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HEDGING THE MARKET: THE PERFORMANCE OF THE FTSE 100 SHARE INDEX

By E. Dimson, Ph.D. and P. R. Marsh, Ph.D.

ABSTRACT

In this paper, we examine the hedging performance of the new FTSE 100 Share Index. First, we reconstruct a back-history of the monthly and daily index values using the London Share Price Database. Then we analyse the statistical characteristics of the Index, comparing it with the FT–Actuaries All Share and FT Ordinary Indexes. Finally, we examine the sources of undiversification within the FTSE Index. We conclude that the market is likely to track the FTSE Index to within a close tolerance, and that the residual tracking error is attributable partly to sector imbalances, but more importantly, to imbalances in exposure to smaller capitalization stocks.

1. INTRODUCTION

The actuarial profession has made a major contribution to the design and development of stock market indexes in the United Kingdom. The original Actuaries Index was designed in 1929, and followed the format and principles described in Haycocks and Plymen (1956) for over 20 years. By the early 1960s, however, its infrequent publication and limited weighting and averaging procedures led the Actuaries to develop, with the *Financial Times*, the All Share Index (see Haycocks and Plymen, 1964). Since 1962, the availability of the FT–Actuaries All Share Index (the FTA) has had a major impact on investment policy and practice. This index is now widely used in setting investment guidelines, in portfolio performance measurement, and as a measure of the overall value of the U.K. equity market.

In 1984, the Stock Exchange introduced a new index, constructed with the involvement of the Actuaries and published in conjunction with the *Financial Times*. *The Financial Times*–Stock Exchange 100 Share Index (the FTSE) is a continuously updated, real time index. As is explained in § 2.1 below, it was developed to facilitate the introduction of stock index futures and options trading into the U.K. market. Since the FTSE Index has been designed for purposes other than investment performance measurement and policy setting, it is complementary to, rather than competitive with, the FTA Index.

Though complementary to the FTA, the FTSE Index is likely to be of importance to actuaries for at least six reasons. First, as a result of the introduction of stock index futures and options in the U.K., the actuaries' traditional interest in matching and immunization will naturally be extended from the fixed interest portfolio to the equity portfolio (see, e.g., Wise, 1984). Second, the elimination in the 1984 Budget of tax relief for life assurance will further accelerate the trend within insurance companies towards developing new products and seeking innovative sources of income (see Gustavson, 1980). Stock index futures and options are likely to play a significant rôle in these developments: guaranteed contracts, for example, become much easier to manage when index-based investments are available. Third, with the advent of dual capacity in the U.K., actuaries, especially those employed by stockbrokers, will be expected to advise on controlling the market maker's exposure to fluctuations in equity prices.

The remaining motivations for taking an interest in the new index are more traditional. The fourth reason is that the actuarial profession carries a joint responsibility for the management of the FTSE Index, as it does for the FTA Index (see Brumwell, 1984 and prior years). Fifth, members of the profession are likely to use the FTSE Index for the analysis of statistical relationships in the equity market (as in MGWP, 1981, and Clarkson, 1983). Finally, actuaries will inevitably be asked to express an opinion about the suitability of the new index for applications other than those for which it was designed—for example, performance measurement and portfolio selection (see Holbrook, 1977 and Moore, 1972). For these reasons, as well as the tradition of documenting the characteristics of U.K. indexes within this journal, it is appropriate that we present an analysis of the properties of the FTSE Index to an actuarial audience.

The structure of our paper is as follows. Section 2 reviews the development of the FTSE Index, and provides an overview of the methodology and data used in the research. The FTSE Index was initiated only at the beginning of 1984, and it was therefore necessary to reconstruct an index history in order to undertake our research. Section 3 provides the detailed methods of calculation and the estimated index values for the FTSE, both monthly from 1978 to 1983 and daily within 1983.

The distributional and statistical characteristics of the reconstructed 100 Share Index (hereafter referred to as the FTSE) are then appraised in § 4. In this section, we discuss the distribution of the returns on the FTSE, FTA and FT Ordinary Share (FTO) indexes; we analyse the regression and correlation relationships between each index and the overall market; and we explore the sensitivity of our results to the length of the period over which returns are measured.

In § 5 the FTSE Index design is studied in greater detail. Evidence is provided about the impact on the index of constituent changes, and on the frequency with which such changes can be expected to occur. We also examine the extent to which industry and sector imbalances cause the FTSE to be excessively undiversified. We present our summary and conclusions in § 6.

2. THE FTSE INDEX

2.1. *Development of the index*

The FTSE Index was initially proposed as a vehicle for stock index futures trading in the U.K. Stock index futures were first traded in the Kansas market in February 1982 and spread swiftly throughout the various financial exchanges in the United States of America. Subsequent developments have included stock index options and contracts on sub-indexes, as well as options on stock index futures, though the latter have met with limited success. While the U.S. stock exchanges have competed to develop contracts on a variety of equity indexes, the Chicago Board Options Exchange (CBOE) contract on their S & P 100 Share Index, introduced in March 1983, has been the most successful. At the time of writing, the CBOE's index option contracts represented more than half of that exchange's total volume. At the same time, the dominant index futures contract, based on the S & P 500 and traded on the Chicago Mercantile Exchange, has become the major feature on that exchange.

Pressure for the introduction of U.K. index futures trading came from the London International Financial Futures Exchange (LIFFE). At the same time, the London Stock Exchange recognized that a continuously updated index was needed as a basis for futures and options trading. The indexes which existed at the time were the FT Actuaries All Share Index (updated daily) and the FT Industrial Ordinary Share Index (calculated hourly). While futures and options contracts might ideally be based on the All Share Index, it was recognized that it would be impractical to compute a 750 share index on a real-time basis. The infrequency with which the smaller index constituents are traded would, in any case, reduce the value of this exercise. Consequently, it was agreed that an alternative real-time index would be developed as a basis for options and futures trading. The two leading contenders for a real-time index were the existing FT Industrial Ordinary Share Index (the FTO), and a new index similar in conception to the CBOE 100 Share Index. Other possibilities included an equally weighted market leaders index, such as the index introduced in 1983 by the European Options Exchange in Amsterdam, and a more broadly based (but continuously rebalanced) index, such as that introduced in the previous year in the Vancouver market. The drawbacks of these methods of index construction are, however, well known (see Cootner, 1966, and Brennan and Schwartz, 1984), and both Amsterdam and Vancouver were in fact obliged to withdraw or modify their indexes early in their lives. These other possibilities were therefore excluded at an early stage.

As a device for hedging fluctuations in the overall equity market, it was desirable that the chosen index should track the market closely; but it would clearly be preferable to develop a contract based on a popular and well-established index. The FTO Index has been published continuously since 1935, is well known to the public, and is even the basis for existing contracts for betting on future movements in the level of the equity market. It was therefore necessary to examine the viability of linking futures and options contracts to the FTO, rather than developing an alternative index for this purpose. At an early stage of the research, however, it became clear that the FTO Index would not track the market sufficiently closely to meet the needs of investors, hedgers and speculators. The decision was taken to investigate the properties of a capitalization-weighted, arithmetic index based on 100 leading shares. This investigation, which supported the establishment of a real-time 100 Share Index in preference to a real-time version of the FTO, forms a part of the research described here. In addition, we examine a number of other characteristics of the new index, now called the FTSE 100 Share Index, and quantify the magnitude and sources of undiversification in the index.

2.2. *Methodological and data considerations*

This paper focuses on three indexes, two published and one specially estimated for this study. The published indexes are the FT–Actuaries All Share Index (the FTA) and the FT Industrial Ordinary Share Index (the FTO). The FTA covers some 750 shares which comprise over 90% of the capitalization of the entire U.K. equity market. It is used as a proxy for the value of equities as a whole. The FTA is a capitalization-weighted, arithmetic index and measures movements in the market value of a passively managed portfolio invested in all but the smallest stocks. It has been calculated daily since its inception in April 1962.

The FTO, on the other hand, is an equally weighted geometric index whose coverage is restricted to industrial companies and one oil company. It is published hourly, and the constituent prices are collected independently of the FTA's price collection process (see Bell and Greenhorn, 1984). Despite its drawbacks, the FTO has been used as a barometer for the U.K. equity market since 1935. The values of the FTA and FTO indexes were obtained from the London Share Price Database, maintained by the London Business School.

