which it would then do exactly). Finally, 77 is not $\frac{2}{3}$ of 88, and it is difficult to imagine that, if Willcox's hypothesis were correct, Petty could have been so widely out, on his last figure, in applying that simple fraction. Similarly, Ptoukha's constant fraction of 0·63 after age 6 does not reproduce Petty's figures, but gives, beginning with age 16, the series (to the nearest whole number) of 444, 280, 176, 111, 70 and 44, instead of 462, 297, 198, 132, 88 and 77.

It is also true that, if applied directly and without error, the method of differencing the deaths will produce Hull's figures and not Petty's. But it is possible, by this method, to reproduce almost all of Petty's results provided that one initial error is allowed. Suppose that, to begin with, Petty obtained his deaths in the first six years of life by taking, as Graunt did, 36% of his radix of 1100, namely 396, and by subtraction found (correctly) 704 survivors at age 6. Following the method suggested for Graunt he would then have taken the 396 deaths and used them to calculate, by differencing, the deaths in subsequent age-intervals. He might have proceeded as in Table 3.

In calculating his survivors he would have had his radix and his survivors at age 6. He would then have subtracted the deaths in successive age-groups. Up to age 56 the results in his table would be reproduced provided he made a mistake in calculating the survivors at age 16 or the number of deaths at

* C. H. Hull, *The Economic Writings of Sir William Petty*, Cambridge, 1899, Vol. 1, pp. 144–145.

ages 6–16. Various possibilities occur. For example, Petty's results would be reproduced if the deaths at ages 6–16 years were calculated at 242 instead of 264, and this would happen if, in multiplying 8×33 (i.e. $\frac{8}{12}$), the initial 24 were put down as 2 instead of 4, with no carry over into the tens. Given this one mistake, the survivors would be 704, 462, 297, 198, 132 and 88 up to and including age 56.

Table 3

Deaths under 6 years	Deaths in subsequent age-groups			
396	$=\frac{12}{12}$			
	$\frac{8}{12}=264$	6–16 years		
	$\frac{5}{12}=165$	16–26 years		
	$\frac{3}{12}=99$	26–36 years		
	$\frac{2}{12}=66$	36–46 years $=\frac{6}{36}$		
			$\frac{4}{36}=44$	46–56 years
			$\frac{3}{36}=33$	56–66 years
			$\frac{2}{36}=22$	66–76 years

This still does not explain the figure of 77 at age 66. Had Graunt's method been applied correctly throughout, the figure would have been 33. Allowing for the initial error suggested above, the figure should still have been 55 (i.e. 88 minus 33). It is possible that, since he was stopping short at age 66, Petty believed it appropriate to add to the 55 he should have obtained the 22 deaths due for the next decade, though this does not seem very plausible. But what I think is suggested is that Petty did not fully understand the method or principle of Graunt's life table. In particular, he did not appear to realize that the table should have been self-checking, since the total deaths, on Graunt's principle, should have amounted to 1100, and would have done if the deaths in the age-group 6–16 years had been 264 instead of Petty's 242, and if the table had been extended correctly to age 86. To me, at least, this error of principle lends additional support to the view of Prof. Greenwood, that Graunt's life table was really constructed by Graunt and not by Petty.