The index which has been constructed specially for this study is an arithmetic 100 Share Index which is capitalization weighted. One option for such an index would be to compute it from the average prices of the stocks which, at the time of this study (New Year 1984), had the top 100 capitalizations. Although this approach was, in fact, used by at least one commercial investment organization, it is incorrect. Such an index would include stocks that had grown bigger than, and would exclude those whose capitalizations had fallen below, the necessary minimum. Consequently, such an index would appear to outperform the market for quite spurious reasons.

To avoid this *ex post* bias, it is necessary to identify index constituents using only data which were available at the time. The 100 Share Index in any particular year was therefore constructed from the largest companies which were components of the FTA index at the previous year-end. Company size was specified so as to be consistent with the definition used in the All Share Index, and was therefore equated to the aggregate market value of all the listed ordinary shares of the company. Non-dividend paying shares, unlisted ordinary shares, oil production units and preference shares were therefore excluded from the index, as was one company (BL) for which 99·7% of the ordinary shares are owned by the State. Following these principles, we identified the 100 stocks which were eligible to be index constituents in each of the years 1978 to 1983 inclusive.

To obtain the necessary stock price and capital change data to construct the index from these constituents, it is necessary to access a share price database which is completely comprehensive: in particular, the database must include all dead companies with their correct terminal values. Most commercial databases delete stocks which subsequently went bankrupt, were acquired, or otherwise lost their stock exchange listing. They are therefore likely to provide biased estimates of the returns on the index. The London Share Price Database (LSPD) does not suffer these drawbacks as it includes all Sterling denominated U.K. ordinary shares, regardless of their current status. The LSPD was therefore used to compute the monthly index values from the end of 1977 to the end of 1983. In addition, daily estimates of the index values were computed for 1983, using data collected by the Stock Exchange. The methods of calculation and the index values themselves are presented and discussed in § 3 below.

3. ESTIMATES OF MONTHLY AND DAILY FTSE INDEX VALUES

3.1. *Estimates of monthly FTSE index values, 1978–83*

The back-history of the FTSE was constructed using stock selection principles which, as explained above, were designed to avoid any *ex post* bias in the index. The type of bias we wished to avoid was the use of information which would not have been available at the time (e.g., whether the stock was an index constituent as of 1984), or the relevance of which might not have been apparent at the time (e.g., government involvement in the company). In addition, when ranking the shares by their aggregate market value at each year-end, we also took particular care to avoid bias from our choice of stock price.

The bias we sought to avoid occurs because stock prices are typically observed with some error, because of price rounding, the jobber's spread, untimely quotations or source document errors. Consequently, a share which had only just crept into the list of constituents might suffer a small price drop when the stock is subsequently valued at the middle-market price. This would not, of course, be offset by the upward price movement of a stock which had only just been excluded from the index. As observed by Roll (1984) and others, this might speciously cause the new index to appear to underperform the market. We minimized the problem caused by potential measurement error in stock prices by ranking stocks by their capitalization *before* the close of business for the calendar year. We did this by calculating their capitalizations using the market prices prior to the close of business on the last trading day of the year, as shown in the Stock Exchange Daily Official List. These capitalizations were used to select the 100 companies that were to be the constituents of the FTSE for the following year.

The year-end and subsequent index values were then computed using middle-market quotations at the close of business for each month end.

The movements in the index were thus estimated on a month-by-month basis for twelve successive months from each year-end. The index constituents were then revised using the procedure described above. Within each year, the weight given to each index constituent was proportional to its capitalization at the previous year-end. In the rare instances where a stock ceased to exist within the year, the terminal proceeds of the stock were reinvested in the surviving constituents of the index.

The actual number of constituent securities in our index varied between 106 and 107. This is because of the second-line shares which were included in our calculations. Although each of these securities was valued using the correct stock price for the share in question, we will continue to refer to them as though there were exactly 100 constituents. This is to maintain consistency with the published indexes, for which the price of second-line shares is always replaced by the price of the more marketable stock.

The value of the index was calculated for the last trading day of the base month (December 1977) and for each subsequent month-end. We used the following formula:

$$I_t = (1000/C_{72}) \prod_{i=1}^{t} W_i \qquad t = 1 \ldots 72 \tag{1}$$

where C_i is the aggregate market capitalization of the constituents of the FTSE at the end of month i, C_{i-1} is the aggregate market capitalization of the same stocks at the start of month i (i.e., using stock prices from the end of month $i-1$), and $W_i = C_i/C_{i-1}$. Note that a company's capitalization is defined here as the number of shares outstanding at the previous year-end multiplied by its capital-change-adjusted share price, and that C_i and C_{i-1} are defined so as to refer to the capitalization of the same set of index constituents.

Figure 1 displays the monthly values which were estimated for the FTSE Index. The FTA and FTO Index values which accompany the FTSE graph have been drawn with the same base value for 31 December 1977 and to the same scale, though the vertical axes record the actual index values for the FTA and FTO. The figure discloses a close association betwen the FTSE and FTA, with slight outperformance by the latter. This similarity between the indexes is especially striking when it is borne in mind that the basic source for the stock prices used in constructing the 100 Share Index (i.e., the LPSD) is independent of that used in assembling the FTA Index (see Bell and Greenhorn, 1984). By contrast with the FTA, the FTO drastically underperforms the FTSE (and FTA) indexes, a characteristic which we examine futher in § 4. The monthly index values for the FTSE and comparative values for the FTA are listed in Appendix 1 at the end of this paper.

3.2. *Estimates of daily FTSE index values for 1983*

In addition to our estimates of monthly index values using the LSPD, we also

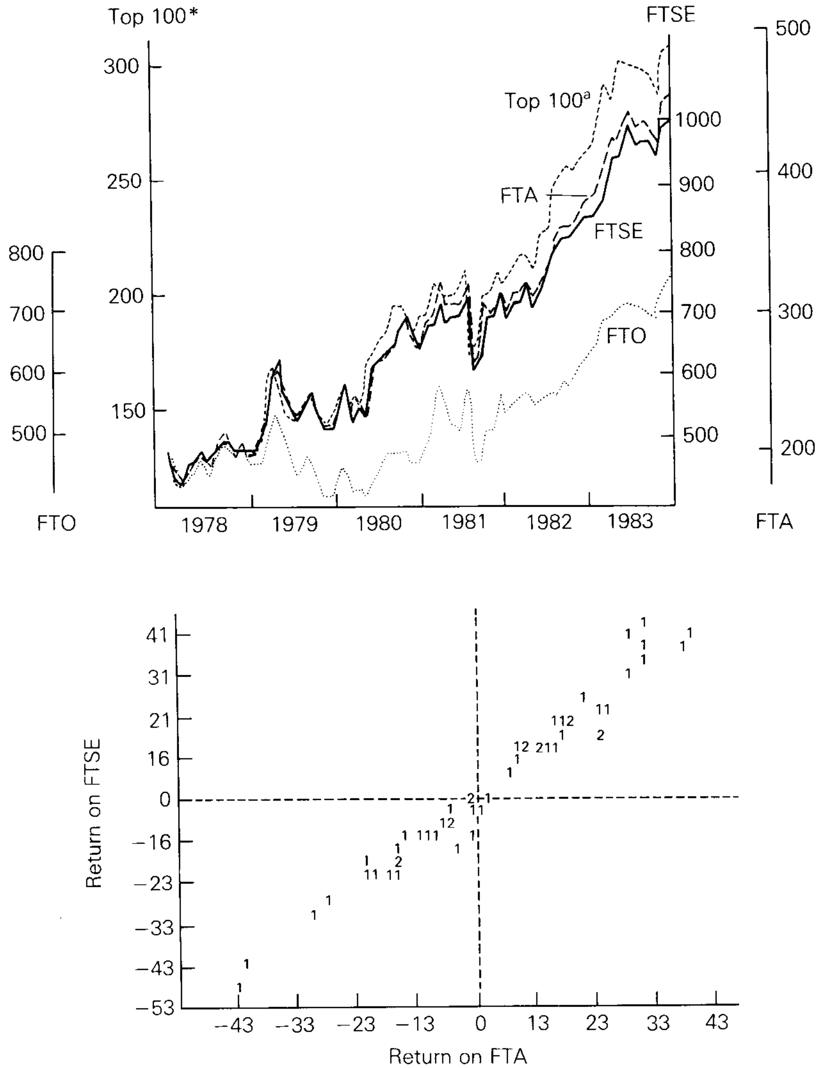

Figure 1. Month-end values for the FTSE, FTA, FTO and Top 100* Indexes, 1978–1983.

* The Datastream-based Top 100 Index is based on 100 of the largest stocks which were listed in 1984. Note that, unlike the other three indexes, which have been estimated on a daily basis and hence plotted at each month-end, the Datastream-based index was computed on a weekly basis using Wednesday's closing prices and is plotted using the Wednesday nearest the month-end.

computed a series of daily index vaues for 1983. For this exercise, the Stock Exchange collected share prices from EPIC, their computer-based information system: these prices were adjusted for capital changes and weighted using each constituent's equity capitalization, so as to produce a sequence of simulated FTSE index values.

Because of the magnitude of the clerical task involved, the data was compiled by the Stock Exchange on a weekly basis, and index values were therefore calculated for the close of business on each Friday during 1983. Fortunately, the association between the weekly changes in the FTSE and in the FTA is very close. Given that daily values of the FTA are readily available, this makes it possible to interpolate accurate daily values for the FTSE. To produce daily estimates from the Friday closing values, we therefore ran the regression

$$R_{st} = a + b\ R_{at} + u_t \qquad t = 1 \ldots 52 \qquad (2)$$

where R_{st} is the continuously compounded return (excluding dividends) on the FTSE Index in week t, R_{at} is the corresponding return on the FTA Index, the slope b is the beta of the FTSE relative to the FTA, the intercept a is the alpha of the FTSE relative to the FTA Index, and u_t is the residual return on the FTSE Index.

Using the parameters a and b and the residuals u_t of regression (2), we then estimated the daily returns, $R_{s\tau}$, on the FTSE Index as follows:

$$R_{s\tau} \approx b\ R_{a\tau} + (a + u_{t(\tau)})/n_{t(\tau)} \qquad \tau = 1 \ldots 255 \qquad (3)$$

where $R_{s\tau}$ is the estimated return on the FTSE Index during trading day τ, $R_{a\tau}$ is the corresponding day's return on the FTA Index, $u_{t(\tau)}$ is the residual return in week t which contains day τ, and $n_{t(\tau)}$ is the number of trading days in the same week t. The estimated index returns $R_{s\tau}$ were then compounded, so that the levels of the FTSE Index could be computed on a daily basis.

The above procedure amounts to equating the daily index values to the Stock Exchange figure for each Friday's close, while the Monday, Tuesday, Wednesday and Thursday figures are estimated in such a way that the week's outperformance or underperformance of the new index, as measured by the week's residual return plus the alpha, is spread evenly over the trading days of the week. Taking account of these daily residual returns, in conjunction with the beta-adjusted daily movements in the FTA, we are therefore able to estimate the daily values of the FTSE. Our estimated values of the FTSE Index are listed in Appendix 2 for each weekday in 1983.

3.3. *Accuracy of the estimates and comparison with alternative estimates*

The monthly values computed using the Stock Exchange figures and presented in Appendix 2 are close, but not identical, to those previously estimated from the LSPD and presented in Figure 1 and Appendix 1. Since the prices and capitalizations used by the Stock Exchange were drawn from alternative sources, and since there were two differences in the actual index constituents for 1983, these discrepancies are not surprising. The actual differences between the two index series, for the six days on which the last trading day of the month was a Friday, are −·02% (on 31 December), +·26% (31 March 1983), +·37% (29 April), +·07% (29 July), +·04% (30 September) and, of course, zero (on 30 December 1983). These differences between the LSPD-based and Stock Exchange-based index values are sufficiently small that, for most purposes, they may be ignored.

There is, in addition, the potential inaccuracy of the daily index estimates for days other than a Friday. However, the residual standard deviation of regression (2) is only ·37%. Hence the standard deviation surrounding the daily estimates of returns, $R_{s\tau}$ in (3), is only ·16%, and this, too, is a small potential error. We conclude that the daily index values presented in Appendix 2 are very close to those which would have been compiled if the FTSE Index had, in fact, gone live at the beginning of 1983.

It is interesting to compare our estimates with the index values which would have been computed if the *ex post* bias, discussed earlier in §2.2, had been allowed to occur. On 9 January 1984, the *Daily Telegraph* published a graph of the 100 Share Index produced to the newspaper's specification by Datastream, the computerized stock market information service. Datastream kindly provided us with the values that were shown in the graph. Their rebased index values for the end of each year from 1977 to 1983 were as follows (our estimates for the FTSE in parentheses): 428·9 (475·5), 442·1 (484·2), 475·3 (509·2), 613·4 (647·4), 656·4 (684·3), 825·2 (834·3) and, of course, 1000 (1000).

Since the Datastream-based version of the FTSE index was estimated using constituents which survived to, and were large at, New Year 1984, it is scarcely surprising that their Top 100 index outperforms the FTSE Index presented in §§3.1 and 3.2. But the magnitude of the outperformance of the Datastream-based index (23% over the last six years, 20% over the last five, 14% over the last four, 9% over the last three and 6% over the last two years) is indicative of the care which is needed in estimating index value histories from a stock price database. The *ex post* bias in the Datastream-based top 100 index transforms the 9% underperformance by the FTSE (relative to the FTA) into a spurious 14% outperformance. The Datastream-based Top 100 index is compared graphically with the FTSE, FTA and FTO indexes in Figure 1.

4. COMPARISON OF THE FTSE, FTA, AND FTO INDEXES

4.1. *Distributional characteristics of the indexes*

We commence, in this section, with an analysis of the statistical characteristics of the FTSE Index values, as estimated in §3. Comparisons are drawn with the capitalization-weighted FTA Index and with the geometric FTO Index. There are small differences in the magnitude and timing of the dividend payments on the constituents of the FTA and FTSE, and larger differences when the FTO is considered. While these differences affect futures and option values, they have little impact on the hedging characteristics of the indexes. Consequently, the returns which are computed in this study exclude dividends.

In Panel A of Table 1 we report on the distributional characteristics of the continuously compounded returns on these three indexes. Over the course of 1983, the FTSE experienced the lowest mean return, ·35% per week or an annualized 19·9%. The performance of the FTA was 5 basis points per week or 3·2% per year better, reflecting the substantial outperformance of the smaller stocks which are omitted from the FTSE Index (see Dimson and Marsh, 1984). The standard deviation of returns on the FTSE, on the other hand, was 1·99% per week, or an annualized 14·4%. This represents rather more volatility than the 1·81% per week (an annualized 13·1%) of the FTA.

The analysis of monthly data over the period 1978 to 1983 repeats the pattern witnessed during 1983. The FTSE is outperformed by the FTA, in this case by some 6 basis points per month. Over the six-year period, this compounds up to a 9% greater appreciation by the FTA than the FTSE. And, as in 1983, the FTSE is marginally more volatile than the FTA. The FTSE has a standard deviation of monthly returns of 4·97% (an annualized 17·2%), as compared to 4·89% (an annualized 16·9%) for the FTA. It is likely that the slightly greater volatility of the FTSE reflects the omission of smaller stocks, which have a below-average beta and add to the overall diversification of the FTA Index (see Dimson and Marsh, 1983). Note, however, that the inclusion of less marketable stocks in the FTA may lead to slight under-estimation of that index's volatility (again, see Dimson and Marsh, 1983).

It is well known that a geometric index will underperform an arithmetic index with the same constituents; see, for example, Marks and Stuart's (1970) analysis of the bias arising from the FTO's method of construction. It should therefore come as no surprise that, despite occasional outperformance by the geometric FTO as in 1983, this index significantly underperforms the FTA over most periods. This is very obvious over the 1978–83 period, when the FTO was outperformed by around ·4% per month.

The third part of Panel A compares the FTA with the FTO over the entire history of the All Share Index, starting in April 1962. Over this period, the return on the FTO is dominated by that on the FTA, with a margin in favour of the latter averaging over ·2% per month. As in the other periods, the FTO also has a higher standard deviation of returns than its broader counterparts.

Finally, we turn to the coefficients of skewness and kurtosis for the index returns. These statistics respectively measure departures from a symmetric distribution of returns and the tendency for extreme values—both high and low—to occur more frequently than in the normal case. The normal distribution has a coefficient of skewness of zero, and an excess kurtosis (relative to the normal distribution) of zero. This provides a standard for comparing the empirical properties of the return distribution.

In fact, the weekly data disclose virtually no departure from normality, whether in terms of skewness or kurtosis. The monthly data for 1978–83 indicate

Table 1. *Comparison of the FTSE index with the FTA and FTO indexes*

Panel A:	Period and frequency	Index name	No. of observations	Mean return	Standard deviation	Coefficient of skewness†	Coefficient of kurtosis†
Moments of the distribution of percentage returns*	Weekly data 1983	FTSE	52	·35	1·99	−·10	−·37
		FTA	52	·40	1·81	−·28	−·45
		FTO	52	·50	2·05	−·27	−·45
	Monthly data 1978–83	FTSE	72	1·03	4·97	−·74	1·95
		FTA	72	1·09	4·89	−·87	2·21
		FTO	72	·65	5·43	−·37	1·45
	Monthly data 1962–83	FTA	260	·59	6·29	·70	8·04
		FTO	260	·37	6·64	·44	4·76

Panel B:	Period and frequency	Dependent variable	Independent variable	Beta (& t-value)‡	Alpha (& t-value)‡	Correlation coefficient	R-Bar squared
Regressions and correlations between percentage returns	Weekly data 1983	FTA	FTSE	·90 (4·3)	·09 (1·8)	·983	·966
		FTA	FTO	·81 (3·8)	−·01 (0·1)	·917	·837
		FTSE	FTO	·87 (2·1)	−·09 (0·7)	·899	·804
	Monthly data 1978–83	FTA	FTSE	·98 (1·6)	·08 (1·1)	·992	·984
		FTA	FTO	·84 (4·1)	·55 (2·5)	·928	·860
		FTSE	FTO	·83 (3·7)	·49 (2·0)	·906	·817
	Monthly data 1962–83	FTA	FTO	·91 (5·6)	·26 (2·4)	·961	·924

* All returns exclude dividends and are continuously compounded. Returns are per period and are not annualized.

† The coefficients of skewness and kurtosis are, under the definitions used here, equal to zero for the normal distribution.

‡ The t-values for beta are for tests against the null hypothesis that the true value of beta is unity, while those for alpha are for tests against a null of zero.

a slight tendency for the distribution to be leptokurtic, i.e. having fat tails, with rather more extreme returns (and slightly fewer moderate returns) than the normal distribution. The monthly data for 1962–83, however, is extremely fat tailed, and consequently there is a risk of over-stating the significance of the t-values obtained in the regression described below. Though we do not report the details in Table 1, it is worth noting that the high kurtosis observed over 1962–83 is largely attributable to the market crash and recovery of 1974/75. Subperiods which exclude 1974/75 display characteristics which are broadly similar to those witnessed over the 1978–83 period.

4.2. *Regressions and correlations between the indexes*

In this section, we present our analysis of the extent to which the FTSE Index can successfully track overall market movements. Our objective here is to determine how well a diversified portfolio of equities, proxied by the FTA Index, can be hedged with a contract based on an alternative index. We therefore estimate regressions (4.1) to (4.3) below:

$$R_{at} = a_1 + b_1 R_{st} + u_{1t} \qquad t = 1 \ldots \mathrm{T} \qquad (4.1)$$

$$R_{at} = a_2 + b_2 R_{ot} + u_{2t} \qquad t = 1 \ldots \mathrm{T} \qquad (4.2)$$

$$R_{st} = a_3 + b_3 R_{ot} + u_{3t} \qquad t = 1 \ldots \mathrm{T} \qquad (4.3)$$

where R_{at} is the continuously compounded return (excluding dividends) on the FTA Index in period t, R_{st} is the corresonding return on the FTSE Index, and R_{ot} is the corresponding return on the FTO Index. The a_i, b_i and u_{it} are respectively the intercept, slope and residual of the regressions.

Regression (4.1) evaluates how well fluctuations in the All Share Index can be hedged using the FTSE Index. The results of this regression may be compared with (4.2), which checks how well the FTO would perform the same function. For completeness, regression (4.3) examines the extent to which movements in the FTSE can be related to movements in the FT Ordinary Index. This final regression can be interpreted as a test of the extent to which a portfolio invested in the 100 largest stocks could be hedged using a contract based on the FTO.

If regression (4.1) is compared with regression (2) in the previous section, it will be seen that the dependent and independent variables have been interchanged. Consequently, the slope coefficient in equation (4.1), for example, can no longer be regarded as a measure of the systematic risk of the FTSE Index relative to the market, as proxied by the FTA. Rather, the beta in (4.1)–(4.3) measures the hedge ratio, i.e. the relative size of the holding in the FTSE (or FTO) which is required to hedge market movements in the FTA (or, in equation 4.3, the FTSE).

Focusing primarily on regressions (4.1) and (4.2), there are a number of inter-related characteristics which would lead us to prefer one of the two narrower indexes as a means of hedging the All Share Index. For convenience, we will specify the conditions under which the FTSE would be more suitable as a vehicle for hedging market movements. First, the beta should be known with as much precision as possible, i.e., have a low standard error, and (ideally, for ease of exposition) be close to unity. Second, the alpha should be known with as much precision as possible and (again, ideally for ease of exposition) be close to zero. Third, the correlation with the All Share Index should be as high as possible. Denoting the correlation between the dependent and independent variables in (4.1) and (4.2) by ρ_1 and ρ_2 respectively, the FTSE would dominate the FTO on *all* criteria if the following conditions were true:

$$\begin{aligned} &\mathrm{Var}(b_1) < \mathrm{Var}(b_2) \quad \text{and ideally,} \\ &\mathrm{Abs}(b_1 - 1) < \mathrm{Abs}(b_2 - 1) \end{aligned} \tag{5.1}$$

$$\begin{aligned} &\mathrm{Var}(a_1) < \mathrm{Var}(a_2) \quad \text{and ideally,} \\ &\mathrm{Abs}(a_1) < \mathrm{Abs}(a_2) \end{aligned} \tag{5.2}$$

$$\rho_1 > \rho_2 \tag{5.3}$$

where Var denotes the estimation variance and Abs denotes the absolute value. Equations (4.1)–(4.3) respectively have been estimated using the weekly data for 1983, the monthly data for 1978–83, and (in the case of the FTA and FTO only) the monthly data for 1962–83. These three sets of results are presented in Panel B of Table 1. First, we note that using weekly data, the beta and alpha of the FTA relative to the FTSE are ·90 and ·09%, the correlation between the two return series is ·983, and thus 96·6% of the variance of FTA returns can be explained by movements in the 100 Share Index. By contrast, when the All Share Index is regressed on the FTO, the beta is ·81 and only 83·7% of the variance can be explained by movements in the FT Ordinary Index. It seems that the FTSE is likely to provide an effective means of hedging fluctuations in the total market, while the FTO would perform very poorly indeed. Finally, note that the FTO would be an extremely poor choice as a device for hedging fluctuations in the top 100 companies, for the R-squared between the FTSE and the FTO is lowest of all, only 80·4%.

The results based on monthly data for 1978–83 are presented in the middle of Panel B. The estimated beta of the monthly FTA relative to the other indexes is higher than with weekly data, and it also has a lower standard error. In the case of the FTA regressed on the FTSE, the beta of ·98 is not significantly different from unity, while it is significantly different in the other regressions. The alpha (·08) of the FTA relative to the FTSE is not significant, while it is significantly positive when other indexes are used. Finally, the R-squared between the FTA and FTSE is 98·4%, whereas the regressions of the FTSE and FTA on the FTO continue to display poor explanatory power, with R-squared values of only 86·0 and 81·7% respectively. Using criteria (5.1) to (5·3), the FTSE is on all counts demonstrably superior to the FTO as a device for hedging fluctuations in the level of the overall market.

4.3. *Impact of the measurement interval on the regression results*

When indexes are based on stock prices that are not truly synchronous, artificial dependencies are introduced into the estimated index returns (though not into returns on index futures). As shown by Dimson (1979), this problem can be acute when U.K. equity prices are involved. In Panel A of Table 2, we see that the broader indexes show no evidence of suffering from significant infrequent trading. In all periods examined, the first order serial correlation is small and insignificant, and only the third-order coefficient using monthly data is statistically significant. If there are measurement errors in the indexes, they arise largely from sources other than thin trading.

The impact of measurement errors in an index (or in a stock or portfolio) is to cause its beta and correlation coefficient, relative to a more accurate index, to be underestimated. Dimson (1979) shows that, even with monthly data, this problem can be very severe in the U.K. Since the dependent variables in regressions (4.1)–(4.3) are broader, and therefore more exposed to possible price measurement errors than the independent variables, it is likely that shortening the measurement interval will attenuate their betas, as well as their correlation coefficients. The results of the regressions, presented previously in Panel B of Table 1 would be consistent with this observation. This motivates us to undertake a more careful scrutiny of the sensitivity of the regression results to the length of the measurement interval.

Panel B of Table 2 therefore provides further evidence on the hedging properties of the FTSE and FTO indexes. The results of a regression of the All Share Index on the FTSE or FTO are presented in this panel in two blocks: the first block is based on the daily data for 1983, and the second is based on the entire 1978–83 period. In each case the regression and correlation coefficients from (4.1) and (4.2) are first computed using daily returns, and then using data which have been aggregated into returns computed over a longer measurement interval.

We cannot meaningfully compare the serial correlation of the daily FTSE with that of the FTA, because the daily FTSE was estimated from the FTA in the first place. For the purposes of this analysis, therefore, the FTSE Index is available with a weekly or longer frequency over 1983, and with a monthly or longer frequency over the 1978–83 period. Using this data over the periods examined in Panel B, the estimated beta of the FTA relative to the FTSE appears to increase as the measurement interval is extended. However, given the lack of an independently estimated daily series for the FTSE, we may also use the FTO to judge whether the measurement interval has a noticeable impact on the regression and correlation coefficients.

We have already noted that the beta of the FTA relative to the FTO is uniformly lower than the figure computed, relative to the FTSE, using the same measurement interval. However, the FTO, which is available on a daily basis, also provides confirmatory evidence that betas and correlation coefficients are indeed attenuated as the measurement interval gets briefer. As might be expected, the bias is at its most severe when daily data is used. This attenuation bias would not, of course, be present if a timely index futures price were

Table 2. *Impact of the measurement interval on characteristics of the FTSE, FTA and FTO indexes*

Panel A:	Period and frequency	Name of index	Serial correlation coefficients: First order	Second order	Third order	Fourth order	Standard error
Serial correlation of the index returns*	Daily 1983	FTSE	na	na	na	na	na
		FTA	·108	·027	·042	−·005	·06
		FTO	·113	·006	−·039	·043	·06
	Weekly 1983	FTSE	·036	−·216	−·240	−·036	·14
		FTA	·004	−·155	−·244	·023	·14
		FTO	−·109	−·119	−·126	·019	·14
	Monthly 1978–83	FTSE	−·092	−·142	−·246	·056	·12
		FTA	−·065	−·142	−·253	·043	·12
		FTO	−·111	−·089	−·254	·069	·12

Panel B:	Period and frequency	FTA regressed on the FTSE: Beta (t-value)	Alpha (t-value)	$\bar{R}$-squared	FTA regressed on the FTO: Beta (t-value)	Alpha (t-value)	$\bar{R}$-squared
Regression results using various measurement intervals*	Daily 1983	na	na	na	·75 (9·2)†	·00 (0·2)	·762
	Weekly 1983	·90 (4·3)†	·09 (1·8)	·966	·81 (3·8)	−·01 (0·1)	·837
	Daily 1978–83	na	na	na	·71 (34·5)	·03 (3·2)	·831
	Weekly 1978–83	na	na	na	·81 (9·2)	·13 (2·7)	·851
	Monthly 1978–83	·98 (1·6)	·08 (1·1)	·984	·84 (4·1)	·55 (2·5)	·860
	Quarterly 1978–83	1·00 (0·1)	·19 (0·8)	·978	·92 (1·0)	1·48 (2·3)	·847

* All returns exclude dividends and are continuously compounded. Returns are per cent per period and are not annualized.

† The t-values for beta are for tests against a null hypothesis of unity, while those for alpha are for tests against a null of zero.

substituted for the index value itself. In evaluating index properties which are relevant to futures and options trading, it is therefore advisable to refer to results based at least on weekly, and preferably on monthly, data.

4.4. *Interpretation of the results*

To interpret the results presented above it is helpful to decompose the tracking error of the FTSE (or the FTO) into three elements. First, there is the alpha or abnormal return, the amount by which the FTSE is outperformed by the FTA. Since this reflects the performance of constituents omitted from the FTSE, one should be cautious in inferring from the non-significance of the estimated alpha that the latter can be expected to be zero. The longer-term outperformance by smaller capitalization stocks is a well-established empirical regularity (see Schwert, 1983, and Dimson and Marsh, 1984), but the magnitude of the expected alpha nevertheless remains very uncertain.

The second element of tracking error arises from the uncertain beta of the FTA relative to the FTSE (or FTO). If the beta differs from unity, and yet an index is used to hedge movements in the FTA using a hedge ratio of one to one, there will still be considerable risk exposure—even if the two indexes are perfectly correlated with each other! For example, consider the beta of the FTA relative to the FTO, estimated with weekly/monthly data in Tables 1 and 2 to be in the region of ·8. Even if the FTA and FTO were perfectly correlated, a holding in one index offset by an equal short position in the other index would still have a beta of ·2. Using current figures for the variability of the market (a 17% annual standard deviation of returns), this supposedly hedged position would have a standard deviation of 3·6%. In this respect, the uncertain beta obtained with the FTO is an especially unwelcome source of incremental risk. The tightly bounded beta obtained with the FTSE is therefore a strong attraction in favour of the latter.

The last element of tracking error, however, is the most important. This is the residual risk of the FTA relative to the FTSE (or FTO). The latter is equal to the standard deviation of the residual of regression 4.1, namely the square root of the error variance $\mathrm{Var}(u_{1t})$. The error variance for equation 4.1, for example, depends on the correlation ρ_1 between returns on the FTA and FTSE and on the variance $\mathrm{Var}(R_{at})$ of returns on the FTA:

$$\mathrm{Var}(u_{1t}) = (1-\rho_1^2)\,\mathrm{Var}(R_{at}) \qquad (6)$$

The residual risk for equations (4.2) and (4.3) may be defined analogously.

For an assumed standard deviation of market index returns of 17%, Table 3 tabulates the residual standard deviation for various levels of the R-squared (i.e. ρ_1^2). Under the assumption that continuously compounded returns are normally distributed, the standard deviation varies with the square root of time; the remaining columns therefore adjust the annual figures to a variety of alternative time horizons.

It can be seen from Table 3 that the R-squared of ·984 estimated for the FTA/FTSE implies an annual standard deviation of residual returns of 2·14%.

Table 3. *Standard deviation of residual returns for various levels of correlation between the FTSE and the FTA*

Level of R-squared	*Standard deviation of residual for alternative investment horizons** 1 year	1 quarter	1 month	1 week	1 day
	%	%	%	%	%
·800	7·6	3·8	2·2	1·1	·48
·850	6·6	3·3	1·9	·9	·41
·900	5·4	2·7	1·6	·7	·34
·950	3·8	1·9	1·1	·5	·24
·970	2·9	1·5	·8	·4	·18
·980	2·4	1·2	·7	·3	·15
·985	2·1	1·0	·6	·3	·13
·990	1·7	·9	·5	·2	·11
·995	1·2	·6	·3	·2	·08

* This table is based on an assumed standard deviation of annualized market returns of 17%. Returns are assumed to be normally distributed, and the indexes are assumed not to suffer from thin trading. Note that if the market were more volatile, all the standard deviations in this table would be increased pro rata. The figure of 17% used here is simply the annualized estimate derived from monthly data for the FTA over the period 1978–83.

Thus a portfolio invested in the constituents of the FTA, and hedged by an appropriate short position in the FTSE Index, would be expected to have a variability of 2·14%. The FTO, with a lower R-squared of ·860, could be expected to generate a residual standard deviation three times that of the FTSE. Given its favourable beta, it can be seen that in about two periods out of three the market is likely to track the FTSE to within plus or minus 2·1% over a year, 1·1% over a quarter, ·6% over a month and ·3% over a week. In about 19 periods out of 20 the tracking error will be within plus or minus twice this range.

5. MANAGEMENT OF THE FTSE INDEX

5.1. *Changes to the index constituents*

The constituents of the FTA are updated continuously to reflect births, deaths, and major changes in company status. The intra-year constituents of our reconstructed FTSE, on the other hand, have been held constant. The 'live' FTSE Index may, therefore, exhibit somewhat better hedging characteristics than the reconstructed index if constituents are periodically revised within the year to ensure that they represent the 100 largest companies as closely as possible. However, it is not clear, *a priori*, whether intra-year revisions would lead to a large enough improvement in hedging properties to justify the additional effort involved.

To gain some insights into how the frequency of revisions might impact on hedging characteristics, we reconstructed the FTSE over the 1978–83 period following a policy of permitting no constituent revisions whatsoever. We will refer to this recalculated, unmanaged index as the FTSE* to distinguish it from the FTSE. Over this six-year period, eight out of the original 100 stocks 'died', and whenever this occurred, the terminal proceeds were reinvested in the surviving index constituents. No new stocks were introduced, however, and no year-end revisions or rebalancing were carried out. Of the 92 stocks which survived throughout the period, 68 still ranked in the largest 100 shares at the end of 1983.

We then re-estimated the relationship between the FTSE* and the FTA. Whereas the correlation coefficient and R-squared had been ·992 and ·984 respectively for the FTSE (see Panel B of Table 1), our estimates for the FTSE* were only slightly lower at ·991 and ·982. Similarly, the beta estimate of ·97 was only fractionally below the original figure of ·98 quoted in Table 1.

Since the hedging properties of the FTSE decline only very marginally when we move from once-a-year to once-every-six-year updates, our evidence suggests that the gain from intra-year updating of the FTSE will, in general, be negligible. Except in rare circumstances (e.g., the flotation of British Telecom), annual revision of the constituents should be perfectly adequate.

Notwithstanding the statistical adequacy of annual updating, the FTSE is nevertheless constrained by its ultimate rationale. Whereas the FTO can (in principle) be left alone until delisting of a stock forces revision of the index, the FTSE must be seen to be a fair representation of the 100 largest companies. For this reason, rather than to maintain its hedging properties, the FTSE must be updated with acceptable expeditiousness. Here, however, there is a logistical problem: the ranking amongst marginal companies (the smallest constituents and largest non-constituents of the FTSE) is likely to be volatile. Yet very frequent changes in composition are undesirable, both from a management viewpoint and because of the possibility of inducing bias into the index (see Roll, 1984, and Blume and Stambaugh, 1984). It is therefore useful to examine the breadth of the FTSE's coverage of the market and the rate at which deletions occur; and to identify the frequency with which constituents might switch their ranking from one year to the next.

Panel A of Table 4 provides some evidence on the FTSE's coverage of the constituents of the FTA All Share Index. The top 20 companies in the FTSE are equal in market value to over one-third of the entire FTA, and in total the 100 FTSE constituents represent over two-thirds of the total value of the FTA. Apart from an average of one to two stocks which lose their Stock Exchange listing during each year, about 10 companies are replaced at the year-end by larger capitalization alternatives. This replacement process involves stocks which amount to under 3% of the value of the FTA.

The transition matrix in Panel B shows that (virtually) all the stocks in quintiles 1–3, and approximately 90% of those in quintile 4, remain in the FTSE

Table 4. *Average coverage, deletions and transition matrix for FTSE 1978–83*

Panel A:	Constituents of FTSE in year t	Coverage of value of FTA Index: % of FTA†	Coverage of value of FTA Index: Cum. % of FTA	Stocks delisted during year: No.	Stocks delisted during year: % of FTA†	Stocks deleted at end of year: No.	Stocks deleted at end of year: % of FTA†
Average coverage and deletions from the FTSE	Quintile 1*	37·4	37·4	—	—	—	—
	Quintile 2	12·4	49·8	·2	·1	—	—
	Quintile 3	8·0	57·8	—	—	·2	·1
	Quintile 4	6·0	63·8	·2	·1	2·0	·6
	Quintile 5	4·2	68·0	1·0	·2	8·2	1·9
	All stocks	68·0	68·0	1·4	·4	10·4	2·6

Panel B:	Constituents of FTSE in year t	Average No. of stocks in each quintile (or not in any quintile) in year $t+1$: Quintile 1	Quintile 2	Quintile 3	Quintile 4	Quintile 5	Not in FTSE
Average transition matrix for the FTSE	Quintile 1*	17·2	2·8	—	—	—	—
	Quintile 2	2·6	12·8	3·6	·8	—	·2
	Quintile 3	·2	3·4	10·2	3·8	2·2	·2
	Quintile 4	—	·8	4·4	8·2	4·4	2·2
	Quintile 5	—	—	·8	3·6	6·4	9·2
	All stocks	20·0	19·6	19·0	16·6	13·0	11·8

* Each quintile contains 20 of the 100 FTSE constituents ranked by capitalization at the beginning of year t (i.e., using market values on the last trading day of year $t-1$).

† As at the beginning of year t (i.e., using market values on the last trading day of year $t-1$).

in the subsequent year. On average, some 12 stocks drop out of the FTSE each year, almost all from the bottom quintile. The majority of these constituent changes arise from rigid adherence to a '101 out, 100 in' rule. Indeed, over the last eight quarters of the sample period, strict application of this rule on a *quarterly* basis led to 22 cases of deletion and replacement per year; switching to a '120 out, 83 in' rule reduced the number of changes to six per year. (We are grateful to John Brumwell for these estimates.) The use of wide tolerances (e.g., 120/83) for changing the FTSE constituents is no easier to justify than relatively infrequent revisions to the index. For this reason, it was eventually decided that, in addition to the replacement of delisted securities, the index would be reviewed on a quarterly basis, using a '110 out, 90 in' rule as the guideline.

5.2. *Industry and sector imbalances in the index*

While the evidence presented in § 4 indicates that the FTSE can be expected to track the FTA quite closely, the R-squared of ·984 is obviously indicative of less than perfect diversification. Some of this undiversification undoubtedly arises from over- or under-exposure to particular industries. Such sector imbalances could clearly result in an unnecessary degree of divergent performance by the FTSE, and hence in allegations that it fails to meet attainable standards for the extent to which it represents the overall equity market. The magnitude and likely impact of sector imbalances are therefore important issues in the construction and management of the index.

Table 5 shows the load ratios for the major industry classifications used in the FT–Actuaries classifications. The load ratio is the percentage of the total market value of the FTSE which is invested in a sector, divided by the corresponding percentage for the market. It is notable that the oil sector (load ratio = 1·4) is considerably over-represented in the FTSE, largely at the expense of the investment trust sector (load ratio = ·1). This is attributable to the large average size of the oil companies, which leads to inclusion in the FTSE of five oil stocks (versus 16 stocks in the FTA), while investment trusts are generally so small that the FTSE included only two of them (versus 107 trusts in the FTA). Indeed, since it went live in 1984, the number of investment trusts included in the FTSE has shrunk to just one stock.

The oil industry has an especially high residual standard deviation. The *London Business School Risk Measurement Service*, for example, estimated the oil sector's residual standard deviation at 23%, as compared to 7–9% for the other U.K. industry sectors (see Table 5) and an average 25% for a single stock. It is, therefore, no surprise that the FTSE's imbalance in the oil sector has triggered proposals for a weighting scheme which restores the weights used in the FTA Index.

To investigate the impact of sector imbalances, we reconstructed the FTSE for 1978–83 in such a way that each sector had a load ratio of 1·0 relative to the FTA at the beginning of each year. To achieve this, we set each stock's start-year weight equal to its market capitalization, divided by the load ratio for the

Table 5. *FTSE load ratios for industry sectors**

FTA industry sector	Percentage of total value		Load ratio relative to†		No. of constituents		Residual standard deviation (%)‡
	FTSE	FTA	FTA	All U.K. shares	FTSE	FTA	
Capital goods	18·0	19·8	·91	·83	21	203	8
Consumer goods	34·8	31·8	1·09	1·14	38	197	7
Other industrials	9·9	10·0	·99	·96	10	84	8
Oils	15·9	11·8	1·35	1·42	5	16	23
Financial group	17·6	17·6	1·00	1·03	21	124	9
Investment trusts	·6	5·9	·10	·10	2	107	9
Commodities and overseas	3·2	3·0	1·07	1·03	3	19	15

* As at 30 December 1983.

† The load ratio relative to the FTA is the percentage of the total market value of the FTSE invested in a sector, divided by the corresponding percentage for the FTA. The load ratio relative to all U.K. shares is the percentage of the total market value of the FTSE invested in a sector, divided by the corresponding percentage for the total universe of all U.K. registered and quoted shares. These figures are derived from usage of the London Business School's Portfolio Analysis Service (for details, see *London Business School Financial Services*, 1983).

‡ These figures are taken from the *London Business School Risk Measurement Service*, **6**, No. 1.

relevant sector. We also examined several variants on this strategy by analysing a series of selective overweight positions for each sector in turn. This involved starting from FTA sector weightings, and increasing the investment in stocks within the sector up to the point where the sector load ratio was 1·25 relative to the FTA. Other shares' load ratios were hence reduced to a level that compensated for the overweight position which had been constructed. This was also repeated for sector load ratios of 1·50.

Table 6 shows the effect of these rebalancing and selective overweighting strategies on the hedging properties of the index. Columns 1 and 4 indicate that when the FTSE is rebalanced to FTA sector proportions, the R-squared from regressing the FTA on the rebalanced FTSE is ·989, while the residual standard deviation is 1·76%. As individual sector load ratios are increased to 1·25 and then 1·50, the hedging properties of the FTSE decline, as would be expected. The extent of the decline is a function both of the size of the sector in question, and also of its residual standard deviation (see Table 5). For example, although the oil sector is smaller in value than the capital goods, consumer goods and financial groups, its residual standard deviation is much higher. Hence a 50% overweight position in oils has virtually the same impact on hedging characteristics as a 50% overweight position in financials, a sector nearly twice its size. However, even the most extreme position considered, namely a 1·5 load ratio on capital goods, resulted in a residual standard deviation of only 2·35%.

These results suggest that although sector imbalances do have an impact on the hedging properties of the index, we should be careful not to overstate their importance. In § 4.4, we reported that the R-squared and residual standard deviation from regressing the FTA on the FTSE were ·984 and 2·14%

Table 6. *Impact of sector imbalances on the hedging properties of the FTSE*

	R-bar squared* with sector load ratio† of			Residual standard deviation* with sector load ratio† of		
FTA industry sector	1·0	1·25	1·50	1·0	1·25	1·50
Capital goods	·989‡	·986	·981	1·76‡	2·00	2·35
Consumer group	·989	·987	·982	1·76	1·91	2·27
Other industrials	·989	·989	·988	1·76	1·79	1·88
Oils	·989	·989	·984	1·76	1·77	2·13
Financial group	·989	·988	·984	1·76	1·89	2·14
Investment trusts	·989	·989	·989	1·76	1·75	1·77
Commodities and overseas	·989	·990	·990	1·76	1·72	1·69

* R-bar squared and percentage standard deviation of residual returns from a regression of the FTA on the rebalanced FTSE.

† The figures for load ratios of 1·00 show the effect of rebalancing the FTSE at the start of each year to have FTA weightings in each sector. The figures for load ratios of 1·25 and 1·50 show the effect of diverging from these FTA sector weightings by assuming overweight positions of 25% and then 50% respectively in each sector in turn.

‡ Note that these figures should be compared with the $\bar{R}$-squared of ·984 and the residual standard deviation of 2·14% (see Table 1 and § 4 above) obtained for the actual, unrebalanced FTSE index.

respectively. The effect of rebalancing the FTSE to FTA sector weightings improved these figures to ·989 and 1·76% respectively. The elimination of sector imbalances thus reduced the residual standard deviation by just under 20%.

5.3. *Imbalances arising from the omission of smaller companies*

Clearly, most of the undiversification of the FTSE stems not from sector imbalances, but from the fact that the index is constructed from the 100 *largest* companies. The FTSE is, of course, equivalent to the FTA, but with the 101st to the smallest companies, ranked by capitalization, being given a loading of zero. As we have already noted, the omitted companies account for some 30% of the value of the FTA.

Table 7 focuses directly on the effects of giving zero weight in the FTSE to stocks whose capitalization ranked 101 or lower. All U.K. quoted and registered equities have been ranked in this Table by each of five characteristics: market capitalization, gross dividend yield, price/earnings ratio, beta and residual risk. For each of these ranked sequences, the stocks have then been subdivided into four quartiles, each containing stocks that comprise one-fourth of the market value of all U.K. equities. Thus, one-quarter of the equity market has a characteristic which falls below the lower limit of the inter-quartile range, and one-quarter has a characteristic which exceeds the upper limit of the range. All of the figures given in this Table are derived from use of the London Business School's Portfolio Analysis Service (for further details, see *London Business School Financial Services*, 1983).

The first line of Table 7 shows the FTSE's load ratio for the market capitalization quartiles explained above. The FTSE excludes all stocks which are in the bottom quarter of the U.K. equity market: this is compensated by the large load ratios (over 1·4) for the top two quartiles. These differences in capitalization

Table 7. *FTSE load ratios for other stock characteristics**

Factor	Inter-quartile range for all U.K. shares†	Load ratio‡ relative to FTA for each quartile of the U.K. market: Quartile 1†	Quartile 2	Quartile 3	Quartile 4
Market Capitalization (£billion)	·17 to 1·8	·00	·88	1·48	1·42
Dividend yield (%)	3·1 to 5·8	·86	1·04	1·07	1·00
Price/earnings ratio	9·9 to 21·7	1·12	1·08	1·04	·74
Beta	·91 to 1·06	·96	1·12	·92	·96
Residual standard deviation (%)	20 to 28	1·04	1·22	·88	·81

* The figures in this table are derived from usage of the London Business School's Portfolio Analysis Service (for details, see *London Business School Financial Services*, 1983) and refer to index constituents as at 30 December 1983.

† Each quartile contains one-fourth of the total market value of all U.K. quoted and registered equities, ranked by the specified characteristic.

‡ The load ratio relative to the FTA is the percentage of the total market value of the FTSE invested in a quartile divided by the corresponding percentage for the FTA.

are reflected in similar disparities in terms of trading frequency and marketability. One-quarter of U.K. shares currently take at least 2·5 days until a transaction in the stock is recorded in the Stock Exchange Daily Official List; one-half experience delays of ·1 to 2·5 days, and only a quarter trade within 0 to ·1 days. For the FTA, only 2% of the constituents take at least 2·5 days to trade, while 58% trade within 0 to ·1 days (i.e., the frequency of days with no trading in the stock is less than one in ten). For the FTSE, every stock trades within 0 to ·1 days; in fact, every stock trades (many times) on each day. (These figures for the marketability of shares are taken from the *London Business School Risk Measurement Service*.)

Some further consequences of the size distribution of the FTSE can be seen in the remaining four lines of Table 7. The FTSE is somewhat light (load ratio = ·9) on low yielding stocks. It is significantly underweight (load ratio = ·7) in the stocks standing on a high price/earnings multiple. It is slightly light (load ratio = ·9) in the higher beta stocks. And it is significantly underweight (load ratio = ·8) in the stocks with high residual risk. These are some of the characteristics, associated with company size, which introduce into the FTSE the prospect of performance which diverges from the FTA.

The evidence on the idiosyncratic behaviour of stock returns on smaller companies is now well documented. Dimson and Marsh (1984), for example, show that the small firm effect in the U.K. has been as pronounced as in the U.S.A. In these circumstances, the only way to improve the tracking error of the FTSE would be to replace some of the larger constituents by smaller capitalization alternatives. In practice, this would be counter-productive. The increased tracking error from removing a large constituent would not be offset by the ability of a single small stock to replicate the behaviour of small companies as a whole, since there are nearly 2,000 stocks in the bottom quartile. Unless a suitable open-ended fund, invested in bottom quartile companies, were to be made available as a constituent for the FTSE, we can see no practicable way of substantially improving on the FTSE's hedging properties. The only solution would be to increase the number of constituents permitted in the FTSE itself.

6. SUMMARY AND CONCLUSION

This paper has researched the qualities of the new FTSE 100 Share Index as a device for hedging movements in the U.K. equity market. The FTSE is a capitalization-weighted index which covers about two-thirds of the value of the entire market. As it was produced only from the start of 1984, it was necessary to reconstruct a history of the FTSE, a history which was not only computationally accurate but untainted by survivorship bias. Historical series for the FTSE were, therefore, estimated using two sources: a monthly history from 1978 through 1983 was estimated using the London Share Price Database (LSPD), and a daily history for 1983 was estimated from underlying weekly data supplied by the Stock Exchange. The FTSE is also compared in this paper with a much older 'market leaders' index, the *Financial Times* Industrial Ordinary Share (FTO) Index.

The FT–Actuaries All Share (FTA) Index was found to have a beta of ·98 relative to the FTSE, a coefficient which is not significantly different from unity. Thus the appropriate hedge ratio for users of the index is approximately 1:1, unlike the FTO Index whose hedge ratio was found to be significantly below this value. The FTSE has been outperformed by the FTA Index by, on average, three-quarters of 1% per year from 1978 to 1983. This is very small by comparison with the FTO, which has been outperformed over the same period at a rate of nearly $\frac{1}{2}$% per *month*. Finally, the monthly movements in the FTA and FTSE during the 1978–83 period were found to have a correlation of over ·99 (an R-squared of 98·4%). This may be compared with the correlation between the FTA and FTO over the same period of under ·93 (an R-squared of 86·0%).

In studying the impact of the choice of measurement interval, we noted that daily data can give misleading results when one index is compared with another. This is because index values are based on stock prices which are measured with a degree of measurement error. It is, therefore, preferable to infer the potential risk exposure of a contract based on the FTSE from analysis of weekly or monthly data. Our results suggest a standard deviation of residual returns of a little above 2% over a one-year horizon, or half this magnitude over a three-month horizon.

Changes to the constituents of the FTSE over the 1978–83 period were found to have little effect on the statistical characteristics of the index. While quarterly reviews could lead to more substantial turnover in index constituents, the annual review cycle used in this study resulted in only ten to twelve revisions per year. Even if no constituent changes were implemented throughout the 1978–83 period, however, the hedging properties of the FTSE were largely unaffected. There is thus no evidence in favour of especially frequent updates to the composition of the FTSE.

Rebalancing the FTSE to the sector weightings of the FTA led to a small improvement in the hedging properties of the index. However, most of the index's undiversification arises from an imbalance in exposure to smaller capitalization stocks. This is an inevitable consequence of the fact that the index is constructed from the 100 largest companies. Thus the extent to which there is divergence between the FTA and the FTSE indexes is likely to depend more on the performance of smaller companies than on any other factor.

We conclude that the market is likely to track the FTSE Index to within an acceptably close tolerance. We believe that there will be few problems in using a contract based on this index for hedging fluctuations in the U.K. equity market.

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APPENDIX 1

Monthly values for FTSE and FTA indexes, 1978–83

	1978		1979		1980		1981		1982		1983	
	FTSE*	FTA†	FTSE	FTA	FTSE	FTA	FTSE	FTA	FTSE	FTA	FTSE	FTA
End Dec	475·5	456·0	484·2	468·1	509·2	488·4	647·4	621·1	684·3	665·5	834·3	812·4
End Jan	448·2	433·4	486·9	475·3	555·8	535·0	637·4	614·2	715·0	703·4	852·5	839·6
End Feb	427·5	411·0	521·1	504·8	582·8	560·7	665·0	646·5	680·1	673·5	853·0	848·8
End Mar	455·9	436·3	588·5	565·9	529·4	510·9	677·7	658·3	704·0	694·1	875·5	875·5
End Apr	459·1	443·0	619·9	594·0	547·6	530·2	713·9	705·8	711·7	697·3	945·6	933·7
End May	477·1	460·7	575·1	555·3	535·1	517·7	677·9	670·7	736·2	717·2	945·4	930·1
End June	462·0	447·8	547·3	526·8	595·0	572·9	690·5	681·3	704·8	686·1	992·5	974·1
End July	488·9	476·4	523·0	504·0	617·2	600·0	693·1	678·9	729·9	709·6	958·1	947·9
End Aug	498·6	488·1	542·1	525·7	621·3	600·4	727·6	711·2	751·4	729·6	967·3	957·2
End Sept	495·1	485·3	561·9	541·4	641·2	616·9	606·8	591·9	797·1	769·0	959·3	946·9
End Oct	476·6	466·3	526·5	504·7	679·2	650·9	626·0	608·5	812·9	785·7	943·1	929·6
End Nov	487·5	473·0	519·9	491·6	683·6	653·0	691·7	669·8	820·2	797·9	991·1	981·7
End Dec	484·2	468·1	509·2	488·4	647·4	621·1	684·3	665·5	834·3	812·4	1,000·0	1,000·0

* The values for the FTSE Index were estimated from the London Share Price Database, as described in §3.1 of the text.
† The FTA All Share Index has been re-based to 1,000 as at the end of 1983.

APPENDIX 2

*Estimated daily values for the FTSE index during 1983**

Week commencing	Monday	Tuesday	Wednesday	Thursday	Friday
3/1	838·9*	840·7	856·2	860·6	869·8
10/1	863·0	848·0	838·5	846·5	857·0
17/1	866·8	859·1	864·0	871·0	866·5
24/1	845·1	849·8	851·6	849·7	860·1
31/1	862·2	863·3	865·1	876·2	873·1
7/2	871·7	878·7	886·4	887·7	897·7
14/2	895·2	897·1	890·0	881·7	882·0
21/2	878·3	875·2	866·3	865·6	865·4
28/2	860·4	866·6	874·8	883·8	882·7
7/3	890·9	889·9	889·9	903·7	895·7
14/3	897·8	906·4	901·4	890·3	888·3
21/3	872·5	882·6	879·3	880·8	885·9
28/3	884·5	877·5	871·8	881·0	881·0*
4/4	881·0*	881·3	892·6	908·1	913·7
11/4	927·7	942·3	942·8	951·0	953·0
18/4	964·8	953·4	942·8	953·4	944·6
25/4	952·0	952·9	957·6	956·7	952·6
2/5	952·6*	947·2	939·3	934·0	930·2
9/5	927·9	911·4	907·9	904·4	904·9
16/5	907·8	909·5	920·3	928·2	921·0
23/5	916·3	927·3	929·1	943·8	954·5
30/5	954·5*	954·9	947·1	943·7	945·5
6/6	950·3	953·8	960·0	961·8	958·4
13/6	966·5	972·2	964·8	970·5	975·2
20/6	990·9	993·1	984·3	990·5	995·1
27/6	996·5	988·1	982·8	991·9	981·1
4/7	969·4	958·6	961·4	951·3	937·7
11/7	932·3	934·7	925·1	936·6	932·6
18/7	936·3	948·3	958·3	966·7	962·4
25/7	955·9	962·8	972·1	972·1	960·4
1/8	953·3	946·4	973·7	976·1	971·9
8/8	973·5	975·5	979·2	985·4	984·4
15/8	998·3	1,002·7	1,003·5	1,006·5	1,001·8
22/8	1,005·8	990·3	979·5	983·0	985·4
29/8	985·4*	979·0	969·5	971·9	967·1
5/9	981·4	970·6	974·0	975·1	964·9
12/9	969·3	954·8	959·5	956·8	949·7
19/9	957·1	959·6	960·0	966·4	971·3
26/9	971·4	958·7	961·0	963·0	960·2
3/10	957·9	958·5	954·5	956·9	953·4
10/10	946·5	943·2	936·5	926·5	916·7
17/10	914·9	916·1	915·8	928·9	926·9
24/10	922·2	927·2	929·1	931·2	928·0
31/10	942·3	944·1	948·0	957·9	963·9
7/11	964·6	965·3	967·2	971·8	980·3
14/11	986·6	986·9	977·0	973·2	970·0
21/11	969·5	972·9	971·2	971·5	980·9
28/11	990·1	989·4	991·1	985·8	985·5
5/12	984·0	988·5	992·7	1,002·6	996·0
12/12	992·0	990·7	989·6	990·1	986·9
19/12	988·6	991·0	990·3	1,001·0	999·0
26/12	999·0*	999·0*	1,000·1	996·9	1,000·0

* Based on Friday values provided by the Stock Exchange on 6/2/84. Bank Holidays are denoted by an asterisk.